P201 Day 17

Quiz This Wednesday? 16

If not, Quiz $\boxed{\text{next. Monday}}$ 21 OK

Topic:

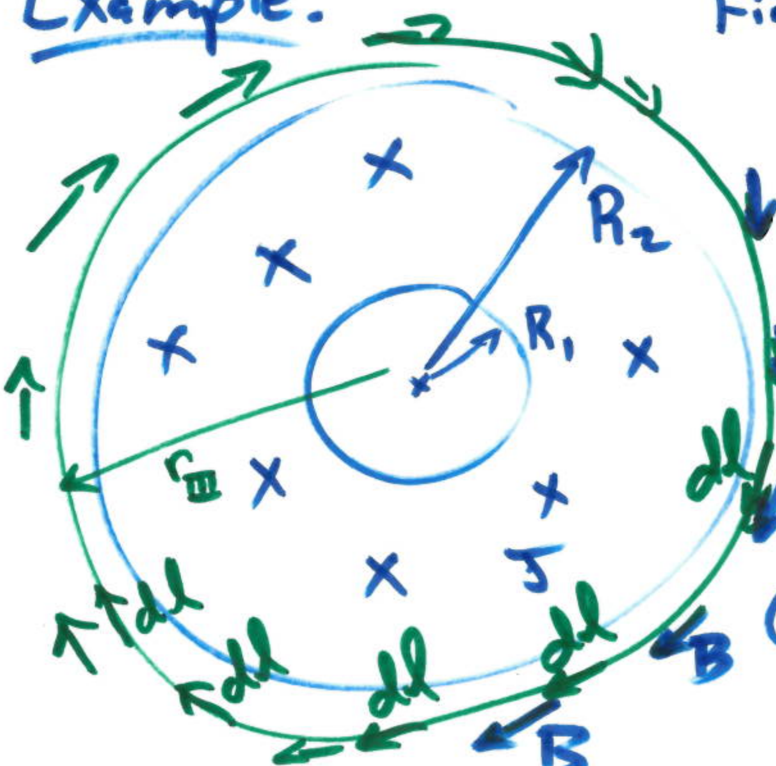
$\boxed{\text{Ampere's Law 8}}$

Faraday's Law 3

Example:

Find $B(r)$

all 3 regions



$$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I_{IN}$$

(I) $r < R_1$ inside

(II) $R_1 < r < R_2$ middle

(III) $R_2 < r$ outside

(III) $\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I_{IN}$

$\oint B\, d\ell =$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

if $\theta = 0$

$$\vec{A}\cdot\vec{B} = AB$$

$$B \oint dl = \mu_0 I_{IN}$$

$$B\, 2\pi r = \mu_0 (J A_{IN})$$

$$J = \frac{I}{\text{area}} \Rightarrow I_{IN} = J A_{IN}$$

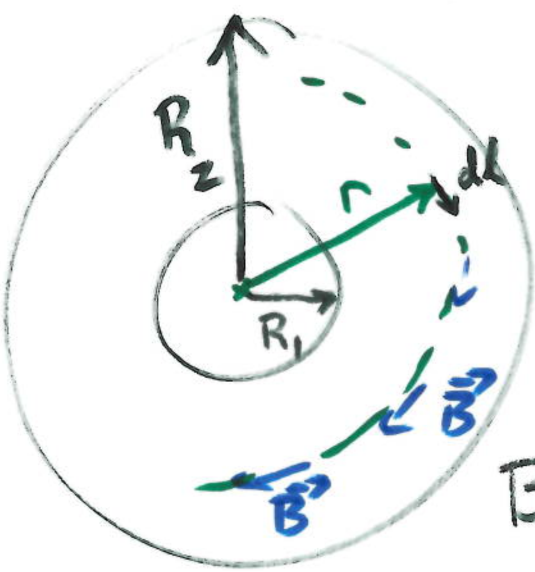
$$A_{IN} = \pi R_2^2 - \pi R_1^2$$ = the area with J in it.

(III) $r > R_2$ outside

$$B = \frac{\mu_0 J(\pi R_2^2 - \pi R_1^2)}{2\pi r}$$

(III)

What changes in (II)?



$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{IN}$$

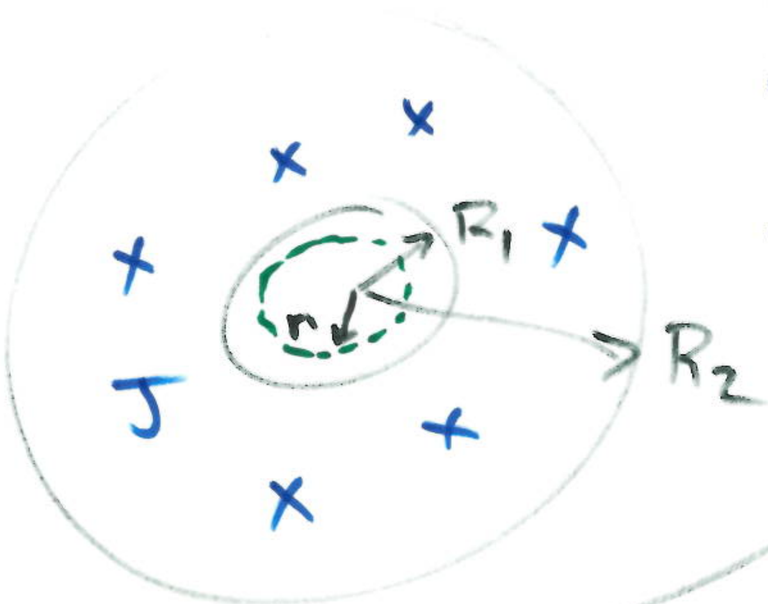
$$B\, 2\pi r = \mu_0 J A_{IN}$$

$$A_{IN} = \pi r^2 - \pi R_1^2$$

$$B = \frac{\mu_0 J(\pi r^2 - \pi R_1^2)}{2\pi r}$$

Region (II) the middle

lastly, region (I)



$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{IN}$ $\; I_{IN} = 0$

$r < R_1$

No I inside.

Last time: AC Circuits:

Inductor L only $V = X_L I$

$X_L = \omega \cdot L$

and for a capacitor only

$\omega = 2\pi f$

angular frequency

frequency (Hz)

$V = X_C I$

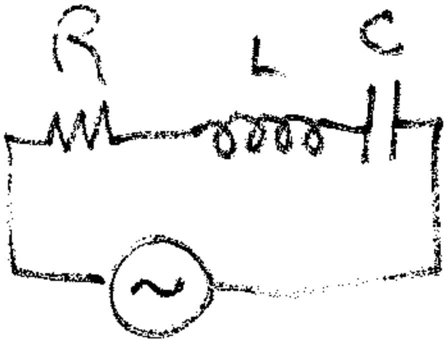
$X_C = \dfrac{1}{\omega C}$

X is called "reactance" unit: ohm
acts like resistance

R only is really boring.

$V = RI$ so $X_R = R$

In series you can combine all 3:

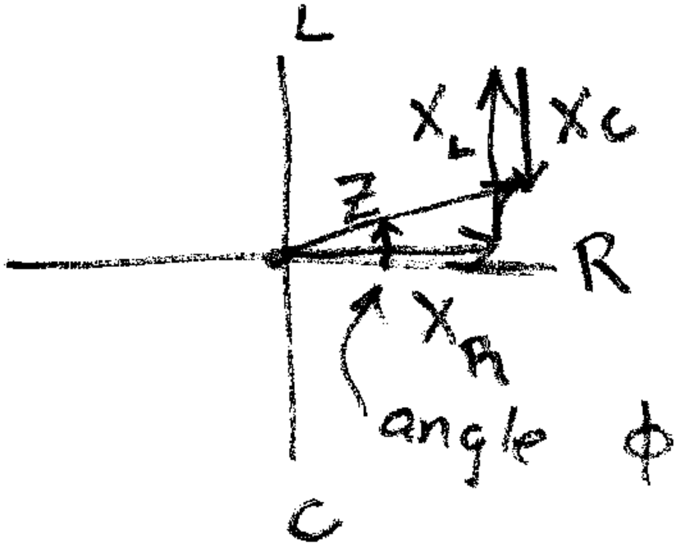


Combine together to make "Impedance"

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

so Z is *minimum* when $X_L = X_C$

we call this resonance.



$$\phi = \mathrm{Tan}^{-1}\left(\frac{X_L - X_C}{R}\right)$$

phase shift

$$V = Z I$$

$X_L = \omega L$

$X_C = \dfrac{1}{\omega C}$

when

$\omega = \omega^*$, $X_L = X_C$ thus: $\omega^* L = \dfrac{1}{\omega^* C}$

$$\omega^{*2} = \frac{1}{LC} \qquad \omega^{*} = \frac{1}{\sqrt{LC}}$$

the resonant frequency. (angular)

$2\pi f^{*} = \omega^{*}$

resonant frequency

$$f^{*} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

If $R = 10.0\,\Omega$, and inductor $L = 1.0\times10^{-6}\,H$ what C do you need to $f^{*} = 90.7\times10^{6}\,Hz$

Then: What is Z at $90.7\times10^{6}\,Hz$?
What is Z at $90.5\times10^{6}\,Hz$

$$f^{*} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{(2\pi f^{*})^{2}L}$$

$$C = \frac{1}{(2\pi\, 90.7\times10^{6})^{2}\cdot 1.0\times10^{-6}} = 3.08\times10^{-12}\,F$$

at resonance, $Z = R$. Duh.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

at resonance, $X_L = X_C$.

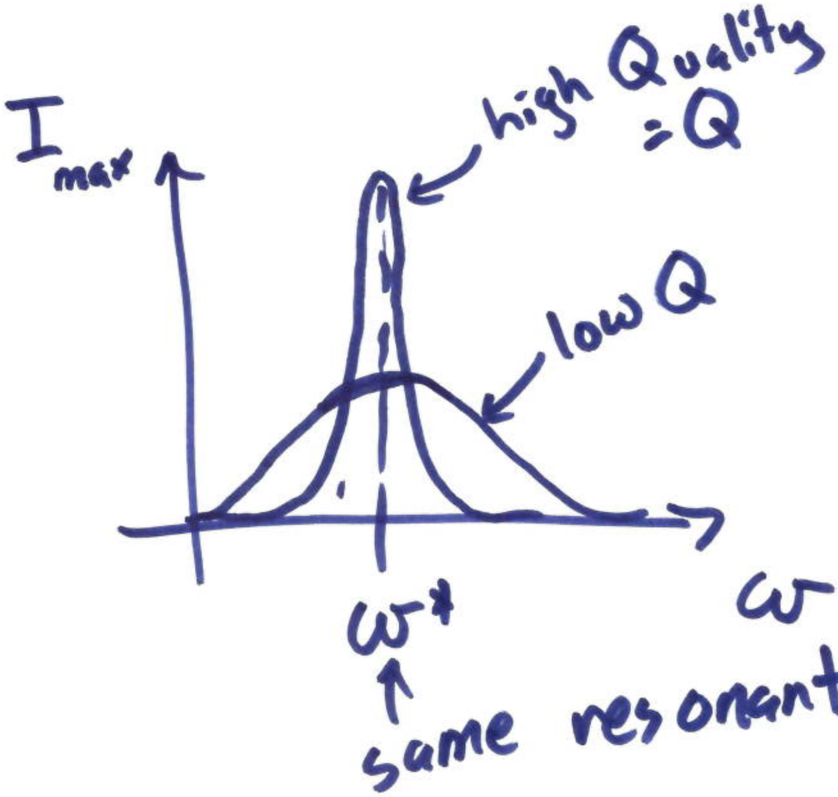
$Z = 10\,\Omega$ at 90.7 MHz.

---

at 90.5 MHz, $X_L = \omega L = 568\,\Omega$

$$X_C = \frac{1}{\omega C} = 572\,\Omega$$

$Z = 10.6\,\Omega$ (very slight difference)



$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega^*}{\Delta\omega}$$

width of peak