Electromagnetic Wave Homework Solutions

1. $B(x,t) = 4\sin(5x-6t)$

$$\frac{\partial B}{\partial x} = \underbrace{4\cdot 5}_{20}\cos(5x-6t)$$

$$\frac{\partial^2 B}{\partial x^2} = -20\cdot 5\sin(5x-6t)$$

$$= -100\sin(5x-6t)$$

$$\frac{\partial B}{\partial t} = 4\cdot(-6)\cos(5x-6t)$$

chain rule

$$\frac{\partial^2 B}{\partial t^2} = -(-24)(-6)\sin(5x-6t)$$

from $\frac{d}{dt}\cos$

$$= -144\sin(5x-6t)$$

$$\frac{\partial^2 B}{\partial x^2} \stackrel{?}{=} \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2}$$

$$-100\sin(5x-6t) = \frac{1}{c^2}(-144)\sin(5x-6t)$$

if

$$c^2 = \frac{144}{100} \qquad c = \frac{12}{10} = \boxed{\frac{6}{5} = c}$$

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2. Try $B(x,t) = 5(6x-8t)^2$

$$\frac{\partial B}{\partial x} = 2\cdot 5(6x-8t)^1\cdot 6$$

power   chain

$$\frac{\partial^2 B}{\partial x^2} = 2\cdot 6\cdot 5\cdot 6 = 360$$

power   chain

$$\frac{\partial B}{\partial t} = 2\cdot 5(6x-8t)^1\cdot(-8)$$

$$\frac{\partial^2 B}{\partial t^2} = 2\cdot 5\cdot(-8)(-8)$$

$$= 640$$

works if:

$$36\cancel{0} = \frac{1}{c^2}\cdot 64\cancel{0}$$

$$36 \qquad c^2 = \frac{64}{36} \qquad \boxed{c = \frac{8}{6}}$$

3. Try: $B(x,t) = 6x^2 - 8t$

$$\frac{\partial B}{\partial x} = 12x \qquad \frac{\partial B}{\partial t} = -8$$

$$\frac{\partial^2 B}{\partial x^2} = 12 \qquad \frac{\partial^2 B}{\partial t^2} = 0$$

works if

$$12 = \frac{1}{c^2}\cdot 0 \rightarrow c = 0$$

No wave speed doesn't work. Wave must move.

4. $$\frac{\partial E_y}{\partial x} = A\cos(kx-\omega t)\cdot \underbrace{k}_{\text{chain}} \overset{\text{must}}{=} -\frac{\partial B_z}{\partial t}$$

so B_z should be of the form

$$B_z = B_0 \sin(kx - \omega t)$$

find a viable B_0 to make this work.

$$-\frac{\partial B_z}{\partial t} = (-1)B_0\cos(kx-\omega t)(-\omega)$$
$$= \omega B_0 \cos(kx - \omega t)$$

Thus: $Ak = \omega B_0$ thus $\boxed{B_0 = A\frac{k}{\omega}}$

5. A has unit $\frac{V}{m}$. k unit $\frac{1}{m}$ ω unit $\frac{1}{s}$

thus: B_0 has unit: $\frac{V}{m}\frac{s}{m} = \frac{Vs}{m^2}$ and this is a Tesla.