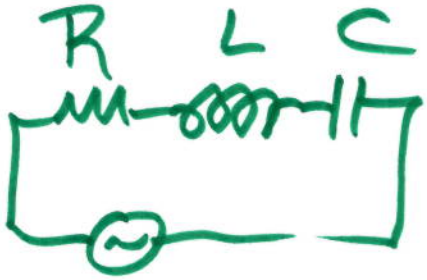
Physics 201 in-lab review for Exam 3

1. 

Find I and V across each element.

$$V = ZI$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L \quad X_C = \frac{1}{\omega C}$$

$$V = XI$$

$R = 110\,\Omega$ $L = 1.257\,\mu H$

$C = 0.0543\,nF$ $V = 3.21\,V$

$f = 2.75 \times 10^7\,Hz$

$\omega = 2\pi f$

$\omega = 1.73 \times 10^8\,\frac{1}{s}$

$X_L = 217\,\Omega$

$X_C = 106\,\Omega$ $Z = 156\,\Omega$

$$I = \frac{V}{Z} = 0.0205\,A = 20.5\,mA$$

$V_R = RI = 2.26\,V$

$V_L = X_L I = 4.45\,V$

$V_C = X_C I = 2.17\,V$

2. $E_A = 2 \frac{kcal}{mol}$

$E_B = -0.5 \frac{kcal}{mol}$ At $T = 300K$;

find P of each.

Note: you can "re-zero" the energies.

$E_B' = E_B + 0.5 = 0 \frac{kcal}{mol}$

$E_A' = E_A + \underbrace{0.5}_{\text{add \underline{same} to all.}} = 2.5 \frac{kcal}{mol}$

$$Z = \sum_i e^{-\beta E_i} \qquad \beta = \frac{1}{K_B T}$$

$K_B = 0.0020 \frac{kcal}{mol \cdot K}$

$\beta = 1.67 \frac{mol}{kcal}$

$$Z = e^{-\beta E_A'} + e^{-\beta E_B'}$$

$$= e^{-1.67 \times 2.5} + e^{0}$$

$$= 0.0154 + \underbrace{1}_{e^{-\beta E_B'}}$$

$$P_A = \frac{e^{-\beta E_A'}}{Z} \qquad P_B = \frac{e^{-\beta E_B'}}{Z} = 0.985$$

$P_A + P_B = 1 \rightarrow P_A = 1 - P_B = 0.015$

3.a. $N = 3.3 \times 10^{22}$ in cube 10 cm side.

$P = 1$ atm. find T.

$PV = N k_B T$ $\qquad k_B = 1.38 \times 10^{-23} \frac{J}{K}$

$V = (0.1 m)^3 = 1 \times 10^{-3} m^3$

$P = 1 atm = 1.013 \times 10^5 Pa \leftarrow \frac{N}{m^2}$

$$\underbrace{101.3 \frac{N}{m^2} m^3}_{N \cdot m = J} = \underbrace{3.3 \times 10^{22} \; 1.38 \times 10^{-23}}_{0.455 \frac{J}{K}} \frac{J}{K} \; T$$

$$\boxed{222 K = T}$$

3.b. Now, T changes to $T = 300 K$. Find new P.

N = const. V = const.

$$\frac{P_0}{T_0} = \frac{N k_B}{V} = \text{const.} = \frac{P_f}{T_f} \rightarrow P_f = P_0 \times \frac{T_f}{T_0}$$

$$P_f = 1.013 \times 10^5 Pa \times \frac{300 K}{222 K} = \boxed{1.37 \times 10^5 Pa}$$

$R = 8.315 \frac{J}{mol \cdot K}$ $\qquad PV = nRT$

$\qquad\qquad\qquad\qquad\qquad\qquad$ # mol

$k_B = 1.38 \times 10^{-23} \frac{J}{K}$ $\qquad PV = N k_B T$

$\qquad\qquad\qquad\qquad\qquad\qquad$ # particles

4.a. Give example where S increases

$$S = k_B \ln \Omega$$

available micro states

quantum

virtually <u>any</u> chemical reaction (fire)
your breathing (resperation)

4.6. When is ideal gas law a "good" approximation?

low pressure (sparse gas)

$\frac{V}{N}$ is big

any noble gas is better - less interaction (bonding)

4.c. In some cases we use G instead of E. Why?

$$G = E - TS$$

$$S = k_B \ln \Omega$$

Use G so that $Z = \Sigma e^{-\beta G}$ is a <u>smaller</u> sum (fewer terms)

4.d. Microscopically, what causes pressure? (What is each particle doing?)

Collisions - <u>many</u> causes pressure.

4.e. In an AC circuit, what does an inductor (L) do? Why might you use one?

$X_L = \omega \cdot L$

It stops high frequencies. (Filters out "spikes").

5. Is $E(x,t) = \frac{5V}{m}(55x^2 + 321t^2)$

a solution to wave equation?

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{5V}{m} \cdot 2 \cdot 321$$

$$\frac{\partial^2 E}{\partial x^2} = \frac{5V}{m} \cdot 2 \cdot 55$$

$321 = c^2 \cdot 55$

$\frac{321}{55} = c^2$

yes, it could work if

$c = \sqrt{\frac{321}{55}}$

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6. $E_A = 0 \quad S_A = 0$

$E_B = -1.11 \ \frac{kcal}{mol}$

at $T = 300K$, state B is twice as likely as state A. Find S_B and T at which $P_A = P_B$.

$P_i \propto e^{-G_i \beta}$

$P_i = e^{-G_i \beta} / Z$

$Z = \sum_i e^{-\beta G_i}$

$\beta = \frac{1}{k_B T}$

$k_B = 0.0020 \ \frac{kcal}{mol \cdot K}$

$$\beta = \frac{1}{300 \cdot 0.002} = \frac{1}{0.6} = 1.67 \frac{\text{mol}}{\text{Kcal}}$$

$$G = E - TS$$

$$G_A = 0$$

$$G_B = -1.11 \frac{\text{Kcal}}{\text{mol}} - 300\text{K}\, S_B^*$$

at 300K $P_B = 2 \cdot P_A$ $P_A = \frac{1}{2} P_B$

$$P_A + P_B = 1$$

$$\frac{1}{2} P_B + P_B = 1 = \frac{3}{2} P_B \qquad P_B = \frac{2}{3}$$

$$P_A = 1 - P_B$$

$$P_A = \frac{1}{3}$$

$$P_A = \frac{e^{-G_A \beta}}{e^{-G_A \beta} + e^{-G_B \beta}} = \frac{1}{1 + e^{-G_B \beta}}$$

$$\frac{1}{3} = \frac{1}{1 + e^{-G_B \beta}} \rightarrow 2 = e^{-G_B \cdot \beta}$$

$$\ln 2 = -G_B \cdot \beta$$

$$0.693 = -G_B \cdot 1.67 \frac{\text{mol}}{\text{Kcal}}$$

$$-0.415 \frac{\text{Kcal}}{\text{mol}} = G_B \quad \text{at } T = 300\text{K}$$

$$-0.415\ \frac{kcal}{mol} = G_B = \underset{\substack{\uparrow \\ -1.11\ \frac{kcal}{mol}}}{E_B} - TS_B$$

$$0.695\ \frac{kcal}{mol} = -\underset{\substack{\uparrow \\ 300K}}{T}S_B$$

$$\boxed{-2.32\times10^{-3}\ \frac{kcal}{mol\cdot K} = S_B}$$ 1st answer.

2nd answer: find T^*, where $P_A = P_B$.

$$\left.\begin{array}{l} P_A \propto e^{-G_A\cdot\beta} \\ P_A \propto e^{-G_B\cdot\beta} \end{array}\right\} \text{set} = \quad e^{-G_A\beta} = e^{-G_B\beta}$$

special answer T.

$$G_A = G_B$$

$$0 = \underbrace{E_B}_{-1.11\ \frac{kcal}{mol}} - T^* \underbrace{S_B}_{-2.32\times10^{-3}\ \frac{kcal}{mol\cdot K}}$$

$$1.11\ \frac{kcal}{mol} = -T^*\left(-2.32\times10^{-3}\ \frac{kcal}{mol\cdot k}\right)$$

$$\frac{1.11}{2.32\times10^{-3}}\ K = \boxed{T^* = 479K}$$

7.

$f = 12.9\,cm$

44.4 cm

Where is image?

magnification?

$d_i > 0$ real image

$d_i < 0$ virtual image

$$\boxed{\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f}} \qquad \boxed{m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}}$$

$d_o = 44.4\,cm$

$d_i =$ dist. to image

$$\frac{1}{d_i} + \underbrace{\frac{1}{44.4\,cm}}_{0.0225} = \underbrace{\frac{1}{12.9\,cm}}_{0.0775}$$

$$\frac{1}{d_i} = 0.0550\,\frac{1}{cm}$$

$$\boxed{d_i = 18.2\,cm}$$

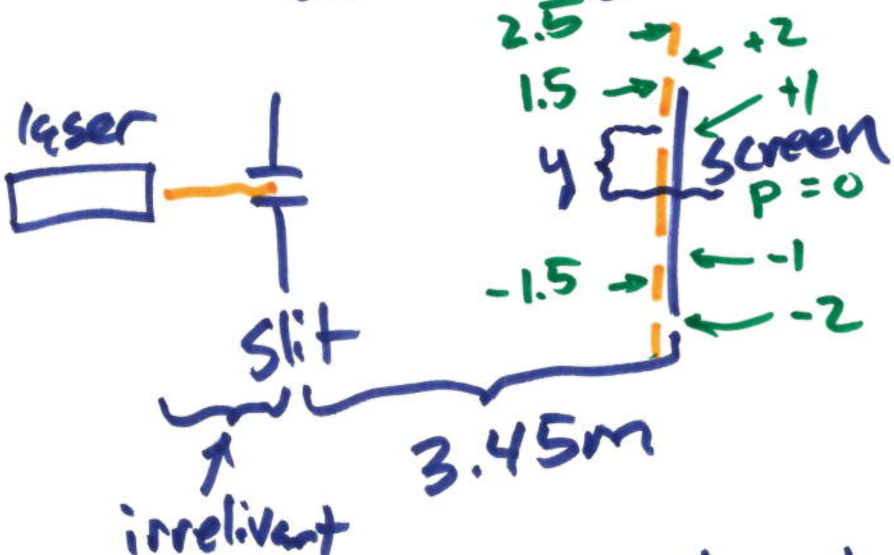
$$m = -\frac{d_i}{d_o} = \frac{-18.2\,cm}{44.4\,cm} = -0.410$$

upside down

8. laser $\lambda = 555\,nm$

slit width $= 0.123\,mm$

onto screen $3.45\,m$



What dist. from center to 1st dark spot? y=?

$$\boxed{\begin{array}{ll} a\sin\theta = p\lambda & p = \pm 1, \pm 2, \ldots \\ d\sin\theta = n\lambda & n = 0, \pm 1, \pm 2, \ldots \end{array}}$$

single Slit minima (dark)

double slit maxima (bright)

$$a \sin\theta = p\lambda$$

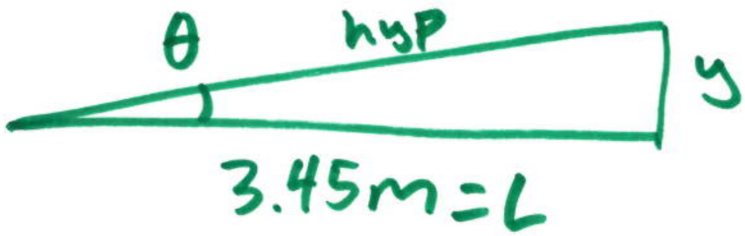
slit width

$$a\sin\theta = 1 \cdot \lambda$$

$$\sin\theta = \frac{\lambda}{a} = \frac{555\times 10^{-9}\,m}{0.123\times 10^{-3}\,m} = 4.51\times 10^{-5}$$

nano

mili



$\tan\theta = \frac{y}{L}$

$\sin\theta = \frac{y}{hyp}$ ✓

$\theta = 2.584\times10^{-3}$ degrees

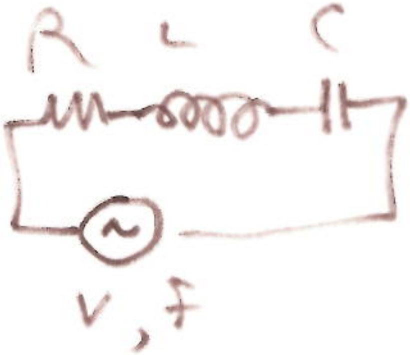
$\tan\theta = 4.51\times10^{-5}$

$y = L\overbrace{\tan\theta}^{\simeq \sin\theta} = 1.56\times10^{-4}\,m$

$\boxed{y = 0.156\,mm}$

P201 Sample Exam 3 Solved!

1.



$R = 110\,\Omega$

$L = 1.257\,\mu H$

$C = 0.0543\,nF$

$V = 3.21\,V$

$f = 2.75 \times 10^7\,Hz$

$V = ZI$

$V = XI$

$\omega = 2\pi f$

$\omega = 1.73 \times 10^8 \frac{1}{sec}$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L = 1.73 \times 10^8 \tfrac{1}{s} \cdot 1.257 \times \underbrace{10^{-6}}_{\mu} H = 217\,\Omega$$

$$X_C = \frac{1}{\omega C} = 106\,\Omega$$

$$Z = \sqrt{(110\,\Omega)^2 + (217 - 106)^2\,\Omega^2} = 156\,\Omega$$

I through <u>all</u> elements: $V = ZI \quad I = \frac{V}{Z}$

$$I_{rms} = \frac{3.21V}{156\,\Omega} = 0.0205\,A = 20.5\,mA$$

The V_{rms} across <u>each</u> element given by:

$V = XI$

$$V_R = RI = 110\,\Omega \cdot 0.0205\,A = 2.26\,V$$

$$V_L = X_L I = 217\,\Omega \times I = 4.46\,V$$

$$V_C = X_C I = 106\,\Omega \times I = 2.18\,V$$

2. 2 states: $E_A = 2$ Kcal/mol

$E_B = -0.5$ Kcal/mol

find P of each at T=300K.

$$P = e^{-\beta E}/Z \qquad Z = \sum_{i\,\text{all}} e^{-\beta E_i}$$

$$\beta = \frac{1}{k_BT} \qquad k_B = 0.0020\ \frac{\text{Kcal}}{\text{mol}\cdot\text{K}}$$

$$k_BT = 0.60\frac{\text{Kcal}}{\text{mol}} \text{ at } T = 300K$$

$$\beta = \frac{1}{k_BT} = 1.67\ \frac{\text{mol}}{\text{K-cal}}$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots$$

Can re-zero the energy:

$$\left.\begin{aligned} E_A' &= E_A + 0.5\ \tfrac{\text{Kcal}}{\text{mol}} = 2.5\ \tfrac{\text{Kcal}}{\text{mol}} \\ E_B' &= E_B + 0.5\ \tfrac{\text{Kcal}}{\text{mol}} = 0 \end{aligned}\right\} \frac{\text{same}}{\text{Prob.}}$$

⋮

$$Z = e^{-\beta E_A'} + \underbrace{e^{-\beta \overset{0}{E_B'}}}$$

$$= \underbrace{e^{-1.67\times 2.5}}_{0.0154} + 1 = 1.0154$$

$$P_B = \frac{e^{-\beta E_B'}}{Z} = \frac{1}{1.0154} = 0.985$$

$$P_A = 1 - P_B = 0.0152$$

3.A. 3.3×10^{22} particles in cube 10cm on a side. If $P = 1\text{atm}$, find T.

$$PV = N k_B T \qquad PV = nRT$$

$$k_B = 1.38 \times 10^{-23} \frac{J}{K} \qquad R = 8.315 \frac{J}{mol \cdot K}$$

$$1\,\text{atm} = 1.013 \times 10^5 \underbrace{Pa}_{N/m^2}$$

n = # mols

N = # particles

N_A = Avogadro's Number

$$n = \frac{N}{N_A}$$

$$T = \frac{PV}{N k_B} = \frac{1.013 \times 10^5 Pa \,(0.1m)^3}{3.3 \times 10^{22} \cdot 1.38 \times 10^{-23} \frac{J}{K}}$$

$$= \frac{101.3}{3.3 \times 0.138} K = \boxed{222 K}$$

3.6. Now, T is raised to 300K keeping V and N fixed. What is new P?

$PV = Nk_BT$ $\quad \dfrac{P}{T} = \dfrac{Nk_B}{V} = \underline{\text{fixed}}$

init = final $= \dfrac{Nk_B}{V}$

$\dfrac{P_o}{T_o} = \dfrac{P_f}{T_f}$ $\quad P_f = P_o \times \dfrac{T_f}{T_o} = \dfrac{1.013\times10^5\,\text{Pa}\; 300}{222}$

$$\boxed{P_f = 1.37\times10^5\,\text{Pa}}$$

4.a.

Give an example of S (entropy) increasing.

$S = k_B \ln \Omega$

microstates (quantum)

answer:
any chemical reaction
(burning, breathing ...)

4.b. When is ideal gas law a good approximation?

Noble gas = good,

Plenty of $\frac{V}{N}$ = good (low density)

4.c. When to use G in place of E?

$G = E - TS$

If S given, use G, if not use E.

(But can always use E ... just $\sum$ over many more states).

4.d. Microscopically, What is Pressure?

Collisions.

4.e. In an AC Circuit, what does L do?

$V = X I$

$X_L = \omega L$

smooth out I, opposes high ω.

5. Does $E(x,t) = E_0\left(55x^2 + 321t^2\right)$ "work" as a solution to wave equ?

$$c^2 \frac{d^2E}{dx^2} = \frac{d^2E}{dt^2}$$

$$\frac{dE}{dx} = E_0 \cdot 2 \cdot 55 \cdot x \qquad \frac{d^2E}{dx^2} = 110\,E_0$$

$$\frac{dE}{dt} = E_0 \cdot 2 \cdot 321 \cdot t \qquad \frac{d^2E}{dt^2} = 642\,E_0$$

$$c^2\, 110 \cancel{E_0} = 642 \cancel{E_0}$$

if $c^2 = \frac{321}{55}$, then it works.

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what if we had found:

$$c^2\, 110\, x\, E_0 = 642\, t\, E_0$$

No this fails. c^2 is a constant not a function of x,t.

6. 2 states $E_A = 0$ and $S_A = 0$.

$E_B = -1.11 \frac{kcal}{mol}$ at $T = 300K$,

$P_B = 2P_A$. Find S_B and T^*, the T at which $P_B = P_A$.

$G = E - TS$

$G_A = 0$

$G_B = -1.11 \frac{kcal}{mol} - TS_B$

$$P_A = \frac{e^{-G_A\beta}}{e^{-G_A\beta} + e^{-G_B\beta}}$$

$$P_A = \frac{1}{1 + e^{-G_B\beta}}$$

$P_A + P_B = 1$

at $\underline{T = 300K}$, $P_B = 2P_A \longrightarrow P_A + \overbrace{2P_A}^{P_B} = 1$

$P_A = 1/3$

$P_B = 2/3$.

$$\frac{1}{3} = \frac{1}{1 + e^{-G_B\beta}} \longrightarrow e^{-G_B \cdot \beta} = 2$$

$$-G_B \cdot \beta = \ln 2 = 0.693$$

$$\beta = 1.67 \frac{\text{mol}}{\text{Kcal}} = \frac{1}{k_B T} = \frac{1}{0.002 \times 300} = \frac{1}{0.6 \frac{\text{Kcal}}{\text{mol}}}$$

$$-G_B = 0.415 \frac{\text{Kcal}}{\text{mol}} \qquad G_B = -0.415 \frac{\text{Kcal}}{\text{mol}}$$

$$G_B = E_B - T S_B = -1.11 \frac{\text{Kcal}}{\text{mol}} - 300\text{K}\, \underset{\substack{\uparrow \\ \text{solve}}}{S_B}$$

$$0.695 \frac{\text{Kcal}}{\text{mol}} = -300\text{K}\, S_B \qquad \boxed{S_B = -2.32 \times 10^{-3} \frac{\text{Kcal}}{\text{mol} \cdot \text{K}}}$$

At T^* $P_A = P_B$

$$\frac{e^{-G_A \beta}}{\cancel{Z}} = \frac{e^{-G_B \beta}}{\cancel{Z}} \quad \text{at } T^*$$

$$G_A = G_B \quad \text{at } T^*$$

$$\underbrace{E_A - T^* S_A}_{} = E_B - T^* S_B$$

$$0 = -1.11 \frac{\text{Kcal}}{\text{mol}} - T^* \left(-2.32 \times 10^{-3} \frac{\text{Kcal}}{\text{mol} \cdot \text{K}}\right)$$

$$\frac{1.11}{2.32 \times 10^{-3}} \text{K} = \boxed{T^* = 479\text{K}}$$

7.

$d_o = 44.4cm$ $\quad$ $f = 12.9cm$

where is image?

Find m.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \qquad m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$\frac{1}{12.9cm} = \frac{1}{d_i} + \frac{1}{44.4cm} \to \frac{1}{d_i} = 0.0550 \frac{1}{cm}$$

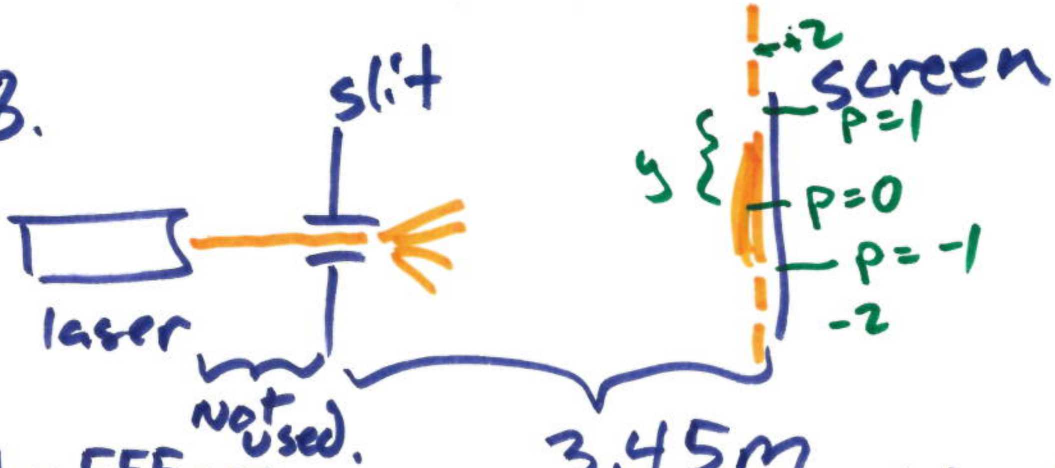
$$d_i = 18.2 \text{ cm}$$

$$m = \frac{-18.2cm}{44.4cm} = -0.410$$

upside down

less than 1? small

8.



$\lambda = 555\,\text{nm}$

0.123 mm slit

double bright spot: $d\sin\theta = n\lambda$, $n = 0, \pm 1, \pm 2, \dots$

single dark spots: $a\sin\theta = p\lambda$, $p = \pm 1, \pm 2, \pm 3, \dots$

Question:

Find y from center to 1^{st} dark spot

$\Rightarrow$ Set $p=1$

$\underset{\substack{\parallel \\ 0.123\,\text{mm}}}{a}\ \sin\theta^{\star} = 1\cdot\lambda$

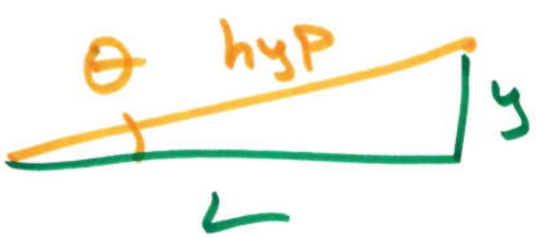
$\sin\theta = \dfrac{555\,\text{nm}}{0.123\,\text{mm}}$

$\sin\theta = 4.51\times 10^{-3}$

find θ

$\theta = 0.2584^\circ$

$\tan\theta = 4.51005\times 10^{-3}$



$\sin\theta = \dfrac{y}{hyp}$

$\tan\theta = \dfrac{y}{L}$

$y = L\tan\theta = 0.0155\,\text{m}$

$= \underline{1.55\,\text{cm}}$