

Energy and Momentum of EM Waves

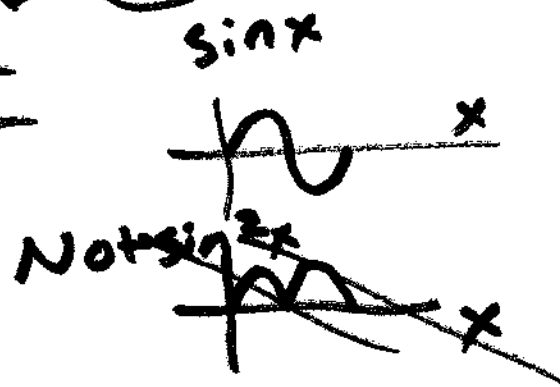
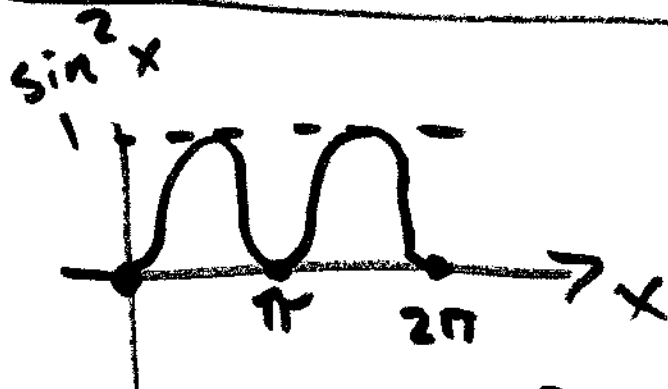
$$\frac{\text{Energy}}{\text{unit Volume}} = u = \underbrace{\epsilon_0 E^2}_{\text{capacitor}} = \frac{B^2}{\mu_0}$$

↑
energy density

EM wave:

$$E(t) = E_0 \sin(kx \pm \omega t)$$

$$\bar{u} = \epsilon_0 \overline{E^2} = \epsilon_0 E_0^2 \underbrace{\frac{1}{2}}_{\frac{1}{2}} \underbrace{\sin^2(\quad)}_{\sin x}$$



$$\sin^2 x + \cos^2 x = 1$$

$$\bar{u} = \frac{\epsilon_0 E_0^2}{2} = \frac{B_0^2}{2\mu_0} = \frac{E_0 B_0}{2\mu_0 c}$$

↑
Joules / m³

$E_0 = \frac{B_0}{c}$

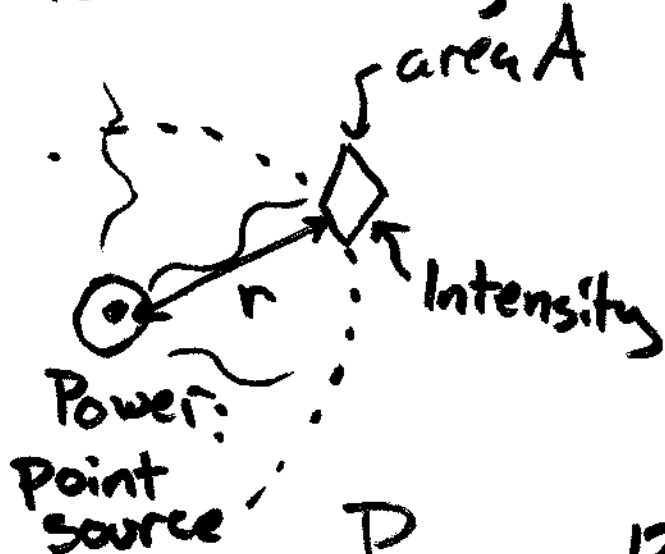
$$P: \text{Power} = \frac{\text{Watts}}{\text{unit}} = \frac{\text{Joule}}{\text{s} \cdot \text{unit}}$$

$$I: \text{Intensity} = \frac{\text{Power}}{\text{Area}} = \frac{\text{Watts}}{\text{m}^2} \leftarrow \text{brightness}$$

side note:

$$\text{decibels} = \text{dB} = 10 \cdot \log_{10} \left(\frac{I}{I_0} \right)$$

Relate to \vec{u} , and thus E_0 , or B_0 .



$$I = \frac{P}{4\pi r^2} \quad \text{for a point source}$$

intensity Sun $\approx 1300 \frac{\text{Watts}}{\text{m}^2}$ at Earth.

$$P_{(\text{sun})} = 1300 \frac{\text{W}}{\text{m}^2} \cdot 4\pi (1.5 \times 10^{11} \text{ m})^2$$

$$150 \times 10^6 \times 10^3 \text{ m} = 1.5 \times 10^{11} \text{ m}$$

$$P = 3.67 \times 10^{26} \text{ Watts (a lot)}$$

Direction of EM waves:

in the $\vec{E} \times \vec{B}$ direction

and Poynting $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

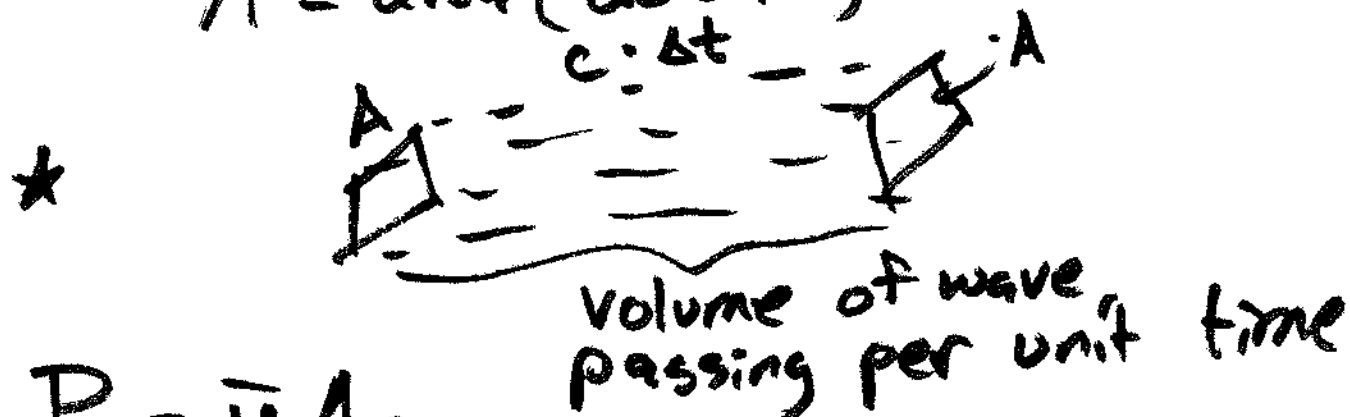
= Intensity (vector)

$$\text{Power} = \bar{u} \cdot A \cdot \text{speed}$$

energy
time = $\frac{\text{energy}}{\text{volume}} \cdot \frac{\text{volume of wave}}{\text{time}}$

$c = \text{speed of light}$

$A = \text{area (detector)}$



$$P = \bar{u} A c$$

intensity $I = \frac{P}{A} = \bar{u} \cdot c$

$$\bar{u} = \frac{E_0 B_0}{2 \mu_0 c}$$

$$I = \frac{E_0 B_0}{2 \mu_0}$$

momentum

$$\vec{F} = \frac{d\vec{p}}{dt} \quad p = mv$$

↑
time average

If EM waves carry p , they can push.

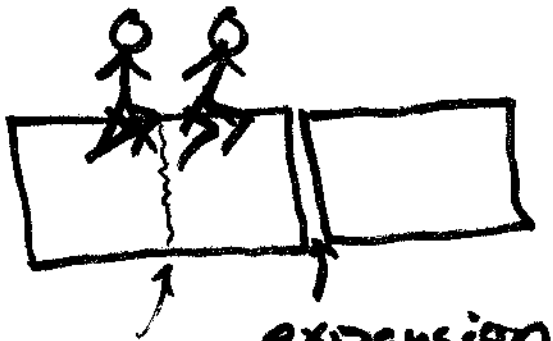
$$F = ma + \left(\frac{dm}{dt} \right) \cdot v$$

(can show (rated X) $p = \frac{\bar{u}}{c}$ momentum
and Pressure $P = \frac{I}{c}$

Last time:

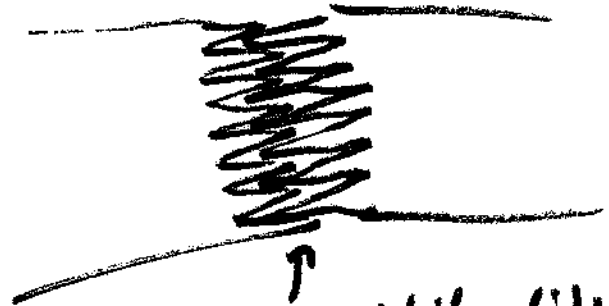
Thermal Expansion

$$L(T) = L_0 (1 + \alpha \Delta T)$$



crack
because
too few
gaps

expansion
gap



"teeth" - like
expansion gap
(bridge)

α is bigger for liquids than solids.

$$\alpha_{\text{steel}} = 12 \times 10^{-6} \frac{1}{^\circ\text{C}} \text{ for steel.}$$

$$\alpha_{\text{mercury}} = 60 \times 10^{-6} \frac{1}{^\circ\text{C}}$$

$$\alpha_{\text{quartz}} = 0.59 \times 10^{-6} \frac{1}{^\circ\text{C}}$$