# Explicit Physics Solutions Part II: Electricity, Magnetism, Waves

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**Abstract**

The need for a large selection of explicit solutions to physics problems is clear to any who teach the subject. Text books give few example problems. Often as many as half are trivial. Class time is limited and students frequently ask for more examples. Existing solution manuals for text books are terse in the extreme. Hand written solutions are well received by students, but do not reproduce well. Further, solutions are only made available after homework is turned in. Thus this work was begun.

For best results, *try each problem before reading the solution.*

## Contents

# 1 Electrostatic Force

### Problems

1. A 0.03 mC charge sits at the origin, a -0.07 mC charge sits on the x-axis 3.1 m away, and a0.061 mC charge sits on the y-axis 9.1 m away. Find the net force vector on the charge at the origin.

2. Three equal charges of 0.213 $\mu$C sit at corners of a right triangle. One at the origin, one along the x-axis at 1.0m and one along the y-axis at y=2.2m. Find the force on the charge at the origin.

3. Two equal charges of charge $q$ sit a distance d apart. Of course, they repel one another. Can a third charge be placed somewhere such that the net force on each and every charge in the problem (now 3) is zero? If so, where must the charge sit, and what magnitude must it have?

### Solutions

1. The force has two components: a positive x component and a negative y component. Each is given in magnitude by: $F = kq_1q_2/r^2$. Computing we find: $F_x = 1.97$N and $F_y = -0.199$N.

2. At the origin, there are two forces, one in negative x direction, one in the negative y direction (like charges repel). The magnitude of each is given by Coulomb's law: 84.4 N in the -y direction and 408N in the -x direction.

3. Yes. A third charge can be placed between the two, a distance d/2 from each. It must be negative to attract back toward itself each of the otherwise repelling charges. Since the distance to each charge q is half as large, to give the same total force, the new charge, let us call it q' must have a value given by: $q' = q/4$.

# 2 Gauss' Law and the Electric Field

### Problems

1. Use Gauss' law to find the electric field inside and outside a uniformly charged coaxial cable. The inner wire has charge per unit length $\lambda_1$ on it and the outer cylindrical wire (the cladding), at radius $b$ has a charge per unit length $-\lambda_1$ on it (total charge is zero). There are two regions of interest: $r < b$ and $r > b$. Find the electric field in each region (as a function of $r$).

2. Use Gauss' law to find the electric field inside and outside a pair of charged spherical shells. The inner shell has charge $Q_1$ on it (evenly distributed over it's surface) and a radius of $a$ and the outer shell, at radius $b$ where $b > a$ has a charge $-Q_1$ on it (total charge is zero). There are three regions of interest: $r < a$, $a < r < b$ and $r > b$. Find the electric field in each region (as a function of $r$).

3. Three equal charges of 0.35 mC sit at corners of a right triangle. One at the origin, one along the x-axis at 1.0m and one along the y-axis at y=2.2m. Find the electric field vector at x=1.0m, y=2.2m, the fourth corner of the rectangle.

### Solutions

1. The total net charge is zero outside the cable, thus the electric field there is zero. Inside the cable, the total charge per unit length is $\lambda_1$. We sketch a cylindrical gaussian surface centered on the cable of radius r with $r < b$ and a length $l$. Note that the result should be independent of $l$, since we could have drawn the surface as large or as small as we like. By symmetry, we know the electric field points radially outward from the inner wire. Thus the flux through the gaussian surface is only through the side of the cylindar, not the endcaps (the dot product between the electric field vector and area vector is zero there). Thus we find the electric field is given by: $E 2\pi l r = \lambda_1 l/\epsilon_O$. As expected, the factors of $l$ cancel and we find: $E = \lambda_1/(2\pi r)$.

2. Again, outside the shells, the total enclosed charge is zero, thus there is no electric field there. Inside the inner shell, there is no enclosed charge, and the field is zero there as well. Between the shells we find the electric field via gauss' law. We construct a gaussian surface of radius r, where $a < r < b$. We find: $E 4\pi r^2 = Q_1/\epsilon_O$ and thus: $E = \frac{Q_1}{4\pi\epsilon_O r^2}$ which is the result for a point charge.

## 3 Electrostatic Potential and Potential Energy

### Problems

1. The Voltage (V, also known as the electrostatic potential, or just the potential) has been measured along the x-axis at several points: at x=0, V=2, x=1, V=1 and at x=2 V=0. If the values of V are in volts, and x in meters, find the electric field vector (x-component) in this region.

2. In a five milimeter region of space, the electric field has a uniform magnitude of 2510 Volts/meter to the right. What is the voltage across this

space? How much work is required to move an electron from the left to right of this space?

3. When two equal charges are 1.0 m away, the energy of the system is 1.0 Joule. Find the charge of each. Find the force between them. Is it attractive or repulsive? Take the zero of energy, as usual, to be at infinity.

### Solutions

3. The energy is $U = kq^2/r$ and so $1J1m = kq^2$ and since we know the value of $k$, we find: $q = 1.05 \times 10^{-5}$.

## 4 Circuits I: Resistor and Resistor/Capacitor

### Problems

1. A resistor R and capacitor C are in series with a battery voltage V, as show below. At t=0 the switch is closed, and at this time there is no charge on the capacitor. For $t > 0$:

(a) Write down the differential equaiton for the charge Q on the capacitor.

(b) Show, by substitution, that $Q(t) = CV(1-e^{-t/\tau})$ is a solution. Find $\tau$.

(c) Find the current as a function of time.

(d) Find the energy stored in the capacitor, as a function of time.

2. A battery of voltage V is connected directly to a resistor of resistance R, followed by, in parallel, a capacitor of value C and (another) resistor of value R. Find the current coming out of the battery in this circuit: (A) at t=0 and (B) as t goes to infinity. The capacitor is initially uncharged.

3. A resistor is made of a rectangle of length L and width W and height H of material with resistivity p. Which of the following will double the resistance: (cirle all that apply):

(i) double the length
(ii) double the width
(iii) half the length
(iv) half the height
(v) decrese the height by an ammount less then halving it (e.g. multiply by 3/4, say)
(vi) double the resistivity
(vii) half the resisitivity

**Solutions**

1. (a) $V - IR - Q/C = 0$.

$V - RdQ(t)/dt - Q(t)/C = 0$

(b) Substitute this into the above and show that $\tau = RC$.

(c) $I(t) = dQ/dt = CV\tau e^{-t/\tau}$.

(d) $E = 1/2CV^2$, but $Q = CV$ and thus: $E(t) = 1/2Q(t)^2/C$ so just substitute the given $Q(t)$ from above into this equation.

2. A battery of voltage V is connected directly to a resistor of resistance R, followed by, in parallel, a capacitor of value C and (another) resistor of value R. Find the current coming out of the battery in this circuit: (A) at t=0 and (B) as t goes to infinity. The capacitor is initially uncharged.

(A) at time zero, the voltage across the resistor is zero, thus all current flows through it and none through the parallel resistor R. The total resistance of the loop is R and thus the current from the battery is $I = V/R$.

(B) at time infinity, the capacitor is fully chaged and no current flows through it. The net resistance of the remaining loop is 2R, and the current is: $I = V/(2R)$.

3. A resistor is made of a rectangle of length L and width W and height H of material with resistivity p. Which of the following will double the resistance: (cirle all that apply):

(i) double the length - yes it will double

(ii) double the width - no it will fall in half

(iii) half the length - no it will fall in half

(iv) half the height - yes halving the area will double the resistance

(v) decrese the height by an ammount less then halving it (e.g. multiply by 3/4, say) no

(vi) double the resistivity - yes

(vii) half the resisitivity - no, will half.

# 5 Magnetic Force and Field

## Problems

1. If a particle with a change of 0.5 C travels at 2.0 m/s along the x axis in a magnetic field of 1.0 T x + 2.0 T y + 3.0 T z, find the magnitude and direction of the resultant force on the particle.

2. For i, ii and iii below, indicate which form or forms of magnetism exhibit each following behivior (faramagnetism, paramagnetism, diamagnetism)?

i. Magnetic field aligns opposite to external field.

ii. Remains magnetized in the absence of external fields.

iii. Magnetic field aligns with external field.

3. You grab a perminant magnet and bring the North pole toward one end of an unknown bar of material. The unknown is repelled. You bring the South pole of the same preminant magnet toward the same end of the unknown bar and it is also repelled.

What kind of material (ferro-, para- or diamagnet) is the unknown? Or is it possible to tell?

What would happen if the North pole of your preminant magnet were to approach the opposite end of the unknown material? Is it possible to tell?

## Solutions

1. If a particle with a change of 0.5 C travels at 2.0 m/s along the x axis in a magnetic field of 1.0 T x + 2.0 T y + 3.0 T z, find the magnitude and direction of the resultant force on the particle.

Here we need to use the vector cross product between v and B vectors, and then multiply by the charge, q:

$$\begin{bmatrix} i & j & k \\ 2 & 0 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

and so vxB becomes: 4k - 6j = (2k - 3j)N of force.

The force is qvxB which is: -3 N in the y-direction, 2 N in the z-direction.

2. For i, ii and iii below, indicate which form or forms of magnetism exhibit each following behivior (faramagnetism, paramagnetism, diamagnetism)?

i. Magnetic field aligns opposite to external field.

Diamagnetic

ii. Remains magnetized in the absence of external fields.

Ferromagnetic

iii. Magnetic field aligns with external field.

Paramagnetic or Ferromagnetic

3. You grab a perminant magnet and bring the North pole toward one end of an unknown bar of material. The unknown is repelled. You bring the South pole of the same preminant magnet toward the same end of the unknown bar and it is also repelled.

What kind of material (ferro-, para- or diamagnet) is the unknown? Or is it possible to tell?

Must be diamagnetic.

What would happen if the North pole of your preminant magnet were to approach the opposite end of the unknown material? Is it possible to tell?

It will also repel.

## 6 Faraday's Law

### Problems

1a. The right hand half of the following circuit is in a magnetic field which has a strength which varies with time as: B(t) = 0.100 T + 0.050 T/s * t. Find the current flowing through the resistor as a function of time, t. Given R=50 ohms.

1b. A 10cm by 10cm region in the middle of the following circuit has a completely isolated magnetic field with strength given by B(t) = 0.222 T - 0.033 T/s * t. Find the current (if any) in the circuit as a function of time, given R=25 ohms.

### Solutions

1a. The right hand half of the following circuit is in a magnetic field which has a strength which varies with time as: B(t) = 0.100 T + 0.050 T/s * t. Find the current flowing through the resistor as a function of time, t. Given R=50 ohms.

Faraday's Law says: $V = -d/dt(BA)$. The magnetic field, B(t) is changing with time. The area is not, and so this term passes through the derivitive, leaving $V = -AdB/dt$. The B(t) has two terms and after the derivitive, only one remains and so we find: $V = -A0.050T/s$, where A is the area of the right hand side of the circuit, the area within the magnetic field, in this case 0.010 $m^2$. To find the current, we set V=RI, and thus I = $1 \times 10^{-5}$A.

1b. A 10cm by 10cm region in the middle of the following circuit has a completely isolated magnetic field with strength given by B(t) = 0.222 T - 0.033 T/s * t. Find the current (if any) in the circuit as a function of time, given R=25 ohms.

Quite similar to 1a. above. $V = -d/dt(BA)$. The area A is just the 10 cm by 10cm square. The current turns out to be: $1.32 \times 10^{-5}$A.

# 7 Circuits II: Inductor Circuits

## Problems

1. The inductance of a solenoid 7.00 cm long is 0.185 mH. a. If its cross-sectional area is 1.00 x $10^{-4}$ $m^2$, how many turns does the solenoid have?

b. If a current of 0.395 amps flows through the solenoid, what is the magnetic field near the center?

c. If you move a bar magnet through this solenoid, which way will the current flow? Draw a schematic to explain your answer, labeling the poles of the magnet, the direction you are moving it, and the direction of the current that is induced in the wire of the solenoid.

## Solutions

1. The inductance of a solenoid 7.00 cm long is 0.185 mH. a. If its cross-sectional area is 1.00 x $10^{-4}$ $m^2$, how many turns does the solenoid have?

We use the physical inductor equation: $L = \mu_0 NnA$, where $n$ is the number of turns per unit length, which we shall call $l$. Solving for N, we find: $N^2 = \frac{Ll}{\mu_0 A}$. Solving for N we find: $N = 321$ turns.

b. If a current of 0.395 amps flows through the solenoid, what is the magnetic field near the center?

$B = \mu_0 nI = 2.28$mT.

c. If you move a bar magnet through this solenoid, which way will the current flow? Draw a schematic to explain your answer, labeling the poles of the magnet, the direction you are moving it, and the direction of the current that is induced in the wire of the solenoid.

It will induce a current to generate a magnetic field which opposes the incoming external field.

# 8 Circuits III: AC Circuits

## Problems

1. An AC power supply, RMS voltage V, is connected to resistor R and then two devices in parallel: an inductor L and capacitor C. In terms of V, L, R and C what is the RMS current through each element, $I_c$, $I_L$, and $I_R$, at: (a) zero frequency and at (b) infinity (or very, very large) frequncy?

## Solutions

1. An AC power supply, RMS voltage V, is connected to resistor R and then two devices in parallel: an inductor L and capacitor C. In terms of V, L, R and C what is the RMS current through each element, $I_c$, $I_L$, and $I_R$, at: (a) zero frequency and at (b) infinity (or very, very large) frequncy?

At zero frequency: $X_L = \omega L$ goes to zero, thus the inductor acts like a wire. The capacitor reactance goes to infinity, and it acts like a broken circuit, thus $I_C = 0$. All current flowing through the resistor goes through the inductor and $I_R = I_L = V/R$.

At infinite frequency the roles are reversed: the inductor acts like a broken circuit (no current) and the capacitor acts like a wire, thus $I_L = 0$ and $I_R = I_C = V/R$.