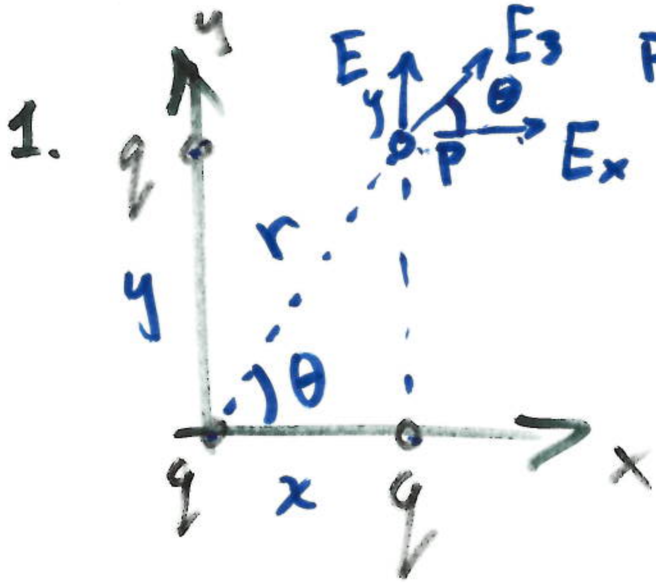
Sample Exam 1 Solution

Noon Lab

1.



Find $\vec{E}$ at P.

$$E = \frac{kq}{r^2}$$

$$E_x = \frac{kq}{(1m)^2} = \frac{9\times10^{9}\,\frac{Nm^2}{C^2} \cdot 0.35\times10^{-9}\,C}{(1m)^2}$$

$$E_x = 3.15\,\frac{N}{C}$$

$x = 1.0m$

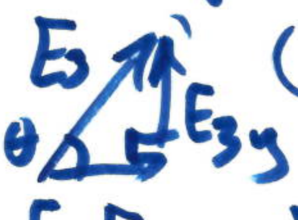
$y = 2.2m$

$q = 0.35nC$

$$E_y = \frac{kq}{(2.2m)^2} = 0.65\,\frac{N}{C}$$

$$E_3 = \frac{kq}{r^2} = 0.54\,\frac{N}{C}$$

$$r^2 = x^2 + y^2 = 1^2 + 2.2^2 = 5.84m^2$$



$$E_{3x} = E_3 \cos\theta$$

$$E_{3y} = E_3 \sin\theta$$

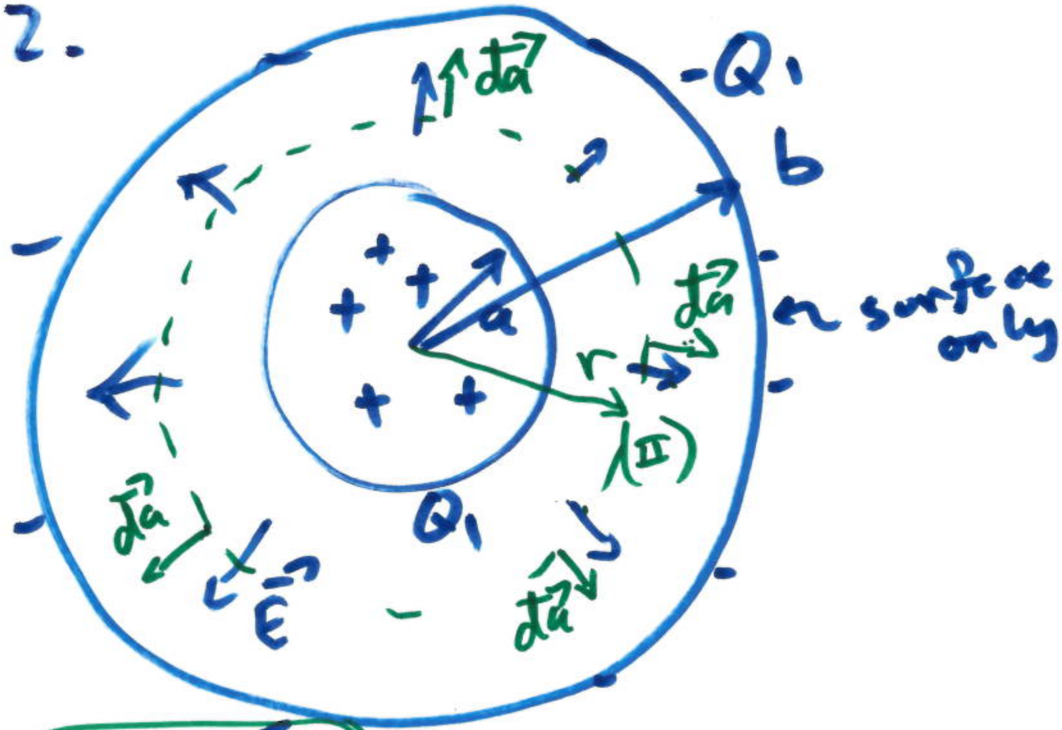
$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = 65.6^\circ$$

$$E_{3x} = 0.54\,\frac{N}{C} \cos 65.6^\circ = 0.22\,\frac{N}{C}$$

$$E_{3y} = 0.54\,\frac{N}{C} \sin 65.6^\circ = 0.49\,\frac{N}{C}$$

$$\vec{E}_{Tot} = \left(E_x + E_{3x},\ E_y + E_{3y}\right) = (3.37,\ 1.14)\,\frac{N}{C}$$

2.



$\boxed{\text{find } E(r)}$ in 3 regions:

I $r<a$

II $a<r<b$

III $b<r$

(II) middle

$$\oint \vec{E}\cdot\vec{da} = \frac{Q_{IN}}{\epsilon_0}$$

$\vec{E} \parallel \vec{da}$

$$\oint E\,da\,\underbrace{\cos\overset{0}{\theta}}_{1} = \frac{Q_1}{\epsilon_0}$$

$$\oint E\,da = \frac{Q_1}{\epsilon_0}$$

E is constant.

$$E \underbrace{\oint da}_{} =$$

$$E\,4\pi r^2 = \frac{Q_1}{\epsilon_0}$$

$$E = \frac{Q_1}{4\pi\epsilon_0 r^2} = \frac{k\,Q_1}{r^2}$$

(III) outside $r > b$

what changes?

$$\oint \vec{E} \cdot \vec{da} = \frac{Q_{IN}}{\epsilon_0}$$

$$E \, 4\pi r^2 = \frac{(Q_1 - Q_1)}{\epsilon_0} = 0$$

$$\boxed{E = 0}$$

No Direction.

(I) ... what is Q_{IN}?

invent $\rho = \frac{\text{charge}}{\text{volume}} = \frac{Q_1}{\frac{4}{3}\pi a^3}$

$$Q_{IN} = \rho \cdot \bar{V}_{IN} = \frac{Q_1}{\cancel{\frac{4}{3}\pi} a^3} \cdot \cancel{\frac{4}{3}\pi} r^3 = Q_1 \frac{r^3}{a^3}$$

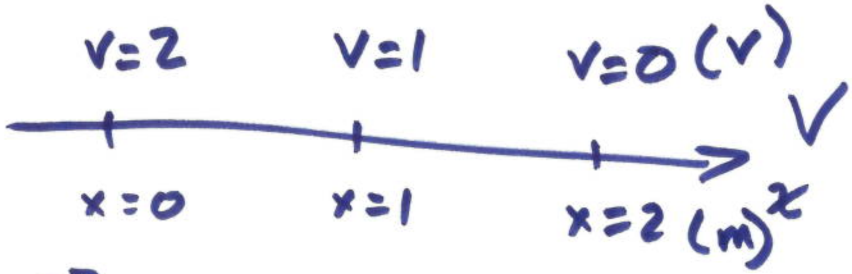
$$\oint \vec{E} \cdot \vec{da} = \frac{Q_{IN}}{\epsilon_0}$$

$$E \, 4\pi \cancel{r^2} = \frac{Q_1}{\epsilon_0} \frac{\cancel{r^3}^{\,1}}{a^3}$$

$$\boxed{E = \frac{Q_1 r}{4\pi \epsilon_0 a^3}}$$

$(r < a)$
region I

3.



$\vec{E}_x = ?$

↑ x-component

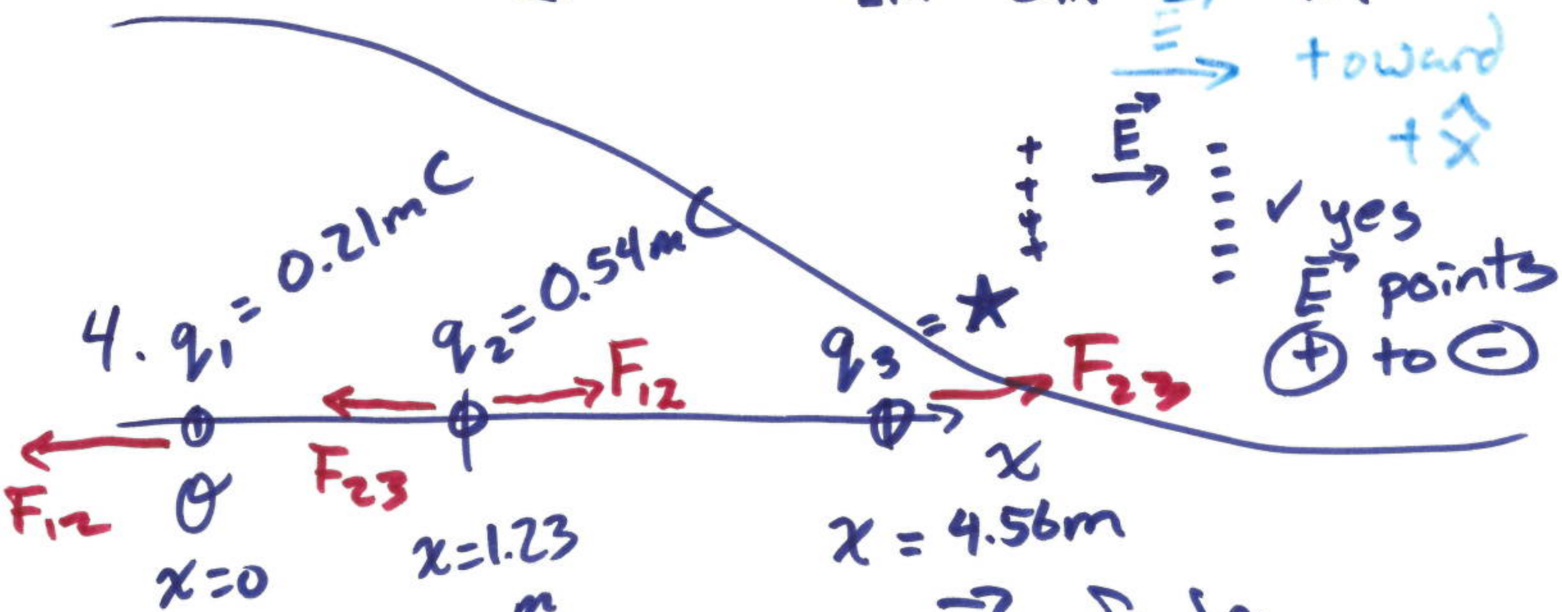
$$E_x = -\frac{dV}{dx}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{l}$$

pick any 2 points

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{(0-2)V}{2m - 0m} = 1\,\frac{V}{m}$$

$\vec{E}$ toward $+\hat{x}$



$\vec{E}$ ✓ yes $\vec{E}$ points ⊕ to ⊖

If q_2 has zero net $\vec{F}$, find q_3.

$$F_E = k\frac{|q_1 q_2|}{r^2}$$

$$F_{23} = F_{12} \quad \text{if net } \vec{F} = 0.$$

$$\frac{\cancel{k}\,\cancel{q_2}\,q_3}{(4.56-1.23)^2\,m^2} = \frac{\cancel{k}\,q_1\,\cancel{q_2}}{(1.23m)^2}$$

$$q_3 = q_1 \frac{(3.33\,m)^2}{(1.23\,m)^2} = 0.21\,mC \left(\frac{3.33}{1.23}\right)^2$$

$$\boxed{q_3 = 1.54\,mC}$$

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5. $V(x) = 1.0V + 0.12\frac{V}{m}x$ along x-axis.

Find $E_x(x)$ and the $\vec{F}_x$ on a q

$q = 0.019\,mC$ at $x = 0.312\,m$.

$$E_x = -\frac{\partial V}{\partial x}\hat{x} = -\left[0 + 0.12\frac{V}{m}\right] = -0.12\frac{V}{m}\hat{x}$$

$$\vec{F}_x \text{ on} q = q'\vec{E} = 0.019\underbrace{mC}_{10^{-3}C} \times \Big(-0.12\underbrace{\tfrac{V}{m}}_{\frac{N}{C}}\Big)\hat{x}$$

$$= -2.28 \times 10^{-6} N\,\hat{x}$$

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6. $U = q'V$ $\qquad U = k\frac{q_1 q_2}{r}$

If $U<0$, $q_1 q_2$ have opposite signs.

(a) is True.

(b) True, opposites attract.

(c) True, they are attracted.

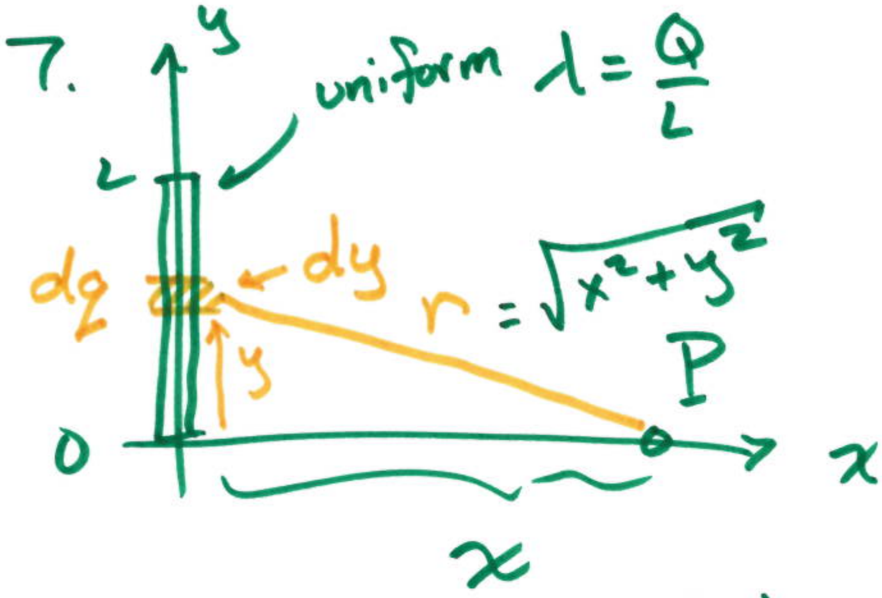
(d) True by 3rd law $\vec{F}_{12} = -\vec{F}_{21}$

(e) Ditto

(f) True they are attracted

(g) FALSE. Although $a_1 = a_2$, if $m_1 = m_2$, the a is a vector and $\vec{a_1} = -\vec{a_2}$.

7.



Set up integral to solve V at P.

Given: L, x, λ, Q, constants find V.

$$V = \frac{kq}{r} \qquad \int dV = \int k \frac{dq}{r}$$

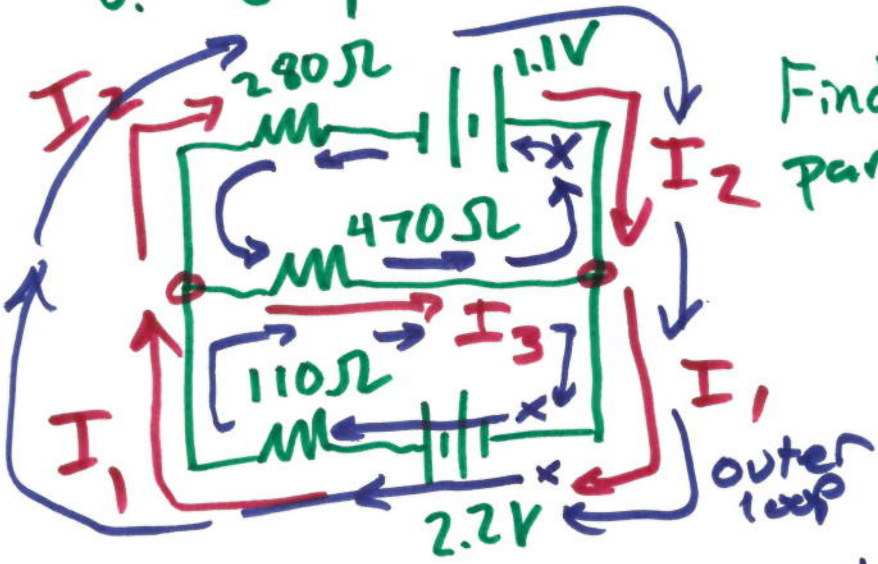
$$V = \int \frac{k\,dq}{r} = k \int \frac{dq}{r} \quad \dashrightarrow \quad \int_0^L dy(\;)$$

$$\lambda = \frac{Q}{L} = \frac{dq}{dy} = \text{constant}$$

$$dq = \lambda\,dy$$

$$V = k \int_{y=0}^{L} \frac{\lambda\,dy}{r} = \boxed{k\lambda \int_0^L \frac{dy}{\sqrt{x^2+y^2}} = V}$$

8. Simple DC Circuit



Find I through each part of circuit.

$$\sum I_{IN} = \sum I_{OUT}$$

$$\sum V_{loop} = 0$$

$$V = IR$$

lower loop: $\sum V_{loop} = 0$

$$\sum I_{IN} = \sum I_{OUT}$$

$$I_1 = I_2 + I_3$$

$$+2.2V - I_1\,110\Omega - I_3\,470\Omega = 0$$

outer

$$+2.2V - 110\Omega\, I_1 - I_2\,280\Omega + 1.1V = 0$$

upper loop:

$$-1.1V + I_2\,280\Omega - I_3\,470\Omega = 0.$$

= lower − outer equation.

lower: $\frac{2.2}{470} - I_1\frac{110}{470} = \frac{470}{470} I_3$

$$0.00468 - 0.23 I_1 = I_3$$

4.68×10^{-3}

1.1+2.2

outer: $\frac{3.3}{280} - \frac{110}{280} I_1 = \frac{280}{280} I_2$

$$0.0118 - 0.393 I_1 = I_2$$

$$I_1 = I_2 + I_3$$

$$I_1 = 0.0118 - 0.393 I_1 + 0.00468 - 0.23 I_1$$

$$I_1 \left(\underbrace{1 + 0.393 + 0.23}_{1.623} \right) = 0.0165$$

$$I_1 (1.623) = 0.0165$$

$$I_1 = 0.0102 \text{ A} = \boxed{10.2 \text{ mA}}$$

$$I_2 = 0.0118 - 0.393(0.0102)$$

$$\boxed{I_2 = 7.80 \text{ mA}}$$

$$I_3 = 0.00468 - 0.23 I_1$$

$$= \cancel{0.885} \text{ mA} \quad \boxed{2.33 \text{ mA}}$$