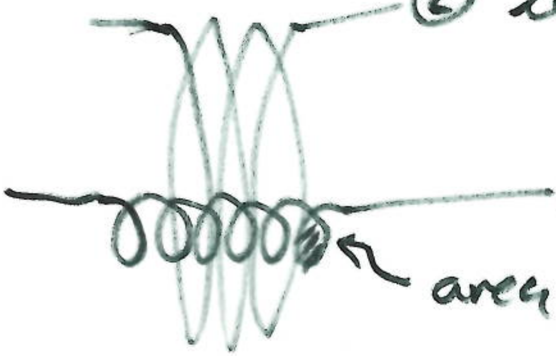
Physics 201 — Day 15

# Inductance:

No Lab This Friday

Mutual Inductance, M

Self Inductance, L




② length $l_2$ # turns $N_2$, area $A_2$

$n_2 = N_2 / l_2$

① length $l_1$, # turns $N_1$, area $A_1$ $\Big\}$ $n_1 = \dfrac{N_1}{l_1}$

There is $V$ in ② due to changes in $I_1$.

$$V_2 = -\frac{\partial \Phi_{B_2}}{\partial t} \quad \text{Faraday's Law}$$

$$B_{coil} = \mu_0 I n \qquad n = \frac{N}{l}$$

We assume: $B$ inside coils is $\mu_0 I n$ and $0$ outside (approx).

$$\Phi_B = BAN$$

what is $\Phi_{B \text{ in } 2}$ due to 1? $= B_1 A_1 N_2$

$$V_2 = \frac{d}{dt}\Phi_{B_2} = A_1 N_2 \frac{\partial B_1}{\partial t}$$

$$B_1 = \mu_0 I_1(t) n_1$$

$$V_2 = \underbrace{A_1 N_2 \mu_0 n_1}_{M} \frac{\partial I_1(t)}{\partial t}$$

the Mutual Inductance $\quad V_2 = M \frac{\partial I_1}{\partial t}$

Can show $V_1 = M \frac{\partial I_2}{\partial t}$ with the <u>same</u> $M$.

$$\cancel{V_1} = \underset{\text{due to 2}}{\Phi_{B_1}} = B_2 A_1 \underbrace{(n_1 l_2)}_{\text{\# of turns of 1 inside 2}}$$

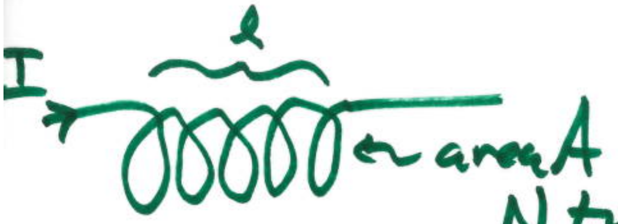
$$B_2 = \mu_0 I_2 n_2$$

$$\Phi_{B_1} = \mu_0 I_2 n_2 A_1 n_1 l_2 \qquad n_2 \ldots l_2 \to N_2$$

$$V_1 = M \frac{\partial I_2}{\partial t_2}$$

$$M = \mu_0 A_1 n_1 N_2 \quad \checkmark \text{ yay}$$

# Self Inductance, $L$




$$\Phi_B = BAN$$

$$B = \mu_0 I n$$

$$V = -L\frac{dI}{dt}$$

$$n = \frac{N}{l}$$

$$V = -\frac{d\Phi_B}{dt} = \underbrace{AN\mu_0 n}_{\text{Self-Inductance } L}\frac{dI}{dt}$$

$$L = \mu_0 AN^2/l = \mu_0 A n N$$

100 turns coil 3 cm length diameter 1.0 cm.

Find $L$ $\qquad \mu_0 = 4\pi \times 10^{-7}\,\frac{Tm}{A}$

$A = \pi r^2$

$L = 3.3\times10^{-5}$ H $\qquad$ H = Henries $= \frac{1\,V\cdot s}{A} = 1\,\Omega\cdot s$

Inductor as circuit element:

In DC: $$V_L = -L\frac{\partial I}{\partial t}$$

resists <u>change</u> in $I$, allows steady current through.

Compare with capacitor: $V_c = \dfrac{Q(t)}{c}$ $\qquad Q = CV$

resists DC (or steady) currents
allows rapidly changing AC

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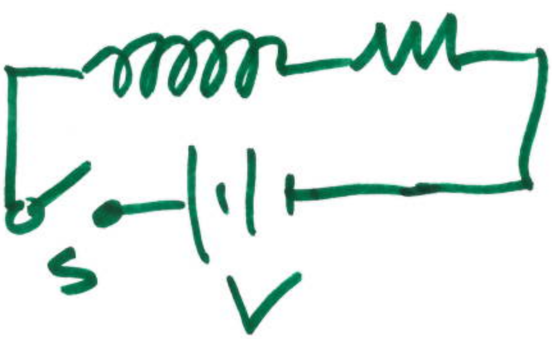
LR Circuit:

at $t=0$, close switch
find: $I(t)$




$$\sum_{loop} V = 0$$

$$V - L\frac{\partial I(t)}{\partial t} - IR = 0$$

<u>Know</u>: at $t=0$, $I=0$.

long time, 

$$I_{\text{long time}} = \frac{V}{R}$$


try: $I(t) = I_\infty\left(1 - e^{-t/\tau}\right)$

show: $I_\infty = V/R$

and find $\tau$.

$$\frac{dI(t)}{dt} = \frac{I_\infty}{\tau} e^{-t/\tau}$$

$$V - L \underbrace{\frac{I_\infty}{\tau} e^{-t/\tau}}_{dI/dt} - \Big(\underbrace{I_\infty\left(1 - e^{-t/\tau}\right)}_{I(t)} R\Big) = 0$$

$$V - I_\infty R - L \frac{I_\infty}{\tau} e^{-t/\tau} + I_\infty R e^{-t/\tau} = 0$$

for any $t$

$I_\infty = \frac{V}{R}$ ✓ yay!

$$\cancel{I_\infty} R = L \frac{\cancel{I_\infty}}{\tau}$$

$$\boxed{\tau = \frac{L}{R}}$$

Compare with $\tau = RC$ for RC Circuit.

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Show: $\frac{\text{energy}}{\text{volume}}$ in $\vec{B}$ is $\frac{B^2}{2\mu_0}$

$$\text{energy} = \int \text{Power}\, dt$$

$$\text{Power } P = IV$$

$$V \text{ inductor } L\frac{dI}{dt}$$

$$\text{energy} = \int I(t) L \frac{dI(t)}{dt} dt$$

clearer ~~chain rule~~ application

$$E = \frac{1}{2} L I^2$$

integration by parts

$$\int u\,dv = uv - \int v\,du$$

$$u = I(t) \quad dv = \frac{dI}{dt} \quad \text{so } v = I$$

$$\int I\,dI = I^2 - \int I\,dI$$

$$2\int I\,dI = I^2$$

$$\int I\,dI = \frac{1}{2} I^2$$

$$E = \frac{1}{2} L I^2 \qquad \left(\text{compare } E = \frac{1}{2} C V^2\right)$$

$$B = \mu_0 n I$$

plug in

Volume of coil

$$E = lA \frac{B^2}{2\mu_0}$$

$$\boxed{\frac{E}{\text{Volume}} = \frac{B^2}{2\mu_0}}$$