Quiz 2: Gauss' Law

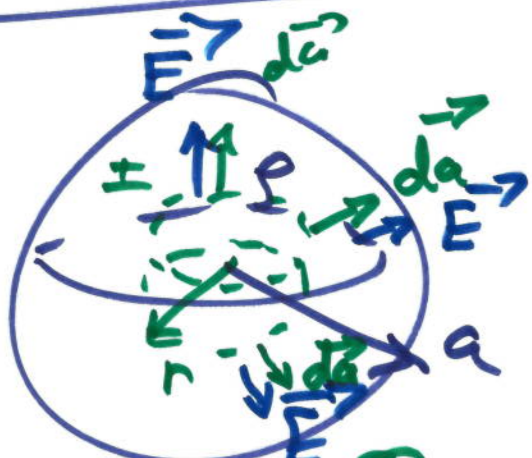
Quiz 2 Solution

Exam 1 on Friday

Today: any questions?

magnetism

- materials
- due to a current
- Ampere's Law



2 regions

$r<a$

$r>a$

$$\oint \vec{E}\cdot\vec{da} = \frac{Q_{IN}}{\epsilon_0}$$

$\vec{E} // \vec{da}$ $\theta = 0°$

$\cos 0 = 1$

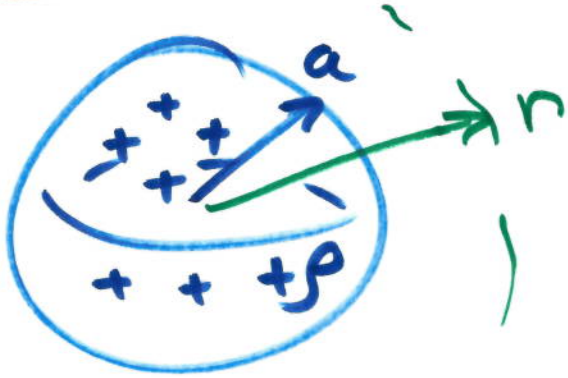
$$\oint E\,da = \frac{Q_{IN}}{\epsilon_0}$$

$$E\underbrace{\oint da}_{} = \frac{Q_{IN}}{\epsilon_0} = \rho\frac{V_{IN}}{\epsilon_0}$$

$$E\,\cancel{4\pi}(\cancel{r})^2 = \rho\frac{\cancel{\tfrac{4}{3}\pi}\, r^{\cancel{3}\,1}}{\epsilon_0}$$

$$\boxed{E = \rho\frac{r}{3\epsilon_0}}$$ $(r<a)$ inside

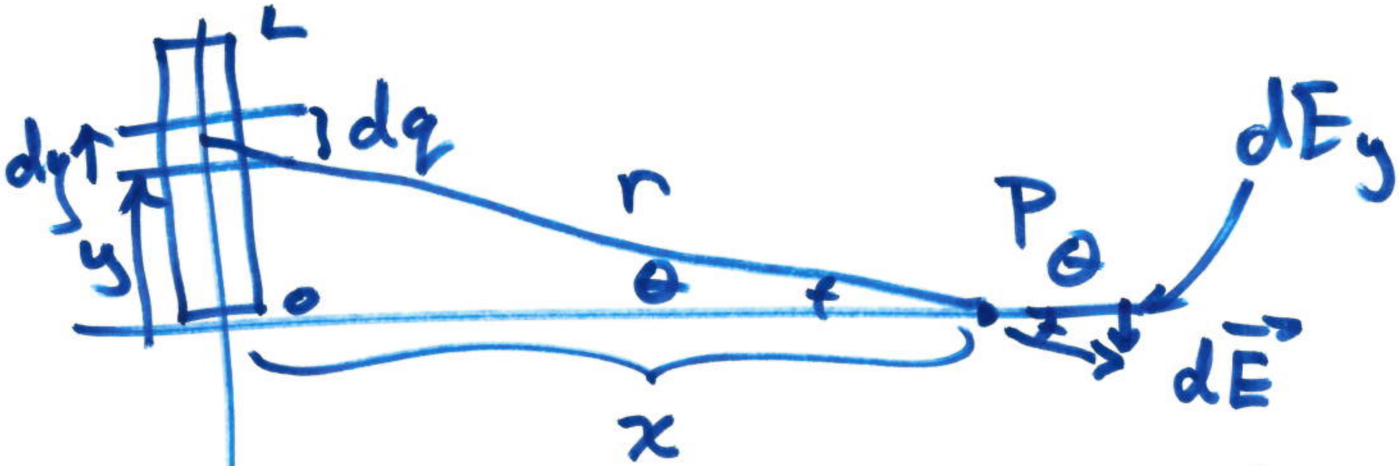
(II)



$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{IN}}{\epsilon_0}$$

$$E\,\cancel{4\pi}(r)^2 = \rho\frac{\cancel{\tfrac{4}{3}}\cancel{\pi}(a)^3}{\epsilon_0}$$

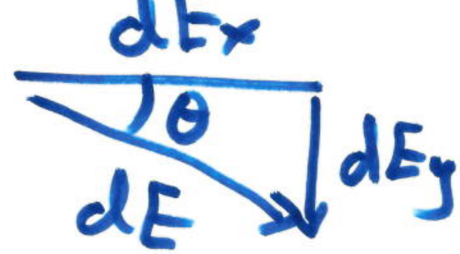
$$\boxed{E = \frac{\rho a^3}{3\epsilon_0 r^2}}$$ $(r>a)$ outside



Set up (but do not solve) integral to find E_y at P.

$$E = \frac{kq}{r^2} \qquad dE = \frac{k\,dq}{r^2}$$

$$\int dE_y = \int \frac{k\,dq}{r^2} \sin\theta$$



$$E_y = k \int \frac{dq}{r^2} \sin\theta$$

Given: x L λ on rod $\left(\frac{Q}{L}\right)$ k

$$\lambda = \frac{Q}{L} = \frac{dq}{dy} \qquad dq = \lambda\, dy$$

$$E_y = k \int_0^L \frac{\lambda\, dy}{r^2} \sin\theta$$

$$r^2 = x^2 + y^2 \qquad \sin\theta = \frac{y}{r}$$

$$E_y = k\lambda \int_0^L \left[\frac{dy}{x^2+y^2} \quad \frac{y}{r} \right]$$

$$r = \sqrt{x^2+y^2}$$

$$E_y = k\lambda \int_0^L \frac{dy \quad y}{\left(x^2+y^2\right)^{3/2}}$$

Magnetism:

2 sources: magnetic materials and electrical currents.

- materials:

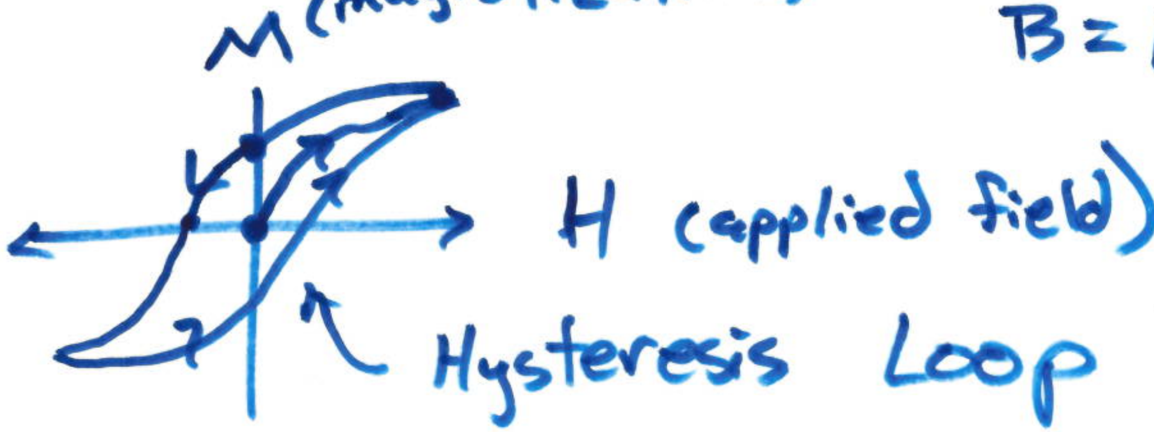
ferromagnet: generates a strong field aligned with external field. Remains when external field is gone (memory).

paramagnet:
generates a weak field aligned with external field. When external field is gone, it demagnetizes.

diamagnet:
generates a weak field opposes any external field. When external field is gone, it demagnetizes.

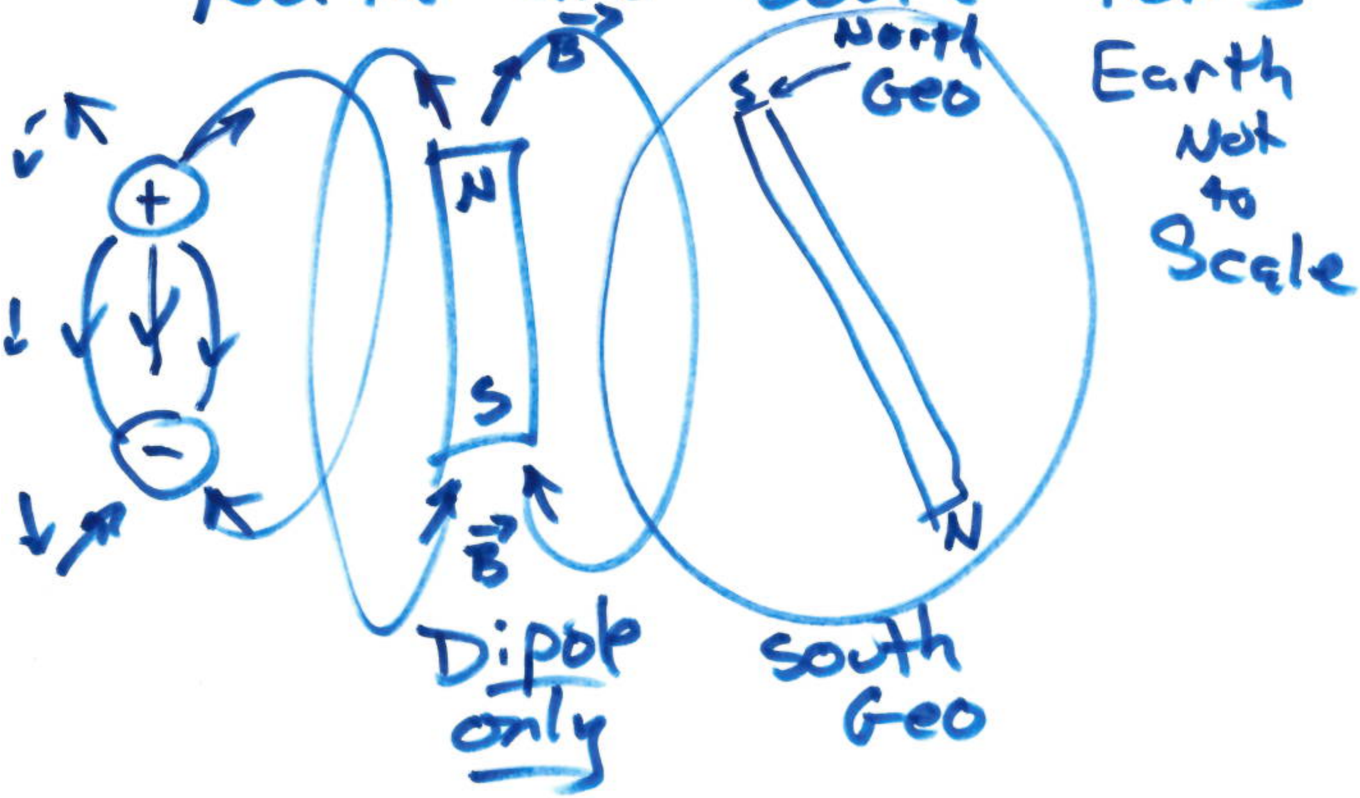
$\vec{B}$ the magnetic field.

$B = M + H$



Hysteresis Loop

ferromagnet has equal & opposite North and South Poles



P201 Exam 1

Exam 1		
90-96%	11	A
80	5	B
70	10	
60	12	
lower	11	

Exam 1 Corrections | Day 11

due Friday
Re-write whole problem
1 per page.
Staple onto front of exam in order.
Neatly.

Magnetic Fields Due to Currents:

Electromagnet:

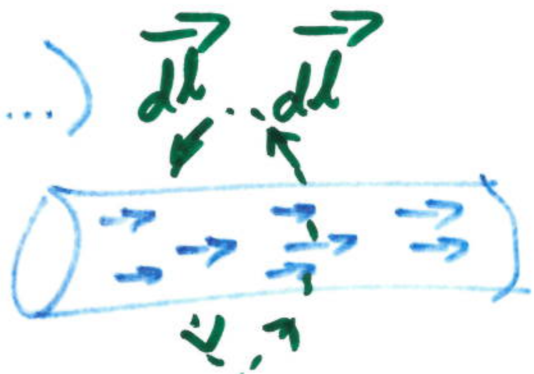
CBW
OK
✓
X

Ampere's Law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{IN}$$

cylindrical (plane...)

$$J = \frac{I}{\text{area}}$$



μ_0 = permiablity of free space

$$\mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$$

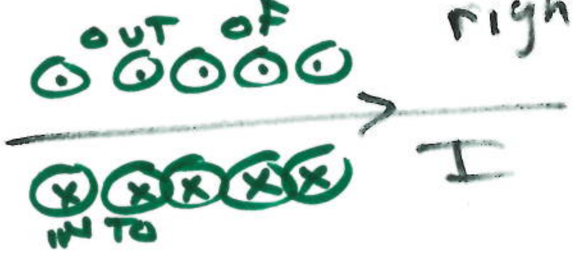
Tesla unit of B (magnetic field)

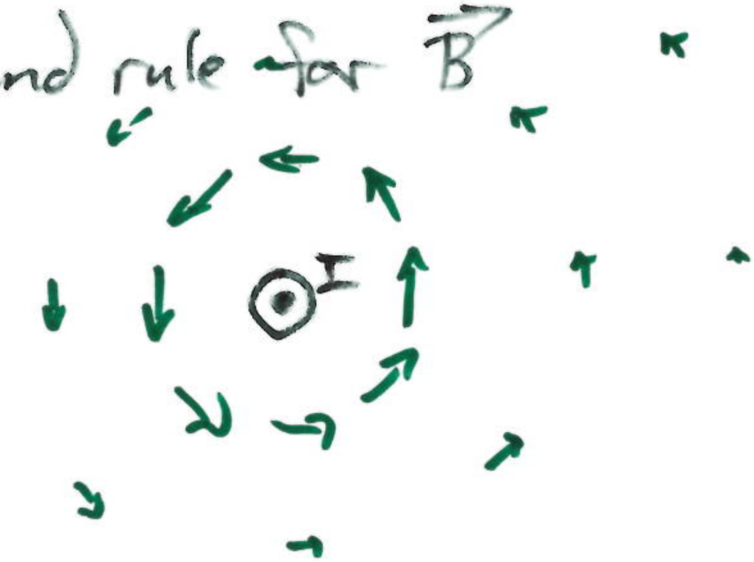
10000 gauss = 1T

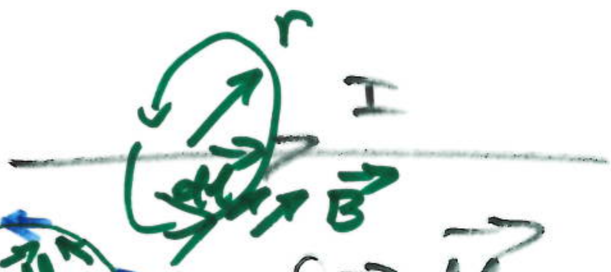
$B_{earth} \simeq \frac{1}{2}$ gauss

Direction of $\vec{B}$ due to current:

right hand rule for $\vec{B}$







closed loop

$$\oint \underbrace{\vec{B}\cdot d\vec{l}}_{\parallel \text{ or } \perp \text{ by construction.}} = \mu_0 I_{IN} = \mu_0 I$$

$$\oint B\,dl = \mu_0 I$$

$$B \oint dl = \mu_0 I$$

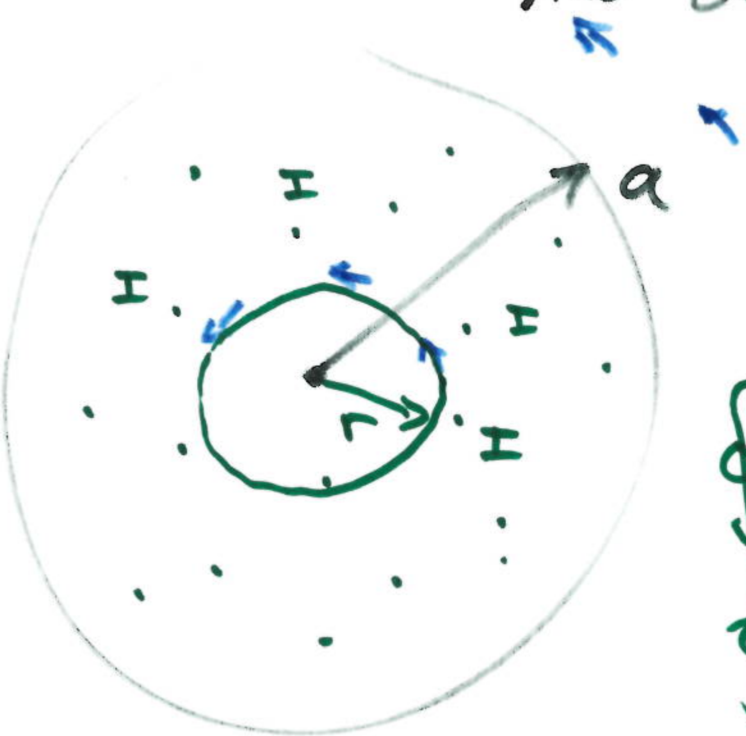
constant by rotational symmetry

$$B\,2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Thick wire radius a
with constant $J = \frac{I}{\text{area}}$

find $B(r)$ inside and outside
$(r < a)$ $(r > a)$



$r < a$ (inside)

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{IN}$$

$$\oint B\, dl = \mu_0 J (A_{IN})$$

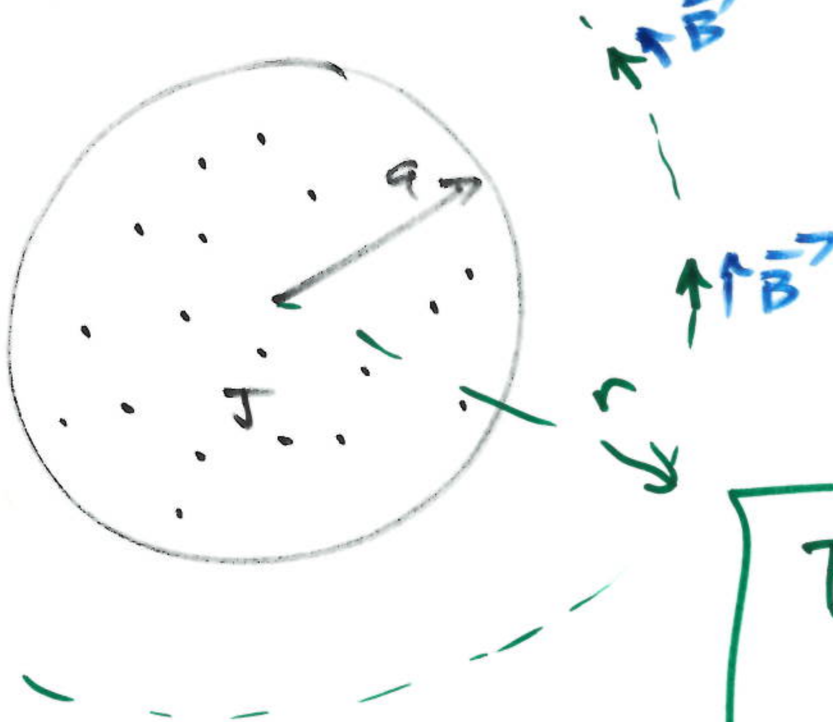
$$B \oint dl = \mu_0 J \pi r^2$$

$$B\, 2\cancel{\pi}\cancel{r} = \mu_0 J \cancel{\pi} r^{\cancel{2}}$$

$$\boxed{B = \frac{\mu_0 J r}{2}}$$

as $r \to 0$ $B \to 0$ ✓

$r > a$ (outside)



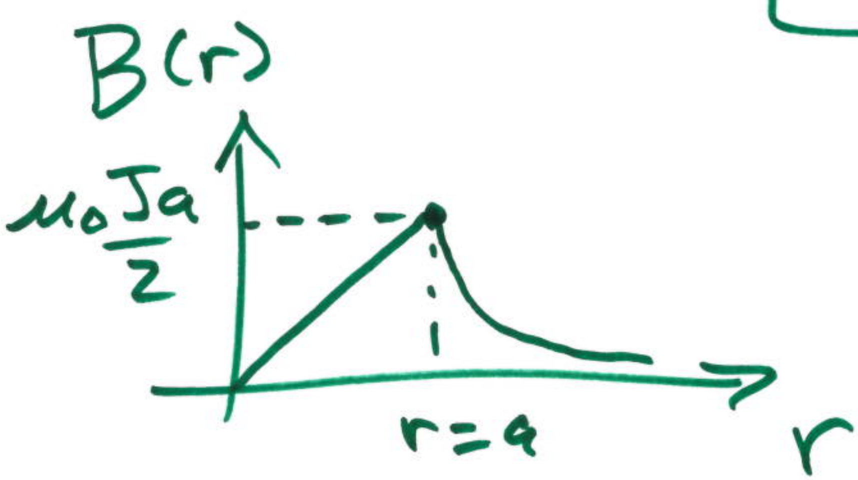
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{IN}$$

$$\oint B\,dl = \mu_0 J A_{IN}$$

$$B \oint dl = \mu_0 J \underbrace{\pi a^2}_{}$$

$$B\,2\cancel{\pi} r = \mu_0 J \cancel{\pi} a^2$$

$$\boxed{B = \frac{\mu_0 J a^2}{2r}}$$

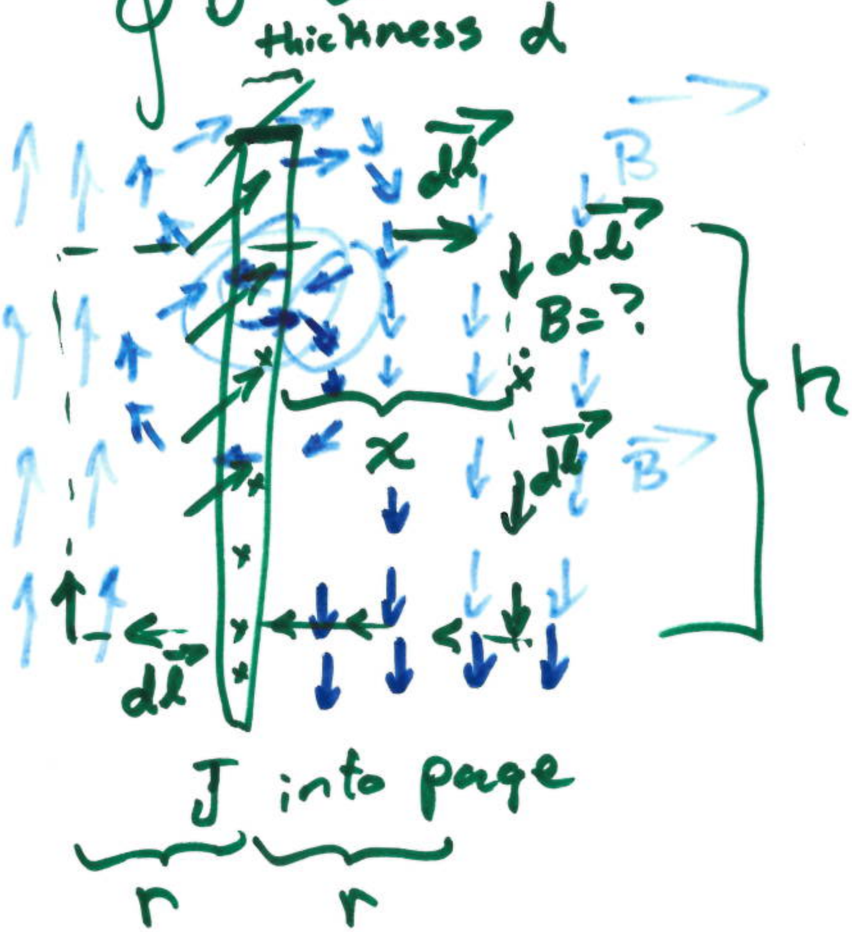


Physics 201 | Day 12

Exam 1 Corrections: due Fri.

Ampere's Law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{IN}$$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{IN}$$

$$Bh + Bh = \mu_0 J (\text{area})$$

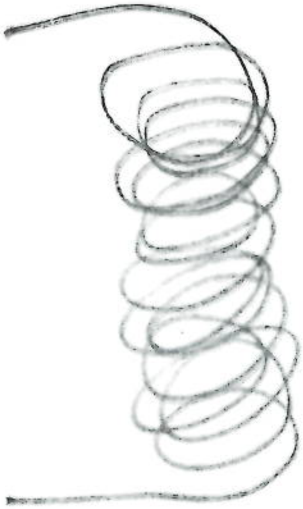
$$2B\cancel{h} = \mu_0 J d \cancel{h}$$

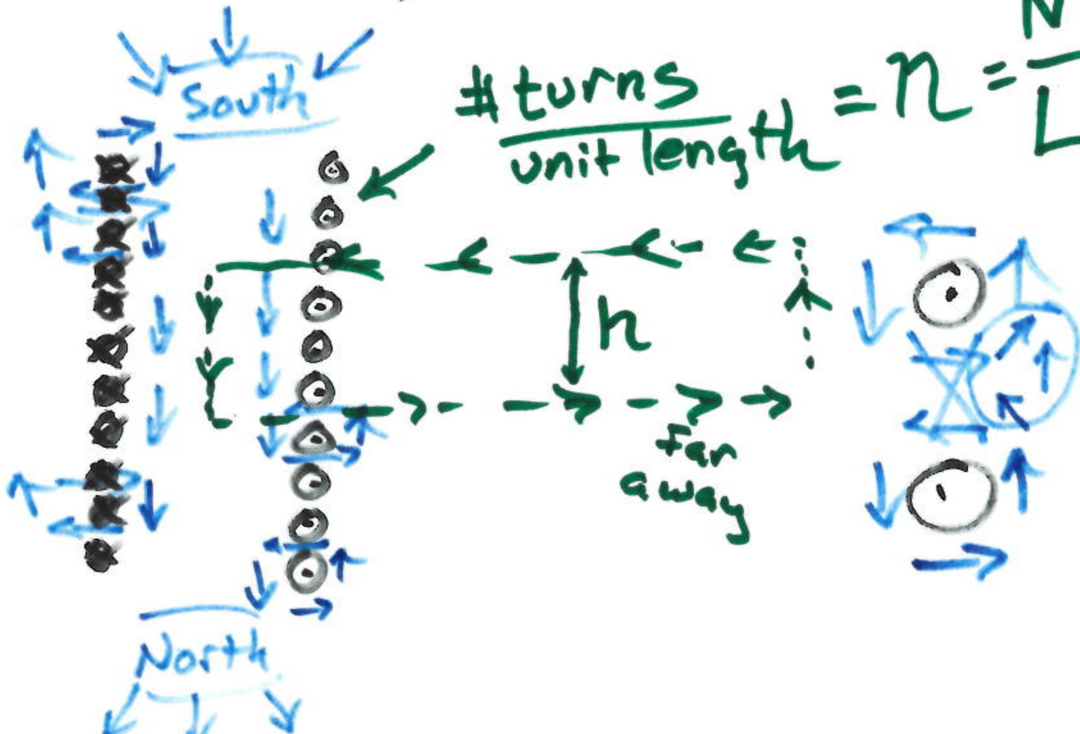
$$\boxed{B = \frac{\mu_0 J d}{2}}$$

due to ∞ sheet.

The Coil of wire: what is B inside?

Solenoid, inductor, coil





$$\frac{\#\text{turns}}{\text{unit length}} = n = \frac{N}{L}$$

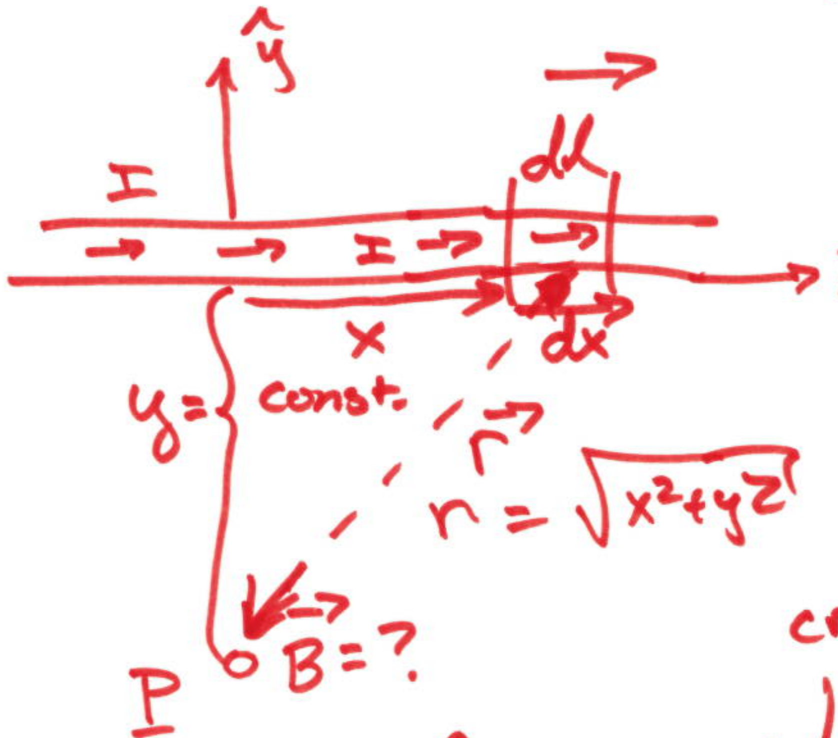
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{IN}$$

$$B\cancel{h} = \mu_0 I \underbrace{n \cancel{h}}_{\#\text{ loops in } h}$$

$$\boxed{B = \mu_0 I n}$$ uniform inside coil.

Biot-Savart Law

$$\int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l} \times \vec{r}}{r^3}$$



$$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

cross product

$$B = \frac{\mu_0}{4\pi}\int \frac{I\; dx\, \hat{x} \times (y\hat{y} + x\hat{x})}{(x^2+y^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi}\, y\ \hat{z} \underbrace{\int_{-\infty}^{\infty} \frac{dx}{(x^2+y^2)^{3/2}}}_{\frac{2}{y^2}}$$

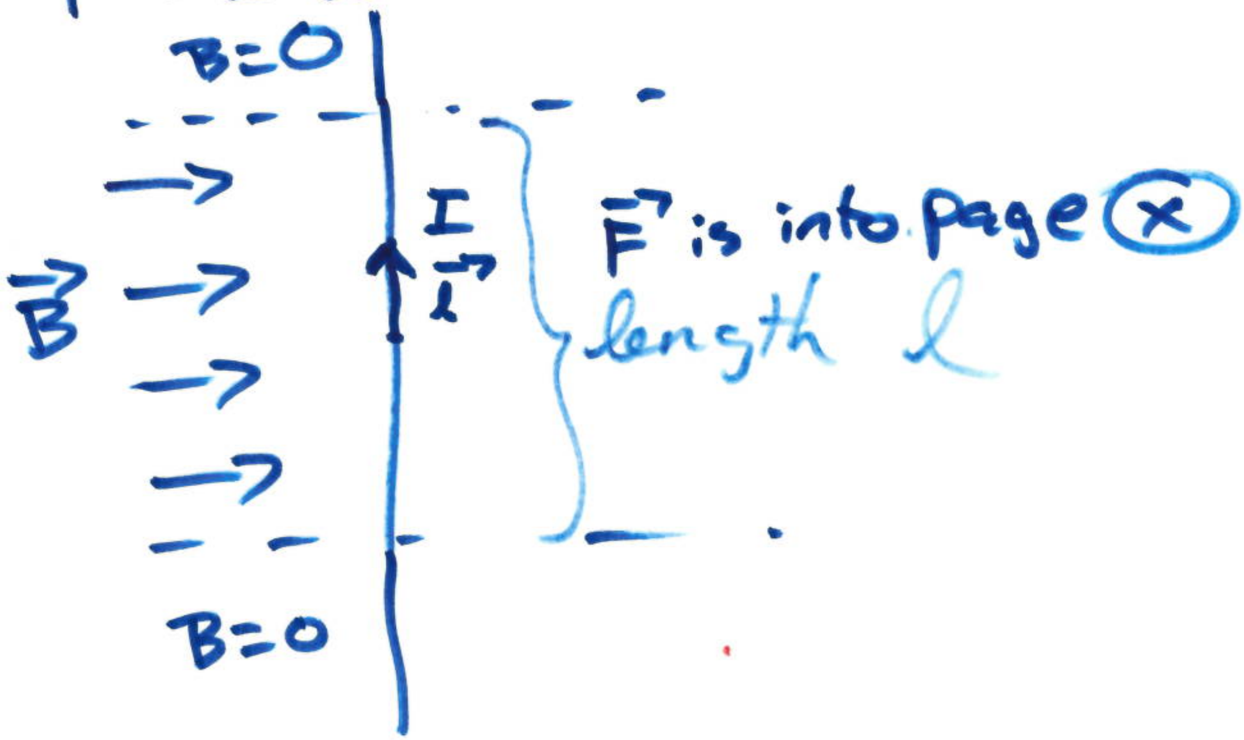
look up?

$$= \left(\frac{\mu_0 I}{2\pi y}\right)\hat{z}$$

Magnetic Force Laws:

Force on: moving charge
current carrying wire

$$\vec{F} = I\,\vec{l} \times \vec{B} = BIl\sin\theta$$

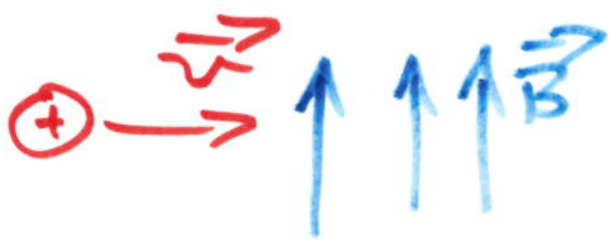


$$\vec{F} = q\,\vec{v} \times \vec{B}$$

charge

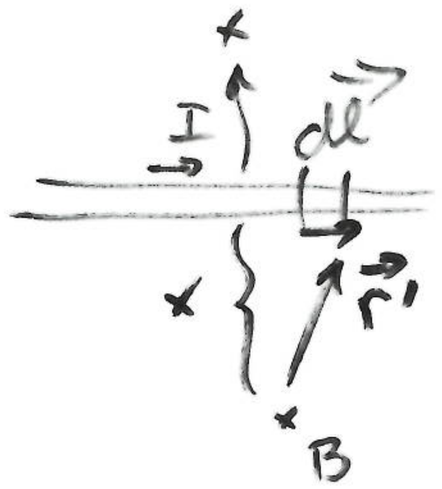
velocity vector of q

B magnetic Field



$\vec{F}$ $\odot$ out of page

$$B(r) = \frac{\mu_0}{4\pi} \int_C \frac{I\,\vec{dl} \times \vec{r}'}{|r'|^2}$$



∞ wire

$$= \frac{\mu_0}{4\pi} I \int_{-\infty}^{\infty} \frac{\vec{dy} \times (x\hat{x} + y\hat{y})}{(x^2+y^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi}\, x\, (-\hat{z}) \underbrace{\int \frac{dy}{(x^2+y^2)^{3/2}}}_{\cancel{2\frac{1}{x^2}} = \cancel{\frac{2}{x^2}}}$$

$$= \cancel{\frac{\mu_0 I}{2\pi x}}$$