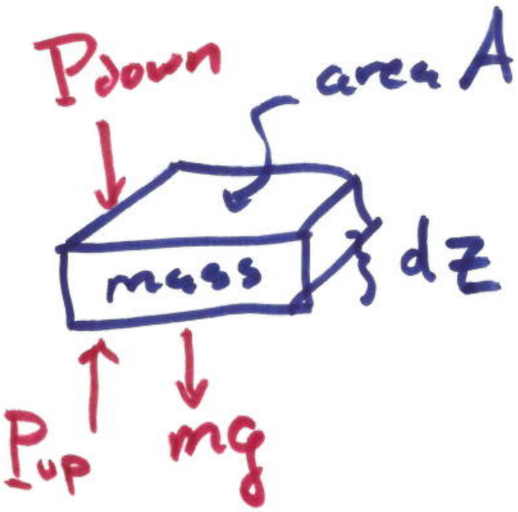
Physics 201 — Day 24

Application: the Atmosphere



$$\frac{F_{up}}{A} = P_{up}$$

$$F_{up} = P_{up}\cdot A$$

$$F_{Down} = P_{Down}\cdot A$$

$$\Sigma F = 0$$

$$F_{up} - mg - F_{Down} = 0$$

$$P_{up}\cdot\cancel{A} - \overbrace{\cancel{A}dz\,\rho g}^{m} - P_{Down}\cancel{A} = 0$$

the mass density

$$\rho = \frac{m}{V} = \frac{m}{A\cdot dz}$$

$$m = V\cdot\rho$$

$$m = A\cdot dz\cdot\rho$$

$$-P_{up} + P_{Down} \doteq dP$$

$$-dz\,\rho g - dP = 0$$

$$-\rho g\,dz = dP$$

$$-\rho g = \frac{dP}{dz}$$

$PV = N k_B T$ $\qquad$ $P = \underbrace{\frac{N}{V}}_{\text{close to } \rho = \frac{\text{mass}}{\text{volume}}} \cdot k_B T$

$$\frac{N}{V} = \left(\frac{k_B T}{P}\right)^{-1}$$

$$\rho = \underset{\substack{\uparrow \\ \text{mass of one air molecule}}}{m} \cdot \frac{N}{V}$$

$$\rho = m \left(\frac{k_B T}{P}\right)^{-1}$$

$$-\rho g = \frac{dP}{dz}$$

$$-m \left(\frac{k_B T}{P}\right)^{-1} g = \frac{dP}{dz}$$

$$-\frac{mg}{k_B T} P = \frac{dP}{dz}$$

If $T \simeq$ constant (Warning: not true)

let $\alpha = \frac{mg}{k_B T}$, $\qquad -\alpha f(x) = \frac{df(x)}{dx}$

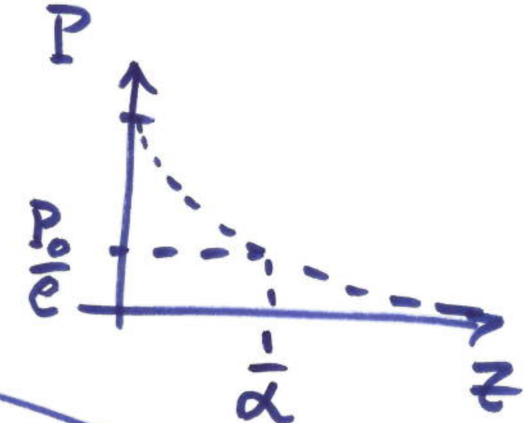
Try: $f(x) = f_0 e^{-\alpha x}$

$$P(z) = \underset{\substack{\uparrow \\ \text{Press at sea level?}}}{P_0} e^{-\alpha z}$$

$$\frac{dP(z)}{dz} = -\alpha P_0 e^{-\alpha z}$$

$$-\frac{mg}{k_B T} \overbrace{\cancel{P_0} \cancel{e^{-\alpha z}}}^{P(z)} \stackrel{?}{=} -\alpha \cancel{P_0} \cancel{e^{-\alpha z}}$$

$$\frac{mg}{k_B T} = \alpha \checkmark$$



There are a few ideal "processes" you can do. to a gas.

One is called Adiabatic → No heat exchanged

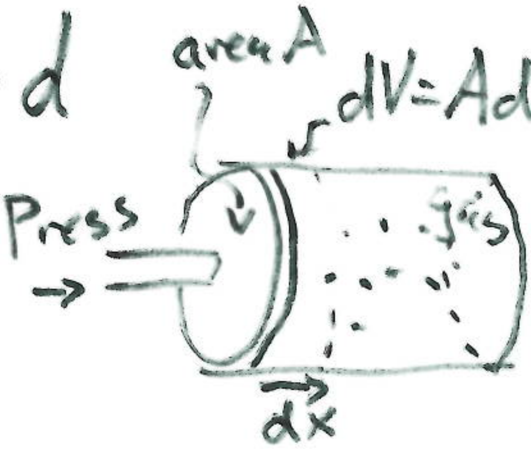
Work done on gas ⟶ internal energy of gas = thermal energy

$W \rightarrow U$

the heat, Q, is energy leaving system by, let's say, conduction.

Work = F · d

$P = \frac{F}{A}$



$dV = A\,dx$

$W = F \cdot dx$

$= P \cdot \underbrace{(A\,dx)}_{dV}$

$$\boxed{W = P\,dV}$$

work $\qquad$ change in internal energy

$$-PdV = dU = \frac{f}{2} N k_B \, dT$$

f = # of degrees of freedom = d.o.f.

$$\frac{f}{2} N k_B = C_V \leftarrow \text{const. volume}$$

specific heat $\qquad$ for: $Q \rightarrow U$

$$P = \frac{N k_B T}{V}$$

$$-\cancel{N k_B} T \frac{dV}{V} = \frac{f}{2} \cancel{N k_B} \, dT$$

$$-\int_{V_0}^{V_f} \frac{dV}{V} = \frac{f}{2} \int_{T_0}^{T_f} \frac{dT}{T}$$

$$-(\ln V_f - \ln V_0) = \frac{f}{2} (\ln T_f - \ln T_0)$$

$$-\ln\left(\frac{V_f}{V_0}\right) = \frac{f}{2} \ln\left(\frac{T_f}{T_0}\right)$$

$$\left(\frac{V_f}{V_0}\right)^{-1} = \left(\frac{T_f}{T_0}\right)^{f/2}$$

$$V_0 T_0^{f/2} = V_f T_f^{f/2}$$

$$VT^{f/2} = \text{const.}$$

want: replace $T \longrightarrow P$ and V

$$PV = N k_B T \qquad T^{f/2} = \frac{(PV)^{f/2}}{(N k_B)^{f/2}}$$

$$V\,(PV)^{f/2} = C_2$$ another constant with N in it.

$$P^{f/2}\, V^{f/2+1} = C_2$$ take $\frac{f}{2}$ root of both sides.

$$P^1\, V^{\gamma} = C_3$$ Divide exponent by $f/2$

$$\gamma = \frac{f/2+1}{f/2} = \frac{f+2}{f}$$

$$PV^{\gamma} = \text{const.} \quad (\text{Adiabatic}).$$

want: $\frac{dT}{dz}$ of atmosphere (Adiabatic) approximation.