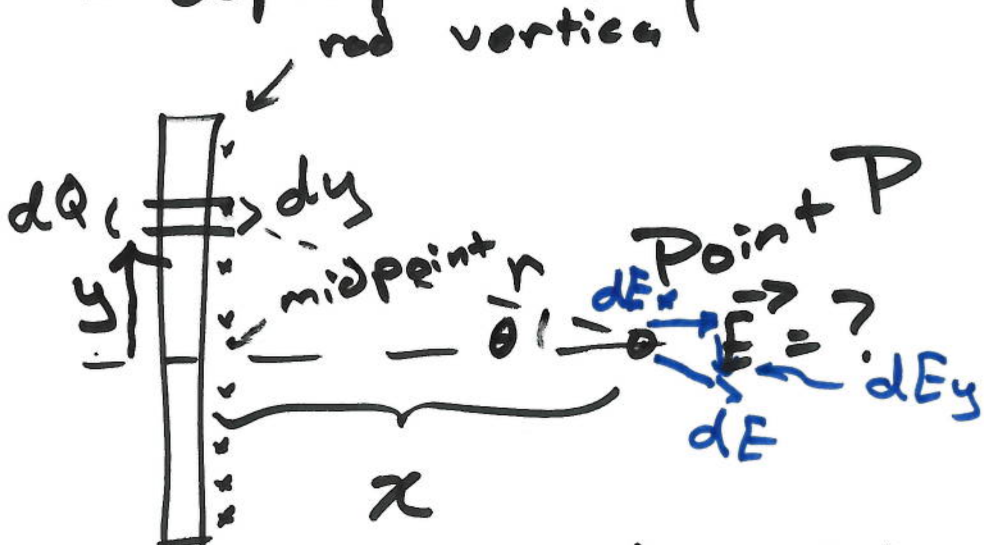
Physics 201 Day 3

various techniques to find $\vec{E}$ by Integration (dun dun DUN)

- Direct Integration
- Gauss' Law
- Superposition



$$E = k\frac{Q}{r^2}$$

rod length L, total charge Q

$$\int dE_x = k\int_{rod} \frac{dQ}{r^2}\cos\theta =$$

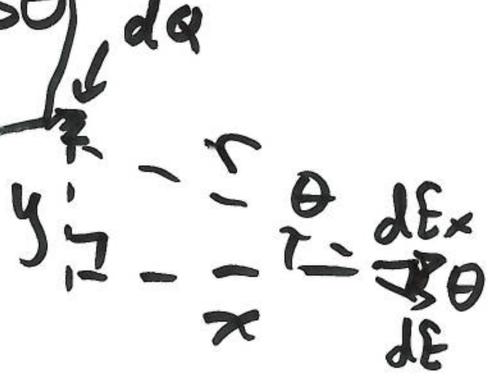
$$dQ = \lambda\, dy \qquad \lambda = \frac{Q}{L} = \frac{dQ}{dy}$$

$$\lambda = \frac{\text{charge}}{\text{unit length}} \overset{\text{const}}{=} \frac{Q}{L}$$

later: $\sigma = \frac{\text{charge}}{\text{area}}$ $\qquad \rho = \frac{\text{charge}}{\text{volume}}$

$$E_x = k\int_{-\frac{L}{2}}^{+\frac{L}{2}} \underbrace{\left[\frac{\lambda\, dy}{(x^2+y^2)}\right]}_{\text{mag. of } d\vec{E}} \cdot \overbrace{\cos\theta}^{\text{makes it } x}$$

const.



$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

$$E_x = k\int_{-\frac{L}{2}}^{\frac{L}{2}} dy\, \frac{\lambda x}{(x^2+y^2)^{3/2}}$$

$$E_x = k\lambda x \int_{-L/2}^{+L/2} \frac{dy}{(x^2+y^2)^{3/2}}$$

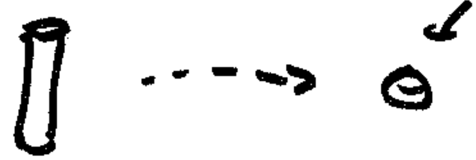
$$\int \frac{dx}{\left(\sqrt{x^2+a^2}\right)^3} = \frac{x}{a^2\sqrt{x^2+a^2}} \qquad \begin{array}{l} x \to y \\ a \to x \end{array}$$

$$E_x = k\lambda x\left[\frac{y}{x^2\sqrt{y^2+x^2}}\right]_{-L/2}^{+L/2 = y}$$

$$E_x = \frac{k\lambda x}{x^2\sqrt{\frac{L^2}{4}+x^2}} \cdot \underbrace{\left[\frac{L}{2} - \frac{-L}{2}\right]}_{L}$$

$$E_x = \frac{k\lambda L}{x\sqrt{\frac{L^2}{4}+x^2}}$$

point charge

If $L \ll x$, this 

point Q: $E = \frac{kQ}{r^2}$ recall $Q = \lambda \cdot L$

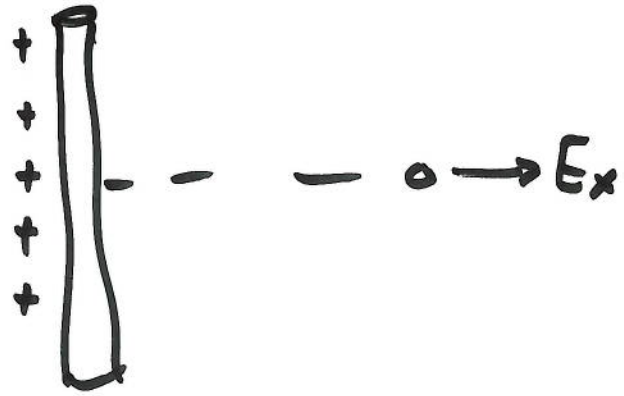
lim $L \ll x$

$$E_x \simeq \frac{k(\overbrace{Q}^{\lambda L})}{x \cdot x} = \frac{kQ}{x^2}$$ ✓ yes!

lim $L \to \infty$: this will agree w/ Gauss' Law.

$L \gg x$

$$E_x \simeq \frac{k\lambda \cancel{L}}{x\frac{\cancel{L}}{2}} = \frac{2k\lambda}{x}$$



$E_y = 0$ by symmetry

also I can show by direct integration.

Gauss' Law

- Sketch $\vec{E}$ near your object (must know the direction).
- Form imaginary surface around your (real) object which is either // or $\perp$ to $\vec{E}$ direction.
- Use Gauss' Law

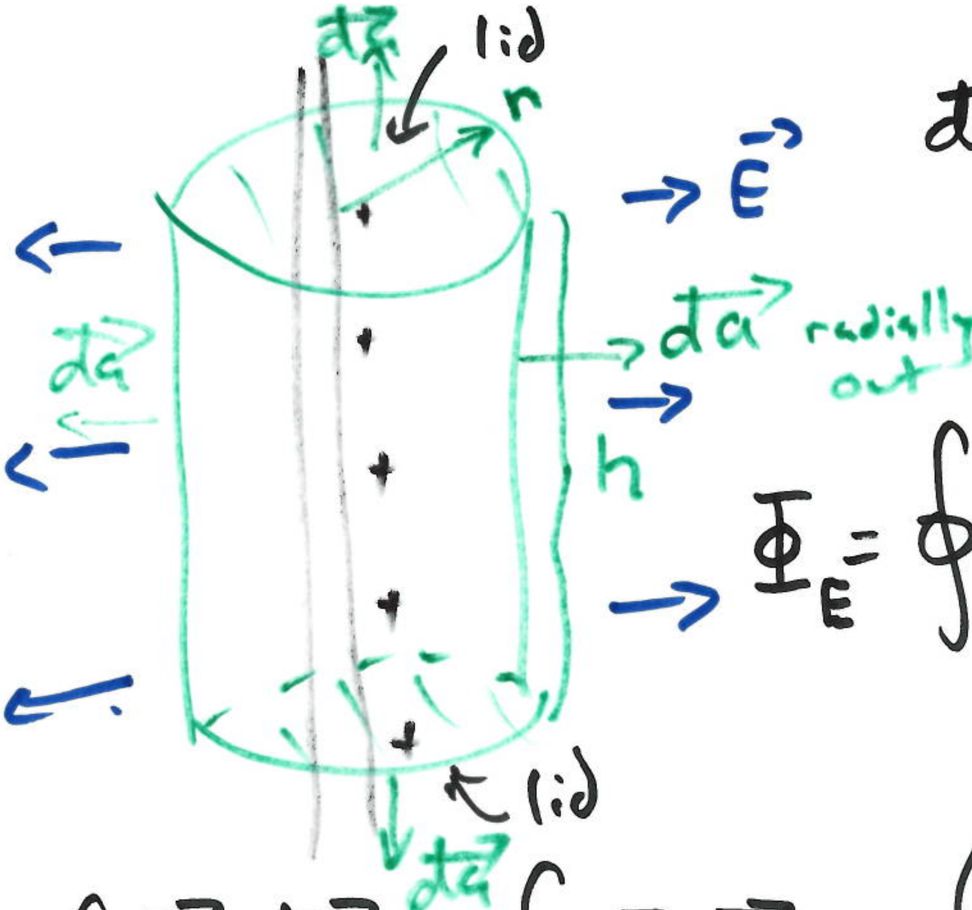
$$\oint \vec{E} \cdot \underbrace{d\vec{a}}_{\text{area vector}} = \frac{Q_{IN}}{\epsilon_0}$$

$\nabla \cdot \vec{E}$

$$= \frac{\int_V dV \cdot \rho}{\epsilon_0}$$

over gaussian surface

closed surface



$d\vec{a}$ is area points $\perp$ to surface points <u>outward</u>

$$\Phi_E = \oint \vec{E} \cdot d\vec{a} = \text{flux or flow of } \vec{E}$$

$$\oint \vec{E} \cdot d\vec{a} = \int_{\text{side}} \vec{E} \cdot d\vec{a} + \int_{\text{lids}} \vec{E} \cdot d\vec{a} = \frac{Q_{IN}}{\epsilon_0}$$

side $\vec{E} \parallel d\vec{a}$ lids $\vec{E} \perp d\vec{a} \rightarrow 0$ $\cos 90° = 0$

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$= \int_{\text{side}} E\,da \overbrace{\cos 0°}^{1} + \underbrace{\int_{\text{lids}} E\,da \cancel{\cos 90°}^{\,0}}_{0} = \frac{Q_{IN}}{\epsilon_0}$$

$$= E \int_{\text{side}} da = \frac{Q_{IN}}{\epsilon_0}$$

$$E\,2\pi r \cdot \cancel{h} = \frac{\lambda \cdot \cancel{h}}{\epsilon_0}$$

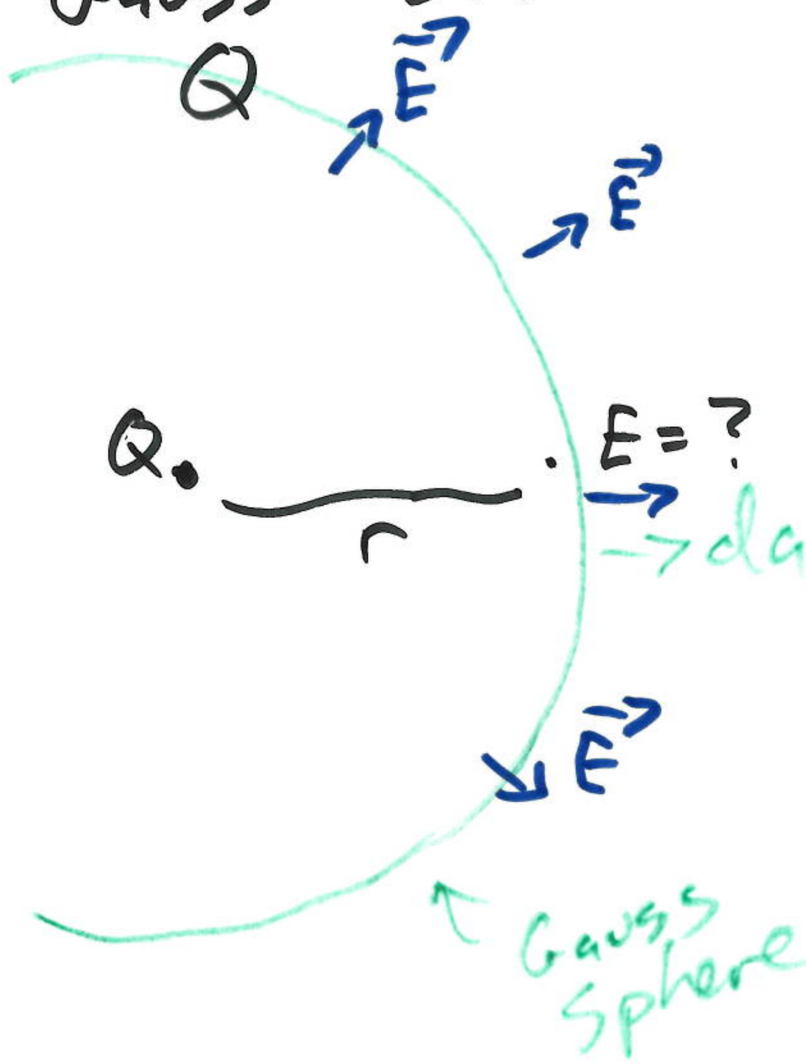
$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$ points outward.

Recall $k = \frac{1}{4\pi\epsilon_0}$

$$E = \frac{2k\lambda}{r}$$ agrees with direct integration

anywhere on side of cylinder.

Gauss' Law for point charge



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{IN}}{\epsilon_0}$$

$$E \oint da = \frac{Q}{\epsilon_0}$$

$$E \, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\boxed{E = \frac{kQ}{r^2}}$$ ✓