Physics 201 Day 27

Ideal Gas Law Quiz

Write Name Front and Back

PV = nRT R=8.315 J/mol-K

1. 5 mols of ideal gas are at 300 K and in a 1.6 m^3 volume. If the temperature is increased to 410 K, without the pressure changing, and no gas leaks in or out, what is the final volume?

$PV = nRT$ ~~= const.~~

$\cancel{P_i V_i = P_f V_f}$

$\frac{V}{T} = \frac{nR}{P} = \text{const.}$

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ $\quad \frac{1.6 m^3}{300 K} = \frac{V_2 *}{410 K} \Rightarrow \boxed{V_2 = 2.19 m^3}$

2. If 9 mols of ideal gas are in a 3.33 m^3 chamber and the pressure is doubled, without changing the temperature, and no gas leaks in or out, what is the final volume?

$P_2 = 2P_1$ $\quad T = \text{const}$

$P_1 V_1 = P_2 V_2$ $\quad V_2 = \frac{1}{2} V_1 = \frac{3.33 m^3}{2} = 1.67 m^3$

$\cancel{P_1} V_1 = 2 \cancel{P_1} V_2$

3. If 7 mols of ideal gas are in a 2.22 m^3 chamber at T=310 K and a pressure of 1.05×10^5 Pa, how many mols of ideal gas must be added to reach a final pressure of 1.55×10^5 Pa? The volume remains unchanged and all gas added is also at T=310 K, so the temperature doesn't change.

$\frac{P}{n} = \frac{RT}{V} = \text{const.}$

$\frac{P_1}{n_1} = \frac{P_2}{n_2} \rightarrow n_2 = n_1 \frac{P_2}{P_1} = 7 mol \times \frac{1.55}{1.05} \ldots$

$n_2 = 10.3$ mols.

Thus we <u>add</u> $n_2 - n_1 = 3.3$ mols.

Average Value Examples:

Zustandsumme

$$\bar{E} = \sum_i \frac{E_i e^{-\beta E_i}}{Z} \qquad \beta = \frac{1}{k_B T}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$$

I will "prove" this.

$$Z = \sum_i e^{-\beta E_i}$$

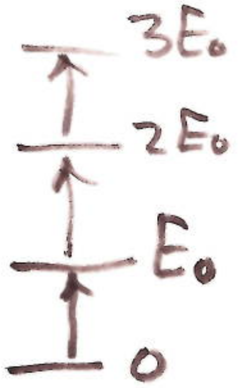
$$\bar{E} = -\frac{\partial \ln Z}{\partial \beta}$$

$$C = \frac{\partial \bar{E}}{\partial T}$$

$$\frac{\partial Z}{\partial \beta} = \sum_i -E_i e^{-\beta E_i}$$

so $-\frac{1}{Z}\frac{\partial Z}{\partial \beta} = \bar{E}$ ✓

The "ladder" system:



$$Z = \sum_i e^{-\beta E_i}$$

$$E_i = i E_0 \qquad i = 0, 1, 2, \ldots$$

$$Z = \sum_{i=0}^{\infty} e^{-\beta i E_0} = \sum_{i=0}^{\infty} \left(e^{-\beta E_0}\right)^i$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

$$Z = \frac{1}{1 - e^{-\beta E_0}} \quad \text{if } e^{-\beta E_0} < 1$$

true if $E_0 > 0$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\overbrace{\left(1 - e^{-\beta E_0}\right)}^{1/Z} \cdot \frac{(-1)}{\left(1 - e^{-\beta E_0}\right)^2} \cdot \underbrace{E_0 e^{-\beta E_0}}_{\frac{\partial}{\partial \beta}\left(1 - e^{-\beta E_0}\right)}$$

$$\boxed{\bar{E} = \frac{E_0 e^{-\beta E_0}}{1 - e^{-\beta E_0}}} = \frac{E_0}{e^{\beta E_0} - 1}$$

$$C = \frac{\partial \bar{E}}{\partial T} = \frac{\partial \bar{E}}{\partial \beta} \cdot \frac{\partial \beta}{\partial T} \qquad \beta = \frac{1}{k_B T}$$

$$= \frac{-E_0}{\left(e^{\beta E_0} - 1\right)^2} \cdot E_0 e^{\beta E_0} \cdot \overbrace{\left(\frac{-1}{k_B T^2}\right)}^{\partial \beta / \partial T}$$

$$\boxed{C = \frac{E_0^2 e^{\beta E_0}}{\left(e^{\beta E_0} - 1\right)^2} k_B \beta^2}$$

What is entropy, S? $= k_B \ln \Omega$

As $T \to 0$ which is more important?

E or S?

Same as $T \to \infty$?

$$G = E - TS$$

as $T \to 0$, $G = E$ ← only

as $T \to \infty$, $G \simeq -TS$ ← most important

Show you can take simple average value (or probability) ← from last class.

$$\overline{X} = \sum_i \frac{x_i e^{-\beta E_i}}{Z}$$