Functional Programming	WS 2018/2019	LVA 703024+703025
Exercises 10		January 25, 2019

All Solutions

2. Let $E = \{f :: \alpha \to \beta, y :: \alpha\}$.

1	$x :: \alpha$	assumption
2	$f :: \alpha \to \beta$	ins E
3	$f\ x :: \beta$	app 2, 1
4	$\lambda x.\ f\ x :: \alpha \to \beta$	abs 1–3
5	$y :: \alpha$	ins E
6	$(\lambda x.\ f\ x)\ y :: \beta$	app 4, 5

3.

$$\underline{\alpha_2 \approx \alpha_1 \to \alpha_0}; \alpha_3 \approx \mathsf{Int} \to \alpha_2; \alpha_2 \approx \alpha_3$$
$$\Rightarrow^{(\mathsf{v}_1)}_{\{\alpha_2 \mapsto \alpha_1 \to \alpha_0\}}$$
$$\underline{\alpha_3 \approx \mathsf{Int} \to \alpha_1 \to \alpha_0}; \alpha_1 \to \alpha_0 \approx \alpha_3$$
$$\Rightarrow^{(\mathsf{v}_1)}_{\{\alpha_3 \mapsto \mathsf{Int} \to \alpha_1 \to \alpha_0\}}$$
$$\underline{\alpha_1 \to \alpha_0 \approx \mathsf{Int} \to \alpha_1 \to \alpha_0}$$
$$\Rightarrow^{(\mathsf{d}_2)}_{\{\}}$$
$$\underline{\alpha_1 \approx \mathsf{Int}}; \alpha_0 \approx \alpha_1 \to \alpha_0$$
$$\Rightarrow^{(\mathsf{v}_1)}_{\{\alpha_1 \mapsto \mathsf{Int}\}}$$
$$\alpha_0 \approx \mathsf{Int} \to \alpha_0$$

At this stage no rule is applicable. As it is not possible to derive $\square$, the unification problem is not solvable.

4.

$$\begin{aligned} \mathsf{Pair}(\mathsf{Bool}, \alpha_0) \approx \mathsf{Pair}(\alpha_1, \mathsf{Int}) &\Rightarrow^{(\mathsf{d}_1)}_{\{\}} & \mathsf{Bool} \approx \alpha_1; \alpha_0 \approx \mathsf{Int} \\ &\Rightarrow^{(\mathsf{v}_2)}_{\{\alpha_1/\mathsf{Bool}\}} & \alpha_0 \approx \mathsf{Int} \\ &\Rightarrow^{(\mathsf{v}_1)}_{\{\alpha_0/\mathsf{Int}\}} & \square \end{aligned}$$

Hence the solution is the composition of $\{\alpha_1/\mathsf{Bool}\}$ and $\{\alpha_0/\mathsf{Int}\}$, that is,

$$\sigma = \{\alpha_0/\mathsf{Int}, \alpha_1/\mathsf{Bool}\}$$

5. First we turn the resulting typing inference problem into a unification problem:

$$\underline{E \rhd map\ (\lambda x.\, x) :: \alpha_0}$$
$$\Rightarrow^{(\mathsf{app})}$$
$$\underline{E \rhd map :: \alpha_1 \to \alpha_0;}$$
$$E \rhd \lambda x.\, x :: \alpha_1$$
$$\Rightarrow^{(\mathsf{cons})}$$
$$(\alpha \to \beta) \to \mathsf{List}(\alpha) \to \mathsf{List}(\beta) \approx \alpha_1 \to \alpha_0;$$
$$\underline{E \rhd \lambda x.\, x :: \alpha_1}$$
$$\Rightarrow^{(\mathsf{abs})}$$
$$(\alpha \to \beta) \to \mathsf{List}(\alpha) \to \mathsf{List}(\beta) \approx \alpha_1 \to \alpha_0; \alpha_1 \approx \alpha_2 \to \alpha_3$$
$$\underline{E, x :: \alpha_2 \rhd x :: \alpha_3}$$
$$\Rightarrow^{(\mathsf{cons})}$$
$$(\alpha \to \beta) \to \mathsf{List}(\alpha) \to \mathsf{List}(\beta) \approx \alpha_1 \to \alpha_0; \alpha_1 \approx \alpha_2 \to \alpha_3; \alpha_2 \approx \alpha_3$$

Then we solve this unification problem:

$$(\alpha \to \beta) \to \mathsf{List}(\alpha) \to \mathsf{List}(\beta) \approx \alpha_1 \to \alpha_0; \alpha_1 \approx \alpha_2 \to \alpha_3; \underline{\alpha_2 \approx \alpha_3}$$
$$\Rightarrow^{(\mathsf{v}_1)}_{\{\alpha_2 \mapsto \alpha_3\}}$$
$$(\alpha \to \beta) \to \mathsf{List}(\alpha) \to \mathsf{List}(\beta) \approx \alpha_1 \to \alpha_0; \underline{\alpha_1 \approx \alpha_3 \to \alpha_3;}$$
$$\Rightarrow^{(\mathsf{v}_1)}_{\{\alpha_1 \mapsto \alpha_3 \to \alpha_3\}}$$
$$\underline{(\alpha \to \beta) \to \mathsf{List}(\alpha) \to \mathsf{List}(\beta) \approx (\alpha_3 \to \alpha_3) \to \alpha_0}$$
$$\Rightarrow^{(\mathsf{d}_2)}_{\{\}}$$
$$\underline{\alpha \to \beta \approx \alpha_3 \to \alpha_3;}\mathsf{List}(\alpha) \to \mathsf{List}(\beta) \approx \alpha_0$$
$$\Rightarrow^{(\mathsf{d}_2)}_{\{\}}$$
$$\underline{\alpha \approx \alpha_3;}\beta \approx \alpha_3; \mathsf{List}(\alpha) \to \mathsf{List}(\beta) \approx \alpha_0$$
$$\Rightarrow^{(\mathsf{v}_1)}_{\{\alpha \mapsto \alpha_3\}}$$
$$\underline{\beta \approx \alpha_3;}\mathsf{List}(\alpha_3) \to \mathsf{List}(\beta) \approx \alpha_0$$
$$\Rightarrow^{(\mathsf{v}_1)}_{\{\beta \mapsto \alpha_3\}}$$
$$\mathsf{List}(\alpha_3) \to \mathsf{List}(\alpha_3) \approx \alpha_0$$
$$\Rightarrow^{(\mathsf{v}_2)}_{\{\alpha_0 \mapsto \mathsf{List}(\alpha_3) \to \mathsf{List}(\alpha_3)\}}$$
$$\square$$

The resulting most general unifier is

$$\mu = \{\alpha \mapsto \alpha_3, \beta \mapsto \alpha_3, \alpha_0 \mapsto \mathsf{List}(\alpha_3) \to \mathsf{List}(\alpha_3), \alpha_1 \mapsto \alpha_3 \to \alpha_3, \alpha_2 \mapsto \alpha_3\}$$

Finally, we conclude that the most general type of $map\ (\lambda x.\, x)$ is $\alpha_0\mu = \mathsf{List}(\alpha_3) \to \mathsf{List}(\alpha_3)$.