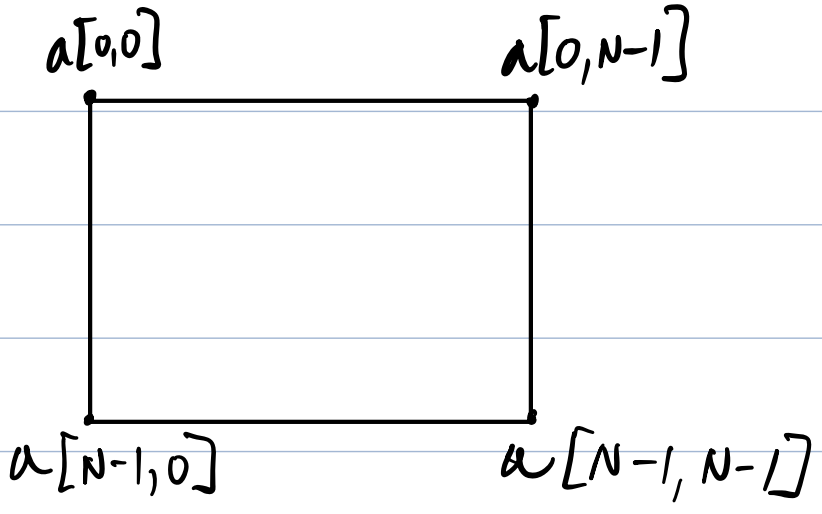
Gravitational Lensing: Mapping Coordinates to Pixel Labels

Task: we need to represent the source and image planes as numpy arrays which are indexed by integers $i, j \in 0, \ldots, N-1$.

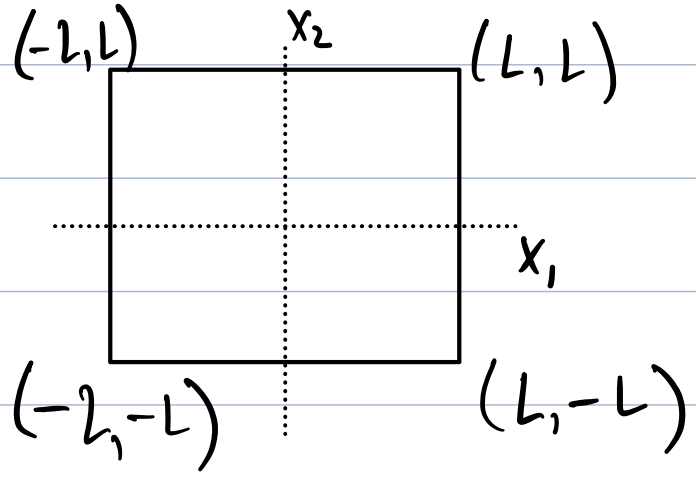
There are many ways to do this but we want to be able to visualize the planes easily using, eg. the imshow function.

imshow takes a 2D array $a[i,j]$ as input and plots its entries as pixels as follows



(for default imshow options)

Our source and lens planes are



Note that the natural order of vertical and horizontal coordinates is exchanged between the physical planes and the pixel labels : eg

$$a[0,0] \leftrightarrow (-L, L)$$

- $a[0,0]$: vertical pixel label; horizontal pixel label.
- $(-L, L)$: horizontal position; vertical position

let $\Delta = L/(N-1)$, then we can map

$$(i, j) \longrightarrow (x_1, x_2)$$ as

follows:

$$x_1 = -L + 2j\Delta, \quad j \in 0, \ldots, N-1$$

$$x_2 = +L - 2i\Delta, \quad i \in 0, \ldots, N-1$$

let's check that this works:

$(i,j) = (0,0) \rightarrow (x_1, x_2) = (-L, L)$ ✓

$(i,j) = (0, N-1) \rightarrow (x_1, x_2) = (L, L)$ ✓

$(i,j) = (N-1, 0) \rightarrow (x_1, x_2) = (-L, -L)$ ✓

$(i,j) = (N-1, N-1) \rightarrow (x_1, x_2) = (L, -L)$ ✓