S7001E Lab 2 - MATLAB Assignments

- For this lab, you may work in groups of maximum two students. Each group should hand in a clear and concise report documenting and explaining the results clearly for each of the exercises. Handwritten computations can be included in the report.
- The MATLAB code should be included as appendices to the report and attached as separate Matlab m-files. The resulting sound files (.wav) should also be attached.
- Some helpful MATLAB commands:

  `help   rand   randn   plot   hist   find`

  type `help` *command* within MATLAB to learn more about them.
- **Please find the deadline in Canvas!**
- *Unclearly or otherwise poorly written reports will be returned without closer examination.*

1. It is common to find software functions capable of generating pseudo-random numbers with a uniform distribution between zero and one (written as $X \sim \mathcal{U}[0, 1]$). The MATLAB function `rand` is an example of this.

   It turns out that this capability also allows us to generate realizations of pseudo-random variables with other distributions, as discussed in the text in Example 3.2-5 on page 161, (p. 172 in soft cover version and page 125 in 3rd. ed). This method may be used to generate samples of random variables with a desired PDF, from samples coming from a uniform PDF.

   (a) Write a short program to generate $N = 10000$ points drawn from the pdf shown in problem 3.22 (3.16 in 3rd ed.) in the textbook.

   (b) Plot the histogram as well as the theoretical pdf, compare and interpret.

2. Perform problem 3.23 (3.17 in 3rd ed.) in the textbook and discuss your results.

3. Write a MATLAB program to generate $N = 10000$ points drawn from a Gaussian distributed $X \sim \mathcal{N}(3, 2)$.

   (a) Generate a histogram as an estimate of the pdf $f_X(x)$.

   (b) Try a variety of values for N and number of bins in the histogram. Note any observations.

4. Read the text about *Jointly Gaussian Random Variables* p. 251–255, (around p. 263 in soft cover version and p. 201–205 in 3rd ed.). Generate a row vector X_1 of 10 000 points drawn from Gaussian distributed $\sim \mathcal{N}(3, 2)$. Generate another row vector (independent from your first) X_2 of 10 000 points drawn from Gaussian distributed as $\sim \mathcal{N}(1, 1)$. Create two new row vectors Y_1, and Y_2, related to X_1 and X_2 by

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}.$$

   (a) What is an expression for the joint pdf $f_{X_1X_2}(x_1, x_2)$? Identify the theoretical mean and variance of X_1, X_2, and their theoretical covariance as well as correlation coefficient.

   (b) From your generated data identify the sample means, and sample covariance, and sample correlation coefficient. Compare these to the theoretical values.

   (c) Produce a scatter plot of X_1 vs. X_2 **and interpret the results**. (Try `plot(X_1, X_2,'.')`).

   (d) Repeat a), b), and c) for Y_1, and Y_2.

   (e) Calculate the theoretical marginal pdfs $f_{Y_1}(y_1), f_{Y_2}(y_2)$.

   (f) Write an expression for the theoretical conditional probability $P\{2 < Y_2 < 3 | Y_1 > 5\}$. Calculate or estimate this theoretical conditional probability by any means you choose.

   (g) Estimate the conditional probability $P\{2 < Y_2 < 3 | Y_1 > 5\}$ from your generated data set. Compare this estimate with your previous theoretical evaluation.

5. **Signal separation**

In this assignment we will briefly see how separation of speech signals may be accomplished. Some years ago, a signal processing student investigated this problem for his master thesis. We will try one of the simplest methods to separate the signals.

As a service to people that have problems reading the printed newspapers, the newspapers in Norrbotten are read and recorded each night and sent (coded) in P1. With a special recorder that decodes the coded version of the newspaper it is possible for people to listen to the newspaper instead. To save transmission time, NSD and Kuriren have been transmitted at the same time, using the two channels in stereo transmission. However, when decoding the signal, the newspapers have been somewhat mixed, which has annoyed people using this service. In this assignment we will try to separate the two speech signals corresponding to different newspapers by minimizing the correlation between the two signals.

Let $X(t)$ and $Y(t)$ be the processes corresponding to NSD and Kuriren respectively. We assume that these processes are mixed linearly without memory in the left (L) and right (R) channels of the received stereo sound. For simplicity we also assume that the amount of mixing is the same for both channels, that is, the mixing may be written with the matrix relation

$$\begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} L \\ R \end{bmatrix}.$$

With this simple model, we can find estimates of X and Y as

$$\hat{X} = L + kR$$

$$\hat{Y} = kL + R$$

with k being a scalar.

In Canvas you find a file named *newspapers.wav* that contains the received signal. You also find *newspaper1.wav* and *newspaper2.wav* that are only the left and right channel respectively. We will start with the received signal and try to separate it into the two different newspapers (better than the left and right channel) by adaptively minimizing the correlation between $\hat{X}$ and $\hat{Y}$.

(a) We choose to minimize the cost function $Q = \left(E\left[\hat{X}\hat{Y}\right]\right)^2$ in order to minimize the correlation. Express Q in terms of k, L and R.

(b) To determine the k that minimizes Q we can find the k that gives

$$\frac{dQ}{dk} = 0$$

Express this derivative in terms of k, L and R.

(c) Load the file *newspapers.wav* to MATLAB using the command
`>>[X, fs, nbits]=wavread('newspapers.wav');`
The first column of X contains the left channel and the second column contains the right channel.

(d) Now we need to estimate the expected values in the expression for the derivative of Q from the signal. We will assume that k may change slightly over time and update the expected values for each sample. The expected values needed may be initialized by estimations from the 100 first samples, e.g.
`>>L2=mean(X(1:100,1).^2);`
where $L2$ is the estimate of $E\left[L^2\right]$.

(e) When all expected values are initialized we can start updating the expected values for each sample. For the expected values we simply keep most of the old value and let the instantaneous values only be a small part of the new expected value, e.g.
`>>L2=(1-lambda)*X(i,1)^2+lambda*L2;`
where λ is the coefficient determining how fast the expected value should be changing ($\lambda \approx 0.99995$) and i is the sample index.

(f) Updating k is usually done by gradient descent methods, but we will use a simpler method (but slower) and only consider the sign of the derivative of Q. The updates of k for each sample (with updated expected values) will be

$$k = k - \mu \cdot sign\left[\frac{dQ}{dk}\right],$$

where μ is the step size which should be small ($\approx 10^{-5}$). Use $k = 0$ as a starting value.

(g) Write a for-loop that goes through the whole signal and updates the expected values and k for each sample. Store the k-value for each index in a vector and use this vector to get the estimates $\hat{X}$ and $\hat{Y}$. Remember that creating a vector of zeros with the same length as X and changing one of the zeros to the correct k-value is much faster than making the k-value vector longer for each sample. Plot the k-vector to see how the estimate of k changes with time.

(h) Write the separated signals to wav-files using
`>>wavwrite(Xhat/max(abs(Xhat)),fs,nbits,'sep_newspaper1.wav');`
The vector `Xhat` is scaled with its maximum amplitude before writing it to file, since `wavwrite` clips everything that exceeds the range $[-1, 1]$. Listen to the separated versions of the newspapers and compare to the versions before separation found in Canvas. What do you think about the result?