Drasil Geometric Algebra Extension

Overview of Requirements

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Overview

1. Einstein Summation Notation
2. Vectors
3. Matrices
4. Geometric Algebra
5. Geometric Algebraic Definition of Vectors and Matrices

The purpose of this slide deck is to define the objects and operations that will be implemented in the project. We will begin with a discussion about vectors and matrices, then generalize with geometric algebra. Finally, we will redefine the operations using geometric algebra.

Einstein Summation Notation

- Used to describe repeated summations and multiplications in a compact notation.
- They behave according to **four rules** [Faculty of Khan, 2023].
  1. Any twice-repeated index in a single term is summed over.
  2. A twice-repeated index is called a dummy index; a once-repeated index is called a free index.
  3. No index may occur 3 or more times in a given term.
  4. In an equation with Einstein notation, the free indices on both sides must match.

Einstein Summation Notation Rules

Here are the rules with some more explanations [Faculty of Khan, 2023]:

1. *Any twice-repeated index in a single term is summed over.*
   For example, $a_{ij}b_i$ represents the term $\sum_{i=1}^{j} a_{ij}b_i$.
2. *A twice-repeated index is called a dummy index; a once-repeated index is called a free index.*
   In the example above, i is a dummy index — it can be renamed however you would like. However, j is a free index and has restrictions on naming.
3. *No index may occur 3 or more times in a given term.*
   For example, $a_{ii}b_i$ is not legal.

We will use this convention from now on.

Einstein Summation Notation Rules

Here are the rules with some more explanations [Faculty of Khan, 2023]:

4. *In an equation with Einstein notation, the free indices on both sides must match.*
   Some examples of correctly-formed equations:
   - $x_i = a_{i,j} b_i$ is valid because i is free on both the LHS and RHS
   - $a_i = A_{k,i} B_{k,j} x_j + C_{i,k} u_k$ is valid because i is a free variable on the LHS, and in every term it is the free variable on the RHS.

   Some examples of incorrectly-formed equations:
   - $x_i = A_{j,i}$ is invalid because i is the only free variable on the LHS, but i and j are both free on the RHS.
   - $x_j = Ai, ku_k$ is invalid because j is free on the LHS, but i is free on the RHS.
   - $x_i = A_{i,k} u_k + c_j$ is invalid because i is free on the LHS, but on the RHS, one term has i free while the other term has j free.

We will use this convention from now on.

Vectors

Blocks of Highlighted Text

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Block

Sample text

Alertblock

Sample text in red box

Examples

Sample text in green box. The title of the block is "Examples".

Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [**?**].

References

Faculty of Khan (2023).
Clrtj7d2flm.
https://www.youtube.com/watch?v=CLrTj7D2fLM.
Accessed: 2025-01-23.

The End