Formalizing and Analyzing the Needham-Schroeder Symmetric-Key Protocol by Rewriting

Monica Nesi Giuseppina Rucci

Dipartimento di Informatica Università di L'Aquila (Italy)

Protocol Verification

- Aim: formally prove properties of security protocols (e.g. authentication, secrecy or confidentiality, freshness, ...)
- Rewriting techniques and strategies
- Case studies
 - the Needham-Schroeder Public-Key protocol (NSPK)
 - the Needham-Schroeder
 Symmetric-Key protocol (NSSK)

Related Work

- Model checking
 - FDR (Lowe 1996)
 - Murphi (Mitchell-Mitchell-Stern 1997) ...
- Theorem proving
 - NRL (Meadows 1996)
 - Isabelle (Paulson 1997, 1998, ...)
 - SPASS (Weidenbach 1999) ...

Related Work

- Rewriting techniques and strategies
 - ELAN (Cirstea 2001)
 - Maude (Denker-Meseguer-Talcott 1998)
 - CASRUL (Jacquemard-Rusinowitch-Vigneron 2000) . . .
- Rewriting + abstract interpretation (Monniaux 1999)

Related Work

- Rewriting + tree automata in Timbuk (Genet-Viet Triem Tong 2001)
- Combination of different approaches
 - the combination of Genet-Klay's approximation technique and Paulson's inductive method (Oehl-Sinclair 2001, 2002)
 - AVISPA project

Outline of the Talk

- The approximation technique by Genet and Klay
- The formalization for NSSK (insecure version) through rewrite systems and tree automata
- The basic ingredients of the rewriting strategy

Outline of the Talk

- The rewriting strategy and its properties
- A verification example: authentication attacks in insecure NSSK
- Conclusions + current and future work

Aim: finding that there are no attacks on a protocol (Genet-Klay 2000).

- The protocol is operationally specified by a TRS \mathcal{R} .
- The initial set E of communication requests and an intruder's initial knowledge are described through a tree automaton A such that $\mathcal{L}(A) \supseteq E$.

- The property p to be proved is given through a tree automaton $\mathcal{A}_{\overline{p}}$ that models the negation of p.
- The approximation technique builds an over-approximation of the set $\mathcal{R}^*(E)$ of all \mathcal{R} -descendants of the set E.
- Result: an approximation automaton $\mathcal{T}_{\mathcal{R}} \uparrow (\mathcal{A})$ such that $\mathcal{L}(\mathcal{T}_{\mathcal{R}} \uparrow (\mathcal{A})) \supseteq \mathcal{R}^*(E)$.

A finite number of tree automata $A_i = \langle \mathcal{F}, \mathcal{Q}, \mathcal{Q}_f, \Delta_i \rangle$ is built as follows:

- 1. $A_0 = A$;
- 2. A_{i+1} is constructed from A_i by computing a critical pair between a rule in \mathcal{R} and the transitions in Δ_i . The rule derived from the critical pair is a new transition that is normalized using an approximation function γ and then added to Δ_i , thus yielding Δ_{i+1} . It follows that $\mathcal{L}(A_i) \subset \mathcal{L}(A_{i+1})$.

Step 2 is repeated until an automaton A_k is obtained such that $\mathcal{L}(A_k) \supseteq \mathcal{R}^*(\mathcal{L}(A_0))$, i.e. $\mathcal{L}(A_k) \supseteq \mathcal{R}^*(E)$.

- Quality of the approximation depends on γ .
- Reachability properties on $\mathcal R$ and E are proved by checking whether

$$\mathcal{L}(\mathcal{T}_{\mathcal{R}}\uparrow(\mathcal{A}))\cap\mathcal{L}(\mathcal{A}_{\overline{p}})=\emptyset.$$

Empty intersection means that property p is satisfied.

Our Approach

As in Genet-Klay's approximation technique,

- the protocol is operationally specified by a TRS $\ensuremath{\mathcal{R}}$
- the intruder's initial knowledge is described through a tree automaton $\mathcal A$

The approximation technique is a particular completion process using an approximation function.

Our Approach

- Aim: prove or disprove properties.
- No approximation function.
- Idea: rewriting strategy simulating the critical pairs computed in the completion process in a bottom-up manner.
- Based on a rewriting strategy for dealing with the divergence of completion (Inverardi-Nesi 1992, 1996).

The NSSK Protocol

Given agents A and B and a server S, the NSSK protocol can be described as follows:

```
1. A \longrightarrow S : A, B, N_A
```

2.
$$S \longrightarrow A : \{N_A, B, K_{AB}, \{K_{AB}, A\}_{K_{BS}}\}_{K_{AS}}$$

3.
$$A \longrightarrow B : \{K_{AB}, A\}_{K_{BS}}$$

4.
$$B \longrightarrow A : \{N_B\}_{K_{AB}}$$

5.
$$A \longrightarrow B : \{N_B - 1\}_{K_{AB}}$$

Insecure version!

The NSSK Protocol

Authentication attack (Denning-Sacco 1981): Hp: an intruder has recorded session (i) and the key K'_{AB} , created in session (i), has been compromised and is known to the intruder.

Session (ii) can develop as follows:

$$ii.1. A \longrightarrow S : A, B, N_A$$
 $ii.2. S \longrightarrow A : \{N_A, B, K_{AB}, \{K_{AB}, A\}_{K_{BS}}\}_{K_{AS}}$
 $ii.3. I(A) \longrightarrow B : \{K'_{AB}, A\}_{K_{BS}}$
 $ii.4. B \longrightarrow I(A) : \{N_B\}_{K'_{AB}}$
 $ii.5. I(A) \longrightarrow B : \{N_B - 1\}_{K'_{AB}}$

Formalizing the Protocol

A protocol is formalized through a rewrite system $\mathcal{R} = \mathcal{R}_P \cup \mathcal{R}_I$, where

- \mathcal{R}_P describes the steps of the protocol and the properties to be verified,
- \mathcal{R}_I defines an intruder's ability of decomposing and decrypting messages.

```
(1)
goal(agt(a), agt(b), r(j))
\rightarrow mesg(agt(a), serv(S), cons(N(agt(a), serv(S), r(j)), cons(agt(a), agt(b))), r(j))
mesg(a_2, a_3, cons(N(agt(a), serv(S), r(j)), cons(agt(a), agt(b))), r(j))
                                                                                              (2)
\rightarrow mesg(serv(S), agt(a),
         encr(ltk(agt(a), serv(S)), serv(S),
            cons(N(aqt(a), serv(S), r(j)),
              cons(agt(b),
                cons(sk(agt(a), agt(b), r(j)),
                   encr(ltk(agt(b), serv(S)), serv(S),
                     cons(sk(agt(a), agt(b), r(j)), agt(a)))))),
         r(j))
```

```
mesg(a_4, a_5, \qquad (3)
encr(ltk(agt(a), serv(S)), a_3, \qquad cons(N(agt(a), serv(S), r(j)), \qquad cons(agt(b), \qquad cons(sk(agt(a), agt(b), r(i_1)), \qquad encr(ltk(agt(b), serv(S)), a_1, \qquad cons(sk(agt(a), agt(b), r(i_2)), agt(a)))))))),
r(j))
\rightarrow mesg(agt(a), agt(b), \qquad encr(ltk(agt(b), serv(S)), a_1, cons(sk(agt(a), agt(b), r(i_2)), agt(a))), \qquad r(j))
```

```
mesg(a_6, a_7,
                                                                                         (4)
       encr(ltk(agt(b), serv(S)), a_5, cons(sk(agt(a), agt(b), r(i)), agt(a))),
       r(j)
\rightarrow mesg(a_7, a_6,
          encr(sk(aqt(a), aqt(b), r(i)), a_7, N(aqt(b), aqt(a), r(j))),
          r(j)
mesg(a_8, a_6,
                                                                                         (5)
          encr(sk(agt(a), agt(b), r(i)), a_7, N(agt(b), agt(a), r(j))),
          r(j)
\rightarrow mesg(a_6, a_8,
          encr(sk(aqt(a), aqt(b), r(i)), a_6, N(aqt(b), aqt(a), r(i))),
          r(j)
```

```
mesg(a_8, a_6, encr(sk(agt(a), agt(b), r(i)), a_7, N(agt(b), agt(a), r(j))),
r(j))
\rightarrow c_{init}(agt(a), agt(b), a_7, r(j))
mesg(a_{10}, a_6, encr(sk(agt(a), agt(b), r(i)), a_9, N(agt(b), agt(a), r(j))),
r(j))
\rightarrow c_{resp}(agt(b), agt(a), a_9, r(j))
(6)
mesg(a_1, a_6, encr(sk(agt(a), agt(b), r(i)), a_7, N(agt(b), agt(a), r(j))),
r(j))
```

A TRS $\mathcal{R}_{\mathcal{I}}$ for NSSK

$$cons(x,y) \rightarrow x \tag{8}$$

$$cons(x,y) \rightarrow y \tag{9}$$

$$encr(sk(agt(0), agt(x), w), y, z) \rightarrow z \tag{10}$$

$$encr(sk(agt(x), agt(0), w), y, z) \rightarrow z \tag{11}$$

$$encr(sk(agt(s(x_1)), agt(x), w), y, z) \rightarrow z \tag{12}$$

$$encr(sk(agt(x), agt(s(x_1)), w), y, z) \rightarrow z \tag{13}$$

$$encr(ltk(agt(0), serv(S)), y, z) \rightarrow z \tag{14}$$

$$encr(ltk(agt(s(x_1)), serv(S)), y, z) \rightarrow z \tag{15}$$

$$mesg(x, y, z, w) \rightarrow z \tag{16}$$

A tree automaton $\mathcal{A} = \langle \mathcal{F}, \mathcal{Q}, \mathcal{Q}_f, \Delta \rangle$, where $\mathcal{Q}_f = \{q_f\}$ and Δ is as follows:

$$0 oup q_{int}$$
 $s(q_{int}) oup q_{int}$
 $agt(q_{int}) oup q_{agtI}$
 $0 oup q_0$
 $A oup q_A$
 $agt(q_A) oup q_{agtA}$
 $s(q_0) oup q_1$
 $B oup q_B$
 $agt(q_B) oup q_{agtB}$
 $r(q_0) oup q_{r_0}$
 $S oup q_S$
 $serv(q_S) oup q_{serv}$
 $r(q_1) oup q_{r_1}$

communication requests

$$goal(q_{agtA}, q_{agtB}, q_f) \rightarrow q_f$$

 $goal(q_{agtB}, q_{agtA}, q_f) \rightarrow q_f$
 $goal(q_{agtA}, q_{agtI}, q_f) \rightarrow q_f$
 $goal(q_{agtB}, q_{agtI}, q_f) \rightarrow q_f$
 $goal(q_{agtI}, q_{agtI}, q_f) \rightarrow q_f$

$$goal(q_{agtA}, q_{agtA}, q_f) \rightarrow q_f$$

 $goal(q_{agtB}, q_{agtB}, q_f) \rightarrow q_f$
 $goal(q_{agtI}, q_{agtA}, q_f) \rightarrow q_f$
 $goal(q_{agtI}, q_{agtB}, q_f) \rightarrow q_f$

intruder's initial knowledge

$$agt(q_{int}) \rightarrow q_f$$

$$agt(q_A) \to q_f$$

$$agt(q_B) \rightarrow q_f$$

$$serv(q_S) \rightarrow q_f$$

$$r(q_0) \to q_f$$

$$r(q_1) \to q_f$$

$$sk(q_{agtI}, q_{agtI}, q_f) \rightarrow q_f$$

 $sk(q_{agtI}, q_{agtA}, q_f) \rightarrow q_f$
 $sk(q_{agtI}, q_{agtB}, q_f) \rightarrow q_f$
 $ltk(q_{agtI}, q_{serv}) \rightarrow q_f$

intruder's initial knowledge

$$mesg(q_f, q_f, q_f, q_f) \rightarrow q_f$$
 $cons(q_f, q_f) \rightarrow q_f$
 $encr(q_f, q_{aqtI}, q_f) \rightarrow q_f$

$$N(q_{agtI}, q_{agtI}, q_f) \rightarrow q_f$$
 $N(q_{agtI}, q_{agtA}, q_f) \rightarrow q_f$
 $N(q_{agtI}, q_{agtB}, q_f) \rightarrow q_f$
 $N(q_{agtI}, q_{serv}, q_f) \rightarrow q_f$

Strategy: Basic Ingredients

- Simulation of critical pairs through a bottom-up strategy
- Expansion of terms
- Well-formedness of terms (to ensure termination of the expansion process)
- Recognizability by the intruder

Expansion

expansion
$$(t, \mathcal{R}) = \{s = \sigma(t[l]_p) \mid \exists l \to r \in \mathcal{R}, p \in Pos'(t) \text{ and } \sigma = mgu(t|_p, r)\}.$$

Expansion step = narrowing step with a reversed rule of \mathcal{R} .

Expansion

Possible introduction of occurrences of "new" variables in s:

- implicitly universally quantified variables
- instantiated by means of a finite set of ground terms Inst, thus getting the instance set

$$\mathcal{I}(t, Inst) = \{ \sigma(t) \mid \sigma : Var(t) \rightarrow Inst \}.$$

In NSSK,
$$Inst = \{A, B, agt(A), agt(B), serv(S), agt(0), 0, s(0)\}.$$

Intuition:

a term t is well-formed if it "agrees" with the syntactic structure of \mathcal{R} .

Examples:

- (i) $t_1 = N(agt(a_1), agt(a_2), w)$ is well-formed for any variables or agent labels a_1, a_2 .
- (ii) $t_2 = N(agt(a_1), sk(agt(a_2), agt(a_3), w'), w)$ is not well-formed.

A term $t \in \mathcal{T}(\mathcal{F}, \mathcal{X})$ is well-formed wf(t) if

(i)
$$t \in \mathcal{X} \cup \mathcal{F}^0$$

or (ii)
$$t = f(t_1, \ldots, t_n)$$
 with $f \in \mathcal{F}^n$ $(n > 0)$

and either $t_i \in \mathcal{X}$ or t_i satisfies the following conditions based on f (i = 1, ..., n):

- f = agt and $t_1 \in L_{agt}$;
- f = serv and $t_1 = S$;
- f = r and $t_1 \in \mathbb{N}$;

- f=goal, $root(t_1)=root(t_2)=agt$, $root(t_3)=r$ and $wf(t_i)$ for i=1,2,3;
- f = mesg, $root(t_1), root(t_2) \in \{agt, serv\}$, $root(t_3) \in \{encr, cons\}$, $root(t_4) = r$ and $wf(t_i)$ for i = 1, 2, 3, 4;
- f = encr, $root(t_1) \in \{sk, ltk\}$, $root(t_2) \in \{agt, serv\}$, $root(t_3) \in \{cons, N\}$ and $wf(t_i)$ for i = 1, 2, 3;

- f = ltk, $root(t_1) = agt$, $root(t_2) = serv$ and $wf(t_i)$ for i = 1, 2;
- f=sk, $root(t_1)=root(t_2)=agt$, $root(t_3)=r$ and $wf(t_i)$ for i=1,2,3;
- f=cons, $root(t_i) \in \{N, agt, sk, cons, encr\}$ and $root(t_i)$ for i=1,2;

- f=N, $root(t_1), root(t_2) \in \{agt, serv\}$, $root(t_3)=r$ and $wf(t_i)$ for i=1,2,3;
- $f \in \{c_{init}, c_{resp}\}$, $root(t_1) = root(t_2) = root(t_3) = agt$, $root(t_4) = r$ and $wf(t_i)$ for i = 1, 2, 3, 4.

Recognizability

A term t is recognizable by the intruder if q_f can be derived from t using Δ .

Proof system $\vdash_{\mathcal{A}}$ for recognizability:

$$\frac{t \xrightarrow{*}_{\Delta} q \quad q \in \{q_f, q_{agtI}\}}{t \vdash_{\mathcal{A}} q}$$

$$\frac{t_1 \vdash_{\mathcal{A}} q_f \quad t_2 \vdash_{\mathcal{A}} q_f}{cons(t_1, t_2) \vdash_{\mathcal{A}} q_f}$$

$$\frac{t_1 \vdash_{\mathcal{A}} q_f \quad t_2 \vdash_{\mathcal{A}} q_f \quad t_3 \vdash_{\mathcal{A}} q_f \quad t_4 \vdash_{\mathcal{A}} q_f}{mesg(t_1, t_2, t_3, t_4) \vdash_{\mathcal{A}} q_f}$$

$$\frac{t_1 \vdash_{\mathcal{A}} q_{agtI} \quad t_2 \vdash_{\mathcal{A}} q_f \quad t_3 \vdash_{\mathcal{A}} q_f}{N(t_1, t_2, t_3) \vdash_{\mathcal{A}} q_f}$$

$$\frac{t_1 \vdash_{\mathcal{A}} q_f \quad t_2 \vdash_{\mathcal{A}} q_{agtI} \quad t_3 \vdash_{\mathcal{A}} q_f}{encr(t_1, t_2, t_3) \vdash_{\mathcal{A}} q_f}$$

$$\frac{t_1 \vdash_{\mathcal{A}} q_{agtI} \quad t_2 \vdash_{\mathcal{A}} q_f \quad t_3 \vdash_{\mathcal{A}} q_f}{sk(t_1, t_2, t_3) \vdash_{\mathcal{A}} q_f}$$

Recognizability

$$\overline{rec}(t) = \emptyset \quad if \quad t \vdash_{\mathcal{A}} q_f$$

otherwise

$$\overline{rec}(t) = \{t_i \mid t = \mathcal{C}[t_i] \text{ and } t_i \not\vdash_{\mathcal{A}} q_f\}$$

subterms labelling the unsolved leaves of the proof tree of t.

The Strategy

Input:

- the rewrite system $\mathcal{R} = \mathcal{R}_P \cup \mathcal{R}_I$,
- the predicate wf,
- the instantiation set Inst,
- the intruder's initial knowledge Δ in \mathcal{A} ,
- the well-formed term t_{in} describing the property under consideration.

The Strategy

Definition: a set of inference rules over configurations.

Configurations: (finite) sets of well-formed terms or elements of the set $\{success, failure\}$.

Initial configuration: $E_0 = \{t_{in}\}.$

Inference Rules

Well-formed Expansion:
$$\frac{t \in E \quad expansion(t, \mathcal{R}) = E'}{E \setminus \{t\} \cup \{t' \in E' \mid wf(t')\}}$$

$$\frac{E = \emptyset}{failure}$$

$$\underbrace{t \in E \quad \exists t'.subterm(t, t') \land \ root(t') = goal}_{success}$$

Inference Rules

$$t \in E \quad expansion(t, \mathcal{R}_P) = \emptyset \quad subterm(t, t_{in})$$

$$\exists t'.subterm(t, t') \land root(t') = mesg$$

$$E \setminus \{t\}$$

$$t \in E$$
 $expansion(t, \mathcal{R}_P) = \emptyset$ $not(subterm(t, t_{in}))$
 $\exists t'.subterm(t, t') \land root(t') = mesg$
 $\mathcal{I}(t, Inst) = E_1 \quad \exists t_1 \in E_1. \overline{rec}(t_1) = \emptyset$

Success₂:

success

Inference Rules

$$t \in E \quad expansion(t, \mathcal{R}_P) = \emptyset \quad not(subterm(t, t_{in}))$$

$$\exists t'.subterm(t, t') \land root(t') = mesg$$

$$\mathcal{I}(t, Inst) = \{t_1, \dots, t_k\} \quad \forall i.\overline{rec}(t_i) \neq \emptyset$$
Split:
$$E \setminus \{t\} \cup \overline{rec}(t_1) \cup \dots \cup \overline{rec}(t_k)$$

The rewriting strategy is:

```
((Well-formed Expansion + Cut)*.

(Failure + Success<sub>1</sub> + Success<sub>2</sub> + Split))*
```

Properties of the Strategy

Given \mathcal{R} , wf, Inst, \mathcal{A} with transitions Δ and $\mathcal{A}_{\overline{p}}$, we have the following (Nesi-Rucci-Verdesca 2003).

Proposition (correctness)
Let $t_{in} \in \mathcal{L}(\mathcal{A}_{\overline{p}})$.

(i) If $\{t_{in}\} \vdash success$, then the transition $t_{in} \rightarrow q_f$ can be generated from critical pairs.

(ii) If $\{t_{in}\} \vdash failure$, then the transition $t_{in} \rightarrow q_f$ cannot be generated from critical

pairs.

Properties of the Strategy

Proposition (termination) The rewriting strategy terminates on any input term $t_{in} \in \mathcal{L}(\mathcal{A}_{\overline{p}})$.

Corollary (completeness)

Let
$$t_{in} \in \mathcal{L}(\mathcal{A}_{\overline{p}})$$
.

- (i) If the transition $t_{in} \rightarrow q_f$ can be generated from critical pairs, then $\{t_{in}\} \vdash success$.
- (ii) If the transition $t_{in} \rightarrow q_f$ cannot be generated from critical pairs, then

$$\{t_{in}\} \vdash failure$$
.

```
t_{in} = c_{resp}(agt(B), agt(A), agt(0), r(s(0))) \in \mathcal{L}(\mathcal{A}_{\overline{a}})
```

By expansion with rules (7), (5) and (4) in \mathcal{R}_P , the last three steps of session (ii) are performed backward:

```
 \begin{aligned} &\{c_{resp}(agt(B), agt(A), agt(0), r(s(0)))\} \\ &\vdash \{mesg(a_{10}, a_{6}, \\ &encr(sk(agt(A), agt(B), r(i)), agt(0), N(agt(B), agt(A), r(s(0))), r(s(0)))\} \\ &\vdash \{mesg(a_{6}, agt(0), \\ &encr(sk(agt(A), agt(B), r(i)), a_{7}, N(agt(B), agt(A), r(s(0))), r(s(0)))\} \\ &\vdash \{mesg(agt(0), a_{6}, \\ &encr(ltk(agt(B), serv(S)), a_{5}, cons(sk(agt(A), agt(B), r(i)), agt(A))), \\ &r(s(0)))\} \end{aligned}
```

```
 \begin{aligned} \{c_{resp}(agt(B), agt(A), agt(0), r(s(0)))\} \\ &\vdash \{mesg(agt(0), a_6, \\ &encr(ltk(agt(B), serv(S)), a_5, cons(sk(agt(A), agt(B), r(i)), agt(A))), \\ &r(s(0)))\} \end{aligned}
```

The last term cannot be further expanded. Using Split, by instantiating with $\sigma=\{agt(B)/a_6,serv(S)/a_5,0/i\}$ and applying \overline{rec} , we have to derive the recognizability of subterm

```
\overline{t} = encr(ltk(agt(B), serv(S)), serv(S), cons(sk(agt(A), agt(B), r(0)), agt(A)))
```

```
\overline{t} = encr(ltk(agt(B), serv(S)), serv(S), cons(sk(agt(A), agt(B), r(0)), agt(A)))
```

By expansion with rules (16), (3), (2) and finally (1), the first three steps of session (i) are executed, thus obtaining success:

```
 \begin{split} \overline{t} \\ &\vdash mesg(x,y,\overline{t},w) \\ &\vdash mesg(a_4,a_5,\\ &encr(ltk(agt(A),serv(S)),a_3,\\ &cons(N(agt(A),serv(S),r(j)),\\ &cons(agt(B),\\ &cons(sk(agt(A),agt(B),r(i_1)),\\ &encr(ltk(agt(B),serv(S)),serv(S),\\ &cons(sk(agt(A),agt(B),r(0)),agt(A)))))),\\ &r(j)) \end{split}
```

```
\vdash mesg(a_2, serv(S), cons(N(agt(A), serv(S), r(0)), cons(agt(A), agt(B))), r(0))\vdash goal(agt(A), agt(B), r(0))\vdash success
```

Thus, $\overline{t} \to q_f$ and hence $t_{in} \to q_f$.

Lowe's multiplicity attack on NSSK

Using Split with substitution

$$\sigma' = \{agt(B)/a_6, serv(S)/a_5, s(0)/i\}$$
 also derives success.

Conclusions

- No approximation function γ
- Property satisfied or not
- Feedback on error location
- Combination of reduction (e.g. narrowing)
 with deduction (e.g. recognizability)
- Compromise between the full efficiency of the approximation technique and the full power of theorem proving based methods

Conclusions

Need more general criteria for

- formalizing the steps of a protocol and the properties to be checked into rules,
- ensuring the termination of the strategy (well-formedness).

Current and Future Work

- Extension of the properties under consideration
- Application of the approach to other (classes of) protocols
- Implementation of the strategy in a theorem proving environment
- Formalization only based on rewrite systems (no tree automata)