${\rm AVISS--Automated\ Verification\ of\ Infinite\ State\ Systems}$

(IST-2000-26410)

Deliverable D3.3

Preliminary definition, implementation and experimentation with the SAT model checker

1 Introduction

The objective of the third work-package WP3 of our project AVISS IST-2000-26410 is to experiment the chosen automated deduction techniques implemented in the prototype verification tool against the verification problems of the corpus [1] of security protocols.

The first step (encoding) is the definition of encodings that translates the protocol verification problems, obtained by applying the translator of WP2 to the corpus, into deduction problems falling into the scope of application of the chosen automated deduction techniques. (The architecture of the prototype is shown again in Figure 1.) These encodings, together with the translation mechanism set up in WP2, allows us to run the available automated deduction engines against the verification problems of the corpus (experiments). The experiments indicate ways to improve the encodings as well as the inference strategies implemented in the available automated deduction engines (tuning).

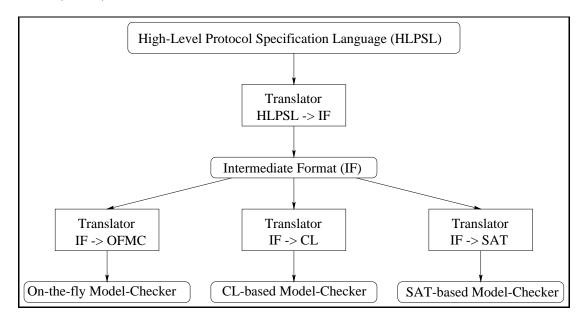


Figure 1: Architecture of the prototype verification tool

This deliverable D3.3 describes our implementation of task T3.3, i.e. define the encoding from the IF to the input format for the model-checker based on propositional satisfiability checking, develop a prototype translator implementing the encoding, and experiment with problems from the corpus, tuning the encoding and/or the model-checker based on propositional satisfiability checking.

2 Overview

More specifically, we describe the SAT Encoder (SATE) we are currently developing and using in the context of the AVISS Project to perform SAT-based model checking of security protocols.

Figure 2 shows the architecture of the SAT-based Model-Checker. As in the other approaches (see Deliverables D3.1 and D3.2) the HLPSL2IF compiler translates the HLPSL specification into the Intermediate Format (IF — see Deliverable D2.2). The parser of the IF, IF2PIF, reads a IF specification and translates it into an equivalent specification in the Prolog Intermediate Format (PIF). The PIF preserves the rewrite rules representation of the IF but it is amenable to further transformations. The PIF2SATE compiler translates the PIF specification into a STRIPS problem expressed in the SATE specification language which is then fed to SATE. SATE takes as input the STRIPS problem and two parameters specifying the bound in the number of operation

applications and the bound on the depth of the terms during expansion, respectively, and generates a propositional formula whose satisfiability guarantees the reachability of a goal state from an initial state. By feeding the propositional formula to a SAT solver (currently Chaff [7] and SATO [9] are interfaced to SATE) it is possible to determine its satisfiability. Since the SAT encoding computed by SATE is constructive, from any model of the propositional formula it is always possible to extract a sequence of rule applications leading from the initial state to a goal state (we call *trace* such a sequence). Therefore whenever a SAT solver finds a model for the formula, SATE extracts and displays the corresponding trace.

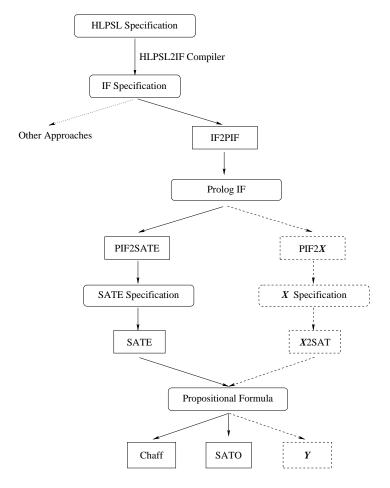


Figure 2: Architecture of the SAT-based Model-Checker

This report is organized in the following way. We start by presenting the IF parser and the difference between the IF and the PIF (section 3). Then we introduce the SATE specification language (section 4) and we discuss the translation from the PIF to the SATE specification language (section 5). We conclude by describing SATE (section 6).

3 The IF Parser

The parser of the IF, IF2PIF, parses specifications in the IF and returns equivalent specifications in the Prolog Intermediate Format (PIF). The parser is written using definite clause grammars (see [2, 8]).¹

¹ Definite clause grammars extend the well-known context-free grammars. They admit both a declarative and a procedural interpretation and are therefore an ideal means to build parsers in a flexible way.

The parser implements two phases. In the first it parses the options lines and in the second it parses the rewrite rules. At the end the PIF file contains the following prolog fact:

```
problem(OPTs, RRs).
```

where OPTs is the prolog list containing the parsed options² and RRs is the prolog list containing the parsed rewrite rules. In order to define the content of the above lists we present the translation from IF to PIF. If t is an IF syntactic construct, then (t) is the corresponding syntactic construct of the PIF.

- Let id be an IF identifier such that $id \neq I$. If the first character of id is an upper-case letter, then (id) is the identifier obtained from id by replacing the first character with the corresponding lower-case letter; otherwise, if id = I (and hence represents the IF intruder constant) then (id) = intruder, otherwise (id) = id. For example, (seo1) = seo1, (I) = intruder, (seo1) = seo1, (I) = intruder, (seo1) = seo1, (seo1) = seo1
- Let end be the IF symbol representing the end of the IF knowledge list and t_1, \ldots, t_n be IF terms, then $\{c(t_1, c(t_2, \ldots c(t_n, end) \ldots))\} = [\{t_1\}, \{t_2\}, \ldots, \{t_n\}]\}$. For example, $\{c(a, c(b, c(ka, etc)))\} = [a, b, ka]$.
- Let t be an IF term, then (pk(t)) = primed(pk((t))).
- Let f be an IF function symbol different from c, then $(f(t_1,\ldots,t_n)) = f((t_1),\ldots,(t_n))$.
- Let t_1, \ldots, t_n be IF terms, then $(t_1, \ldots, t_n) = [(t_1), \ldots, (t_n)]$.
- Let label and category be two identifiers, l, r be two dotted terms, and eol be the end-of-line character, then (# lb=label, type=category eol l=>r) = lrr(([label]), ([category]), ([l]), ([r])), and lrr(([label]), ([category]), ([l]), ([r])) is member of the list RRs.
- Let label and category be two identifiers and l a dotted term, then (# lb=label, type=category eol l) = lrr(([label]), ([category]), ([l), []), and lrr(([label]), ([category]), ([l), []) is member of the list RRs.
- Let op be an IF option identifier, then (# option=op eol) = ([op]) and ([op]) is member of the list OPTs.

4 STRIPS Problems and the SATE Specification Language

The SATE specification language is inspired by STRIPS [5]. A STRIPS problem is a tuple $\langle \mathcal{F}, \mathcal{A}, Ops, I, G \rangle$, where \mathcal{F} and \mathcal{A} are disjoint sets of variable-free atomic formulae of a sorted first-order language called fluents and actions respectively; Ops is a set of expressions of the form

where $Action \in \mathcal{A}$ and Pre, Add, and Del are finite sets of fluents; I is a finite set of fluents and G is a boolean combination of fluents representing the initial and the final states respectively.

The SATE specification language allows us to specify SATE problems and is defined by the grammar of Figure 3.

The main constructs of the language are:

• Sort declaration (sort decl). For example, the declarations

```
sort(fluent).
sort(action).
sort(time).
```

²At now OPTs = [typed] or OPTs = [untyped].

```
(assertion.)^*
                    sate spec ::=
                     assertion
                                       sort decl \mid super sort decl \mid constant decl \mid
                                       invariant \mid initial \mid state \mid goal \mid operator
                    sort \ decl \ ::=
                                       sort(sort id)
             super \ sort \ decl \ ::=
                                       super_sort(sort id,sort id)
                constant \ decl \ ::=
                                       constant(arity, sort id)
                                       invariant(composite fluent)
                     invariant ::=
                 initial\_state ::=
                                       facts(init)
                                       goal(goal)
                          goal ::=
                     operator ::=
                                       action(action, condition, pre, add, del)
                                       const\_id \mid const\_id(sort\_id^*)
                         arity
                                ::=
                                ::=
                                       const\_id \mid const\_id(term^*)
                fluent, action
                                       var \mid const \mid id \mid const \mid id(term^*)
                         term
                                ::=
      init, goal, pre, add, del
                                ::=
                                       [fluent^*]
                                       term = term | fluent | ~ fluent | fluent <=> fluent
            composite fluent
                                ::=
            sort id, const id ::=
                                       /a-z,A-Z//_,a-z,A-Z,0-9/*
                                ::= /_{A-Z}//_{A-Z,A-Z,0-9}
Proviso:
            In the production rule for operator:
            Vars(cond), Vars(prec), Vars(add), Vars(del) \subseteq Vars(actions).
            X^* abbreviates (X(X)^*), and X_1, \ldots, X_n := E abbreviates X_1 := E \ldots X_n := E.
Legenda:
```

Figure 3: Grammar for the SATE specification language

declare fluent, action, and time to be sorts.

• Super-Sort declaration (super sort decl). For example, the declarations

```
super_sort(atom,mr).
super_sort(atom,pk).
```

declare atom to be super-sort of both mr (sort representing the agents) and pk (sort representing the public and private keys).

• Constant declaration (constant decl). For example, the constant declarations

```
constant(0,time).
constant(s(time),time).
constant(mr(user),mr).
constant(a,user).
```

declare O and a to be individual constants of sort time and user, respectively, and s and mr to be function symbols mapping elements of sort time into elements of sort time and of sort user into elements of sort mr, respectively.

• Invariant (invariant). For example, the invariant

```
invariant(~(m(_,_,A,A,_,_))).
```

states, by means of the logical negation, that (at each time instant) the fields third and fourth of the above fluent are different.

• Initial State (initial state). For example, the assertion

asserts that the listed fluents hold at time 0.

• Goal (goal state).³ For example, the assertion

```
goal([m(1,mr(a),mr(a),mr(b),crypt(pk(kb),c(nonce(na,s(s(0))),mr(a))),1)]).
states that all states containing the fluent
    m(1,mr(a),mr(a),mr(b),crypt(pk(kb),c(nonce(na,s(s(0))),mr(a))),1)
```

• Operator (operator). For example, the assertion

are goal states.

```
action(step_O(Xc,XKb,XKa,XB,XA,XTime),
        true,
        [h(s(XTime)),
         inknw(XA,XA,1,Xc),
         inknw(XA,XB,2,Xc),
         inknw(XA,XKa,3,Xc),
         inknw(XA, primed(XKa), 4, Xc),
         inknw(XA,XKb,5,Xc),
         wk(0,mr(intruder),XA,etc,1,true,Xc)],
        [h(XTime),
         m(1,XA,XA,XB,crypt(XKb,c(nonce(c(na,XTime)),XA)),Xc),
         inknw(XA,XA,1,Xc),
         inknw(XA,XB,2,Xc),
         inknw(XA,XKa,3,Xc),
         inknw(XA, primed(XKa), 4, Xc),
         inknw(XA,XKb,5,Xc),
         wk(2,XB,XA,nonce(c(na,XTime)),1,true,Xc)],
        [h(s(XTime)),
         inknw(XA,XA,1,Xc),
         inknw(XA,XB,2,Xc),
         inknw(XA,XKa,3,Xc),
         inknw(XA, primed(XKa), 4, Xc),
         inknw(XA,XKb,5,Xc),
         wk(0,mr(intruder),XA,etc,1,true,Xc)]).
```

³Currently we only allow a list of fluents as argument to goal, whereas in principle it is possible to have an arbitrary boolean combination of fluents. This restriction will be lifted in the next deliverable.

states that if S is a state and σ a substitution such that $\{h(s(XTime))\sigma, wk(0,mr(intruder),XA,etc,1,true,Xc)\sigma, inknw(XA,XA,1,Xc)\sigma, inknw(XA,XB,2,Xc)\sigma, inknw(XA,XKa,3,Xc)\sigma, inknw(XA,primed(XKa),4,Xc)\sigma, inknw(XA,XKb,5,Xc)\sigma\} \subseteq S$, then $(S \setminus \{h(s(XTime))\sigma, wk(0,mr(intruder),XA,etc,1,true,Xc)\sigma, inknw(XA,XA,1,Xc)\sigma, inknw(XA,XB,2,Xc)\sigma, inknw(XA,XKb,5,Xc)\sigma\}) \cup \{\{h(XTime)\sigma, wk(2,XB,XA,nonce(c(na,XTime)),1,true,Xc)\sigma, inknw(XA,XB,crypt(XKb,c(nonce(c(na,XTime)),XA)),Xc)\sigma, inknw(XA,XB,2,Xc)\sigma, inknw(XA,XB,2,Xc)\sigma, inknw(XA,XKa,3,Xc)\sigma, inknw(XA,XB,2,Xc)\sigma, inknw(XA,XKa,3,Xc)\sigma, inknw(XA,XKb,5,Xc)\sigma\}\}$ is a successor state of S.

5 Translating PIF into the SATE Specification Language

We now define the translation from PIF to the SATE specification language. First we present a simple translation and then we show some optimizations.

Our goal is to convert a PIF file representing a security protocol into a SATE file representing the same protocol. In order to do this we have to declare the SATE constructs (sorts, super-sorts, constants, invariants, initial state, goal and operators) characterizing a protocol. Some of these declarations are independent from the protocol. For example, the following declarations,

```
sort(bool).
constant(true,bool).
constant(false,bool).
```

are independent from the protocol, because the sort bool representing the boolean data type is used in each protocol. Figure 4 shows the protocol independent super-sort and constant declarations.⁴

In addition to these we have to define the protocol dependent declarations. Let S be the PIF specification and D_S be the set of protocol dependent declarations; then:

- if t is a PIF syntactic construct, then [t] is the corresponding syntactic construct of SATE;
- if t is a PIF term, then VAR(t) is the function returning the set of PIF variables occurring in t; for example, $VAR(h(xTime)) = \{xTime\};$
- if x is a PIF variable occurring in S, then SORT(x, S) is the function returning the SATE sort of x on the base of the position of it inside the specification S; for example, if h(xTime) occurs in S, then SORT(xTime, S) = time;
- for each PIF constant symbol s (hence the first character of s is not the x character) if

```
- mr(s) occurs in \mathcal{S}, then constant(s,user) \in D_{\mathcal{S}};

- pk(s) occurs in \mathcal{S}, then constant(s,public_key) \in D_{\mathcal{S}};

- primed(pk(s)) occurs in \mathcal{S}, then constant(s,private_key) \in D_{\mathcal{S}};

- sk(s) occurs in \mathcal{S}, then constant(s,private_key) \in D_{\mathcal{S}};

- fu(s) occurs in \mathcal{S}, then constant(s,fu_symbol) \in D_{\mathcal{S}};

- nonce(c(s,xTime)) occurs in \mathcal{S}, then constant(s,fresh_nonce_id) \in D_{\mathcal{S}};

- pk(c(s,xTime)) occurs in \mathcal{S}, then constant(s,fresh_public_id) \in D_{\mathcal{S}};

- primed(pk(c(s,xTime))) occurs in \mathcal{S}, then constant(s,fresh_symmetric_id) \in D_{\mathcal{S}};

- sk(c(s,xTime)) occurs in \mathcal{S}, then constant(s,fresh_symmetric_id) \in D_{\mathcal{S}};

- c(s,s) occurs in \mathcal{S}, then constant(s,intruder_const) \in D_{\mathcal{S}};
```

⁴To increase the readability, the sort declarations are not given, but they can be derived by the other declarations. For example, super_sort(knw_el,msg) implies that knw_el and msg are sorts and constant(crypt(msg,msg),crypt) implies that crypt and msg are sorts.

```
constant(h(time),fluent).
super_sort(knw_el,msg).
                                    constant(w(wstep,mr,mr,knw,knw,bool,session),fluent).
super_sort(msg,binary).
                                    constant(m(mstep,mr,mr,mr,msg,session),fluent).
super_sort(msg,unary).
                                    constant(i(knw_el),fluent).
                                    constant(secret(knw_el,fsecrecy),fluent).
super_sort(binary,crypt).
super_sort(binary,scrypt).
                                    constant(etc,knw).
super_sort(binary,pair).
                                    constant(c(knw_el,knw),knw).
super_sort(binary, funct).
                                    constant(etc,etc).
super_sort(binary,rcrypt).
                                    constant(crypt(msg,msg),crypt).
super_sort(unary, atom).
                                    constant(scrypt(msg,msg),scrypt).
                                    constant(funct(msg,msg),funct).
super_sort(atom, mr).
                                    constant(c(msg,msg),pair).
                                    constant(rcrypt(msg,msg),rcrypt).
super_sort(atom,pk).
super_sort(atom, sk).
super_sort(atom, nonce).
                                    constant(pk(public_key),pk).
super_sort(atom,fu).
                                    constant(pk(private_key),pkinv).
                                    constant(primed(pkinv),pk).
                                    constant(sk(symmetric\_key), sk).
constant(0,time).
                                    constant(fu(fu_symbol),fu).
constant(s(time),time).
                                    constant(mr(user),mr).
constant(true,bool).
                                    constant(nonce(fresh_nonce),nonce).
constant(false,bool).
constant(s(session),session).
                                    constant(pk(fresh_public_key),pk).
constant(f(session),fsecrecy).
                                    constant(pk(fresh_private_key),pkinv).
                                    constant(sk(fresh_symmetric_key),sk).
                                    constant(pk(fresh_intruder_const),pk).
                                    constant(pk(fresh_intruder_const),pkinv).
                                    constant(sk(fresh_intruder_const),sk).
                                    constant(nonce(fresh_intruder_const),nonce).
                                    constant(c(fresh_public_id,time),fresh_public_key).
                                    constant(c(fresh_private_id,time),fresh_private_key).
                                    constant(c(fresh_symmetric_id,time),fresh_symmetric_key).
                                    constant(c(fresh_nonce_id, time), fresh_nonce).
                                    constant(c(intruder_const,intruder_const),fresh_intruder_const).
```

Figure 4: Protocol independent declarations

- for each lrr(label, cat, [t₁,...,t_n], [t_{n+1},...,t_{n+m}]) occurring in $\mathcal S$ such that $cat \in \{\texttt{protocol_Rules}\}$ and for each $k,j=1,\ldots,n+m$ such that $t_k = \texttt{m}(mi,_,_,_,_,_)$, $t_j = \texttt{w}(wi,_,_,_,_,_)$, then $\{\texttt{constant}(mi,\texttt{mstep}),\texttt{constant}(wi,\texttt{wstep})\} \in \mathcal D_{\mathcal S}$;
- for each $lrr(_,init,[t_1,\ldots,t_n],[])$ occurring in S, then $facts([[t_1]],\ldots,[[t_n]]) \in D_S$;
- for each lrr(label, cat, $[lt_1, \ldots, lt_n]$, $[rt_1, \ldots, rt_m]$) occurring in \mathcal{S} such that $cat \in \{\text{protocol_Rules,intruder_Rules,decomposition_Rules}\}$ (see Deliverable D2.2), then

and constant (label (s_1, \ldots, s_k)), action) $\in D_S$, where $x_1, \ldots, x_k = VAR(lt_1) \cup \cdots \cup VAR(lt_n) \cup VAR(rt_m)$ and $s_i = SORT(x_i, S)$ for each $i = 1, \ldots, k$;

- for each $lrr(_,goal,[t_1,\ldots,t_n],[])$ occurring in S then $goal([[t_1]],\ldots,[t_n]]) \in D_S$;
- $[\![\mathbf{w}(ws, s, r, [t_1, \dots, t_n], [t_{n+1}, \dots, t_{n+m}], b, ses)]\!] = \mathbf{w}([\![ws]\!], [\![s]\!], [\![r]\!], \mathbf{c}([\![t_1]\!], \dots \mathbf{c}([\![t_n]\!], etc) \dots), \mathbf{c}([\![t_{n+1}]\!], \dots \mathbf{c}([\![t_{n+m}]\!], etc) \dots), [\![b]\!], [\![ses]\!]);$
- $[f(t_1,\ldots,t_n)] = f([t_1],\ldots,[t_n]);$
- if v be a PIF variable (hence it begins with the character \mathbf{x}) then $\llbracket v \rrbracket$ is the variable obtained from v by replacing the first character with \mathbf{X} ; for example, $\llbracket \mathbf{x} \mathbf{A} \rrbracket = \mathbf{X} \mathbf{A}$.

The number of ground instances of fluents and of ground instances of actions directly affect the number of clauses and of propositional variables generated by the SATE module (see section 6 and [4]). The ground instances of fluents (actions) are given by expansion of the constant declarations relative to the sort fluent (action). For example the expansion of the declaration

```
constant(m(mstep,mr,mr,mr,msg,session),fluent)
```

generates a number of fluents equal to

$$|\mathtt{mstep}| * |\mathtt{mr}|^3 * |\mathtt{msg}| * |\mathtt{session}|$$

where |s| returns the number of expanded terms relative to the sort s. In order to decrease the number of ground fluents and actions we apply the optimizations presented in the following subsections.

5.1 Invariants

This is an useful way to decrease the number of ground instances of fluents. The idea is to use some invariants for describing what ground instances of fluents are allowed and what are not. For example, to state that the fields official sender and receiver in a message term must be different we can use:

```
invariant(~(m(_,_,A,A,_,_)))
```

where the symbol ~ is the logic negation. In the same way, to avoid the ground instances of fluents in which the fields sender and receiver of a principal term are equal, we can use:

```
invariant(~(w(_,A,A,_,_,_,)))
```

Applying the above invariants the number of ground instances of fluents generated by the declarations

```
 \begin{split} & \text{constant} \left( \texttt{m} (\texttt{mstep,mr,mr,mr,msg,session}), \texttt{fluent} \right) \\ & \text{constant} \left( \texttt{w} (\texttt{wstep,mr,mr,knw,knw,bool,session}), \texttt{fluent} \right) \\ & \text{become respectively:} \\ & & | \texttt{mstep} | * | \texttt{mr} |^2 * (|\texttt{mr}| - 1) * |\texttt{msg}| * |\texttt{session}| \\ & | \texttt{wstep} | * |\texttt{mr}| * (|\texttt{mr}| - 1) * |\texttt{knw}|^2 * |\texttt{bool}| * |\texttt{session}| \end{split}
```

5.2 Protocol dependent messages

Note that in Figure 4 the declarations about the messages (msg) are too general. In fact the expansion of these declarations generates a lot of useless terms i.e. those messages which are not allowed by the protocol. The idea is to restrict the generated terms on the base of the messages allowed by the protocol. In order to do this we avoid to express the declarations about the messages as independent from the protocol and we analyze the PIF specification S to extract the patterns of the possible exchanged messages allowed by the protocol.

In particular, for each lrr (label, cat,_,[rt_1,...,rt_m]) occurring in S such that $cat \in \{\text{protocol_Rules}\}$ and for each k = 1, ..., m such that $rt_k = m(i, _, _, _, msg, _)$ then $\{\text{sort}(\text{msg}_i), \text{sort}(\text{mstep}_i), \text{super_sort}(\text{msg}_i, sort), \text{constant}(\text{m}(\text{mstep}_i,\text{mr,mr,mr,msg}_i,\text{session}),\text{fluent})\} \cup Cs \in D_S \text{ where } Cs \text{ and } sort \text{ are computed by the procedure PREPROCESS_MSG}(msg,i,sort,Cs). This procedure (partially given in Figure 5) executes a syntactical analysis of the PIF message <math>msg$ computing sort, i.e. the sort of msg (previous a concatenation with i), and Cs, i.e. the list of constant declarations useful to define the SATE sub-language relative to msg.

Let us consider the example of the Needham-Schroeder Public-Key (NSPK) protocol (see [1]). The PIF rewrite rules relative to the NSPK protocol steps are given in Figure 6. For each of these rewrite rule we analyze the right-hand-side (fourth parameter of the term 1rr) and if it contains a message term of the form $m(i, _, _, _, msg, _)$ then we extract the field msg. Finally we call the procedure PREPROCESS_MSG(msg, i, sort, Cs) returning the SATE sort of msg and the set of constant declarations relative to msg. In this case we call the procedure three times and the results are the following:

```
PREPROCESS_MSG(crypt(xKb,c(xNa,xA)),1,
                crypt_1,
                [ constant(crypt(pk,pair_1),crypt_1),constant(c(atom,atom),pair_1),
                   super_sort(knw_el,pair_1),super_sort(knw_el,crypt_1)
               1
)
PREPROCESS_MSG(crypt(xKa,c(xNa,nonce(c(nb,xTime)))),2,
                [ constant(crypt(pk,pair_2),crypt_2),constant(c(atom,nonce),pair_2),
                   super_sort(knw_el,pair_2),super_sort(knw_el,crypt_2)
               1
)
PREPROCESS_MSG(crypt(xKb,xNb),3,
                crypt_3,
                [constant(crypt(pk,atom),crypt_3),super_sort(knw_el,crypt_3)]
)
Hence at the end of the above process D_{\mathcal{S}} will contain the following declarations:
sort(msg_1).
sort(mstep_1).
```

```
super_sort(msg_1,crypt_1).
constant(m(mstep_1,mr,mr,mr,msg_1,session),fluent).
sort(msg_2).
sort(mstep 2).
super_sort(msg_2,crypt_2).
constant(m(mstep_2,mr,mr,mr,msg_2,session),fluent).
sort(msg_3).
sort(mstep_3).
super_sort(msg_3,crypt_3).
constant(m(mstep_3,mr,mr,mr,msg_3,session),fluent).
constant(crypt(pk,pair_1),crypt_1).
constant(c(atom,atom),pair_1).
constant(crypt(pk,pair_2),crypt_2).
constant(c(atom,nonce),pair_2).
constant(crypt(pk,atom),crypt_3).
super_sort(knw_el,pair_1).
super_sort(knw_el,crypt_1).
super_sort(knw_el,pair_2).
super_sort(knw_el,crypt_2).
super_sort(knw_el,crypt_3).
```

Using the general declaration of Figure 4, the number of ground instances of fluents relative to the message terms was:

$$|mstep| * |mr|^3 * |msg| * |session|$$

while using this optimization that number decreases to the value:

$$(|mstep_1| * |msg_1| + |mstep_2| * |msg_2| + |mstep_3| * |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_2| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_2| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_2| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_2| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_2| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_2| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_2| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_3| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_3| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_3| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_3| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_3| + |msg_3| + |msg_3|) * |mr|^3 * |session| = (|msg_1| + |msg_3| + |msg_3|$$

Since

$$|mstep| * |msg| \gg |msg_1| + |msg_2| + |msg_3|$$

the number of ground instances of fluents decreases considerably. If we assume that in the NSPK protocol is executed by three agents with the respective public and private keys and two nonces, then we have $|\mathtt{mstep}| * |\mathtt{msg}| \simeq 5,690,000$, while $|\mathtt{msg}_{-}1| + |\mathtt{msg}_{-}2| + |\mathtt{msg}_{-}3| \simeq 1,000$.

5.3 Knowledge splitting

The idea of the knowledge splitting comes from some features of the acquired and initial knowledge. Specifically we can note that:

- 1. the domain of the initial knowledge is fixed by the initial state and this domain is a subset of the domain relative to the acquired knowledge;
- 2. the initial knowledge is a static⁵ property of an agent involved in a protocol session;
- 3. the ordering of the terms in both kinds of knowledge is relevant.

⁵It does not change during the simulation of the protocol session.

```
procedure PREPROCESS MSG(msg,lb,sort,Cs)
  inputs: msg, a PIF message
            lb, a label
            sort, the sort relative to the message msg
            Cs, a set of SATE declarations
     msg == crypt(\_, submsg):
            PREPROCESS MSG(submsg,lb,submsgsort,Cs)
            	exttt{constant(crypt(pk,} submsgsort), crypt\_lb) } \in Cs
            \verb"super_sort"(\verb"knw_el, \verb"crypt_" lb") \ \in \mathit{Cs}
            sort \leftarrow \texttt{crypt\_}lb
            break
      msg == c(submsg_1, submsg_2):
            {\tt PREPROCESS\_MSG}(submsg_1, lb, submsgsort_1, Cs)
            PREPROCESS\_MSG(submsg_2, lb, submsgsort_2, Cs)
            constant(c(submsgsort_1, submsgsort_2), pair\_lb) \in Cs
            	exttt{super_sort(knw_el,pair_}lb) \in Cs
            sort \leftarrow \texttt{pair\_}lb
            break
      msg == pk(submsg) and submsg is an atom:
            sort \leftarrow \mathtt{pk}
            break
      msg == primed(pk(submsg)) and submsg is an atom :
            sort \leftarrow pk
      msg == nonce(c(submsg, \_)) and submsg is an atom:
            sort \leftarrow \texttt{nonce}
            break
      msg is an atom:
            sort \leftarrow \texttt{atom}
            break
esac
```

Figure 5: Pseudo-code of the procedure PREPROCESS_MSG

Feature 1 suggests to use two different sorts for the initial and the acquired knowledge, feature 2 suggests to define a new fluent relative to the initial knowledge, and feature 3 suggests to use a sequence of fluents containing one single knowledge term and an index stating the position of this term in the knowledge rather than to use lists.

The protocol dependent declarations change on the base of the following:

```
• for each w(_, _, _, [ak_1, \ldots, ak_k], [ik_1, \ldots, ik_l], _, _) occurring in S then {constant(1,acq_index),...,constant(k,acq_index)} \cup {constant(1,in_index),...,constant(l,in_index)} \in D_S;
```

```
• for each lrr(_,init,[t_1,...,t_n],[]) occurring in S and for each k=1,...,n such that t_k = w(\_,\_,\_,\_,[ik_1,...,ik_l],\_,\_) then { super_sort(inknw_el,sort_1),...,super_sort(inknw_el,sort_l) } \cup Cs_1 \cup \cdots \cup Cs_l \in D_S where Cs_i and sort_i (i=1,...,l) are computed by the procedure calling PREPROCESS MSG(ik_i, inknw, sort_i, Cs_i);
```

```
• for each principal term such that the initial and acquired knowledge are empty then [w(ws, s, r, [], [], ses)] = wk([ws], [s], [r], etc, 1, [ses]),

inknw([r], etc, 1, [ses])
```

```
• for each principal term such that the initial knowledge is empty and k \geq 1 then \llbracket w(ws, s, r, \lceil ak_1, \ldots, ak_k \rceil, \lceil \rceil, ses) \rrbracket = wk(\llbracket ws \rrbracket, \llbracket s \rrbracket, \llbracket r \rrbracket, \llbracket ak_1 \rrbracket, 1, \llbracket ses \rrbracket),
\vdots
wk(\llbracket ws \rrbracket, \llbracket s \rrbracket, \llbracket r \rrbracket, \llbracket ak_k \rrbracket, n, \llbracket ses \rrbracket),
inknw(\llbracket r \rrbracket, etc, 1, \llbracket ses \rrbracket)
```

```
• for each principal term such that the acquired knowledge is empty and l \geq 1 then [\![w(ws,s,r,[\!],[ik_1,\ldots,ik_l],ses)]\!] = wk([\![ws]\!],[\![s]\!],[\![r]\!],etc,1,[\![ses]\!]), inknw([\![r]\!],[\![ik_1]\!],1,[\![ses]\!]))
• for each principal term such that k \geq 1 and l \geq 1 then [\![w(ws,s,r,[ak_1,\ldots,ak_k],[ik_1,\ldots,ik_l],ses)]\!] = wk([\![ws]\!],[\![s]\!],[\![r]\!],[\![ak_1]\!],1,[\![ses]\!]), \vdots wk([\![ws]\!],[\![s]\!],[\![r]\!],[\![ak_k]\!],n,[\![ses]\!]), inknw([\![r]\!],[\![ik_1]\!],1,[\![ses]\!]), \vdots inknw([\![r]\!],[\![ik_1]\!],n,[\![ses]\!])
```

Note that the expansion of constant (w(wstep,mr,mr,knw,knw,bool,session),fluent) generates a number of ground instances of fluents equal to $|wstep| * |mr|^2 * |knw|^2 * |bool| * |session|$ where $|knw| = |knw_el|^{|knw_el|}$. In fact let us suppose that $\{x,y,z\}$ is the set of knowledge elements then the sort knw is characterized by 3^3 terms showed in the following set:⁶ $\{[], [x], [y], [z], [x,x], [x,y], [y,x], ..., [z,z,z]]\}$. If we use the knowledge splitting then we expand the declarations constant (wk(wstep,mr,mr,knw_el,acq_index,bool,session),fluent) and constant (inknw(mr,inknw_el,in_index,session),fluent) and since $|acq_index| \le |knw_el|$ and $|in_index| \le |inknw_el|$, the number of generated ground instances of fluents is equal to or less than $|wstep| * |mr|^2 * |knw_el|^2 * |bool| * |session| + |mr| * |inknw_el|^2 * |session|$. Exploiting the fact that $|inknw_el| \le |knw_el|$ and making some simplification, the advantage by using the knowledge splitting is given by the fact that $|wstep| * |mr| * |knw_el|^2 * |bool| + |inknw_el|^2 \ll |wstep| * |mr| * (|knw_el|^{|knw_el|})^2 * |bool|$.

In conclusion, by applying the above optimization the number of ground fluents is reduced by many orders of magnitude. In Figure 7 we present the protocol independent declarations we have to use together with the above optimizations.

6 SATE

We recall that SATE takes as input a STRIPS problem and two parameters n and δ specifying the bound in the number of operator applications and the bound on the depth of the terms depth during expansion, respectively, and generates a propositional formula whose satisfiability guarantees the reachability of a goal state from an initial state. The encoding of a STRIP problem into a SAT formula can be done in a variety of ways (see [6, 4] for a survey.) In the first of phase of the project we focused on the linear encodings, i.e. encodings in which only one action can occur at each time step. More sophisticated encodings which relax this condition (as, e.g., the parallelized encodings) will be considered in the future.

The encodings we consider are based on the idea of adding an additional time-index to the actions and fluents, to indicate the state at which the action begins or the fluent holds. Fluents are indexed by 0 through n and actions by 0 through n-1. If p is a fluent or an action and i is an index in the appropriate range, then i:p is the corresponding time-indexed propositional formula.

We have considered and experimented with two encodings. The first one is based on the classical frame axioms, the second on the explanatory frame axioms.

⁶We use the prolog list representation to increase the readability. Note that [] is equivalent to etc,[x] is equivalent to c(x,etc) and so on.

⁷The parameters corresponding to n and δ in the implementation are steps and term_depth_bound respectively.

6.1 Encoding based on the Classical Frame Axioms

Let S be a SATE specification. The encoding of S based on the Classical Frame Axioms is the smallest set of propositional formulae, S^* , such that:

UNIVERSAL AXIOMS Let facts (list_of_fluents) $\in S$, then $0: p \in S^*$ for all $p \in list_of_fluents$ and $\neg(0:p) \in S^*$ for all $p \notin list_of_fluents$. Let goal(list_of_fluents) $\in S$, then $n: p \in S^*$ for all $p \in list_of_fluents$. Let action(α, c, pre, add, del) $\in S$, then

$$(i:\alpha\sigma \supset \bigwedge\{i:p\sigma \mid p \in pre\}) \in S^*$$
$$(i:\alpha\sigma \supset \bigwedge\{(i+1):p\sigma \mid p \in add\}) \in S^*$$
$$(i:\alpha\sigma \supset \bigwedge\{\neg(i+1):p\sigma \mid p \in del\}) \in S^*$$

for all substitutions σ such that $\alpha \sigma$ is ground, $c\sigma$ holds, and $0 \le i \le n-1$.

CLASSICAL FRAME AXIOMS Let action $(\alpha, c, pre, add, del) \in S$, then

$$i: \alpha\sigma \supset ((i+1): f\sigma \leftrightarrow i: f\sigma) \in S^*$$

for all substitutions σ such that $\alpha \sigma$ is ground, $c\sigma$ holds, $f \notin add \cup del$, and 0 < i < n-1.

AT-LEAST-ONE AXIOMS

$$\bigvee \{i : \alpha \sigma \mid \mathtt{action}(\alpha, c, \mathit{pre}, \mathit{add}, \mathit{del}) \in S, \alpha \sigma \text{ is ground and } c \sigma \text{ holds}\} \in S^*$$

for
$$0 < i < n - 1$$
.

To instruct SATE to use the encoding based on the explanatory frame axioms the parameter encoding must be set to the list [ape_axiom,classical_frame_axiom,at_least_one_axiom].

6.2 Encoding based on the Explanatory Frame Axioms

Let S be a SATE specification. The encoding of S based on the Explanatory Frame Axioms is the smallest set of propositional formulae, S^* , such that:

UNIVERSAL AXIOMS As in the encoding based on the classical frame axioms (in the previous section).

EXPLANATORY FRAME AXIOMS

$$(i:f \land \neg(i+1):f) \supset \bigvee \{i: \alpha\sigma \mid \mathtt{action}(\alpha,c,pre,add,del) \in S, \alpha\sigma \text{ is ground}, c\sigma \text{ holds, and } f \in del\sigma\} \in S^*$$
 and

$$(\neg i: f \land (i+1): f) \supset \bigvee \{i: \alpha\sigma \mid \mathtt{action}(\alpha, c, pre, add, del) \in S, \alpha\sigma \text{ is ground, } c\sigma \text{ holds, and } f \in add\sigma\} \in S^*$$

for all ground instances of fluents f and $0 \le i \le n-1$.

EXCLUSION AXIOMS

$$\neg(i:\alpha_1\sigma_1\wedge i:\alpha_2\sigma_2)\in S^*$$

for all α_1 , α_2 such that $\operatorname{action}(\alpha_1, c_1, pre_1, add_1, del_1) \in S$, $\operatorname{action}(\alpha_2, c_2, pre_2, add_2, del_2) \in S$, $\alpha_1\sigma_1$ and $\alpha_2\sigma_2$ are ground and such that $\alpha_1\sigma_1 \neq \alpha_2\sigma_2$, $c_1\sigma_1$ and $c_2\sigma_2$ hold, and $0 \leq i \leq n-1$. To avoid the generation of redundant formulae it is convenient to include the additional requirement $\alpha_1\sigma_1 \prec \alpha_2\sigma_2$, where \prec is a lexicographic ordering over actions.

To instruct SATE to use the encoding based on the explanatory frame axioms the parameter encoding must be set to the list [ape_axiom,explanatory_frame_axiom,conflict_exclusion_axiom]).

6.3 Complexity Analysis

The number of literals is in $O(n|\mathcal{F}| + n|\mathcal{A}|)$ for both encodings. The number of formulae is in $O(n|\mathcal{A}||\mathcal{F}|)$ in the encoding based on the classical frame axioms, whereas it is in $O(n|\mathcal{F}| + n|\mathcal{A}|^2)$ in the encoding based on the explanatory frame axioms. Finally, the total size of the encoding is in $O(n|\mathcal{A}||\mathcal{F}|)$ in the encoding based on the classical frame axioms, and it is in $O(n|\mathcal{A}||\mathcal{F}| + n|\mathcal{A}|^2)$ in the encoding based on the explanatory frame axioms.

If we consider the number of formulae, we can conclude that the encoding based on the explanatory frame axioms is better than that based on the classical frame axioms when $|\mathcal{A}| \ll |\mathcal{F}|$. This observation seems to be contradicted when we consider the total size of the encoding. However it must be noted that the worst case analysis in the encoding based on the explanatory frame axioms assumes that all the actions occur in each explanatory frame axioms. However this situation rarely occurs in practice, as usually only a few actions affect the truth value of any given fluent. If the number of actions affecting the truth value of fluents is bounded by a number k (which is usually small), then the total size of the encoding based on the explanatory frame axioms becomes $O(nk|\mathcal{F}|+n|\mathcal{A}|^2)$, thereby confirming the conclusion that the encoding based on the explanatory frame axioms is better than that based on the classical frame axioms if $|\mathcal{A}| \ll |\mathcal{F}|$.

6.4 Invariant-based Simplification

Let S be a SATE specification we define R_S to be the following set of rewrite rules:

$$\begin{array}{ll} R_S = & \{p \rightarrow \mathtt{true} \mid \mathtt{invariant}(p) \in S\} \cup \\ & \{p \rightarrow \mathtt{false} \mid \mathtt{invariant}(\@ifnextcharpoonup place{1mu} \mid S\} \cup \\ & \{p \rightarrow q \mid \mathtt{invariant}(p < = > q) \in S\} \cup \\ & \{s \rightarrow t \mid \mathtt{invariant}(s = t) \in S\} \end{array}$$

The formulae generated by the encoding algorithms are simplified by normalizing their atomic formulae w.r.t. R_S and then performing standard boolean simplifications.⁸ This may drastically reduce the size of the formulae.

6.5 Clausification

The simplified formulae are then clausified using a standard clausification technique.

6.6 SAT Solvers

The current version of SATE supports the interface to two state-of-the-art SAT solvers: SATO [9] and Chaff [7]. Interfacing SATE to other solvers is straightforward but it is left for the future work. The selection of the SAT solver is done by setting the parameter solver to either sato or chaff.

⁸To ensure termination of the normalization process we use ordered rewriting (see, e.g., [3]) using the Prolog @< ordering relation to compare the rewritten atomic formula with the original one.

```
lrr(step_0,protocol_Rules,
[
    h(s(xTime)),
    w(0,xSe0,xA,[],[xA,xB,xKa,primed(xKa),xKb],xbool,xc)
],
Γ
    h(xTime),
    m(1,xA,xA,xB,crypt(xKb,c(nonce(c(na,xTime)),xA)),xc),
    w(2,xB,xA,[nonce(c(na,xTime))],[xA,xB,xKa,primed(xKa),xKb],true,xc)
]
lrr(step_1,protocol_Rules,
    h(s(xTime)),
    m(1,xr,xA,xB,crypt(xKb,c(xNa,xA)),xc2),
    w(1,xA,xB,[],[xB,xKa,xKb,primed(xKb)],xbool,xc)
],
Γ
    h(xTime),
    m(2,xB,xB,xA,crypt(xKa,c(xNa,nonce(c(nb,xTime)))),xc2),
    w(3,xA,xB,[xA,crypt(xKb,c(xNa,xA)),nonce(c(nb,xTime))],[xB,xKa,xKb,primed(xKb)],true,xc)
]
lrr(step_2,protocol_Rules,
h(s(xTime)),
    m(2,xr,xB,xA,crypt(xKa,c(xNa,xNb)),xc2),
    w(2,xB,xA,[xNa],[xA,xB,xKa,primed(xKa),xKb],xbool,xc)
],
h(xTime),
    m(3,xA,xA,xB,crypt(xKb,xNb),xc2),
    \texttt{w(0,xSe0,xA,[],[xA,xB,xKa,primed(xKa),xKb],true,s(xc))}
]
)
lrr(step_3,protocol_Rules,
Ε
    h(s(xTime)),
    m(3,xr,xA,xB,crypt(xKb,xNb),xc2),
    w(3,xA,xB,[xA,crypt(xKb,c(xNa,xA)),xNb],[xB,xKa,xKb,primed(xKb)],xbool,xc)
],
    h(xTime),
    w(1,xA,xB,[],[xB,xKa,xKb,primed(xKb)],true,s(xc))
]
)
```

Figure 6: Protocol rewrite rules of the NSPK protocol

```
super_sort(knw_el,inknw_el).
                                      constant(h(time),fluent).
                                      constant(wk(wstep,mr,mr,knw_el,acq_index,bool,session),fluent).
super_sort(atom,mr).
                                      constant(inknw(mr,inknw_el,in_index,session),fluent).
                                      constant(i(knw_el),fluent).
super_sort(atom,pk).
                                      constant(secret(knw_el,fsecrecy),fluent).
super_sort(atom, sk).
super_sort(atom, nonce).
super_sort(atom,fu).
                                      constant(etc,knw_el).
                                      constant(etc,inknw_el).
constant(0,time).
                                      constant(pk(public_key),pk).
constant(s(time),time).
                                      constant(pk(private_key),pkinv).
constant(true,bool).
                                      constant(primed(pkinv),pk).
constant(false, bool).
                                      constant(sk(symmetric_key),sk).
constant(s(session), session).
                                      constant(fu(fu_symbol),fu).
constant(f(session),fsecrecy).
                                      constant(mr(user),mr).
                                      constant(nonce(fresh_nonce),nonce).
                                      constant(pk(fresh_public_key),pk).
                                      constant(pk(fresh_private_key),pkinv).
invariant( (wk(_,A,A,_,_,_,))).
                                      constant(sk(fresh_symmetric_key),sk).
invariant( (m(_,_,A,A,_,_))).
                                      constant(pk(fresh_intruder_const),pk).
                                      constant(pk(fresh_intruder_const),pkinv).
                                      constant(sk(fresh_intruder_const),sk).
                                      constant(nonce(fresh_intruder_const),nonce).
                                      constant(c(fresh_public_id,time),fresh_public_key).
                                      constant(c(fresh_private_id, time), fresh_private_key).
                                      constant(c(fresh_symmetric_id, time), fresh_symmetric_key).
                                      constant(c(fresh_nonce_id,time),fresh_nonce).
                                      constant(c(intruder_const,intruder_const),fresh_intruder_const).
```

Figure 7: Protocol independent declarations using the optimizations

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