An Optimized Intruder Model for SAT-based Model-Checking of Security Protocols

Luca Compagna
joint work with Alessandro Armando

AI-Lab - MRG-DIST - University of Genova

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Motivations

- **Context:** Dramatic speed-up of SAT solvers in the last decade: problems with thousands of variables are now solved routinely in milliseconds.

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- **Optimization:** In this work we propose an optimized intruder model that leads in many cases to shorter attacks which can be detected in our framework by generating smaller propositional formulae.

Roadmap

- Protocol Analysis
- Modeling via a simple example:
  - Standard Model
  - Axioms and the Optimized Intruder
- Protocol Insecurity Problems with Axioms
- Encoding Protocol Insecurity Problems with Axioms into SAT
- Implementations and Results
- Conclusions and Perspectives

Protocol Analysis: Modeling

- Protocol as a state transition system in which states correspond to information possessed by participating agents.

- Perfect cryptography: an encrypted message can be neither altered nor read without the appropriate key.

- The Dolev-Yao intruder:
  - controls all the traffic in the network;
  - can compose and send fraudulent messages from the knowledge he can glean from the observed traffic and his own initial knowledge.

Protocol Analysis: Security Problems

- Specified by means of the IF rule-based language suitable for security protocols:
  - state: set of facts;
  - transition relation: labeled rewrite rules.
- Security requirements such as authentication and secrecy are reduced to reachability problems on this model.
- We focus on reachability problem with finite number of sessions.
- This is adequate in practice as attacks on well-known protocols often exploit a small number of sessions.

Modeling: Needham-Schroeder authentication prot. (1)

Let us consider the well known NSPK protocol:

$$\begin{array}{ll} 1. & A \rightarrow B: \ \{A, N_A\}_{K_B} \\ 2. & B \rightarrow A: \ \{N_A, N_B\}_{K_A} \\ 3. & A \rightarrow B: \ \{N_B\}_{K_B} \end{array}$$

Scenario: two concurrent sessions of the protocol

session 1: a talks to the intruder i;

session 2: a talks to b.

Security Requirement: B authenticates A on N_A.

Modeling: Needham-Schroeder authentication prot. (2)

States are represented as sets of the following facts:

- $fresh(N)$ means that the nonce N has not been used yet.
- $ik(T)$ means that the intruder knows T.
- $m(J, S, R, T)$ means that sender S has (supposedly) sent message T to principal R at protocol step J.
- $w(J, S, R, [T_1, \ldots, T_k], C)$ represents the state of principal R at step J of session C; it means that R
  - knows the terms stored in the lists $[T_1, \ldots, T_k]$, and
  - is waiting for a message from S (if $J \neq 0$).

Modeling: Needham-Schroeder authentication prot. (3)

- Initial State:

$$w(0, a, a, [a, i, ka, ka^{-1}, ki], 1).$$
$$w(0, a, a, [a, b, ka, ka^{-1}, kb], 2).w(1, b, a, [b, a, kb, kb^{-1}, ka], 2).$$
$$fresh(nc(n1, 1)).fresh(nc(n1, 2)).fresh(nc(n2, 2)).$$
$$ik(i).ik(a).ik(b).ik(ki).ik(ki^{-1}).ik(ka).ik(kb)$$

- Bad States:

$$w(0, a, a, [], [a, b, ka, kb, ka^{-1}], s(1)).$$
$$w(1, a, b, [], [b, a, kb, ka, kb^{-1}], 1)$$

Modeling: Needham-Schroeder authentication prot. (4)

- Labeled Rewrite Rules:
  - Behaviour of Honest Participants:

$$fresh(nc(n1,S)).w(0,A,A,[A,B,Ka,Ka^{-1},Kb],S) \xrightarrow{step_0(A,B,Ka,Kb,S)} w(2,B,A,[nc(n1,S),A,B,Ka,Ka^{-1},Kb],S).m(1,A,B,\{A,nc(n1,S)\}_{Kb})$$

  - Behaviour of the Intruder:

$$m(J,S,R,M) \xrightarrow{divert(J,M,R,S)} ik(S)$$

$$ik(\{M\}_K).ik(K^{-1}) \xrightarrow{decrypt(K,M)} ik(M).ik(\{M\}_K).ik(K^{-1})$$

Modeling: Needham-Schroeder authentication prot. (5)

The attack on the simple NSPK protocol

$$
\begin{array}{llllll}
(1.1) & a & \rightarrow & i & : & \{a, na\}_{ki} & snd \\
(2.1) & i(a) & \rightarrow & b & : & \{a, na\}_{kb} & rc + dec + dec + snd \\
(2.2) & b & \rightarrow & i(a) & : & \{na, nb\}_{ka} & rc\_snd \\
(1.2) & i & \rightarrow & a & : & \{na, nb\}_{ka} & \\
(1.3) & a & \rightarrow & i & : & \{nb\}_{ki} & rc\_snd \\
(2.3) & i(a) & \rightarrow & b & : & \{nb\}_{kb} & rc + dec + snd
\end{array}
$$

requires 3 intruder knowledge manipulations (dec) to be executed.

For industrial-scale security protocols in which messages can have a complex structure, such a number can be much more significant.

Question: can we save such decomposing transitions?

Modeling: Axioms and the Optimized Intruder

Axiom: formula that states a relation between facts of the transition system and that holds at each state of the transition system.

Axioms are particularly suited to represent relations between intruder knowledge facts. E.g.

$$ik(\{M\}_K) \wedge ik(K^{-1}) \supset ik(M)$$

"Every time the intruder knows $\{M\}_K$ and K^{-1}, then it knows instantaneously also M."

Idea: optimize the intruder by replacing decomposing rules with appropriate decomposing axioms.

Protocol Insecurity Problems with Axioms (1)

A Protocol Insecurity Problem (PIP) with axioms is a tuple $\Xi = \langle \mathcal{F}, \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{I}, \mathcal{G} \rangle$ where:

- $\mathcal{F}$ and $\mathcal{L}$ are sets of atomic formula of sorted 1^{st}-order languages called *facts* and *rule labels*, respectively;
- $\mathcal{R}$ is a set of labeled rewrite rules of the form $L \xrightarrow{\lambda} R$, where $L, R \subseteq \mathcal{F}$, and $\lambda \in \mathcal{L}$;
- $\mathcal{A}$ is a set of axioms of the form $\bigwedge_{i=1}^{j} p_i \supset c$, where $p_1, \ldots, p_j, c \in \mathcal{F}$
- $\mathcal{I}$ and $\mathcal{G}$ are respectively the initial state and a boolean formula representing the bad states.

Protocol Insecurity Problems with Axioms (2)

A PIP with axioms represents a state transition system in which:

- States: set of facts S (i.e. $S \subseteq \mathcal{F}$) such that $S \models \mathcal{A}$;
- Transition Relation: let S be a state and $L \xrightarrow{\lambda} R$ be a rewrite rule, then $S \overset{\lambda}{\leadsto} S'$ iff $L \subseteq S$ and $S' = (S \setminus L) \cup R$ is such that $S' \models \mathcal{A}$.

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An attack to a PIP with axioms is a sequence of rules $\lambda_1, \ldots, \lambda_n$ such that $S_i \overset{\lambda_i}{\rightsquigarrow} S_{i+1}$ for $i = 1, \ldots, n$ with $S_1 = \mathcal{I}$ and $S_n \models \mathcal{G}$.

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Attacks to a PIP with axioms can be compactly represented by means of partial-order attack.

Encoding PIP with axioms into SAT (1)

Given a PIP with axioms (without equivalence cycles) Ξ and a positive integer n, we build a propositional formula Φ^n_Ξ such that any model of Φ^n_Ξ corresponds to a partial-order attack of Ξ.

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To do so, we:

1. add an additional time-index parameter to each rule λ or fact p, to indicate the state at which time the rule begins or the fact holds.

2. build Φ^n_Ξ by unfolding n times the transition relation:

$$\Phi^n_\Xi = I(p^0) \wedge \bigwedge_{i=0}^{n-1} T_i(p^i, \lambda^i, p^{i+1}) \wedge G(p^n)$$

   where I, T and G are formulae defining the initial state, the transition relation and the goal states, respectively.

Encoding PIP with axioms into SAT (2)

The encoding of PIP with axioms into a SAT formulae can be done in a variety of ways (see [1,2]).

The main differences between them are reflected in the formula representing the transition relation: $\bigwedge_{k=0}^{n-1} T_i(p^i, \lambda^i, p^{i+1})$.

By introducing axioms, significant changes must be done on the encodings.

We have adapted and extended the following two for supporting axioms:

- Linear encoding, and
- Graphplan-based encoding.

[1] Armando, Compagna. *Abstraction-driven SAT-based Analysis of Security Prot.* (*SAT'03*)
[2] Armando, Compagna, Ganty.
SAT-based Model-Checking of Security Prot. using Planning Graph Analysis (*FME'03*)

Linear Encoding with Axioms (1)

The formula $T_i(p^i, \lambda^i, p^{i+1})$ for $i = 0, \ldots, n-1$ is given by the conjunction of the following:

Universal Formulae: for each rewrite rule $\lambda \in \mathcal{L}$ s.t. $(L \xrightarrow{\lambda} R) \in \mathcal{R}$

$$\begin{array}{rcl} \lambda^i & \supset & \bigwedge\{p^i \mid p \in L\} \\ \lambda^i & \supset & \bigwedge\{p^{i+1} \mid p \in R \backslash L\} \\ \lambda^i & \supset & \bigwedge\{\neg p^{i+1} \mid p \in L \backslash R\} \end{array}$$

Cardinality: $O(n|\mathcal{L}|r)$, where r max #facts in a rule (usually small).

Axioms Formulae: for each $(p_1 \wedge \cdots \wedge p_j \supset c) \in \mathcal{A}$

$$(p_1^i \wedge \cdots \wedge p_j^i) \supset c^i$$

Cardinality: $O(n|\mathcal{A}|)$.

Linear Encoding with Axioms (2)

Explanatory Frame Formulae with Axioms: for all facts $f \in \mathcal{F}$

$$(\neg f^i \wedge f^{i+1}) \supset \left(\bigvee \left\{ \lambda^i \mid (L \xrightarrow{\lambda} R) \in \mathcal{R}, f \in (R \setminus L) \right\} \vee \bigvee \{ p_1^{i+1} \wedge \cdots \wedge p_j^{i+1} \mid (p_1 \wedge \cdots \wedge p_j \supset f) \in \mathcal{A} \} \right)$$

$$(f^i \wedge \neg f^{i+1}) \supset \left(\bigvee \left\{ \lambda^i \mid (L \xrightarrow{\lambda} R) \in \mathcal{R}, f \in (L \setminus R) \right\} \vee \bigvee \{ \neg p_1^{i+1} \wedge p_2^{i+1} \wedge \cdots \wedge p_j^{i+1} \mid (\neg p_1 \wedge p_2 \wedge \cdots \wedge p_j \supset \neg f) \in \hat{\mathcal{A}} \} \right)$$

where $\hat{\mathcal{A}}$ is the set of contraposed axioms. E.g. $\neg b \supset \neg a$ is the contraposed of $a \supset b$. **Cardinality:** $O(n|\mathcal{F}| + nt|\mathcal{A}|)$, where t is the max number of preconditions in an axiom (usually small).

Linear Encoding with Axioms (3)

Conflict Exclusion Formulae with Axioms: for all distinct rule λ_1, λ_2 such that $(L_1 \xrightarrow{\lambda_1} R_1) \in \mathcal{R}, (L_2 \xrightarrow{\lambda_2} R_2) \in \mathcal{R}$ with $L_1 \cap dep_{\mathcal{A}}(L_2 \setminus R_2) \neq \emptyset$ or $L_2 \cap dep_{\mathcal{A}}(L_1 \setminus R_1) \neq \emptyset$

$$\neg(\lambda_1^i \wedge \lambda_2^i)$$

where $dep_{\mathcal{A}}(L_j \setminus R_j)$ $(j = 1, 2)$ is the set of facts from which all the facts deleted by λ_i possibly depend wrt $\mathcal{A}$. E.g. let $\mathcal{A} = \{a \supset b, b \wedge c \supset d\}$, then

$$dep_{\mathcal{A}}(\{b\}) = \{a, b\}$$
$$dep_{\mathcal{A}}(\{c\}) = \{c\}$$
$$dep_{\mathcal{A}}(\{d\}) = \{a, b, c, d\}$$

Cardinality: $O(n|\mathcal{L}|^2)$.

Implementation: SATMC

SATMC v1.0:

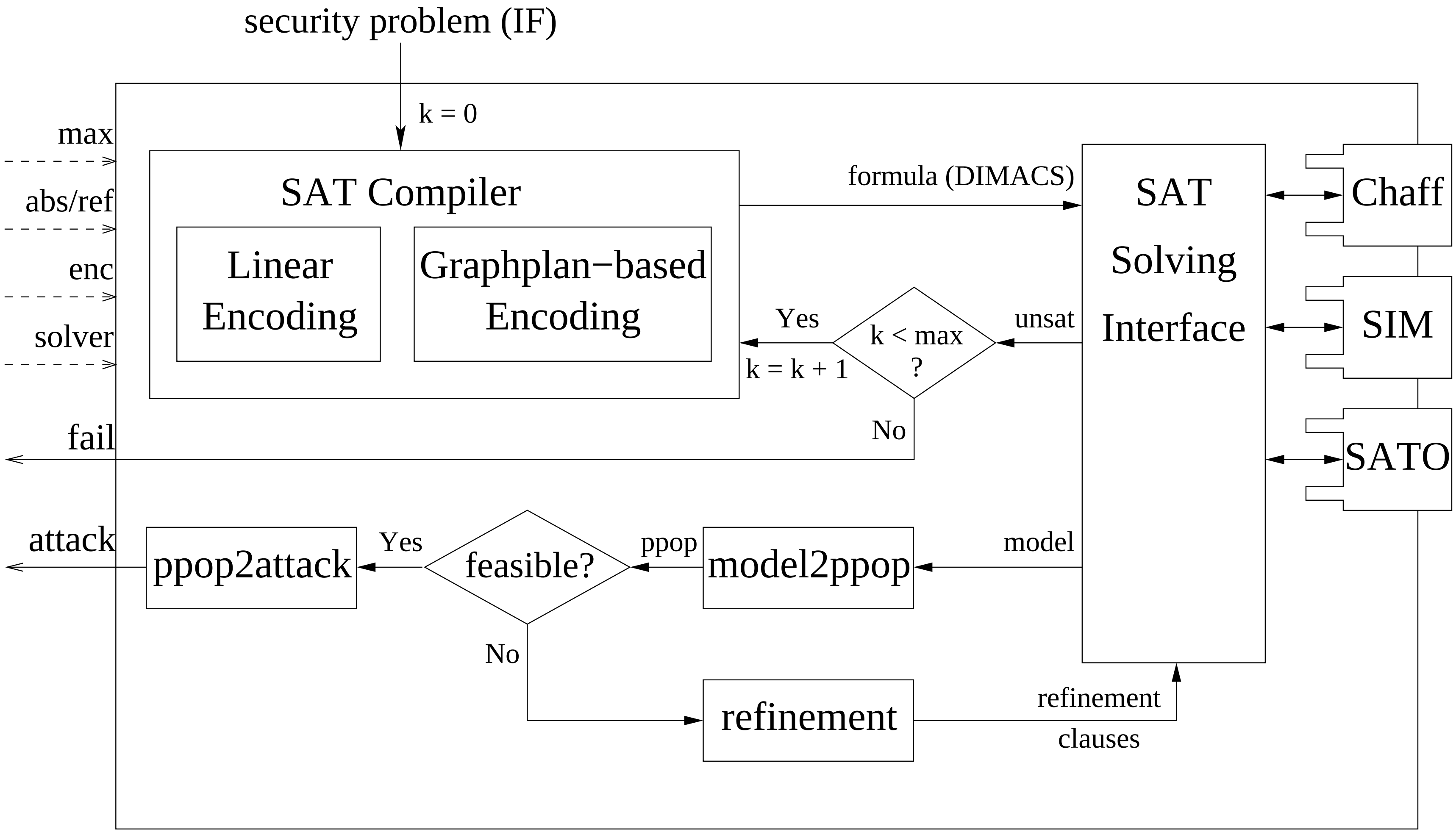
- input specification in IF v.1 language;
- set of optimizing transformations to get encodings of manageable size;
- linear encoding with iterative deepening on the number of steps.

SATMC v2.0:

- input specification in IF v.2 language;
- abstraction/refinement strategy based on neglecting mutex relations;
- an optimized graphplan-based encoding;
- support axioms.

Download it at: `http://www.mrg.dist.unige.it/satmc`

Implementation: Architecture



Experimental Results on C/J

Protocol	DY N	DY Atoms	DY Clauses	Optimized DY N	Optimized DY Atoms	Optimized DY Clauses
KaoChow 2	9	530,726	1,804,005	7	414,536	1,489,121
KaoChow 3	9	995,323	5,736,662	7	776,805	4,590,268
NSCK	9	114,530	334,086	8	88,343	298,491
NSPK	7	6,612	33,326	4	3,714	19,242
NSPK-server	8	9,157	53,741	5	5,600	33,835
Woo-Lam M	6	481,394	2,498,382	5	409,114	2,133,265

Linear Encoding

Protocol	DY N	DY Atoms	DY Clauses	Optimized DY N	Optimized DY Atoms	Optimized DY Clauses
KaoChow 2	9	726	3,065	7	458	1,784
KaoChow 3	9	990	5,019	7	587	2,606
NSCK	9	435	1,392	8	348	1,105
NSPK	7	411	1,249	4	199	549
NSPK-server	8	847	2,688	5	380	1,177
Woo-Lam M	6	481	1,518	5	358	1,137

Graphplan-based Encoding

Conclusions and Perspectives

- Proposed an optimized intruder model for SAT-based model-checking of security protocols.
- Encodings schemes extended for supporting the specification of set of axioms (without equivalence cycles).
- Up to 40% shorter attacks and up to 50% smaller SAT formulae.

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- Encodings schemes extended for supporting the specification of set of axioms (without equivalence cycles).
- Up to 40% shorter attacks and up to 50% smaller SAT formulae.
- Investigate and extend our approach for encoding generic set of axioms also specifying equivalence cycles: algebraic equations (e.g. exponentiation in the Diffie-Hellman protocol).
- Experiment such an optimization against industrial-scale security protocols: a considerable number of intruder knowledge manipulations can be required.

Thanks for you attention