CS 212 **DUE: Thursday, April 18 at 11:59 PM**
Spring 2019
Assignment 6
4/11/2019 **10 points**

Introduction: This assignment will give you practice working with functions.

This is an individual effort homework assignment. You must write up your solutions in LaTeX. Use the `a6.tex` template that I provide and be sure to replace each "Put your answer for ___ here." with your answers but leave everything else alone. Your solutions must be written up in a clear, concise and rigorous manner.

When you are done, zip up your .TEX file and corresponding .PDF file. Upload your .ZIP file to the **a6** dropbox on d2l. After you have uploaded the file, double-check to ensure your file was uploaded correctly. It is your responsibility to ensure your submission was done correctly. Assignments that are not uploaded correctly are worth 0 points.

Going forward, assume that

$$\mathbb{N} = \{0, 1, 2, \ldots\}$$

and

$$\mathbb{Q}^{\geq 0} = \left\{ \frac{p}{q} \;\middle|\; p \in \mathbb{N} \wedge q \in \mathbb{N} \wedge q \neq 0 \wedge \gcd(p, q) = 1 \right\}.$$

In this homework, you will study functions with domain and co-domain $\mathbb{N} \times \mathbb{N} \times \mathbb{Q}^{\geq 0}$ and $\mathbb{N}$, respectively.

1. (7 points) Let $f : \mathbb{N} \times \mathbb{N} \times \mathbb{Q}^{\geq 0} \longrightarrow \mathbb{N}$.

   (a) (1 point) Give an example of two distinct elements of the domain of f.

   $(0, 1, \frac{1}{1})$ and $(1, 1, \frac{1}{1})$

(b) (1 point) Give an example of two distinct elements of the co-domain of f.

0 and 1

(c) (1 point) Explain why the rule $f\left(x, y, \frac{p}{q}\right) = x + y + p + q$ is not a one-to-one function.

Let $x = 1$, $y = 2$, $p = 1$, and $q = 1$. We can now evaluate $f\left(x, y, \frac{p}{q}\right)$:
$f\left(1, 2, \frac{1}{1}\right) = 1 + 2 + 1 + 1 = 5$. Now let $x = 2$, $y = 1$, $p = 1$, and $q = 1$. With these different inputs, let's now evaluate $f\left(x, y, \frac{p}{q}\right)$:
$f\left(2, 1, \frac{1}{1}\right) = 2 + 1 + 1 + 1 = 5$. Since we have arrived at the same output despite having 2 different inputs, this function is not one-to-one. $\square$

(d) (1 point) Specify a rule for f that is one-to-one.

$f\left(x, y, \frac{p}{q}\right) = a \cdot b \cdot c \cdot d$ where a, b, c, and d are all unique prime numbers

(e) (1 point) Take the elements (of the domain of f) that you specified in (a) and give the elements (of the co-domain of f) to which they map under your rule for f from (d).

Since all prime numbers are natural numbers, a product of them will also be a natural number, therefore the codomain of f is $\mathbb{N}$.

(f) (1 point) Prove that the rule that you specified in (d) is one-to-one. If your rule is not one-to-one, then you will automatically receive 0 for this part.

By the fundamental theorem of arithmetic, each value of f in the domain would map to a unique value in the codomain of f, as a, b, c, and d are all unique prime numbers.

2. (3 points) Let $g : \mathbb{N} \times \mathbb{N} \times \mathbb{Q}^{\geq 0} \longrightarrow \mathbb{N}$.

   (a) (1 point) Specify a rule for g that is onto.

$$g\left(x, y, \frac{p}{q}\right) = x + 0y + 0p + 0q$$

(b) (1 point) What element of the domain of g maps to 23571113171923 under the rule you specified in (a)?

If we take any x to be 23571113171923, and y, p, and q to be any arbitrary elements a, b, and c respectively within their respective domains, we have: $g\left(x, y, \frac{p}{q}\right) = g\left(23571113171923, a, \frac{b}{c}\right) 23571113171923 + 0a + 0b + 0c =$ 23571113171923. As we can see, when $x = 23571113171923$, $y = a$, $p = b$, and $q = c$, g maps to 23571113171923.

(c) (1 point) Prove that the rule that you specified in (a) is onto. If your rule is not onto, then you will automatically receive 0 for this part.

Here is our rule: $g\left(x, y, \frac{p}{q}\right) = x + 0y + 0p + 0q$. As we can see, this is the same as $g\left(x, y, \frac{p}{q}\right) = x$ by the multiplication property of zero. Since the values of y, p, and q do not matter, the only numbers that this function can output are elements from the domain of x. Since the domain of x is $\mathbb{N}$, and the codomain is also $\mathbb{N}$, the rule specified in (a) is onto, because each element of the codomain can be mapped to from (an) element(s) from the domain. $\square$

3. (1 point) Finally, is there a bijection from $\mathbb{N} \times \mathbb{N} \times \mathbb{Q}^{\geq 0}$ into $\mathbb{N}$? Either way, justify your answer.

No, there is no bijection. I cannot even find a one-to-one function, even though finding an onto function is trivial. 4 variables makes it even more difficult to find unique outputs.