CS 212 **DUE: Tuesday, May 7 at 11:59 PM**
Spring 2019
Assignment 8
4/30/2019 **10 points**

Introduction: This assignment will give you practice working with summations and recurrence relations.

This is an individual effort homework assignment. You must write up your solutions in LaTeX. Use the `a8.tex` template that I provide and be sure to replace each "Put your answer for ___ here." with your answers but leave everything else alone. Your solutions must be written up in a clear, concise and rigorous manner.

When you are done, zip up your .TEX file and corresponding .PDF file. Upload your .ZIP file to the **a8** dropbox on d2l. After you have uploaded the file, double-check to ensure your file was uploaded correctly. It is your responsibility to ensure your submission was done correctly. Assignments that are not uploaded correctly are worth 0 points.

Consider the following recursive Java method:

```
// Assumes n is a non-negative integer, e.g., 0, 1, ...
public static int myMethod(int n) {
   if (n == 0) return 1;
   int sum = 0;
   for (int i = 0; i <= n - 1; i++) {
      sum += myMethod(i) + myMethod(i);
   }
   return 1 + sum;
}
```

We would like to study `myMethod` and determine a closed-form expression for its return value, for all non-negative `int` values. We will do this by first formulating a recurrence relation for the value returned by `myMethod`, and then we will use iteration to solve for a closed-form expression. Going forward, let $T(n)$ denote the value returned by `myMethod`.

1. (1 point) What is the base case for for $T(n)$?

   A. $T(0) = 0$

   B. $T(0) = 1$

   C. $T(1) = 0$

   D. $T(1) = 1$

B. $T(0) = 1$

2. (1 point) What is the recursive case for $T(n)$?

   A. $T(n) = 1 + 2\sum_{i=0}^{n-1} T(i)$

   B. $T(n) = 1 + \sum_{i=1}^{n-1} (T(i) + T(i))$

   C. $T(n) = \sum_{i=0}^{n-1} 2 \cdot T(i)$

   D. $T(n) = 1 + 2\sum_{i=0}^{n} T(i)$

A. $T(n) = 1 + 2\sum_{i=0}^{n-1} T(i)$

Work the rest of the problems assuming your choices for questions 1 and 2.

3. (1 point) Compute $T(1)$:

$$
\begin{aligned}
T(1) &= 1 + 2\sum_{i=0}^{1-1} T(i) \\
&= 1 + 2(1) \\
&= \boxed{3}
\end{aligned}
$$

4. (1 point) Compute $T(2)$:

$$\begin{aligned} T(2) &= 1 + 2\sum_{i=0}^{2-1} T(i) \\ &= 1 + 2(1+3) \\ &= \boxed{9} \end{aligned}$$

5. (1 point) Compute $T(3)$:

$$\begin{aligned} T(3) &= 1 + 2\sum_{i=0}^{3-1} T(i) \\ &= 1 + 2(1 + 3 + 9) \\ &= \boxed{27} \end{aligned}$$

6. (1 point) Using $T(0)$, $T(1)$, $T(2)$, and $T(3)$, guess a closed-form expression for $T(n)$.

$$T(0) = 1$$
$$T(1) = 3$$
$$T(2) = 9$$
$$T(3) = 27$$

$$\boxed{V_n = 3^n}$$

7. (4 points) Verify your guessed solution from question 6 is correct.

$$\begin{aligned} V_0 &= 3^0 \\ &= 1 \\ &\overset{\checkmark}{=} T(0) \end{aligned}$$

Now we'll prove the recursive case. First, let's start by looking at the definition of $T(n)$:

$$T(n) = 1 + 2\sum_{i=0}^{n-1} T(i)$$

Just like with weak induction, let's assume that $T(m) = V(m), \forall m \leq n-1$. That means that we can swap out $T(i)$ for $V(i)$ since the upper limit of the summation is $n-1$:

$$T(n) = 1 + 2\sum_{i=0}^{n-1} V(i)$$

Since $V(n) = 3^n$, (or in this case, $V(i) = 3^i$), we can swap that out in the summation:

$$T(n) = 1 + 2\sum_{i=0}^{n-1} 3^i$$

Now let's rewrite the expression by getting rid of the summation notation:

$$T(n) = 1 + 2(3^0 + 3^1 + 3^2 + \cdots + 3^{n-1})$$

As an aside, let's set $2(3^0 + 3^1 + 3^2 + \cdots + 3^{n-1})$ equal to S. Now let's see what happens if we multiply S by 3:

$$3 \cdot 2(3^0 + 3^1 + 3^2 + \cdots + 3^{n-1}) = 2(3^1 + 3^2 + 3^3 + \cdots + 3^n)$$

Now we can subtract S from $3S$:

$$\begin{aligned} 3S - S &= 2(3^1 + 3^2 + 3^3 + \cdots + 3^n) - 2(3^0 + 3^1 + 3^2 + \cdots + 3^{n-1}) \\ 2S &= 2(3^n - 3^0) \\ S &= 3^n - 1 \end{aligned}$$

Here we have a closed form expression for S. We can take this and plug it back into the formula for $T(n)$:

$$
\begin{aligned}
T(n) &= 1 + 3^n - 1 \\
&= 3^n \\
&\overset{\checkmark}{=} V(n)
\end{aligned}
$$