CS 331 - A5

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2.

(a)
$$(\lambda \underline{\mathbf{u}}.(\lambda z.\underline{\mathbf{u}}))$$
 $(\lambda \underline{\mathbf{u}}.\underline{\mathbf{u}}.\lambda \underline{\mathbf{u}}.))$

(b)
$$(\lambda \mathbf{x}. \mathbf{y} \mathbf{x}) \lambda \mathbf{z}. \mathbf{w} \lambda \mathbf{w}. ((\mathbf{w} \mathbf{z}) \mathbf{w}))$$

(c)
$$\lambda x.(\lambda x.\lambda z.(\underline{z} \underline{x}) \lambda z.\lambda z.(\underline{x} \underline{z}))$$

(d)
$$(\lambda x. \lambda y. \lambda z. ((\underline{x} (\underline{z} \lambda x. (\underline{y} \underline{x}))) \otimes) \lambda y. \lambda x. (\underline{y} \lambda x.$$

3. (a)
$$subst(u, v, \lambda u.v) = \lambda a.u$$
 [case 2c]

(b)
$$subst(u, v, \lambda v.u) = \lambda v.u$$
 [case 2a]

(c)
$$subst(u, v, \lambda v.w) = \lambda v.w$$
 [case 2a]

(d)
$$subst(u, v, \lambda w.v) = \lambda w.u$$
 [case 2b]

(e)
$$subst(u, v, \lambda u.u) = \lambda a.a [case 2c]$$

(f)
$$subst(u, v, \lambda v.v) = \lambda v.v$$
 [case 2a]

(g)
$$subst(u, v, \lambda w.w) = \lambda w.w$$
 [case 2b]

- 4. (a) $subst((g y), x, (\lambda f.\lambda y.(g x) y)) = (\lambda f.\lambda a.(g (g y)) y)$
 - (b) $subst(w, x, ((x \lambda x.(x w)) \lambda z.((x w) z))) = ((w \lambda x.(w w)) \lambda z.((w w) z))$
 - (c) $subst(v, u, \lambda v.\lambda u.((g v) u)) = \lambda a.\lambda u.((g a) u)$
 - $(d) \ \mathit{subst}(\ (f\ x),\ g,\ (\lambda x.(\lambda f.(f\ g)\ (g\ x))\ \lambda g.(f\ x))\) = (\lambda a.(\lambda b.(b\ (f\ x))\ ((f\ x)\ a))\ \lambda g.(f\ x))$
 - (e) $subst(\lambda x.(y y), x, \lambda y.\lambda x.\lambda y.z) = \lambda a.\lambda x.\lambda y.z$