CS 381 - A10

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1 Problem

Assume $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ is a standard Turing machine. We say that M has a double state $q \in Q$, if it transitions to q exactly twice on every input $w \in \Sigma^*$.

Prove that the set of all (encodings of) TMs $\langle M \rangle$, such that M has a double state, is undecidable using one of the techniques that we've talked about.

For the purpose of just this problem, the only other problem that you may assume is undecidable is A_{TM}.

Justify, at least briefly, all aspects of your proof.

2 Proof

Assume for the sake of obtaining a contradiction that $L(M) = \{\langle M \rangle \mid M \text{ is a TM and has a double state}\}$ is decidable. Let D be a TM that decides $L(M)$. We can now construct a TM S to decide A_{TM}:

$S =$ "On input $\langle M, w \rangle$, where M is a TM and w is a string:

1. Use M and w to construct the following TM M_2:

   $M_2 =$ "On any input

   (a) Simulate M on w. Record how many times each state is entered.

   (b) If M entered any state q exactly twice, then *accept*; else *reject*"

2. Run D on input $\langle M_2 \rangle$.

3. If D accepts, *accept*; if D rejects, *reject*"

If D decides $L(M)$, then S decides A_{TM}. Because A_{TM} is undecidable, $L(M)$ must also be undecidable.