Homework 11

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Due: April 26^{th}, 2019

Chapter 7

2. (10 points) The director of quality at alight bulb factory needs to estimate the mean life of a large shipment of light bulbs. The standard deviation is known to be 100 hours. A random sample of 64 light bulbs has a sample average life of 350 hours.

(a) Calculate a 98% confidence interval for the true mean life of light bulbs.

$$\alpha/2 = .01$$
$$z_{\alpha/2} = 2.33$$
$$\bar{x} = 350$$
$$\sigma = 100$$
$$n = 64$$

Large sample CI ($n > 40$): $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$$\begin{aligned}\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} &= 350 \pm 2.33\frac{100}{\sqrt{64}} \\ &= 350 \pm 29.13\end{aligned}$$

$$\boxed{(320.87, 379.13)}$$

(b) Suppose it is desired to estimate the true mean life of light bulbs within 10 hours with a 98% confidence, how large a sample size is required?

An interval accurate within 10 hours would mean that we're looking for an $\bar{x} \pm 5$:

$$
\begin{aligned}
5 &= z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \\
5 &= 2.33\frac{100}{\sqrt{n}} \\
\sqrt{n} &= 2.33\frac{100}{5} \\
\sqrt{n} &= 46.6 \\
n &= 2171.56
\end{aligned}
$$

Since we can't have a non-integer amount of samples, we'll round this up to $\boxed{2172}$.

4. (10 points) The lifetime of an electric component is known to be normally distributed with standard deviation 40 hours. A random sample of 64 electric components yields an average lifetime of 1280 hours.

(a) Calculate a 96% confidence interval for the mean lifetime of the electric component.

$$
\begin{gathered}
\alpha/2 = .02 \\
z_{\alpha/2} = 2.05 \\
\bar{x} = 1280 \\
\sigma = 40 \\
n = 64
\end{gathered}
$$

Large sample CI ($n > 40$): $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$$
\begin{aligned}
\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} &= 1280 \pm 2.05\frac{40}{\sqrt{64}} \\
&= 1280 \pm 10.25
\end{aligned}
$$

$$\boxed{(1269.75, 1290.25)}$$

(b) Suppose it is desired to estimate the mean lifetime of the electric component with an error of 12 hours with a 96% confidence, how large a sample size is required?

An interval accurate within 12 hours would mean that we're looking for an $\bar{x} \pm 6$:

$$\begin{aligned}
6 &= z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \\
6 &= 2.05\frac{40}{\sqrt{n}} \\
\sqrt{n} &= 2.05\frac{40}{6} \\
\sqrt{n} &= 13.67 \\
n &= 186.87
\end{aligned}$$

Since we can't have a non-integer amount of samples, we'll round this up to $\boxed{187}$.

6. (5 points) Health care workers who use latex gloves with glove powder on a daily basis are particularly susceptible to developing latex allergy. In a sample of 46 hospital employees who were diagnosed with latex allergy, it was found that the number of gloves used per week had a mean of 19.3 gloves and a standard deviation of 11.9 gloves. Calculate a 94% confidence interval for the mean number of latex gloves used per week by all health care workers with latex allergy.

$$\alpha/2 = .03$$
$$z_{\alpha/2} = 1.88$$
$$\bar{x} = 19.3$$
$$s = 11.9$$
$$n = 46$$

Large sample CI ($n > 40$): $\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$

$$\begin{aligned}
\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}} &= 19.3 \pm 1.88\frac{11.9}{\sqrt{46}} \\
&= 19.3 \pm 3.30
\end{aligned}$$

$$\boxed{(16.0, 22.6)}$$

7. (5 points) A manufacture of ceramic pistons for an experimental diesel engine selects 100 pistons for test, and find 15 pistons were cracked. Calculate a 94% score confidence interval for the true proportion of cracked pistons.

$$\alpha/2 = .03 \qquad z_{\alpha/2} = 1.88$$

$$\hat{p} = \frac{15}{100} \qquad n = 100$$

$$\tilde{p} = \frac{\hat{p} + z_{\alpha/2}^2/2n}{1 + z_{\alpha/2}^2/n}$$

Large sample CI (n > 40): $\tilde{p} \pm z_{\alpha/2} \dfrac{\sqrt{\hat{p}(1-\hat{p})/n + z_{\alpha/2}^2/4n^2}}{1 + z_{\alpha/2}^2/n}$

$$\begin{aligned}\tilde{p} &= \frac{\hat{p} + z_{\alpha/2}^2/2n}{1 + z_{\alpha/2}^2/n} \\ &= \frac{.15 + 1.88^2/2(100)}{1 + 1.88^2/100} \\ &= .1619\end{aligned}$$

$$\begin{aligned}\tilde{p} \pm z_{\alpha/2} \frac{\sqrt{\hat{p}(1-\hat{p})/n + z_{\alpha/2}^2/4n^2}}{1 + z_{\alpha/2}^2/n} &= .1619 \pm 1.88 \frac{\sqrt{.15(1-.15)/100 + 1.88^2/4(100)^2}}{1 + 1.88^2/100} \\ &= .1619 \pm .0357\end{aligned}$$

$$\boxed{(.1262, .1976)}$$

9. (5 points) When Mendel conducted his famous genetics experiments with peas, one sample of offspring consisted of 428 green peas and 152 yellow peas. Calculate a 92% large sample confidence interval for the proportion of yellow peas.

$$\alpha/2 = .04 \qquad z_{\alpha/2} = 1.75$$

$$\hat{p} = \frac{152}{580} \qquad n = 580$$

$$\text{Large sample CI} = \hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n}$$

$$\begin{aligned}\hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n} &= .2621 \pm 1.75\sqrt{.2621(1-.2621)/580}\\ &= .2621 \pm .0320\end{aligned}$$

$$\boxed{(.2301, .2941)}$$

10. (5 points) A simple random sample of 16 Top Taste cereal boxes produced a mean net weight of 31.98 ounces with a standard deviation of .26 ounces. Assume that the net weights of all Top Taste cereal boxes have a normal distribution. Find a 98% confidence interval for the mean net weight of Top Taste cereal boxes.

Since the sample size is less than 40, we'll use the t distribution:

$$\bar{x} = 31.98 \qquad \alpha/2 = .01$$

$$s = .26 \qquad n = 16$$

$$t_{\alpha/2,n-1} = 2.602$$

$$\text{Prediction interval: } \bar{x} \pm t_{\alpha/2,n-1} \cdot s\sqrt{1+\frac{1}{n}}$$

$$\begin{aligned}\bar{x} \pm t_{\alpha/2,n-1} \cdot s\sqrt{1+\frac{1}{n}} &= 31.98 \pm 2.602 \cdot .26\sqrt{1+\frac{1}{16}}\\ &= 31.98 \pm .6973\end{aligned}$$

$$\boxed{(31.2827, 32.6773)}$$

12. (10 points) The wall thickness of 16 glass bottles was measured by a quality control engineer. The sample mean was 1.10 mm, and the sample standard deviation was 0.08 mm.

(a) Calculate a 90% confidence interval for the true mean wall thickness.

Once again, the small sample size lends itself to the t distribution:

$$\bar{x} = 1.10 \qquad \alpha/2 = .05$$
$$s = 0.08 \qquad n = 16$$
$$t_{\alpha/2,n-1} = 1.753$$

Prediction interval: $\bar{x} \pm t_{\alpha/2,n-1} \cdot s\sqrt{1 + \frac{1}{n}}$

$$\begin{aligned} \bar{x} \pm t_{\alpha/2,n-1} \cdot s\sqrt{1 + \frac{1}{n}} &= 1.10 \pm 1.753 \cdot 0.08\sqrt{1 + \frac{1}{16}} \\ &= 1.10 \pm .1446 \end{aligned}$$

$$\boxed{(0.9554, 1.2446)}$$

(b) Calculate a 90% prediction interval for the wall thickness of the 17^{th} bottle selected from this population.

This is the same thing, but we need $t_{\alpha/2,n}$ instead: $t_{\alpha/2,n} = 1.746$

$$\begin{aligned} \bar{x} \pm t_{\alpha/2,n} \cdot s\sqrt{1 + \frac{1}{n}} &= 1.10 \pm 1.746 \cdot 0.08\sqrt{1 + \frac{1}{16}} \\ &= 1.10 \pm .1440 \end{aligned}$$

$$\boxed{(0.956, 1.244)}$$