Homework 2

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Due: February 15^{th}, 2019

Chapter 1

1. [The unit for all numbers below is miles.]

d) Below are the calculated values for the lower fourth, median, and upper fourth:

$$\text{Lower fourth} = 12^{th} \text{ number in the set} = \boxed{31.4}$$

$$\text{Median} = 23^{rd} \text{ number in the set} = \boxed{33.2}$$

$$\text{Upper fourth} = 34^{th} \text{ number in the set} = \boxed{34.8}$$

e) Below is a box plot constructed with the values from above. A few extra values also used to draw the box plot are given as well:

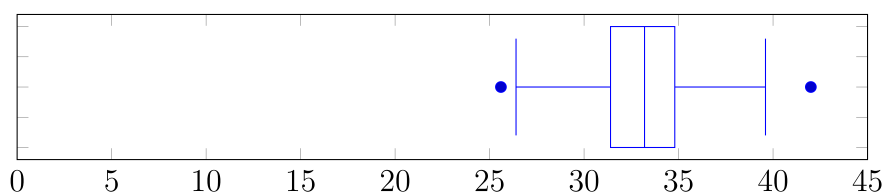
$$\text{Fourth spread} = \text{Upper fourth} - \text{Lower fourth} = 34.8 - 31.4 = \boxed{3.4}$$

$$\text{Lower outlier bound} = \text{Lower fourth}-1.5(\text{Fourth spread}) = 31.4-1.5(3.4) = \boxed{26.3}$$

$$\text{Lower extreme outlier bound} = \text{Lower fourth}-3(\text{Fourth spread}) = 31.4-3(3.4) = \boxed{21.2}$$

$$\text{Upper outlier bound} = \text{Upper fourth}+1.5(\text{Fourth spread}) = 34.8+1.5(3.4) = \boxed{39.9}$$

$$\text{Upper extreme outlier bound} = \text{Upper fourth}+3(\text{Fourth spread}) = 34.8+3(3.4) = \boxed{45.0}$$



As you can see, there are two mild outliers in the data set: 25.6 on the low end and 42.0 on the high end.

2. [The unit for all numbers below is minutes.]

d) Below are the calculated values for the lower fourth, median, and upper fourth:

$$\text{Lower fourth} = 13^{th} \text{ number in the set} = \boxed{17.5}$$

$$\text{Median} = \frac{25^{th}\text{ number} + 26^{th}\text{ number}}{2} = \frac{19.2 + 19.4}{2} = \boxed{19.3}$$

$$\text{Upper fourth} = 38^{th} \text{ number in the set} = \boxed{20.6}$$

e) Below is a box plot constructed with the values from above. A few extra values also used to draw the box plot are given as well:

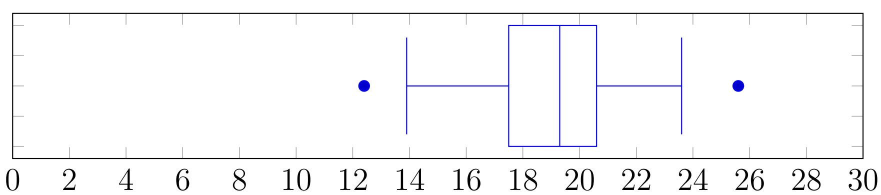
$$\text{Fourth spread} = \text{Upper fourth} - \text{Lower fourth} = 20.6 - 17.5 = \boxed{3.1}$$

$$\text{Lower outlier bound} = \text{Lower fourth} - 1.5(\text{Fourth spread}) = 17.5 - 1.5(3.1) = \boxed{12.9}$$

$$\text{Lower extreme outlier bound} = \text{Lower fourth} - 3(\text{Fourth spread}) = 17.5 - 3(3.1) = \boxed{8.2}$$

$$\text{Upper outlier bound} = \text{Upper fourth} + 1.5(\text{Fourth spread}) = 20.6 + 1.5(3.1) = \boxed{25.2}$$

$$\text{Upper extreme outlier bound} = \text{Upper fourth} + 3(\text{Fourth spread}) = 20.6 + 3(3.1) = \boxed{29.9}$$



As you can see, there are two mild outliers in the data set: 12.4 on the low end and 25.6 on the high end.

Chapter 2

1. Consider an experiment involving tossing a fair coin twice, and then rolling a fair once.

a) Listed below are all the possible outcomes of tossing a fair coin twice and rolling a fair die once; (Note: The outcomes of the coin or die landing on its side have been excluded):

$$S = \{ \begin{array}{llll} TT1, & TH1, & HT1, & HH1, \\ TT2, & TH2, & HT2, & HH2, \\ TT3, & TH3, & HT3, & HH3, \\ TT4, & TH4, & HT4, & HH4, \\ TT5, & TH5, & HT5, & HH5, \\ TT6, & TH6, & HT6, & HH6 \end{array} \}$$

b) The only outcomes that are listed here are the ones where only one coin comes up heads and the number rolled on the die is a multiple of 2:

$$S = \{ \begin{array}{ll} TH2, & HT2, \\ TH4, & HT4, \\ TH6, & HT6 \end{array} \}$$

2. A sample of 3 batteries is selected from a manufacturing line, and each battery is classified as either defective or non-defective. Let A, B and C denote the events that the first, second and third battery, respectively, are defective.

a) Listed below are all possible outcomes in the sample space for the above experiment:

$$S = \{ A'B'C',\ A'B'C,\ A'BC',\ A'BC,\ AB'C',\ AB'C,\ ABC',\ ABC \}$$

b) Listed below are all possible outcomes in A:

$$S = \{ AB'C',\ AB'C,\ ABC',\ ABC \}$$

c) Listed below are all possible outcomes in $A \cup B$:

$$S = \{\ A'BC',\ A'BC,\ AB'C',\ AB'C,\ ABC',\ ABC\ \}$$

d) Listed below are all possible outcomes in $A \cap B$:

$$S = \{\ ABC',\ ABC\ \}$$

e) Listed below are all possible outcomes in $B \cup C$:

$$S = \{\ A'B'C,\ A'BC',\ A'BC,\ AB'C,\ ABC',\ ABC\ \}$$

3. A survey of 1000 students at a large university shows that 750 students own stereos, 450 own cars, and 350 own cars and stereos. Let event A mean that a student owns a stereo, and let event B mean that a student owns a car. If a student at the university is selected at random, find the probability that...

a) ...the student owns either a car or a stereo.

$$\begin{aligned} P(A) &= \frac{750 \text{ students}}{1000 \text{ students}} = 0.75 \\ P(B) &= \frac{450 \text{ students}}{1000 \text{ students}} = 0.45 \\ P(A \cap B) &= \frac{350 \text{ students}}{1000 \text{ students}} = 0.35 \end{aligned}$$

$$\begin{aligned} \text{Addition Rule: } A \cup B &= P(A) + P(B) - P(A \cap B) \\ &= 0.75 + 0.45 - 0.35 \\ &= \boxed{0.85} \end{aligned}$$

However, that is if ”or” is taken here to be inclusive. If it is exclusive in this context, then we would need to subtract an additional $P(A \cap B)$:

$$0.85 - 0.35 = \boxed{0.50}$$

b) ...the student owns neither a car nor a stereo.

Since this is the compliment of the event above (in the inclusive case), I can simply use the fact that the compliment of a given event is simply 1 minus the probability of that event. I will call the event above event C.

$$P(C') = 1 - P(C) = 1 - 0.85 = \boxed{0.15}$$

c) ...the student owns only a stereo.

I will use some of the results obtained in a) to get an answer here:

$$\begin{aligned} P(A) &= 0.75 \\ P(A \cap B) &= 0.35 \end{aligned}$$

$$\begin{aligned} P(\text{the student only owns a stereo}) &= P(\text{students owns a stereo}) \\ &\quad - P(\text{student also owns a car}) \\ &= P(A) - P(A \cap B) \\ &= 0.75 - 0.35 \\ &= \boxed{0.40} \end{aligned}$$

4. The probability that an integrated circuit chip will have defective etching is 0.12, the probability it will have a crack defect is 0.29, and the probability that it has both defects is 0.07. Let A be the event where the chip has defective etching, and let B be the event where the chip has a crack defect. Find the probability that a newly manufactured chip will...

a) ...have either an etching or a crack defect.

Just like in 3a, we can use the addition rule and substitute in the values of the three given probabilities:

$$\begin{aligned} P(A) &= 0.12 \\ P(B) &= 0.29 \\ P(A \cap B) &= 0.07 \end{aligned}$$

$$\begin{aligned} \text{Addition Rule: } A \cup B &= P(A) + P(B) - P(A \cap B) \\ &= 0.12 + 0.29 - 0.07 \\ &= \boxed{0.34} \end{aligned}$$

However, that is if "or" is taken here to be inclusive. If it is exclusive in this context, then we would need to subtract an additional $P(A \cap B)$:

$$0.34 - 0.07 = \boxed{0.27}$$

b) ...have neither defect.

Since this is the compliment of the event above (in the inclusive case), I can simply use the fact that the compliment of a given event is simply 1 minus the probability of that event. I will call the event above event C.

$$P(C') = 1 - P(C) = 1 - 0.34 = \boxed{0.66}$$

c) ...have an etching defect only.

I will use some of the results used in a) to get an answer here:

$$\begin{aligned} P(A) &= 0.12 \\ P(A \cap B) &= 0.07 \end{aligned}$$

$$\begin{aligned} P(\text{the chip on has an etching defect}) &= P(\text{chip has an etching defect}) \\ &\quad - P(\text{the chip also has a crack defect}) \\ &= P(A) - P(A \cap B) \\ &= 0.12 - 0.07 \\ &= \boxed{0.05} \end{aligned}$$