UNIVERSITY OF SOUTHAMPTON

Faculty of Physical Engineering and Science
School of Electronics and Computer Science

A project report submitted for the award of
MEng Computer Science

Supervisor: Dr Tim Norman

Auctions for online elastic resource allocation in cloud computing

by **Mark Towers**

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University of Southampton

Abstract

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Edge clouds enable computational tasks to be completed at the edge of the network, without relying on access to remote data centres. A key challenge in these settings is that limited computational resources often need to be allocated to many self-interested users. Here, existing resource allocation approaches usually assume that tasks have inelastic resource requirements (i.e., a fixed amount of compute time, bandwidth and storage), which may result in inefficient resource use. In this paper, we expand previous work to an online setting such that job will arrive over time with the task prices and resource allocation determined through training an agent using reinforcement learning.

Contents

Declaration of Authorship

I declare that this thesis and the work presented in it is my own and has been generated by me as the result of my own original research.

I confirm that:

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Acknowledgements

I want to thank my parents for all of the support they have given me because I would not be where I am now without them. Also to my housemates for surviving with me pestering them about proof reading this project and for dealing with me stressing about this project.

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Also to Professor Tim Norman for this constant guidance over the project and the wisdom to know what to investigate and implement.

Chapter 1

Project problem

Cloud computing is a rapidly growing service with competition from Google, Amazon, Microsoft and more that aims to allow users to run computer programs that are too large, difficult or time consuming for users to run locally. These services provide the computational resources, e.g. cpu cores, RAM, hard drive space, bandwidth, etc to be able to run such programs. However, as these resources are limited, bottlenecks can occur when numerous users all require large amounts of these resources, limiting the number of tasks [1] that can be run on the cloud servers simultaneously.

For Google Cloud Services (GCP), Microsoft Azure or Amazon Web Services, their cloud computing facilities contain huge server nodes limiting the probability of such bottlenecking occurring and if such an event does occur. Users also have a range of data centres across the global to use instead if a single data centre becomes overloaded. Therefore this work considers still a developing paradigm (**?**) called Edge/Mobile cloud computing. Edge cloud computing is believed to have a wide range of application where traditional cloud computing would be impractical. This could be due to tasks being highly delay-sensitive, intermittent internet connectivity or high operational security that prevent or limit the effectiveness of traditional cloud computing.

Currently disaster response **?**, smart cities **?** and internet-of-things (**?**) (IoT) are all area that utilise edge cloud computing due to its ability to process computationally small tasks locally with low latency. For example, in smart cities, this allows for smart intersection systems using of road-side sensors or smart traffic

[1]Tasks, Programs and Jobs will be used interchangeable to refer to the same idea of a computer programs that has a fixed amount of resources required to compute.

lights based on cameras to minimise the waiting times (?). Or for the police to analysis CCTV footage to spot suspicious behaviour or to track people between cameras (?). In the case of disaster response, maps can be produced using data from autonomous vehicles sensors that can then be used in the search for potential victims and support responders (?).

However the problem of bottlenecking is of particular relevant in edge cloud computing, as instead of large server farms that can be geographically distant from the users. Edge cloud computing server are significantly smaller, being just high powered desktop computers and single server nodes. This results in greater demand on server resources, meaning that efficient allocation of resources is extremely important. Because of this, resource allocation in edge cloud computing is an important and interesting research area with edge cloud computing.

However it is believed that there is a shortcoming of existing work in resource allocation within edge cloud computing (??) due to the nature of task resource requirements. Traditional, to compute a task, several types of resource are required to be allocated for the task including communication bandwidth, computational power and data storage resources (?). However this locks away these resources, preventing any other users have using these resources till the task is finished with them. But this has the disadvantage bottlenecking occurring due to the inflexible nature of resources that cannot allow rebalancing of resources between tasks. The reason for the continued use of fixed resource allocation methods is that in traditional cloud computing such bottlenecking is rare and it allows for simpler pricing mechanism as the price can be proportional to the quantity of resources required.

Previous work by this author (?) proposed a novel resource allocation method to allow for significantly more flexibility in resource allocation with the aims of reducing possible bottlenecking. This was done by requesting that users provide a total resource usage over a tasks lifetime instead of the required resource usage. With this change of how much of each resource to be allocated could be decided by the server instead of the user. This is possible as the time taken for an operation is complete is generally proportional to the resources provided for the operation. An example for this is downloading an image, the time taken is proportional to the bandwidth allocated. This sort of flexibility is similarly true for computing a task or sending results back to the user as well. Therefore using a deadline, provided by the user, it is possible to reallocate resources around tasks to reduce the overall strain on certain resources while still finishing the task with its deadline. Therefore Using this alternative resource allocation procedure, bottlenecks can be prevent through proper balancing of resources that in turn will allow more tasks to run simultaneously and to reduce the price for user to run a task.

But in previous work (**?**), this resource allocation method was only considered in a static or one-shot approach where all tasks were presented at the first time step. At which point tasks would be auctioned and resource allocated. As a result while tasks would be processed in batches such that servers would bid on all tasks submitted every 5 minutes or so. Therefore previous work could also not dynamically change the resources allocated between batches making it impractical to be used commercially, this work aims to address these problems in previous work.

These problems are addressed by introducing time into the optimisation problem (outlined in section 3.1) but due to this addition, all previous mechanism proposed in **?** are incompatible with the new optimisation problem. Therefore using a standard auction mechanism, this project investigates different methods of learning how to bid on tasks based on their resource requirements and to efficiently allocate resources to tasks by a server.

This report is set out is following chapters. Chapter 3 proposed a solution to the project outline in this chapter with chapter **??** justifying why this approach as taken over alternative. Chapter 2 investigates the previous research that this project builds upon within both resource allocation in cloud computing and reinforcement learning methods. The proposed solution is then implemented in chapter **??** with testing and evaluation in chapters **??** and **??** respectively.

Chapter 2

Related Work

There is a considerable amount of research in the area of pricing and resource allocation in cloud computing, of which some use auction mechanisms to deal with competition (**????**) as this project does. Therefore section 2.1 presents the similar approaches to resource allocation in cloud computing and edge cloud computing to the one taken in this project.

The proposed solution of the project (presented in chapter 3) uses a form of machine learning, called reinforcement learning. Section 2.2 explores the research and the current state of the art algorithms of deep Q learning and policy gradient that are used in this project.

2.1 Related Work in Cloud Computing

A majority of approaches for pricing and resource allocation in cloud computing use a fixed resource allocation mechanism such that user request a fixed amount of certain resource from the cloud provider. However this mechanism provides no control over the resources quantity allocated to the server, only to which server these resources are allocated to. Therefore a majority of this resources is focusing on designing efficient and incentive compatible auction mechanism with **?**, survey of these approaches.

Other closely related work on resource allocation in edge clouds **?** considers both the placement of code/data needed to run a specific task, as well as the scheduling of tasks to different edge clouds. The goal there is to maximize the expected rate of successfully accomplished tasks over time. Our work is different both in the setup and the objective function. Our objective is to maximize the value over all tasks. In terms of the setup, they assume that data/code can be shared and they do not consider the elasticity of resources.

Previous work by this author in **?** proposed the novel resource allocation (explained in chapter 1) along with an optimisation problem mathematically describing the resource allocation. The goal of the problem is to maximise the social welfare, the sum of all task values that are run and completed successfully within the task deadline. The paper presents three mechanism for the optimisation problem, one to maximise the social welfare and two auction mechanisms. A greedy algorithm presented allows for quick approximation of a solution through the use of several heuristics in order to maximise the social welfare. Results found that this mechanism could achieves over 90% of the optimal solution given certain heuristics compared to fixed resource allocatiom methods with only 70% of the solution solution. The algorithm is a polynomial time algorithm that will find solution within $\frac{1}{n}$ of the optimal social welfare. The heuristics were for ordering the task by density then for each task, selecting a server based on available resource on each servers then to allocate resources that minimises the final resource heuristics. The first of the auction mechanisms is a novel distributed iterative auction developed using a reverse vcg principle (**???**) to calculate a task price. That meant that a task doesnt need to reveal its private value also that the auction could be run in a decentralised way. This means that the auction is budget balanced however it is not economically efficient or incentive compatible. The mechanism achieves over 90% of the optimal solution due to its solving of the optimisation problem allowing it to find optimal resource allocations. The third algorithm is an implementation of a single parameter auctions (**?**) using the greedy algorithm (explained previously) to find the critical value of a task. Using this mechanism with a monotonic value density heuristic results in the auction being incentive compatible also well a inheriénting the social welfare properties of the greedy mechanism.

2.2 Related Work in Reinforcement learning

Computer scientists have always been interested in testing computers against humans (**?**) and a key element of humans is the ability to learn from rewards. For computers this ability is much more complex and researchers have found

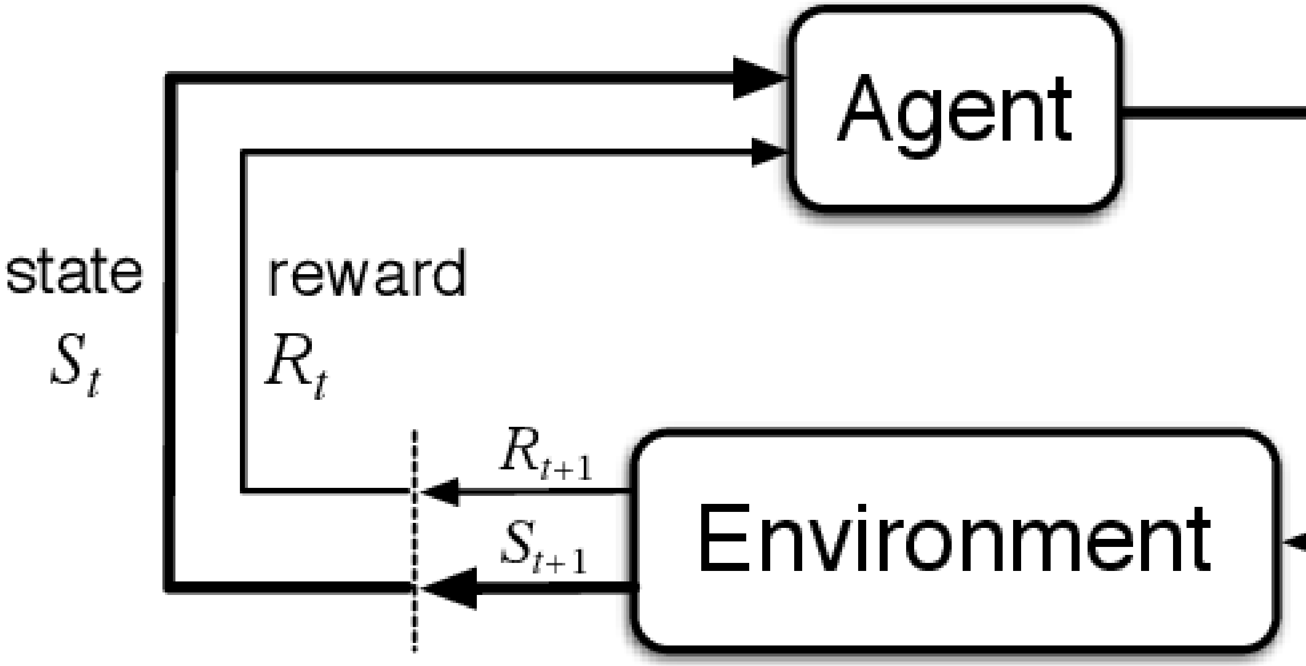

FIGURE 2.1: Reinforcement learning model (Source: **?**)

a variety of ways to allow computers to do this. These methods are broadly grouped into three categories of supervised, unsupervised and reinforcement learning. Supervised learning uses pairs of inputs to true outputs like in case of image classifications where each image has a correct category for the image to be mapped to. Unsupervised learning instead doesn't have a true output meaning that algorithms tries to find links between similar data.

However both of these methods are not effective for real world interactions as agents must make a series of actions that result in rewards. Algorithms designed for these problems with a series of actions fall into the category of reinforcement learning which aims to maximise the agent rewards over time (an example environment has the format 2.1). This is the area of machine learning that this project utilises as the problem can be modelled as a markov decision process (explained in section 3.3) that allows agents to interact with the environment to learn over time. Reinforcement learning is a rapidly growing field of research within AI due to its real world applications like driving a car, playing games, etc.

Q-learning algorithm **?** is a learning method used for estimating the action-value function, one of the bases on for modern reinforcement learning. As the series of actions can be formed into a tree of actions, an agent is interested in which actions will result in the largest reward in the future. This is formulated as in equation (2.2).

$$Q(s_t, a_t) = E[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+2} + \cdots] \tag{2.1}$$

$$Q(s_t, a_t) = Q(s_t, a_t) + \alpha \cdot (r_t + \lambda \cdot \max_a Q(s_{t+1}, a) - Q(s_t, a_t)) \tag{2.2}$$

$$\tag{2.3}$$

However the curse of dimensionally was found to be a major problem as to use Q learning required forming a table of state-actions in order to calculate and as the number of dimensions increase, the number of state-actions will increase exponentially. Therefore making the method impractical for problem that had large state space. Therefore use of a function approximate is used to circumvent this problem, traditionally done using a neural network. Work by **?** using a deep convolution neural network was able to achieve state of the art in six of seven games tried on atari with three of these scores being superhuman. This work was followed up by **?** and found that with no modifications to the hyper-parameters, neural network and training method; state of the art results were achieved in almost all 49 atari games and superhuman results in 29 of these games. Additional heuristics have been proposed for deep Q learning: double DQN (**?**), prioritized experience replay (**?**), dueling network architecture (**?**), multi-step bootstrap targets (**??**), A3C **?**, distributional Q-learning (**?**) and noisy DQN (**?**). These methods were combined to together **?**, called rainbow DQN, achieving over 200% of the original DQN algorithm and over 50% than any optimisation on its own in a quarter of the observations.

Using the base of Q-learning, policy gradient 2.2 separate the action selection policy to the q-value policy. In Q-learning, the selection the action is based on the maximum Q-value of the actions however policy gradient separates. This has the advantages of being able to deal with both discrete and continuous action spaces where Q-learning can only deal with discrete action space. Also the learning method doesn't require ϵ-greedy action selection that can for Q-learning cause the resulting policy to differ from the optimal policy. Therefore policy gradient has been used to master the game of Go (**?**) and achieve top 1% in Dota 2 (**?**) and Starcraft 2 (**?**).

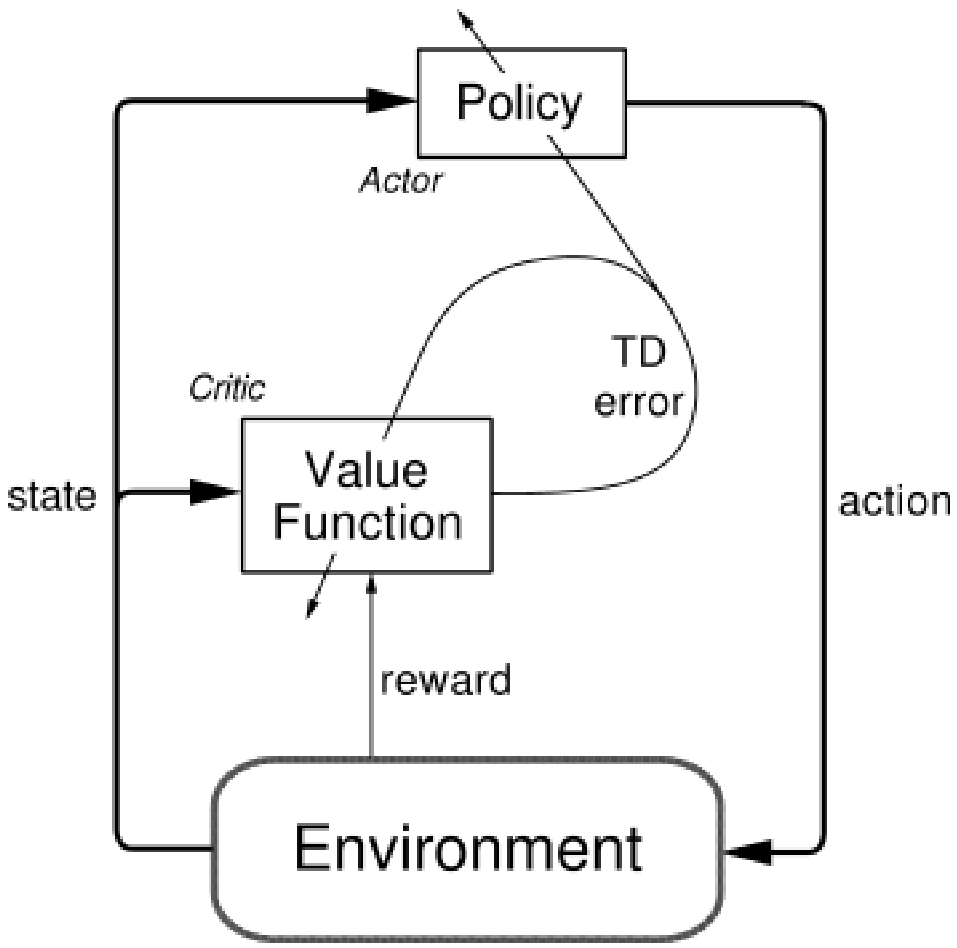

FIGURE 2.2: Actor Critic model (Source: **?**)

Chapter 3

Proposed solution to problem

In chapter 1, the problem that this project address was outlined along with the reasoning for the project. This chapter builds upon that giving a format mathematical model of the problem (section 3.1). While the overall problem is about resource allocation, we believe that cloud provider would wish to be paid for the use of their services so this additional problem is addressed in section 3.2. Using these two problems from the previous two sections, section 3.3 proposed agents who can learn how to maximise their profit over time.

3.1 Resource Allocation Optimisation problem

Using the flexible resource allocation principle presented in **?**, a format mathematical description of the model can developed. The principle is that for certain resources, the time taken for a operation to occur, e.g. loading of a program, computing the program and sending of results, etc, is proportional to the amount of resources allocated to completed the operation. [1] Therefore instead of allowing the user to requesting a fixed amount of resources for loading, computing and sending back the results of a program, the user would instead inform the server the total amount of bandwidth required, computational power, etc for the task to be completed. Then the server can dynamically allocate its available resources to all of its allocated tasks. This is believed to be effective compared to standard resource allocation mechanism as bottleneck

[1] This principle is not always true, for example, video decompression is a generally a single thread operation and cannot be effectively multi-threaded. However for this project we only consider tasks that can be parallelised effectively.

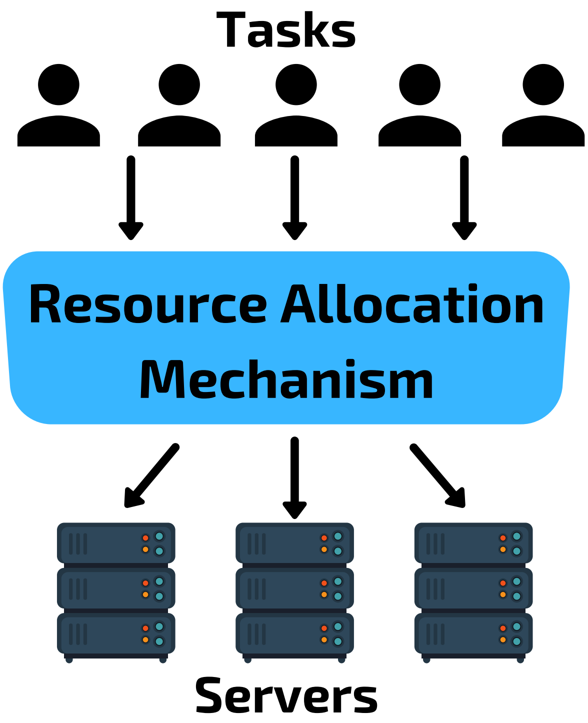

FIGURE 3.1: System model

can occur on certain resource preventing additional tasks from being allocated due servers not having the requested resources available for the task. Therefore using this principle, a modified version of a standard resource allocation formulation can be described to maximise social welfare.

A sketch of the system is shown in Fig. 3.1. We assume that in the system there is a set of $I = \{1, 2, \ldots, |I|\}$ servers are heterogeneous in all characters. Each server has a fixed availability of resources: storage for the code/data needed to run a task (e.g., measured in GB), computation capacity in terms of CPU cycles per time interval (e.g., measured in FLOP/s), and communication bandwidth to receive the data and to send back the results of the task after execution (e.g., measured in Mbit/s). We denote these resources for server i: the storage capacity as S_i, computation capacity as W_i, and the communication capacity as R_i.

There is a set $J = \{1, 2, \ldots, |J|\}$ of different tasks that require service from one of the servers in set $I = \{1, 2, \ldots, |I|\}$. To run any of these tasks on a server requires storing the appropriate code/data on the same server. These could be, for example, a set of images, videos or CNN layers in identification tasks. The storage size of task j is denoted as s_j with the rate at which the program is transferred to the server i at time t being $s'_{i,j,t}$. For a task to be computed successfully, it must fetch and execute instructions on a CPU. We consider the total number of CPU cycles required for the program to be w_j, where the rate at which the CPU cycles are assigned to the task on server i at time t is $w^{'}_{i,j,t}$. Finally, after the task is run and the results obtained, the latter need to be sent back to the user. The size of the results for task j is denoted with r_j, and the rate at which they are sent back to the user is $r^{'}_{i,j,t}$ on server i at time t. Every task has a beginning time, denoted by b_j and a deadline, denoted by d_j. This is the maximum time for the task to be completed in order for the user to derive its value. This time includes: the time required to send the data/code to the server, run it on the server, and get back the results. Therefore for the task to be successfully completed, it must completed fulfill the constraint in equation (3.1). These operations must occur in order (loading, computing then sending of results) as a server couldn't compute a task that was not fully loaded on the machine.

$$\frac{s_j}{\sum_{t=b_j}^{d_j} s^{'}_{i,j,t}} + \frac{w_j}{\sum_{t=b_j}^{d_j} w^{'}_{i,j,t}} + \frac{r_j}{\sum_{t=b_j}^{d_j} r^{'}_{i,j,t}} \leq d_j \qquad \forall j \in J \tag{3.1}$$

As server have limited capacity, the total resource usages for all tasks running on a server must be capped. The storage constraint (equation (3.2)) is unique as the previous amount loaded in kept till the end of a program on server. While the computation capacity (equation (3.3) is the sum of compute used by all of the tasks on a server i at time t and the bandwidth capacity (equation (3.4)) is the sum of loading and sending usages by tasks.

$$\sum_{j \in J} \left(\sum_{t=b_j}^{d_j} s^{'}_{i,j,t} \right) \leq S_i, \qquad \forall i \in I \tag{3.2}$$

$$\sum_{j \in J} w^{'}_{i,j,t} \leq W_i, \qquad \forall i \in I, t \in T \tag{3.3}$$

$$\sum_{j \in J} s^{'}_{i,j,t} + r^{'}_{i,j,t} \leq R_i, \qquad \forall i \in I, t \in T \tag{3.4}$$

$$\tag{3.5}$$

3.2 Auctioning of Tasks

While the mathematically description of the problem presented above doesn't contain any auctioning properties, in real life cloud providers normally wish to be paid to the use of their services. However due to the modifications that this project has to make to the optimisation problems compared to a traditional cloud computing optimisation problem. All traditional auction mechanisms that have been discussed in section 2.1 cannot be used as the user is not requesting a fixed amount of resources nor can the available resources be easily computed as this is dynamic depending on the currently allocated tasks to a server. This means that a novel or modified auction mechanism must be used to deal with these changes. Due to the complexities of devising new auction mechanism and the large corpus of research on auctions already, this project has chosen to use the Vickrey auction (**?**). This decision is justified in section **??** on why auction over other alternatives was chosen.

The modification that is made to the Vickrey auction is to reverse the auction as server are bidding on tasks instead of a task paying servers. Because of this, the auction is referenced to as the reverse Vickrey auction. The auction works by allowing servers all submit their bid for the task with the winner being the server with the lowest price but actually only gains second lowest price. The advantage of using the Vickrey auction is that it is incentive compatible meaning that the dominant strategy for bidding on a task is to bid your truthful value. This should help server as they dont need to learn how to outbid another agent as it only needs to consider its own evaluation. This also allows agents to learn through self-play effectively. The auction also has only a single round of bidding compared to alternative auctions like English or Dutch auctions. This makes auctioning fast no matter the number of servers and it also allows for tasks to set a reserve price.

3.3 Proposed Agents

Using the optimisation formulation and auction problem from the previous two sections, the problem can explained using the markov decision process format in figure **??**. The format separates out the auction parts of the problem and the resource allocation part of the problem with separate agents to act during these parts of the problem. While the state and action spaces of the agents can be

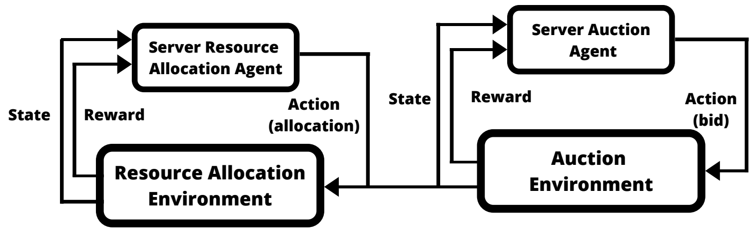

FIGURE 3.2: Markov Decision process system model

the same, the policies that the agents will be following are completely different therefore making it impossible to merge the two agents into one. In subsection **??** and **??** agents are proposed for the auction environment and resource allocation environment respectively.

The overall problem is of particularly interesting for multi-agent reinforcement learning environment as aim of the environment is cooperative (to maximise the social welfare) but the servers in competition during the auctions in order to maximise their profit. But then the server must be cooperative in allocating of resources to each task as the server wish to complete as many tasks as they can.

3.3.1 Proposed auction agents

Traditionally pricing mechanisms (**?**) rely on mixture of metrics; resource availability, resource demand, quality of service, task resource requirements, task resource allocation quantity, etc. However these values are difficult to approximate at the point of server bidding. Due to the complexity of a function to this, reinforcement learning will be used to learn this function in order to maximise the profits of the server. Simple heuristics will also be implemented in order compare the effectiveness of the reinforcement learning to untrained heuristics.

As the space of prices, the action space of the agent, is continuous value greater than zero. Therefore a deep deterministic policy gradient (**?**) agent that allows for discrete state spaces and continuous action spaces will be implemented. Also as the action space can be discretized then several deep Q learning agents will be trained that use different heuristics in order to compare the results.

These agents will use neural networks, as they function as universal function approximators (**?**), for the agents to use to learn. Because of this, a long/short term memory (LSTM) layer will be used as it allows for multiple inputs and provides a single output that will have several additional layers to allow additional complexity. With the network ending at a single ReLU neuron for DDPG or multiple linear activation neurons for DQN agents. The justification for this network over other neural network models is explained in section **??**

3.3.2 Proposed resource allocation agents

When a new task is allocated to the server or a task completes a stage then the server resource will be redistributed. As the problem of how to allocation resources isn't as complex as the agent pricing in section **??**, both simple heuristics and reinforcement learning agents will be implemented in order to compare effectiveness.

However a similar problem exists to the proposed auction agents (in subsection **??**) as to know how to allocate resources to be single task is be aware of the other tasks that the server currently contains. Therefore a similar network is proposed to allow for the other tasks to be passed in in order to compare the current task having resources allocated to and the other tasks also allocated to the server. The only other change is that the action space doesn't represent the equivalent amount of a certain type of resources but instead the weighting for that resources. This is as if the network wanted to allocate as many resources as possible while having to keep the total amount of resource under the total available then this is extremely difficult. Therefore the action space instead represents the weight of that resource with the equivalent amount of resources can then be determined by the server. This aims to reduce the needed complexity of the resource allocation function and speed up the learning time of the agent.

Chapter 4

Justification of the solution

The proposed solutions to the problem were outlined in chapter 3, this chapter explains the reasoning for the chooses made for the auction mechanism in section **??** and the network architecture in section **??**.

4.1 Justification of Vickrey auction mechanisms

In auction theory, there are numerous types of auctions that have different properties and uses in different areas. The area in which this project is interested in is single indivisible items as while the item has multiple resource requires, a server is required to buy the task as a single unit. Therefore combinatorial auctions are not considered here. Table **??** outlines a description of possible applicable auction mechanisms while table **??** outline the properties of each auction mechanisms in the previous table.

Auction type	**Description**
English auction	A traditional auction where all participant can bid on a single item with the price slowing ascending till only a single participant is left who pays the final bid price. Due to the number of rounds, this requires a large amount of communication and requires tasks to be auctioned in series.

Dutch auction	The reverse of the English auction where the starting price is higher than anyone is willing to pay with the price slowly dropping till the first participant "jumps in". This can result in sub-optimal pricing if the starting price is not highest enough and the latency can have a large effect on the winner.
Japanese auction	Similar to the English auction except that the auction occurs over a set period of time with the last highest bid being the winner. This means that it has the same disadvantages as the English auction except that there is no guarantee that the price will converge to the maximum. Plus additional factors like latency can have a large effect on the winner that will have a larger affect in the application of this project, edge cloud computing. But this time limit results in the auction taking a fixed amount of time unlike the English or Dutch auctions.
Blind auction	Also known as a First-price sealed-bid auction, all participants submit a single secret bid for an item with the highest bid winning and pays their bid value. As a result there is no dominant strategy (not incentive compatible) as an agent would not wish to bid higher than their task evaluation but if all other agents bid significantly lower then it would have been beneficial for the agent to bid much lower than their true evaluation. Due there being a single round of biding, latency doesn't affect an agent and many more auctions could occur within the same time a English, Dutch or Japanese auction would take to run.
Vickrey auction (**?**)	Also known as a second-price sealed bid auction, all participants submit a single secret bid for an item with the highest bid winning but it only pays the price of the second highest bid. Because of this, it is a dominant strategy for an agent to bid its true value as even if the bid is much higher than all other participants its doesn't matter.

TABLE 4.1: Descriptions of auctions

Auction	Incentive compatible	Iterative	Fixed time length
English	False	True	False
Japanese	False	True	True
Dutch	False	True	False
Blind	False	False	True
Vickrey	True	False	True

TABLE 4.2: Properties of the auctions described in Table **??**

Due to the properties that the Vickrey auction has in compared to the other auctions, of incentive compatibility and single round being of most importance means that this auction is believe the most appropriate for this project. The auction has additional properties for learning with agents due to the auction being strategyproof meaning that hte dominant strategy is to truthfully bid the agent's value of the task. This has the advantage of allowing agent to self-train as the agents don't need to learn how to out price other agents. Another advantage of the auction is that it is not iterative, making the auction fast with only a single round of bidding required. This means that the auction can be certain to complete within a fixed amount of time as server must submit a bid within the time frame or loss out on the task.

4.2 Justification of Auctioning and Resource allocation network architectures

Neural network can be made up an unlimited number of layers therefore the choice of layer to allow for efficient and fast network training. In table **??**, a range of primary layer of agent neural networks are explored and explained with there difference.

Neural Network	**Description**
Artificial neural networks (**?**)	Originally developed as a theoretically approximation for the brain, it was found that for networks with at least one hidden layer that a neural network could approximate any function (**?**). This made neural networks extremely helpful for cases where it would normally be difficult for a human to specify the exact function, artificial neural network can be trained through gradient descent to find a close approximation to the true function.

Recurrent neural network (?)	A major weakness of artificial neural networks is that it must use a fixed number of inputs and outputs making it unusable with text, sound or video where previous data is important for understanding the inputs. Recurrent neural network's extend neural networks to allow for connections to neurons again so that the network is not stateless. This means that individual letters of a words can be passed in with the network "remembering" the previous letter.
Long/Short Term Memory **?**	While recurrent neural network's can "remember" previous inputs to the network, it also struggles from the vanishing or exploding gradient problem where gradient tends to zero or infinity making it unusable. This network aims to prevent this by using forget gates that determines how much information the next state will get, allowing for more complexity information to be learnt compared to recurrent neural networks.
Gated Recurrent unit **?**	Gated recurrent unit are very similar to long/short term memory, except for the use of a different wiring mechanisms and use one less gate with an update date instead of forgot gates. These changes mean that gated recurrent units allow for them to run faster and are easier to code than long/short term memory however are not as expressive allowing for less complex functions to be encoded.
Neural Turing Machine **?**	Inspired by computers, neural turing machines build on long/short term memory by using an external memory module that instead of memory being inbuild to the network. This allows for external observers to understand what is going on much better than other networks due to their black-box nature.
Differentiable neural computer **?**	This is an expansion to the neural turing machine that allows the memory module to scalable in size allowing for additional memory to be added if needed.

TABLE 4.3: Neural network layer descriptions

4.2.1 Justification for Auctioning networks

Outlined in Table **??** is the properties of popular neural network layer architectures that would allow for a variable amount of inputs (except for artificial neural networks). This is due to there being multiple inputs with each of the current allocated tasks to a server that till compute time is of unknown length. Of the available architecture, long/Short term memory model is the simplest model but still with the complexity to encode the policy. With the neural turing machine and differentiable neural network, these networks are extremely complex and require a large amount of data to train the networks. Also the ability of these networks to be able to store data in external storage is not important as the data doesn't need to be store for future inputs. The opposite problem exists for the recurrent neural network or the gated recurrent unit that they are possibly not complex enough to encode the policy.

4.2.2 Justification for Resource allocation networks

The justification for the resource allocation agent neural network is very similar justification to the previous subsections **??**. The long/short term memory architecture should be complex enough but it is possible that the ability to use external storage of neural turing machine and differentiable neural network to store the allocation of resource to previous tasks. It is believed that this additional complexity will not allow for the heuristic to do better but it could be investigated in future work.

The reason for the output to be the resource weighting rather than the actual resources is it would require the network to learn a less complex function in comparison. This means that the network learns instead how important the allocation of resources are for a task instead of the exact amount of resources allocated.

Chapter 5

Implementation of the solution

Using the proposed solution in chapter 3, there are several stages in implementing the problem. These stages are: the environment, in order to simulate edge cloud computing servers (explained in section ??), defining server auction agents and resource allocation agents (explained in section ??) and finally to train the agents on the environment (explained in section ??).

The implementation discussed below as written in Python and available to download from Github at https://github.com/stringtheorys/Online-Flexible-Resource

5.1 Simulating edge cloud computing services

As the aim of the environment is to train agents to learn how to action optimally then the way that agents interact with the environments matters. OpenAI have create a large number of open source popular reinforcement learning environment using the python module gym (?). Due to the wide scale use of the environment specification by reinforcement learning researchers, this project follows the specification with three primary functions: make, step and reset. An example of using the environment is given in in listing ?? with generating of the actions.

```
env = FlexibleResourceEnv.make('basic_settings')
state = env.reset()
done = False
while not done:
    if state.auction_task:
        actions = {
            server: server_auction_agent.bid(state)
```

Function name	Function purpose
make(settings)	Accept a list of environment settings filenames in order to setup the environment with the specifications of how to generate new and random environments. The function returns a new flexible resource allocation environment with the settings assigned as an attribute.
step(actions)	Accepts a dictionary of actions for each of the servers with the step type (auction or resource allocation) being determined by the state having an auction task specified or not. The function returns a tuple of updated state, reward, is environment done and information.
reset()	Resets the environment using the environment specifications settings set from the make function arguments and returns the new state.

TABLE 5.1: Table of flexible resource environment functions

```
            for server in state
        }
    else:
        actions = {
            server: {
                task: server_resource_agent.weight(state)
            }
        }
    state, reward, done, info = env.step(actions)

    if done:
        obs = env.reset()
env.close()
\label{listing:example_flexible_resource_env}
```

5.2 Defining the agents

Each server has to have a unique policy for bidding on tasks and allocating resources that is explained in section 3.3. Even those the state space and action space are exactly the same, the optimal policies for task pricing and resource allocation are completely different.

These policies are referred to as the auction agents, for bidding on tasks and the resource weighting agent, for allocating of resources. However determining these policies is a difficult due to the complexity and the interaction of task attributes and the server resources. Because of this, the ability of reinforcement

Policy Type	Explanation
Dqn (?)	A standard deep Q learning agent that discretizes the action space
Distributional Dqn (?)	Standard deep Q learning agents return a scalar value (representing the q value) for each action. Distributional Dqn changes the output value to a probability distribution over the possible values that the agent returns.
Rainbow Dqn (?)	Using a vary of additional heuristic this will expand the previous distributional dqn and dqn agents. The additional that the rainbow uses is explained in section 2.2
Deep deterministic policy gradient (?)	As the action space is continuous, DDPG allows for investigation of the difference between continuous and discrete action spaces and policy gradient can be more effective at learning a policy where the reward function is too complex.

TABLE 5.2: Table of policy types

learning to interact an environment and learn a policy in order to maximise its reward over time is helpful. Therefore a range of different reinforcement learning policies have been implemented as Table **??** in order to evaluate the performance of the different policies learning methods.

5.3 Training of agents

This is currently unknown as this training being investigated currently.

Chapter 6

Testing

To test the effectiveness of the implemented policies from chapter **??**, a range of tests are proposed in order to compare the solutions. These tests aim to compare the effectiveness of training techniques, policy hyperparameters and to fixed heuristics. These tests fall into several distinct sections that are explained in Table **??**.

Test Name	Explanation
Multi vs Single environment training settings	There are huge ranges of possible environment settings that agents could be trained for. This test investigates how good single environment trained agents react to a new environment and how agents compare on an environment when an agent is only trained on that environment or on multiple environments.
Multi vs Single agent environment training settings	As the Vickrey auction is incentive compatible then it is possible to train both the auction agents and resource allocation against itself. This test investigates the difference between the agent policies when self-trained or train with other agents.
Reinforcement Learning Policy testing	As multiple different reinforcement learning policy outlined in table **??**, this test compares the results of the different agents against each other.
Fixed heuristic policies testing	In order to compare to the reinforcement learning policies, a number of fixed heuristic policies are investigated both against other heuristics and reinforcement learning policies.

TABLE 6.1: Table of Testing Areas

Chapter 7

Evaluation of the implementation

Chapter 8

Conclusion and future work

Appendices

Appendix A: Paper

This paper has been submitted to the International Conference on Autonomous Agents and Multiagent Systems (AAMAS) 2020 at the University of Auckland. The paper is under peer-review with the authors being myself, Dr Sebastian Stein, Professor Tim Norman from Southampton University, Dr Fidan Mehmeti, Professor Tom La Porta, Caroline Rubein from Penn State University and Dr Geeth Demel from IBM and within this project is referred to as **?**. A copy of the paper is found below.

Auction-based Mechanisms for Allocating Elastic Resources in Edge Clouds

Paper #1263

ABSTRACT

Edge clouds enable computational tasks to be completed at the edge of the network, without relying on access to remote data centres. A key challenge in these settings is the limited computational resources that need to be allocated to many self-interested users. Here, existing resource allocation approaches usually assume that tasks have inelastic resource requirements (i.e., a fixed amount of compute time, bandwidth and storage), that may result in inefficient resource use due to unbalanced requirements. To address this, we propose a novel approach that takes advantage of the elastic nature of some of the resources, e.g., to trade-off computation speed with bandwidth allowing a server to execute more tasks by their deadlines. We describe this problem formally, show that it is NP-hard and then propose a scalable approximation algorithm. To deal with the self-interested nature of users, we show how to design a centralized auction that incentives truthful reporting of task requirements and values. Moreover, we propose novel auction-based decentralized approaches that are not always truthful, but that limit the information required from users and that can be adjusted to trade off convergence speed with solution quality. In extensive simulations, we show that considering the elasticity of resources leads to a gain in utility of around 20% compared to existing fixed approaches and that our novel auction-based approaches typically achieve 95% of the theoretical optimal.

KEYWORDS

Edge clouds; elastic resources; auctions

1 INTRODUCTION

In the last few years, cloud computing [2] has become a popular solution to run data-intensive applications remotely. However, in some application domains, it is not feasible to rely a remote cloud, for example when running highly delay-sensitive and computationally-intensive tasks, or when connectivity to the cloud is intermittent. To deal with such domains, *mobile edge computing* [13] has emerged as a complementary paradigm, where computational tasks are executed at the edge of mobile networks at small data-centers, known as *edge clouds*.

Mobile edge computing is a key enabling technology for the Internet-of-Things (IoT) [6] and in particular applications in smart cities [19] and disaster response scenarios [9]. In these applications, low-powered devices generate computational tasks and data that have to be processed quickly on local edge cloud servers. More specifically, in smart cities, these devices could be smart intersections that collect data from road-side sensors and vehicles to produce an efficient traffic light sequence to minimize waiting times [14]; or it could be CCTV cameras that analyse video feeds for suspicious behaviour, e.g., to detect a stabbing or other crime in progress [20]. In disaster response, sensor data from autonomous vehicles (including video, sonar and LIDAR) can be aggregated in real time to produce maps of a devastated area, search for potential victims and help first responders in focusing their efforts to save lives [1].

To accomplish these tasks, there are typically several types of resources that are needed, including communication bandwidth, computational power and data storage resources [7], and tasks are generally delay-sensitive, i.e., have a specific completion deadline. When accomplished, different tasks carry different values for their owners (e.g., the users of IoT devices or other stakeholders such as the police or traffic authority). This value will depend on the importance of the task, e.g., analysing current levels of air pollution may be less important than preventing a large-scale traffic jam at peak times or tracking a terrorist on the run. Given that edge clouds are often highly constrained in their resources [12], we are interested in allocating tasks to edge cloud servers to maximize the overall social welfare achieved (i.e., the sum of completed task values). This is particularly challenging, because users in edge clouds are typically self-interested and may behave strategically [3] or may prefer not to reveal private information about their values to a central allocation mechanism [18].

An important shortcoming of existing work of resource allocation in edge clouds, e.g., [3, 7], is that it assumes tasks have strict resource requirements — that is, each task consumes a fixed amount of computation (CPU cycles per time), takes up a fixed amount of bandwidth to transfer data and uses up a fixed amount of storage on the server. However, in practice, edge cloud servers have some flexibility in how they allocate limited resources to each task. In more detail, to execute a task, the corresponding data and/or code first has to be transferred to the server it is assigned to, requiring some bandwidth. This then takes up storage on the server. Next, the task needs computing power from the server in terms of CPU cycles per time. Once computation is complete, the results have to be transferred back to the user, requiring further bandwidth. Now, while the the storage capacity at the server for every task is *strict*, since the task cannot be run unless all the data is stored, the bandwidth and compute speed allocated to the task can be *elastic*. This allows flexibility in the resource allocation process enabling resources to be shared evenly, prevent resource self-interested users and for more task to receive service simultaneously.

Against this background, we make the following novel contributions to the state of the art:

- We formulate an optimization problem for assigning the tasks to the servers, whose objective is to maximize total

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social welfare, taking into account resource limitations and elastic allocation of resources.
- We prove that the problem is NP-hard and propose an approximation algorithm with a performance guarantee of $\frac{1}{n}$, where n is the number of tasks, and a linearithmic computational complexity, i.e., $O(n \log(n))$.
- We propose a range of auction-based mechanisms to deal with the self-interested nature of users. These offer various trade-offs regarding truthfulness, optimality, scalability, information requirements from users, communication overheads and decentralization.
- Using extensive realistic simulations, we compare the performance of our algorithm against other benchmark algorithms, and show that our algorithm outperforms all of them, while at the same time being within 95% to the optimal solution.

The paper is organized as follows. In the next section we discuss related work. This is followed by the problem formulation in Section 3. Our novel resource allocation mechanisms are presented in Section 4. In Section 5, we evaluate the performance of our mechanisms and compare them against the optimal solution and other benchmarks. Finally, Section 6 concludes the work.

2 RELATED WORK

There is a considerable amount of research in the area of resource allocation and pricing in cloud computing, some of which use auction mechanisms to deal with competition [3, 4, 11, 22]. However, these approaches assume that users request a fixed amount of resources system resources and processing rates, with the cloud provider having no control over the speeds, only the servers that the task was allocated to. In our work, tasks' owners report deadlines and overall data and computation requirements, allowing the edge cloud server to distribute its resources more efficiently based on each task's requirements.

Our problem is related to multidimensional knapsack problems. In particular, Nip et al. [15] consider flexibility in the allocation, with linear constraints that are used for elastic weights. The paper provides a pseudo-polynomial time complexity algorithm for solving this problem to maximize the values in the knapsack. Our problem case is similar to their problem, but our problem has non-linear constraints due to the deadline constraint, so their algorithm cannot be applied here.

Other closely related work on resource allocation in edge clouds [7] considers both the placement of code/data needed to run a specific task, as well as the scheduling of tasks to different edge clouds. The goal there is to maximize the expected rate of successfully accomplished tasks over time. Our work is different both in the setup and the objective function. Our objective is to maximize the value over all tasks. In terms of the setup, they assume that data/code can be shared and they do not consider the elasticity of resources.

3 PROBLEM FORMULATION

In this section we first describe the system model. Then, we present the optimization problem and prove its NP-hardness.

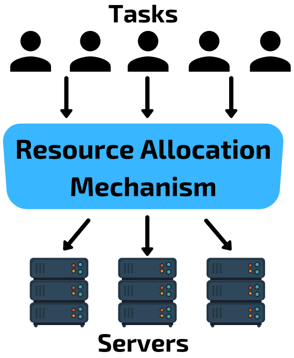

Figure 1: System Model

3.1 System model

A sketch of the system is shown in Fig. 1. We assume that in the system there is a set of servers $I = \{1, 2, \ldots, |I|\}$ servers, which could be edge clouds that can be accessed either through cellular base stations or WiFi access points (APs). Servers have different types of resources: storage for the code/data needed to run a task (e.g., measured in GB), computation capacity in terms of CPU cycles per time interval (e.g., measured in FLOP/s), and communication bandwidth to receive the data and to send back the results of the task after execution (e.g., measured in Mbit/s). We assume that the servers are heterogeneous in all their characteristics. More formally, we denote the storage capacity of server i with S_i, computation capacity with W_i, and the communication capacity with R_i.

There is a set $J = \{1, 2, \ldots, |J|\}$ of different tasks that require service from one of the servers.[1] Every task $j \in J$ has a value v_j that represents the value of running the task to its owner. To run any of these tasks on a server requires storing the appropriate code/data on the same server. These could be, for example, a set of images, videos or CNN layers in identification tasks. The storage size of task j is denoted as s_j with the rate at which the program is transferred to the server being s_j'. For a task to be computed successfully, it must fetch and execute instructions on a CPU. We consider the total number of CPU cycles required for the program to be w_j, where the rate at which the CPU cycles are assigned to the task per unit of time is w_j'. Finally, after the task is run and the results obtained, the latter need to be sent back to the user. The size of the results for task j is denoted with r_j, and the rate at which they are sent back to the user is r_j'. Every task has its deadline, denoted by d_j. This is the maximum time for the task to be completed in order for the user to derive its value. This time includes: the time required to send the data/code to the server, run it on the server, and get back the results. We assume that there is an *all* or *nothing* task execution reward scheme, meaning that for the task value to be awarded the entire task must be run and the results sent back within the deadline.

[1] We focus on a single-shot setting in this paper. In practice, an allocation mechanism would repeat the allocation decisions described here over regular time intervals, with longer-running tasks re-appearing on consecutive time intervals. We leave a detailed study of this to future work.

3.2 Optimization problem

Given the aforementioned assumptions, the optimal assignment of tasks to servers and optimal allocation of resources in a server to the tasks assigned to that server is obtained as a solution to the following optimization problem. Here, the decision variables are $x_{i,j} \in \{0,1\}$ (whether to run task j on server i) as well as $s_j^{'}$, $r_j^{'}$ and $w_j^{'}$ (indicating the bandwidth rates for transferring the code, for returning the results and the CPU cycles per unit of time, respectively).

$$\max \sum_{\forall j \in J} v_j \left(\sum_{\forall i \in I} x_{i,j} \right) \tag{1}$$

s.t.

$$\sum_{\forall j \in J} s_j x_{i,j} \leq S_i, \qquad \forall i \in I, \tag{2}$$

$$\sum_{\forall j \in J} w_j^{'} x_{i,j} \leq W_i, \qquad \forall i \in I, \tag{3}$$

$$\sum_{\forall j \in J} (r_j^{'} + s_j^{'}) \cdot x_{i,j} \leq R_i, \qquad \forall i \in I, \tag{4}$$

$$\frac{s_j}{s_j^{'}} + \frac{w_j}{w_j^{'}} + \frac{r_j}{r_j^{'}} \leq d_j, \qquad \forall j \in J, \tag{5}$$

$$0 \leq s_j^{'} \leq \infty, \qquad \forall j \in J, \tag{6}$$

$$0 \leq w_j^{'} \leq \infty, \qquad \forall j \in J, \tag{7}$$

$$0 \leq r_j^{'} \leq \infty, \qquad \forall j \in J, \tag{8}$$

$$\sum_{\forall i \in I} x_{i,j} \leq 1, \qquad \forall j \in J, \tag{9}$$

$$x_{i,j} \in \{0,1\}, \qquad \forall i \in I, \forall j \in J. \tag{10}$$

The objective (Eq.(1)) is to maximize the total value over all tasks (i.e., the social welfare). Task j will receive the full value v_j only if it is executed entirely and the results are obtained within the deadline for that task. Constraint (Eq.(2)) relates to the finite storage capacity of every server to store code/data for the tasks that are to be run. The finite computation capacity of every server is expressed through Eq.(3), whereas Eq.(4) denotes the constraint on the communication capacity of the servers. As can be seen, the communication bandwidth comprises two parts: one part to send the data/code or request to the server, and the other part to get the results back to the user.[2] Constraint Eq.(5) is the deadline associated with every task, where the total time of the task in the system is the sum of the time to send the request and code/data to the server, time to run the task, and the time it takes the server to send all the results to the user. Note that if a task is not run on any server, this constraint can be satisfied by choosing arbitrarily high bandwidth and CPU rates (without being constrained by the resource limits of any server). The rates at which the code is sent, run and the results are sent back are all positive and finite (Eqs. (6), (7), (8)). Further, every task is served by at most one server (Eq.(9)). Finally, a task is either served or not (Eq.(10)).

[2]Not that sending and receiving data will not always overlap, but for tractability we assume they deplete a common limited bandwidth resource per time step. This ensures that the bandwidth constraint is always satisfied in practice.

Complexity: In the following we show that this optimization problem is NP-hard.

Theorem 3.1. *The optimization problem* (1)-(10) *is NP-hard.*

Proof. The optimization problem without constraint (5) is a 0-1 multidimensional knapsack problem [10], which is a generalization of a simple 0-1 knapsack problem. The latter is an NP-hard problem [10]. Given this, it follows that the 0-1 multidimensional knapsack problem is also NP-hard. Since optimization problem (1)-(10) is a generalization of a 0-1 multidimensional knapsack problem, it follows that it is NP-hard as well. □

Before we propose our novel allocation mechanisms for the allocation problem with elastic resources, we briefly outline an example that illustrates why considering this elasticity is important. In this example, there are 12 potential tasks and 3 servers (the exact settings can be found in table 2 for the tasks and table 1 for the servers).

Figure 2 shows the best possible allocation if tasks have fixed resource requirements. The resource speeds were chosen such to the minimum total resource usage that the task would require from the deadline. Here, 9 of the tasks are run, resulting in a total social welfare of 980 due to the limitation of the server's computation and the task requirement not being balanced.

In contrast to this, Figure 3 depicts the optimal allocation if elastic resources are considered. Here, it is evident that all of the resources are used by the servers whereas the fixed (in figure 2) cant do this. In total, the elastic approach manages to schedule all 12 tasks within the resource constraints, achieving a total social welfare of 1200 (an 19% improvement over the fixed approach).

The figures represent resource usage of the servers by the three bars relating to each of this resources (storage, CPU and bandwidth). For each task that is allocated to the server, the percentage of the resource's used is bar size. Then, for the tasks that are assigned to corresponding servers, the percentage of used resources are also depicted.

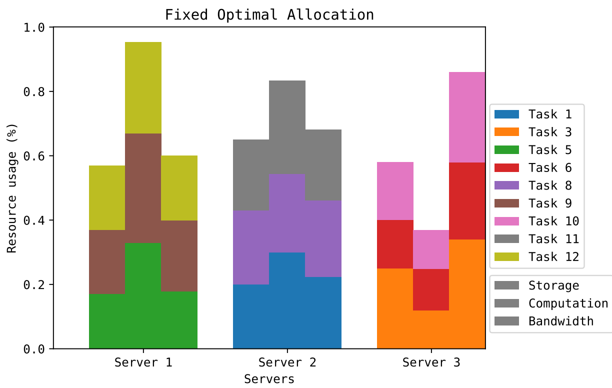

Figure 2: Optimal solution with fixed resources. Due to not being able to balance out the resources, bottlenecks on the server 1 and 2's computation have occurred

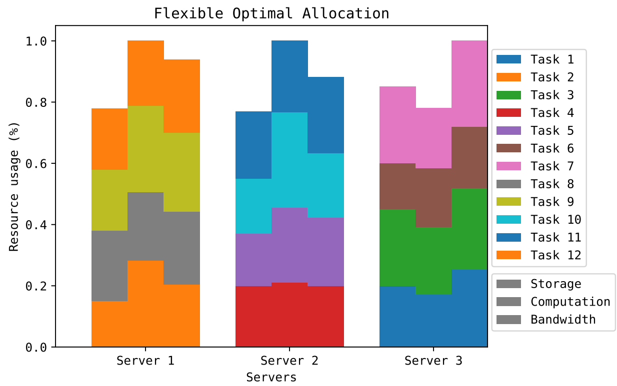

Figure 3: Optimal solution with elastic resources. Compared to the fixed allocation, the elastic allocation is able to fully use all of its resources

Name	S_i	W_i	R_i
Server 1	400	100	220
Server 2	450	100	210
Server 3	375	90	250

Table 1: Servers - Table of server attributes

Name	v_j	s_j	w_j	r_j	d_j	s_j'	w_j'	r_j'
Task 1	100	100	100	50	10	30	27	17
Task 2	90	75	125	40	10	22	32	15
Task 3	110	125	110	45	10	34	30	17
Task 4	75	100	75	35	10	27	21	13
Task 5	125	85	90	55	10	24	28	17
Task 6	100	75	120	40	10	20	32	16
Task 7	80	125	100	50	10	31	30	19
Task 8	110	115	75	55	10	30	22	20
Task 9	120	100	110	60	10	27	29	24
Task 10	90	90	120	40	10	25	30	17
Task 11	100	110	90	45	10	30	26	16
Task 12	100	100	80	55	10	24	24	22

Table 2: Tasks - Table of task attributes, the columns for resource speeds (s_j', w_j', r_j') is for fixed speeds which the flexible allocation does not take into account. The fixed speeds is the minimum required resources to complete the task within the deadline constraint.

4 FLEXIBLE RESOURCE ALLOCATION MECHANISMS

In this section, we propose several mechanisms for solving the resource allocation problem with elastic resources. First, we discuss a centralized greedy algorithm (detailed in Section 4.1) with a $\frac{1}{|J|}$ performance guarantee and polynomial run-time. Then, we consider settings where task users are self-interested and may either report their task values and requirements strategically or may wish to limit the information they reveal to the mechanism. To deal with such cases, we propose two auction-based mechanisms, one of which can be executed in a decentralized manner (in Sections 4.2 and 4.3).

4.1 Greedy Mechanism

As solving the allocation problem with elastic resources is NP-hard, we here propose a greedy algorithm (Algorithm 1) that considers tasks individually, based on an appropriate prioritisation function.

More specifically, the greedy algorithm does this in two stages; the first sorts the tasks and the second allocates them to servers. A value density function is applied to each of the task based on its attributes: value, required resources and deadlines. Stage one uses this function to sort the list of tasks. The second stage then iterates through the tasks in the given order, applying two heuristics to each task: one to select the server and another to allocate resources. The first of these heuristics, called the server selection heuristic, works by checking if a server could run the task if all of its resources were to be used for meeting the deadline constraint (eq 5) then calculating how good it would be for the job to be allocated to the server. The second heuristic, called the resource allocation heuristic, finds the best permutations of resources to minimise a formula, i.e., the total percentage of server resources used by the task.

In this paper we prove that the lower bound of the algorithm is $\frac{1}{|J|}$ (where $|J|$ is the number of jobs) using the value of a task as the value density function and using any feasible server selection and resource allocation heuristic. However we found that the task value heuristic is not the best heuristic as it does not consider the effect of the deadline or resources used for a job. In practice, the following heuristic often works better: $\frac{v_j \cdot (s_j + w_j + r_j)}{d_j}$. For the server selection heuristic we use $argmin_{\forall i \in I} S_i' + W_i' + R_i'$, where S_i', W_i', R_i' are the server's available storage, computation and bandwidth resources respectively. While for the resource allocation heuristic we use $min \frac{W_i'}{w_j'} + \frac{R_i'}{s_j' + r_j'}$.

Theorem 4.1. *The lower bound of the greedy mechanism is $\frac{1}{n}$ of the optimal social welfare*

Proof. Taking the value of a task as the value density function, the first task allocated will have a value of at least $\frac{1}{n}$ total values of all jobs. As the allocation of resources for a task is not optimal, allocation of subsequent tasks is not guaranteed. Therefore, as the optimal social welfare must be the total values of all jobs or lower then the lower bound of the mechanism must be $\frac{1}{n}$ of the optimal social welfare. □

In figure 4, an example allocation using the algorithm is shown using the model from tables 1 and 2. The algorithm uses the recommend heuristic proposed above and allows for all tasks to be allocated achieving 100% of the flexible optimal in figure 3.

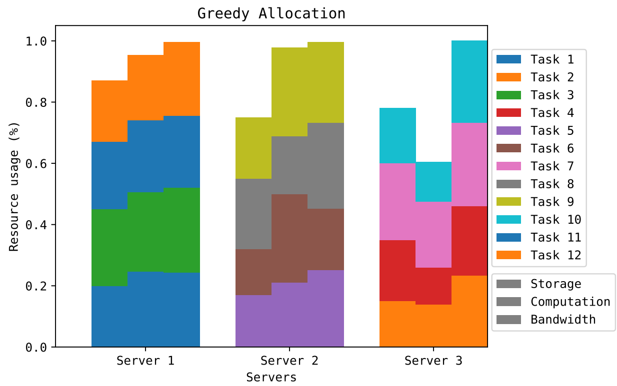

Figure 4: Example Greedy allocation using model from table 2 and 1

Algorithm 1 Greedy Mechanism

Require: J is the set of tasks and I is the set of servers
Require: $S_i^{'}$, $W_i^{'}$ and $R_i^{'}$ is the available resources (storage, computation and bandwidth respectively) for server i.
Require: $\alpha(j)$ is the value density function of a task
Require: $\beta(j, I)$ is the server selection function of a task and set of servers returning the best server, or $\emptyset$ if the task is not able to be run on any server
Require: $\gamma(j, i)$ is the resource allocation function of a task and server returning the loading, compute and sending speeds
Require: $sort(X, f)$ is a function that returns a sorted list of elements in descending order, based on a set of elements and a function for comparing elements
  $J^{'} \leftarrow sort(J, \alpha)$
  for all $j \in J^{'}$ **do**
    $i \leftarrow \beta(j, I)$
    if $i \neq \emptyset$ **then**
      $s_j^{'}, w_j^{'}, r_j^{'} \leftarrow \gamma(j, i)$
      $x_{i,j} \leftarrow 1$
    end if
  end for

Theorem 4.2. *The time complexity of the greedy algorithm is $O(|J|\,|I|)$, where $|J|$ is the number of tasks and $|I|$ is the number of servers. Assuming that the value density and resource allocation heuristics have constant time complexity and the server selection function is $O(|I|)$.*

Proof. The time complexity of the stage 1 of the mechanism is $O(|J|\log(|J|))$ due to sorting the tasks and stage 2 has complexity $O(|J|\,|I|)$ due to looping over all of the tasks and applying the server selection and resource allocation heuristics. Therefore the overall time complexity is $O(|J|\,|I| + |J|\log(|J|) = O(|J|\,|I|)$. □

4.2 Critical Value Auction

Due to the problem case being non-cooperative, if the greedy mechanism was used to allocate resources such that the value is the price paid. This is open to manipulation and misreporting of task attributes like the value, deadline or resource requirements. Therefore in this section we propose an auction that is weakly-dominant for tasks to truthfully report it attributes.

Single-Parameter domain auctions are extensively studied in mechanism design [16] and are used where an agent's valuation function can be represented as single value. The task price is calculated by finding the task's value such that if the value were any smaller, the task could not be allocated. This value is called the critical value. This has been shown to be a strategyproof [17] (weakly-dominant incentive compatible) auction so it is a weakly-dominant strategy for a task to honestly reveal its value.

The auction is implemented using the greedy mechanism from section 4.1 to find an allocation of tasks using the reported value. Then for each task allocated, the last position in the ordered the task list such that the task would still allocated is found. The critical value of the task is then equal to the inverse of the value density function where the density is the density of the next task in the list after that position.

In order that the auction is strategyproof, the value density function is required to be monotonic so that misreporting of any task attributes will result in the value density decreasing. Therefore a value density function of the form $\frac{v_j d_j}{\alpha(s_j, w_j, r_j)}$ must be used so that the auction is strategyproof.

Theorem 4.3. *The value density function $\frac{v_j d_j}{\alpha(s_j, w_j, r_j)}$ is monotonic for task j assuming the function $\alpha(s_j, w_j, r_j)$ is monotonic decreasing.*

Proof. In order to misreport the task private value and deadline must be less than the true value. The opposite is true for the required resources (storage, compute and result data) with the misreported value being greater than the true value. Therefore the α function will increase as the resource requirements increase as well, meaning that density will decrease. □

4.3 Decentralised Iterative Auction

VCG (Vickrey-Clark-Grove) auction [21] [5] [8] is proven to be economically efficient, budget balanced and incentive compatible. A task's price is found by the difference of the social welfare for when the task exists compared to the social welfare when the task doesn't exist. Our auction uses the same principle for pricing by finding the difference between the current server revenue and the revenue when the task is allocated (at £0).

The auction iteratively lets a task advertise its requirements to all of the servers who respond with their price for the task. This price is equal to the server's current revenue minus the solution to the the problem in section 4.3.1 plus a small value called the price change variable. Being the reverse of the VCG mechanism, such that the price is found for when the task exists rather than when it doesn't exist. The price change variable allows for the increase in the revenue of the server and is can be chosen by the server. Once all of the server have responded, the task can compare the minimum server price to its private value. If the price is less then the task will accept the servers with the minimum price offer, otherwise the task will stop looking as the price for the task to run on any server is greater than its reserve price.

To find the optimal revenue for a server m given a new task p and set of currently allocated tasks N has a similar formulation to section 3.2. With an additional variable is considered, a task's price being p_n for task n.

4.3.1 Server problem case.

$$\max \sum_{\forall n \in N} p_n x_n \tag{11}$$

$$\text{s.t.} \tag{12}$$

$$\sum_{\forall n \in N} s_n x_n + s_p \leq S_m, \tag{13}$$

$$\sum_{\forall n \in N} w_n' x_n + w_p \leq W_m, \tag{14}$$

$$\sum_{\forall n \in N} (r_n' + s_n') \cdot x_n + (r_p' + s_p') \leq R_m, \tag{15}$$

$$\frac{s_n}{s_n'} + \frac{w_n}{w_n'} + \frac{r_n}{r_n'} \leq d_n, \qquad \forall n \in N \cup \{p\}, \tag{16}$$

$$0 \leq s_n' \leq \infty, \qquad \forall n \in N \cup \{p\} \tag{17}$$

$$0 \leq w_n' \leq \infty, \qquad \forall n \in N \cup \{p\} \tag{18}$$

$$0 \leq r_n' \leq \infty, \qquad \forall n \in N \cup \{p\} \tag{19}$$

$$x_n \in \{0, 1\}, \qquad \forall n \in N \tag{20}$$

The objective (Eq.(11)) is to maximize the price of all tasks (not including the new task as the price is zero). The server resource capacity constraints are similar to the constraints in the standard model set out in section 3.2 however with the assumption that the task k is running so there is no need to consider if the task is running or not. The deadline and non-negative resource speeds constraints (5, 6, 7 and 8) are all the same equation with the new task included with all of the other tasks. The equation to check that a task is only allocated to a single server is not included as only server i considers the task k's price.

In auction theory, four properties are considered: Incentive compatible, budget balanced, economically efficient and individual rationality.

- Budget balanced - Since the auction is run without an auctioneer, this allows for the auction to be run in a decentralised way resulting in no "middlemen" taking some money so all revenue goes straight to the servers from the tasks
- Individually Rational - As the server need to confirm with the task if it is willing to pay an amount to be allocated, the task can check this against its secret reserved price preventing the task from ever paying more than it is willing
- Incentive Compatible - Misreporting can give a task as if the task can predict the allocation of resources from server to tasks then tasks can misreport so to be allocate to a certain server that otherwise would result in the task being unallocated.
- Economic efficiency - At the begin then task are almost randomly assigned in till server become full and require kicking tasks off, this means that allocation can fall into a local price maxima meaning that the server will sometime not be 100% economically efficient.

Algorithm 2 Decentralised Iterative Auction

Require: I is the set of servers
Require: J is the set of unallocated tasks, which initial is the set of all tasks to be allocated
Require: $P(i, k)$ is solution to the problem in section 4.3.1 using the server i and new task k. The server's current tasks is known to itself and its current revenue from tasks so not passed as arguments.
Require: $R(i, k)$ is a function returning the list of tasks not able to run if task k is allocated to server i
Require: $\leftarrow_R$ will randomly select an element from a set
  while $|J| > 0$ **do**
    $j \leftarrow_R J$
    $p, i \leftarrow argmin_{i \in I} P(i, j)$
    if $p \leq v_j$ **then**
      $p_j \leftarrow p$
      $x_{i,j} \leftarrow 1$
      for all $j' \in R(i, j)$ **do**
        $x_{i,j'} \leftarrow 0$
        $p_j' \leftarrow 0$
        $J \leftarrow J \cup j'$
      end for
    end if
    $J \leftarrow J \setminus \{j\}$
  end while

The algorithm 2 is a centralised version of the decentralised iterative auction. It works through iteratively checking a currently unallocated job to find the price if the job was currently allocated on a server. This is done through first solving the program in section 4.3.1 which calculates the new revenue if the task was forced to be allocated with a price of zero. The task price is equal to the current server revenue − new revenue with the task allocated + a price change variable to increase the revenue of the server. The minimum price returned by $P(i, k)$ is then compared to the job's maximum reserve price (that would be private in the equivalent decentralised algorithm) to confirm if the job is willing to pay at that price. If the job is willing then the job is allocated to the minimum price server and the job price set to the agreed price. However in the process of allocating a job then the currently allocated jobs on the server could be unallocated so these jobs allocation's and price's are reset then appended to the set of unallocated jobs.

4.4 Attributes of proposed algorithms

In table 3, the important attributes for the proposed algorithm

Attribute	GM	CVA	DIA
Truthfulness		Yes	No
Optimality	No	No	No
Scalability	Yes	Yes	No
Information requirements from users	All	All	Not the reserve value
Communication over heads	Low	Low	High
Decentralisation	No	No	Yes

Table 3: Attributes of the proposed algorithms: Greedy mechanism (GM), Critical Value auction(CVA) and Decentralised Iterative auction (DIA)

5 EMPIRICAL EVALUATION

To test the algorithms presented in section 4, synthetic models have been used to generate a list of tasks and servers.

The synthetic models have been handcrafted with each attribute being generated from a gaussian distribution with a mean and standard deviation.

To compare the greedy algorithm to the optimal elastic allocation, a branch and bound was implemented to solve the problem in section 3.2. In order to compare to fixed speed equivalent models, the minimum total resource required to run the job is found and set as the resource speeds for all of the tasks, with the optimal solution for running the job with the fixed speeds is found as well. To implement the greedy mechanism, the value density function was $\frac{v_j}{s_j+w_j+r_j}$, server selection was $argmin_{\forall i \in I} S_i' + W_i' + R_i'$ and the resource allocation was $min s_j' + w_j' + r_j'$ for job j and servers I.

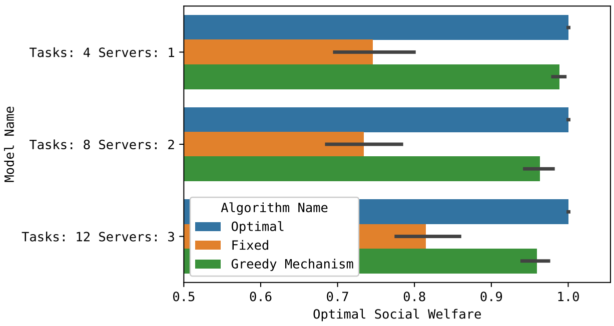

Figure 5: Comparison of the social welfare for the greedy mechanism, optimal, relaxed problem, time limited branch and bound

As figure 5 shows, the greedy mechanism achieves 98% of the optimal solution for the small models, the mechanism achieves within 95% for larger models. In comparison, the fixed allocation achieves 80% of the optimal solution and always does worse than the social welfare of the greedy mechanism.

Figure 6 compares the social welfare of the auction mechanisms: vcg, fixed resource speed vcg, critical value auction and the decentralised iterative auction with different price change variables.

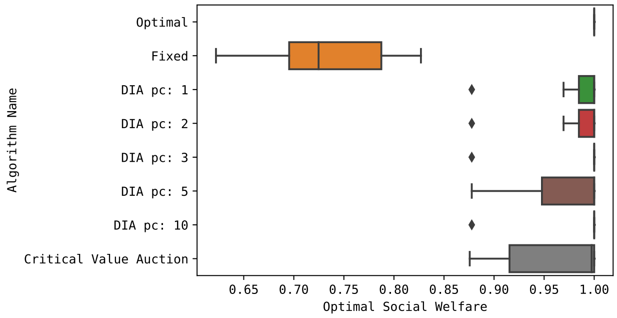

Figure 6: Comparison of the social welfare for the auction mechanisms

VCG is an economically efficient auction that requires the optimal solution to the problem in section 3.2.

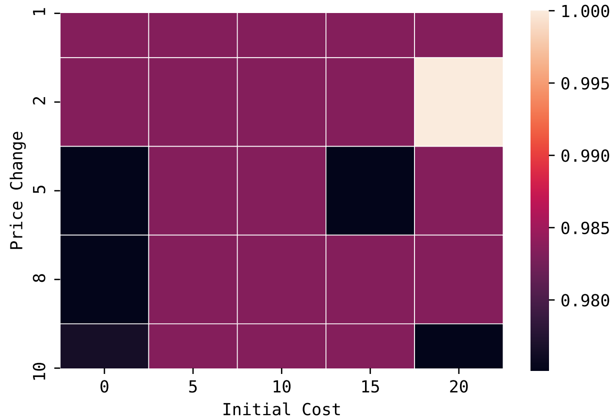

Figure 7: Average number of rounds with a price change variables and task initial cost

Within the context of edge cloud computing, the number of rounds for the decentralised iterative auction is important to making it a feasible auction as it is proportional to the time required to run. We investigated the effect of two heuristic on the number of rounds and social welfare of the auction; the price change variable and initial cost heuristic. With an auction using as minimum heuristic values for the price change and initial cost, figure 7, on average 400 rounds were required for the price to converge while an auction using a price change of 10 and initial cost of 20 means that only on average 80 rounds are required, 5x less. But by using high initial cost and price change heuristics, this can prevent tasks from being allocated, figure 8, shows that the difference in social welfare is only 2% from minimum to maximum heuristics.

6 CONCLUSIONS

In this paper, we studied a resource allocation problem in edge clouds, where resources are elastic and can be allocated to tasks at varying speeds to satisfy heterogeneous requirements and deadlines.

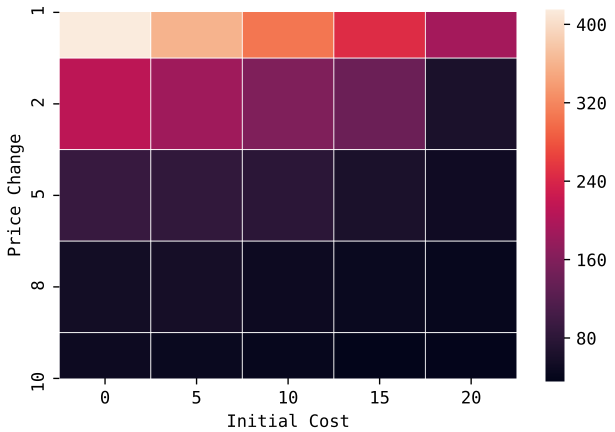

Figure 8: Average social welfare with a price change variables and task initial cost

To solve the problem, we proposed a centralized greedy mechanism with a guaranteed performance bound, and a number of auction-based mechanisms that also consider the elasticity of resources and limit the potential for strategic manipulation. We show that explicitly taking advantage of resource elasticity leads to significantly better performance than current approaches that assume fixed resources.

In future work, we plan to consider the dynamic scenario where tasks arrive and depart from the system over time, and to also consider the case where task preemption is allowed.

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Appendix B: Progress Report

This is the progress report submitted on the 10th of December 2019.

University of Southampton

Faculty of Physical Engineering and Science
School of Electronics and Computer Science

A project report submitted for the award of
MEng Electronic Engineering

Supervisor: Dr Tim Norman

Auctions for online elastic resource allocation in cloud computing

by **Mark Towers**

March 21, 2020

University of Southampton

Abstract

Faculty of Physical Engineering and Science
School of Electronics and Computer Science

roject report submitted for the award of MEng Electronic Engineering

Auctions for online elastic resource allocation in cloud computing

by Mark Towers

louds enable computational tasks to be completed at the edge of the net-
without relying on access to remote data centres. A key challenge in these
s is that limited computational resources often need to be allocated to
self-interested users. Here, existing resource allocation approaches usu-
sume that tasks have inelastic resource requirements (i.e., a fixed amount
npute time, bandwidth and storage), which may result in inefficient re-
use. In this paper, we expand previous work to an online setting such
o will arrive over time with the task prices and resource allocation deter-
through training an agent using reinforcement learning.

ntents

Declaration of Authorship

re that this thesis and the work presented in it is my own and has been ted by me as the result of my own original research.

rm that:

his work was done wholly or mainly while in candidature for a degree t this University;

Vhere any part of this thesis has previously been submitted for any other ualification at this University or any other institution, this has been clearly tated;

Vhere I have consulted the published work of others, this is always clearly ttributed;

Vhere I have quoted from the work of others, the source is always given. Vith the exception of such quotations, this thesis is entirely my own work;

have acknowledged all main sources of help;

Vhere the thesis is based on work done by myself jointly with others, I ave made clear exactly what was done by others and what I have con- ributed myself;

arts of this work have been published as: S.R. Gunn. Pdflatex instruc- ions, 2001. URL http://www.ecs.soton.ac.uk/~srg/softwaretools/document/
. J. Lovell. Updated templates, 2011
.R. Gunn and C. J. Lovell. Updated templates reference 2, 2011

l:.. Date:..................

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roject wouldn’t have started without Dr Sebastian Stein and a team of ylvania State University that has produced a paper investigating the static f this problem. So I am grateful for the support they gave in kick starting oject.

usemates for surviving with me pestering them about proof reading my and this project, all of the time.

apter 1

roduction

e Cloud Platform, Amazon Web Service and Microsoft Azure provide a e to users with computer programs that are too large, difficult or time con-g to be run on standard computer. User can request a fixed amount of re-s to run the program, e.g. cpu cores, RAM, hard drive space, bandwidth, owever, this can create bottlenecks on certain resources due to large num-f resource requests preventing other jobs from running. This problem is ularly relevant in edge cloud computing as servers are small thus making mand on resources much greater. This project considers the case where er states the total resource requirements for the program instead of the rd procedure that user request a fixed amount of resources. This allows oud provider the ability to balance resource demand as it has complete edge of all user's requirements and can flexibly change the amount of ces allocated to each task. This can prevent bottlenecks through proper es of resources allowing more tasks to run simultaneously and can also the price due to there being a lower overall demand on resources.

ly, cloud computing (Bahrami , 2015) has become a popular solution for ely running data-intensive applications. But for some problem domains, t possible to use large cloud providers, for example running highly delay-ve tasks or where connectivity to the cloud is intermittent. Mobile edge ting (Mao et al., 2017) has emerged as a complementary paradigm to for small data-centers, close to users, to execute tasks. These data centers own as edge clouds.

er response, smart cities and Internet-of-things (IoT) are popular tech-es that utilise mobile edge computing due to the use of ability to process

ms locally with low latency. For smart cities, this allows for the
nart intersections with the use of road-side sensors or smart traffic
on cameras to minimise the waiting times (Mustapha et al., 2018).
lice to analysis CCTV footage to spot suspicious behaviour or to
between cameras (Sreenu and Saleem Durai, 2019). In the case of
onse, maps can be produced using autonomous vehicles sensors
the search for potential victims and support responders (Alazawi

hese task, several types of resources are required included commu-
width, computational power and data storage resources (Farhadi
Tasks will have a deadline such that the program must be com-
this point and a private value. This value is depend on the pro-
nd its value to the owner, .e.g analysis air pollution is less impor-
venting traffic jams at rush hour or tracking a criminal on the run.
s interested in allocated task to servers to maximise the social wel-
all allocated task values) over time. But due to users being self-
ey may behave strategically (Bi et al., 2019) or prefer to not reveal
ublicity (Pai and Roth, 2013).

ning of existing work for resource allocation in edge cloud com-
adi et al., 2019; Bi et al., 2019) has the assumption that tasks have
e requirements. However, flexibility is possible in practise with
s are allocaed to each task. For example, the allocated bandwidth
ne program is proportional to the time taken to load the program.
also the computational requirements and for sending results back
his project investigates flexible allocation of resource and pricing
when task arrive over time and have private values.

apter 2

lated Works

the novel approach for resource allocation in cloud computing, there is apers that allow for flexible resource allocation. However there is a con- ble amount of research in the area of resource allocation and pricing in computing, some of which use auction mechanisms to deal with com- n Kumar et al. (2017); Zhang et al. (2017); Bingqian Du (2019); Bi et al. In Section 2.1 considers the previously related work for flexible resource ion in cloud computing and Section 2.2 consider recent work in the field forcement learning.

Related work in Cloud computing

rity of the approaches for pricing and resource allocation in cloud com- require users to request a fixed amount of certain resource with the cloud er having no control over the resources only the servers that the task was ed to (Kumar et al., 2017; Zhang et al., 2017; Bingqian Du, 2019; Bi et al., The flexible approach that this project assumed has only been considered ers et al. that allows the server to distribute its resources more efficiently on each task's requirements. The primary difference between this project at paper is that this project considers the addition of time allowing for ce speed to change over time and that there are task stages.

us work by Towers et al. considers three solutions to a single-shot prob- se, a greedy algorithm to quickly approximate a solution to maximise cial welfare and two auction mechanisms as server are normally paid their resources. The greedy algorithm is a polynomial time al- will find solution within $\frac{1}{n}$ of the optimal social welfare. This is n the use of modular heuristics for ordering the task by density task, select a server based on available resource on each servers ate resources that minimises a resource heuristics. Using certain e greedy algorithm achieves at least 90% of the optimal solution e than optimal solution for fixed resource equivalent problems. A ted iterative auction was developed that use a reverse vcg prin- late a task price that meant that a task didnt need to reveal its also that the auction could be run in a decentralised way. This ne auction is budget balanced however it is not economically ef- entive compatible. The third algorithm is an implementation of meter auctions (Nisan et al., 2007) using the greedy algorithm to al value of a task. Using this mechanism with a monotonic value stic means that the auction is incentive compatible.

related work on resource allocation in edge clouds Farhadi et al. ers both the placement of code/data needed to run a specific task, scheduling of tasks to different edge clouds. The goal there is to expected rate of successfully accomplished tasks over time. Our rent both in the setup and the objective function. Our objective is the value over all tasks. In terms of the setup, they assume that n be shared and they do not consider the elasticity of resources.

ated work in Reinforcement learning

earning allows for the training of agents to converge towards a while unsupervised learning allows for the training of agents find ata where no-truth value exist. Reinforcement learning works in ound where truth value exist but are unknown so agent will inter- nvironment that depending on certain actions will result in being sing this resulted in the first successful "machine-learning" agent TD-Checking (Samuel, 1988) where the truth value was the differ- neighbouring" checkers boards. Temporal difference, Q-learning, other were early training methods for agents using reinforcement ork of Mnih et al. (2013) developed the usage of these methods much r by coupling them with deep neural network allowing an agent to be d using the same algorithm to achieve state-of-the-art in 6 of 7 games nd superhuman scores in 3. This recent work has reinvigorated to the rimarily due to the availability of data to be used and the computational available. This has allowed Silver et al. (2017) to achieve mastery of the of Go learning from no human expert to beat the world champion 4 games With following work expanding to other games like DOTA 2 (OpenAI) g the world champions and Starcraft 2 (Vinyals et al., 2017) becoming in o 2% world wide.

apter 3

posed solution

oblem case presented in Chapter 1 has two stages: auction and resource ion. These stages are discussed in sections 3.2 and 3.3 respectively.

Optimisation problem

ch of the system is shown in Fig. 3.1. We assume that in the system there t of $I = \{1, 2, \ldots, |I|\}$ servers are heterogeneous in all characters. Each has a fixed availability of resources: storage for the code/data needed to ask (e.g., measured in GB), computation capacity in terms of CPU cycles ne interval (e.g., measured in FLOP/s), and communication bandwidth ive the data and to send back the results of the task after execution (e.g., red in Mbit/s). We denote these resources for server i: the storage ca- as S_i, computation capacity as W_i, and the communication capacity as

is a set $J = \{1, 2, \ldots, |J|\}$ of different tasks that require service from one servers in set $I = \{1, 2, \ldots, |I|\}$. To run any of these tasks on a server es storing the appropriate code/data on the same server. These could be, mple, a set of images, videos or CNN layers in identification tasks. The e size of task j is denoted as s_j with the rate at which the program is trans- to the server i at time t being $s^{'}_{i,j,t}$. For a task to be computed successfully, t fetch and execute instructions on a CPU. We consider the total number J cycles required for the program to be w_j, where the rate at which the ycles are assigned to the task on server i at time t is $w^{'}_{i,j,t}$. Finally, after

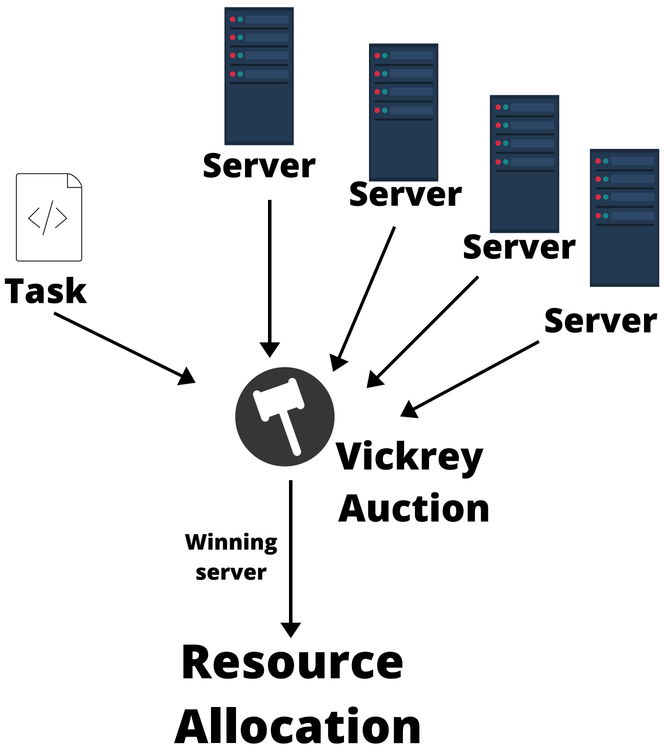


FIGURE 3.1: System model

n and the results obtained, the latter need to be sent back to the e of the results for task j is denoted with r_j, and the rate at which back to the user is $r^{'}_{i,j,t}$ on server i at time t. Every task has a begin- noted by b_j and a deadline, denoted by d_j. This is the maximum ask to be completed in order for the user to derive its value. This : the time required to send the data/code to the server, run it on nd get back the results. Therefore for the task to be successfully must completed fulfill the constraint in equation (3.1). These op- t occur in order (loading, computing then sending of results) as a 't compute a task that was not fully loaded on the machine.

$$\frac{s_j}{\sum_{t=b_j}^{d_j} s^{'}_{i,j,t}} + \frac{w_j}{\sum_{t=b_j}^{d_j} w^{'}_{i,j,t}} + \frac{r_j}{\sum_{t=b_j}^{d_j} r^{'}_{i,j,t}} \leq d_j \qquad \forall j \in J \tag{3.1}$$

ve limited capacity, the total resource usages for all tasks running ust be capped. The storage constraint (equation (3.2)) is unique as amount loaded in kept till the end of a program on server. While mputation capacity (equation (3.3) is the sum of compute used by all of ks on a server i at time t and the bandwidth capacity (equation (3.4)) is n of loading and sending usages by tasks.

$$\sum_{j \in J} \left(\sum_{t=b_j}^{d_j} s'_{i,j,t} \right) \leq S_i, \qquad \forall i \in I \tag{3.2}$$

$$\sum_{j \in J} w'_{i,j,t} \leq W_i, \qquad \forall i \in I, t \in T \tag{3.3}$$

$$\sum_{j \in J} s'_{i,j,t} + r'_{i,j,t} \leq R_i, \qquad \forall i \in I, t \in T \tag{3.4}$$

$$\tag{3.5}$$

Auction solution

gent wish to run on task on the cloud, the task can be put forward with uirements of required storage, computation, results data and deadline. er for fast and truthful, a reverse Vickrey auction (Vickrey, 1961) will be mented where servers all submit their bid for the task with the winner the server with the lowest price but actually only gains second lowest The Vickrey auction is incentive compatible meaning that the dominant y for bidding on a task is to bid your truthful value for a task. This l help server as they dont need to learn how to outbid another agent as it eeds to consider its own evaluation. As there is also only a single round ding compared to alternative auctions like English or Dutch auctions, this auctioning fast no matter the number of servers and it also allows for a e price to be used.

er to calculate the price of the task for a server requires a understand- e resource requirements of the task, the future supply and demand for and the resource requirements of currently allocated tasks. Due to the exity in creating a heuristic that can accurately use this information and nount of memory required for a table based approach. Because of this, a hort term memory (LSTM) will be implemented (Hochreiter and Schmid- 1997) for evaluating the price of a task. The justified for the use of this rk over other neural network models is explained in Section 4.1. The rk would take as input, the currently allocated tasks requirements, the requirements and the server resource capacity, outputting just a epresenting the price of the task, normalised between 0 and 100.

ource allocation solution

ork (Towers et al.), that utilised a single shot problem case where arrive over time, the resource speeds set were fixed and assumed ading, computing and sending result occurred concurrently. With of time, results in these assumptions not to hold anymore as tasks s for the loading, computing and sending of results thus requiring ource speeds to change over time. Therefore at each time step, a to reallocate all of its resource to its currently allocated tasks as ill have completely one of its stages.

ect how to allocate resource to tasks, this problem doesn't seem as e pricing in section 3.2 therefore simple heuristic and long/short neural network will be implemented and compared. This is juson 4.2. The LSTM will take as input, all of the currently allocated at a particular stages resource requirements and the task's reement returning a single value between 1 to 100. Once this is comch job, the percentage of the total values will be assigned to each

ning and reward schemes

ee popular types of training methods for neural networks: superervised and reinforcement learning. This project will utilise reinrning as supervised learning requires truth labels for data that for case is too difficult to compute. While Unsupervised learning is d for grouping data together in groups making it not appropriate ct. Therefore reinforcement learning will be utilised as the agent with the environment resulting in actions and can earn rewards in actions.

cheme for the pricing heuristic is equivalent to the winning bid e task fails to be completed then the negative bid is the reward at the time the task at auction. This aims to force the heuristic to only bid ks that it can complete but not to penalise if the agent fails to win a task uction. The agent's future discount variable will stop after the deadline task as the reward of the agent winning a task has the largest affect now affects shouldn't continue when the task is not allocated.

rce allocation uses a reward scheme similar to the pricing heuristics exat the reward will be awarded at the point that the task is completed. If k fails to complete then the reward is negative of the task price and the s future discount variable is also similar pricing reward scheme.

apter 4

tification of the approach

roposed solution in Chapter 3 as two parts explained in section 3.2 and
is chapter explains the reason for why each section is being solved in its
ular way.

approaches to pricing and resource allocation heuristics are using neural
rks to find the optimal function, table 4.1 has a description of how the
rks architectures differ.

al Network	Description
cial neural orks (ANN) ulloch and (1943)	Originally developed as a theoretically approximation for the brain, it was found that for networks with at least one hidden layer that a neural network could approximate any function (Csáji, 2001). This made neural networks extremely helpful for cases where a function would normally to difficult to find the exact function, an ANN could be trained through supervised learning to be a close approximation to the true function.
rrent neural ork (RNN) n (1990)	A major weakness of ANN's is that it must use a fixed input and output making it unusable with text, sound or video where the previous data in important in understanding an input. RNN's extend ANN's to allow for connections to neurons again so that the network is not stateless compared to ANN. This means that individual letters of a words can be passed in with the network "remembering" the previous letter.

Term LSTM) and er	While RNN's can "remember" previous inputs to the network, it also struggles from the vanishing or exploding gradient problem where gradient tends to zero or infinity making it unuseable. LSTM aims to prevent this by using forget gates that determines how much information the next state will get, allowing for more complexity information to be learnt compared to RNN's
current Chung	GRU are very similar to LSTM, except that they use different wiring and a single less gate, using an update gate instead of a forgot gate. These additional mean that the they run faster and are easier to code than LSTM however are not as expressive allowing for less complex functions to be encoded.
Turing (NTM) (2014)	Inspired by computers, neural turing machines build on LSTM by using an external memory module that instead of memory being inbuild in a neuron. This allows for external observers to understand what is going on much better than LSTM due to its black-box nature.
le nputer Graves	An expansion to the NTM where the memory module is scalable in size allowing for additional memory to be added if needed.

TABLE 4.1: Neural network descriptions

ification for the auction

tage (discussed in Section 3.2) has two considerations, the auction pricing method.

ory, there are numerous types of auctions that have different properes in different areas. The area in which this project is interested in visible items as while the item has multiple resource requires, a ired to buy the task as a single unit. Table 4.2 outlines a descriple auctions while table 4.3 outline the most important properties n has.

ction type	Description
glish auction	A traditional auction where all participant can bid on a single item with the price slowing ascending till only a single participant is left who pays the final bid price. Due to the number of rounds, this requires a large amount of communication and requires tasks to be auctioned in series.
tch auction	The reverse of the English auction where the starting price is higher than anyone is willing to pay with the price slowly dropping till the first participant "jumps in". This can result in sub-optimal pricing if the starting price is not highest enough and the latency can have a large effect on the winner.
anese auc- n	Similar to the English auction except that the auction occurs over a set period of time with the last highest bid being the winner. This means that it has the same disadvantages as the English auction except that there is no guarantee that the price will converge to the maximum. Plus additional factors like latency can have a large effect on the winner that will have a larger affect in the application of this project, edge cloud computing. But this time limit results in the auction taking a fixed amount of time unlike the English or Dutch auctions.
nd auction	Also known as a First-price sealed-bid auction, all participants submit a single secret bid for an item with the highest bid winning and pays their bid value. As a result there is no dominant strategy (not incentive compatible) as an agent would not wish to bid higher than their task evaluation but if all other agents bid significantly lower then it would have been beneficial for the agent to bid much lower than their true evaluation. Due there being a single round of biding, latency doesn't affect an agent and many more auctions could occur within the same time a English, Dutch or Japanese auction would take to run.

auc- ckrey,	Also known as a second-price sealed bid auction, all participants submit a single secret bid for an item with the highest bid winning but it only pays the price of the second highest bid. Because of this, it is a dominant strategy for an agent to bid its true value as even if the bid is much higher than all other participants its doesn't matter.

TABLE 4.2: Descriptions of auctions

ion	Incentive compatible	Iterative	Fixed time length
sh	False	True	False
nese	False	True	True
n	False	True	False
	False	False	True
ey	True	False	True

ABLE 4.3: Properties of the auctions described in Table 4.2

roperties of the Vickrey auction (table 4.3), I believe that it is the o be used. The greatest advantage of the auction is that it is strate-ning the dominant strategy is to truthful bid its price. This means on't have to learn a strategy as with the blind auction where the arn to bid only just lower than other agents. Another advantage of that it is not iterative, making the auction fast with only a single n give a fixed time limit from the task being published to all server omitted.

standard Vickrey auction will not be used as the task is buying the m a server not a server buying the task. But due to resource allo-rver must bid on the task so the Vickrey auction implemented will rse so the lowest bid will win and the task must pay the second-n the final report, a proof will be provided to show that a reverse on is still incentive compatible.

art of the auction solution is the pricing heuristic. I believe that the stic would be too complex to encoded into an algorithm if by hand to understand: future resource allocation of currently allocated esource requirements of the task. Therefore due to neural network approximate any function (Csáji, 2001) and reinforcement learning ds to training without truth data (Section 2.2). I have outlined in Table 4.1 operties of popular neural network architectures that would allow for a le amount of inputs (except for ANN). This is due to having to input to twork the currently allocated tasks to a server that till compute time is of wn length. Of the available architecture, I predict the Long/Short term ry model is the simplest model that will require the least training but still ne complexity to encode the heuristic. With the Neural Turing Machine fferentiable Neural Network, these networks are extremely complex and e a large amount of data to train the networks. Also the ability of these rks to be able to store data in external storage is not important as the data t need to be store for future inputs. The opposite problem exists for the rent neural network or the Gated Recurrent unit that they are possibly not ex enough for the pricing heuristic.

Justification for resource allocation

stification for the resource allocation neural network choice is very simi- tification to the previous section (section 4.1). Long/short term memory ecture should be complex enough for the resource allocation but it is pos- nat the abilily to use external storage of Neural Turing machine and Dif- able Neural network to store the allocation of resource to previous tasks. on't believe that this additional complexity will allow for the heuristic to r better but it could be investigated in future work.

ason that the output of the neural network is normalised is done as it require the network to learn less compared to if the network has output ount of the available resources for a task. Whereas in a normalised value, twork can output how "important" allocation of resources are for a task e exact amount of resources allocated.

apter 5

rk requirements

e project, the additional support I will require is more compute power for g of the neural networks. Because of this, I will request access to Iridis 4 GPUs.

Work to date

s project is an extension to previous work done in the Agent, Interaction omplexity research labs that has produced the paper in Section 5.2. The ty of this research occurred over the summer of 2019 with the paper that ently under peer review done from October 2019 to 15th November 2019. aper produced was done with support from Dr Fidan Mehmeti and Dr ian Stein with myself being the primary author.

e remaining time, I have studied reinforcement learning that is the pri- echnological additional that will be used in the proposed solution (sec- and described in Section 2.2.

Plan of the remaining work

this term having been completing the paper (Section 5.2), I have not ny programming towards the project. Therefore the begin of the next ill be spend building the framework for which different pricing and re- allocation heuristics can be applied and compared. Once this has been

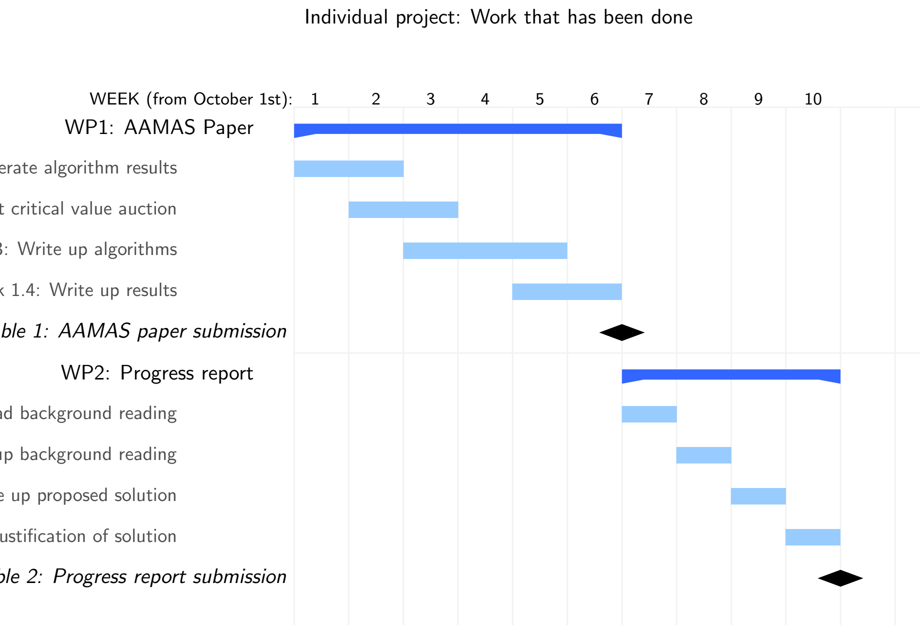


FIGURE 5.1: Work that has been done to date

nalysis and comparison of the heuristic will be done with different sk models. Resulting in a final paper.

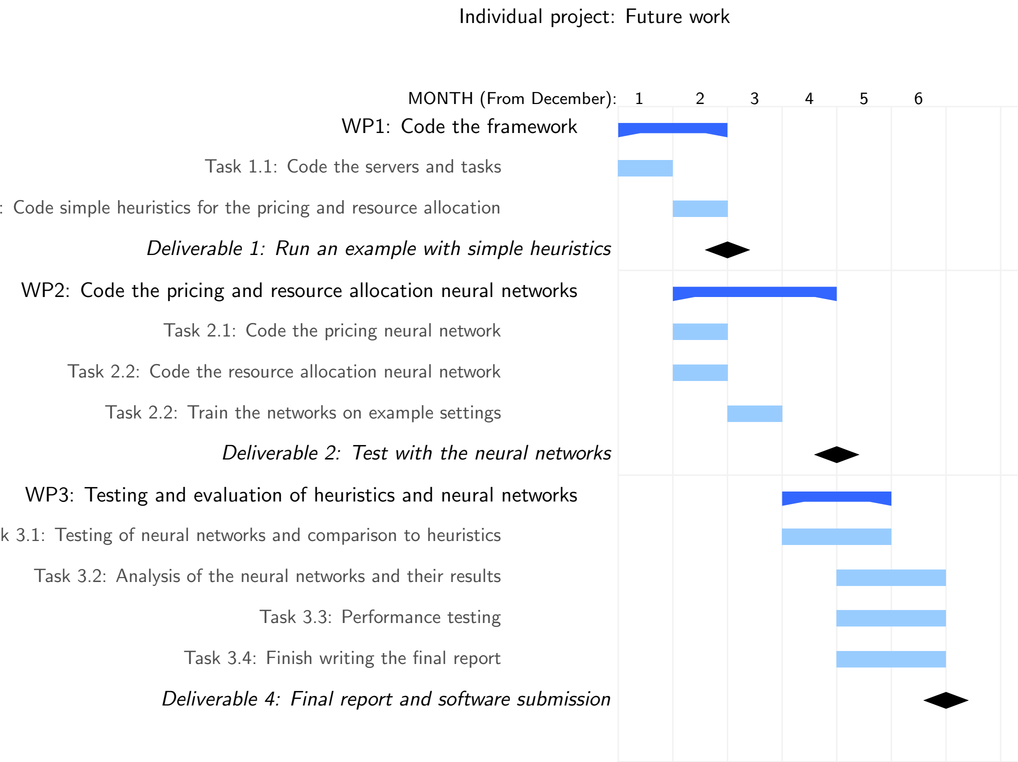


FIGURE 5.2: Work that will be done in the future

pendices

endix A: Paper

aper has been submitted to the International Conference on Autonomous s and Multiagent Systems (AAMAS) 2020 at the University of Auckland. per is under peer-review with the authors being myself, Sebastian Stein, orman, Fidan Mehmeti, Tom La Porta, Caroline Rubein and Geeth Demel ithin this project is referred to as Towers et al.. A copy of the paper is below.

sed Mechanisms for Allocating Elastic Resources in Edge Clouds

Paper #1263

tational tasks to be completed at the edge
elying on access to remote data centres.
settings is that limited computational re-
allocated to many self-interested users.
location approaches usually assume that
urce requirements (i.e., a fixed amount
idth and storage), which may result in
To address this, we propose a novel ap-
age of the elastic nature of some of the
f computation speed with bandwidth if
ecute more tasks by their deadlines. We
mally, show that it is NP-hard and then
imation algorithm. To deal with the self-
s, we show how to design a centralized
thful reporting of task requirements and
pose novel auction-based decentralized
lways truthful, but that limit the infor-
rs and that can be adjusted to trade off
olution quality. In extensive simulations,
the elasticity of resources leads to a gain
mpared to existing fixed approaches and
sed approaches typically achieve 95% of

rces; auctions

N

computing [2] has become a popular solu-
applications remotely. However, in some
not feasible to rely a remote cloud, for
ghly delay-sensitive and computationally-
connectivity to the cloud is intermittent.
, *mobile edge computing* [13] has emerged
igm, where computational tasks are exe-
e networks at small data-centers, known

g is a key enabling technology for the
] and in particular applications in smart
ponse scenarios [9]. In these applications,
erate computational tasks and data that
ckly on local edge cloud servers. More

erence on Autonomous Agents and Multiagent Systems
mith, A. El Fallah Seghrouchni, G. Sukthankar (eds.),
d

specifically, in smart cities, these devices could be smart intersections that collect data from road-side sensors and vehicles to produce an efficient traffic light sequence to minimize waiting times [14]; or it could be CCTV cameras that analyse video feeds for suspicious behaviour, e.g., to detect a stabbing or other crime in progress [20]. In disaster response, sensor data from autonomous vehicles (including video, sonar and LIDAR) can be aggregated in real time to produce maps of a devastated area, search for potential victims and help first responders in focusing their efforts to save lives [1].

To accomplish these tasks, there are typically several types of resources that are needed, including communication bandwidth, computational power and data storage resources [7], and tasks are generally delay-sensitive, i.e., have a specific completion deadline. When accomplished, different tasks carry different values for their owners (e.g., the users of IoT devices or other stakeholders such as the police or traffic authority). This value will depend on the importance of the task, e.g., analysing current levels of air pollution may be less important than preventing a large-scale traffic jam at peak times or tracking a terrorist on the run. Given that edge clouds are often highly constrained in their resources [12], we are interested in allocating tasks to edge cloud servers to maximize the overall social welfare achieved (i.e., the sum of all task values). This is particularly challenging, because users in edge clouds are typically self-interested and may behave strategically [3] or may prefer not to reveal private information about their values to a central allocation mechanism [18].

An important shortcoming of existing work looking at resource allocation in edge clouds, e.g., [3, 7], is that it assumes tasks have strict resource requirements — that is, each task consumes a fixed amount of computation (CPU cycles per time), takes up a fixed amount of bandwidth to transfer data and uses up a fixed amount of storage on the server. However, in practice, edge cloud servers have some flexibility in how they allocate limited resources to each task. In more detail, to execute a task, the corresponding data and/or code first has to be transferred to the server it is assigned to, requiring some bandwidth. This then takes up storage on the server. Next, the task needs computing power from the server in terms of CPU cycles per time. Once computation is complete, the results have to be transferred back to the user, requiring further bandwidth. Now, while the the storage capacity at the server for every task is *strict*, since the task cannot be run unless all the data are stored, the bandwidth allocation and the speed at which the task is run on the server are *elastic*. The latter two depend on how tight the task's deadline is, and can be adjusted accordingly, so that more tasks can receive service simultaneously. Allocating the elastic resources optimally is the focus of this paper.

Against this background, we make the following novel contributions to the state of the art:

ptimization problem for assigning the
s, whose objective is to maximize total
ng into account resource limitations and
ocation of resources.
problem is NP-hard and propose an ap-
thm with a performance guarantee of $\frac{1}{n}$,
ber of tasks, and a linearithmic computa-
.e., $O(n \log(n))$.
e of auction-based mechanisms to deal
ested nature of users. These offer vari-
rding truthfulness, optimality, scalability,
ements from users, communication over-
lization.
listic simulations, we compare the perfor-
thm against other benchmark algorithms,
algorithm outperforms all of them, while
eing within 95% to the optimal solution.

as follows. In the next section we discuss
wed by the problem formulation in Sec-
allocation mechanisms are presented in
e evaluate the performance of our mech-
n against the optimal solution and other
ion 6 concludes the work.

K

ount of research in the area of resource al-
ud computing, some of which use auction
competition [3, 4, 11, 22]. However, these
sers request a fixed amount of resources
essing rates, with the cloud provider hav-
eeds, only the servers that the task was
asks' owners report deadlines and overall
irements, allowing the edge cloud server
s more efficiently based on each task's

to multidimensional knapsack problems.
5] consider flexibility in the allocation,
at are used for elastic weights. The pa-
lynomial time complexity algorithm for
aximize the values in the knapsack. Our
their problem, but our problem has non-
he deadline constraint, so their algorithm

ork on resource allocation in edge clouds
cement of code/data needed to run a spe-
heduling of tasks to different edge clouds.
nize the expected rate of successfully ac-
e. Our work is different both in the setup
. Our objective is to maximize the value
he setup, they assume that data/code can
consider the elasticity of resources.

MULATION

ribe the system model. Then, we present
and prove its NP-hardness.

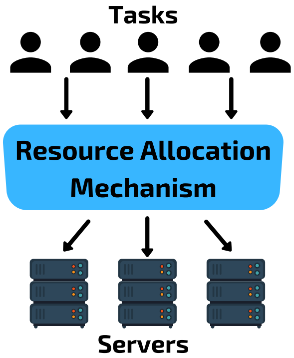


Figure 1: System Model

3.1 System model

A sketch of the system is shown in Fig. 1. We assume that in the system there is a set of servers $I = \{1, 2, \ldots, |I|\}$ servers, which could be edge clouds that can be accessed either through cellular base stations or WiFi access points (APs). Servers have different types of resources: storage for the code/data needed to run a task (e.g., measured in GB), computation capacity in terms of CPU cycles per time interval (e.g., measured in FLOP/s), and communication bandwidth to receive the data and to send back the results of the task after execution (e.g., measured in Mbit/s). We assume that the servers are heterogeneous in all their characteristics. More formally, we denote the storage capacity of server i with S_i, computation capacity with W_i, and the communication capacity with R_i.

There is a set $J = \{1, 2, \ldots, |J|\}$ of different tasks that require service from one of the servers.[1] Every task $j \in J$ has a value v_j that represents the value of running the task to its owner. To run any of these tasks on a server requires storing the appropriate code/data on the same server. These could be, for example, a set of images, videos or CNN layers in identification tasks. The storage size of task j is denoted as s_j with the rate at which the program is transferred to the server being $s_j^{'}$. For a task to be computed successfully, it must fetch and execute instructions on a CPU. We consider the total number of CPU cycles required for the program to be w_j, where the rate at which the CPU cycles are assigned to the task per unit of time is $w_j^{'}$. Finally, after the task is run and the results obtained, the latter need to be sent back to the user. The size of the results for task j is denoted with r_j, and the rate at which they are sent back to the user is $r_j^{'}$. Every task has its deadline, denoted by d_j. This is the maximum time for the task to be completed in order for the user to derive its value. This time includes: the time required to send the data/code to the server, run it on the server, and get back the results. We assume that there is an *all* or *nothing* task execution reward scheme, meaning that for the task value to be awarded the entire task must be run and the results sent back within the deadline.

[1] We focus on a single-shot setting in this paper. In practice, an allocation mechanism would repeat the allocation decisions described here over regular time intervals, with longer-running tasks re-appearing on consecutive time intervals. We leave a detailed study of this to future work.

problem

d assumptions, the optimal assignment
timal allocation of resources in a server
hat server is obtained as a solution to
n problem. Here, the decision variables
er to run task j on server i) as well as
he bandwidth rates for transferring the
ults and the CPU cycles per unit of time,

$$x_{i,j}\Big) \qquad (1)$$

$$S_i, \qquad \forall i \in I, \qquad (2)$$

$$W_i, \qquad \forall i \in I, \qquad (3)$$

$$x_{i,j} \leq R_i, \qquad \forall i \in I, \qquad (4)$$

$$\leq d_j, \qquad \forall j \in J, \qquad (5)$$

$$\forall j \in J, \qquad (6)$$

$$\forall j \in J, \qquad (7)$$

$$\forall j \in J, \qquad (8)$$

$$\forall j \in J, \qquad (9)$$

$$\forall i \in I, \forall j \in J. \qquad (10)$$

is to maximize the total value over all
are). Task j will receive the full value v_j
rely and the results are obtained within
. Constraint (Eq.(2)) relates to the finite
server to store code/data for the tasks
ite computation capacity of every server
), whereas Eq.(4) denotes the constraint
pacity of the servers. As can be seen, the
h comprises two parts: one part to send
the server, and the other part to get the
onstraint Eq.(5) is the deadline associated
e total time of the task in the system is
d the request and code/data to the server,
he time it takes the server to send all the
at if a task is not run on any server, this
by choosing arbitrarily high bandwidth
eing constrained by the resource limits of
hich the code is sent, run and the results
ive and finite (Eqs. (6), (7), (8)). Further,
most one server (Eq.(9)). Finally, a task is
0)).

data will not always overlap, but for tractability we
nited bandwidth resource per time step. This ensures
always satisfied in practice.

Complexity: In the following we show that this optimization problem is NP-hard.

Theorem 3.1. *The optimization problem* (1)-(10) *is NP-hard.*

Proof. The optimization problem without constraint (5) is a 0-1 multidimensional knapsack problem [10], which is a generalization of a simple 0-1 knapsack problem. The latter is an NP-hard problem [10]. Given this, it follows that the 0-1 multidimensional knapsack problem is also NP-hard. Since optimization problem (1)-(10) is a generalization of a 0-1 multidimensional knapsack problem, it follows that it is NP-hard as well. □

Before we propose our novel allocation mechanisms for the allocation problem with elastic resources, we briefly outline an example that illustrates why considering this elasticity is important. In this example, there are 12 potential tasks and 3 servers (the exact settings can be found in table 2 for the tasks and table 1 for the servers).

Figure 2 shows the best possible allocation if tasks have fixed resource requirements. The resource speeds were chosen such to the minimum total resource usage that the task would require from the deadline. Here, 9 of the tasks are run, resulting in a total social welfare of 980 due to the limitation of the server's computation and the task requirement not being balanced.

In contrast to this, Figure 3 depicts the optimal allocation if elastic resources are considered. Here, it is evident that all of the resources are used by the servers whereas the fixed (in figure 2) cant do this. In total, the elastic approach manages to schedule all 12 tasks within the resource constraints, achieving a total social welfare of 1200 (an 19% improvement over the fixed approach).

The figures represent resource usage of the servers by the three bars relating to each of this resources (storage, CPU and bandwidth). For each task that is allocated to the server, the percentage of the resource's used is bar size. Then, for the tasks that are assigned to corresponding servers, the percentage of used resources are also depicted.

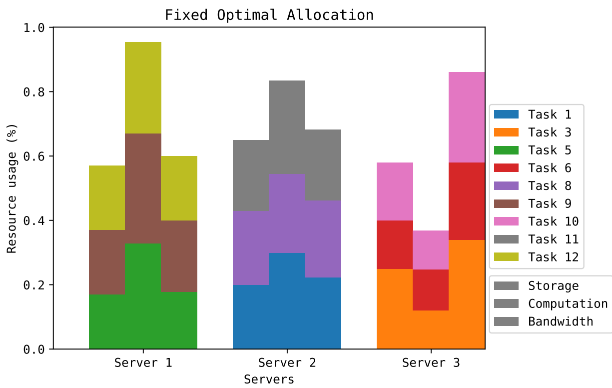
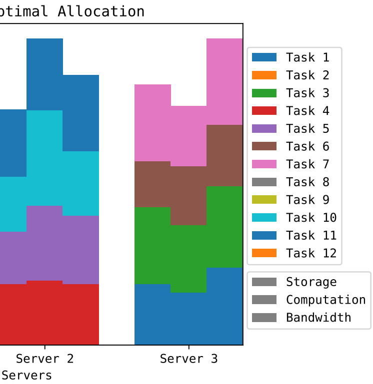


Figure 2: Optimal solution with fixed resources. Due to not being able to balance out the resources, bottlenecks on the server 1 and 2's computation have occurred



on with elastic resources. Compared the elastic allocation is able to fully

e	S_i	W_i	R_i
1	400	100	220
2	450	100	210
3	375	90	250

s - Table of server attributes

	w_j	r_j	d_j	s_j'	w_j'	r_j'
0	100	50	10	30	27	17
5	125	40	10	22	32	15
5	110	45	10	34	30	17
0	75	35	10	27	21	13
5	90	55	10	24	28	17
5	120	40	10	20	32	16
5	100	50	10	31	30	19
5	75	55	10	30	22	20
0	110	60	10	27	29	24
)	120	40	10	25	30	17
0	90	45	10	30	26	16
0	80	55	10	24	24	22

f task attributes, the columns for re- is for fixed speeds which the flexible into account. The fixed speeds is the ources to complete the task within .

OURCE ALLOCATION

se several mechanisms for solving the em with elastic resources. First, we dis- algorithm (detailed in Section 4.1) with tee and polynomial run-time. Then, we ask users are self-interested and may ei- es and requirements strategically or may wish to limit the information they reveal to the mechanism. To deal with such cases, we propose two auction-based mechanisms, one of which can be executed in a decentralized manner (in Sections 4.2 and 4.3).

4.1 Greedy Mechanism

As solving the allocation problem with elastic resources is NP-hard, we here propose a greedy algorithm (Algorithm 1) that considers tasks individually, based on an appropriate prioritisation function.

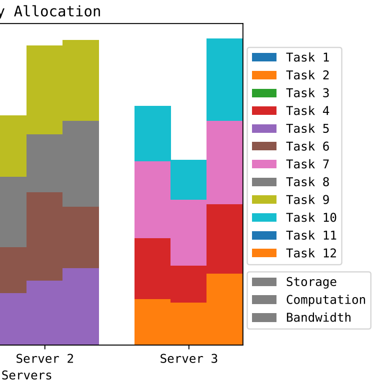
More specifically, the greedy algorithm does this in two stages; the first sorts the tasks and the second allocates them to servers. A value density function is applied to each of the task based on its attributes: value, required resources and deadlines. Stage one uses this function to sort the list of tasks. The second stage then iterates through the tasks in the given order, applying two heuristics to each task: one to select the server and another to allocate resources. The first of these heuristics, called the server selection heuristic, works by checking if a server could run the task if all of its resources were to be used for meeting the deadline constraint (eq 5) then calculating how good it would be for the job to be allocated to the server. The second heuristic, called the resource allocation heuristic, finds the best permutations of resources to minimise a formula, i.e., the total percentage of server resources used by the task.

In this paper we prove that the lower bound of the algorithm is $\frac{1}{|J|}$ (where $|J|$ is the number of jobs) using the value of a task as the value density function and using any feasible server selection and resource allocation heuristic. However we found that the task value heuristic is not the best heuristic as it does not consider the effect of the deadline or resources used for a job. In practice, the following heuristic often works better: $\frac{v_j \cdot (s_j + w_j + r_j)}{d_j}$. For the server selection heuristic we use $argmin_{\forall i \in I} S_i' + W_i' + R_i'$, where S_i', W_i', R_i' are the server's available storage, computation and bandwidth resources respectively. While for the resource allocation heuristic we use $min \frac{W_i'}{w_j'} + \frac{R_i'}{s_j' + r_j'}$.

Theorem 4.1. *The lower bound of the greedy mechanism is $\frac{1}{n}$ of the optimal social welfare*

Proof. Taking the value of a task as the value density function, the first task allocated will have a value of at least $\frac{1}{n}$ total values of all jobs. As the allocation of resources for a task is not optimal, allocation of subsequent tasks is not guaranteed. Therefore, as the optimal social welfare must be the total values of all jobs or lower then the lower bound of the mechanism must be $\frac{1}{n}$ of the optimal social welfare. □

In figure 4, an example allocation using the algorithm is shown using the model from tables 1 and 2. The algorithm uses the recommend heuristic proposed above and allows for all tasks to be allocated achieving 100% of the flexible optimal in figure 3.



dy allocation using model from table

chanism

asks and I is the set of servers
s the available resources (storage, compu-
espectively) for server i.
e density function of a task
rver selection function of a task and set
best server, or $\emptyset$ if the task is not able to

source allocation function of a task and
ding, compute and sending speeds
function that returns a sorted list of el-
order, based on a set of elements and a
elements

*e complexity of the greedy algorithm is
number of tasks and $|I|$ is the number
the value density and resource allocation
ime complexity and the server selection*

lexity of the stage 1 of the mechanism is
ng the tasks and stage 2 has complexity
er all of the tasks and applying the server
ocation heuristics. Therefore the overall
$|+|J|\log(|J|) = O(|J||I|)$. □

Auction

eing non-cooperative, if the greedy mech-
te resources such that the value is the

price paid. This is open to manipulation and misreporting of task attributes like the value, deadline or resource requirements. Therefore in this section we propose an auction that is weakly-dominant for tasks to truthfully report it attributes.

Single-Parameter domain auctions are extensively studied in mechanism design [16] and are used where an agent's valuation function can be represented as single value. The task price is calculated by finding the task's value such that if the value were any smaller, the task could not be allocated. This value is called the critical value. This has been shown to be a strategyproof [17] (weakly-dominant incentive compatible) auction so it is a weakly-dominant strategy for a task to honestly reveal its value.

The auction is implemented using the greedy mechanism from section 4.1 to find an allocation of tasks using the reported value. Then for each task allocated, the last position in the ordered the task list such that the task would still allocated is found. The critical value of the task is then equal to the inverse of the value density function where the density is the density of the next task in the list after that position.

In order that the auction is strategyproof, the value density function is required to be monotonic so that misreporting of any task attributes will result in the value density decreasing. Therefore a value density function of the form $\frac{v_j d_j}{\alpha(s_j, w_j, r_j)}$ must be used so that the auction is strategyproof.

Theorem 4.3. *The value density function $\frac{v_j d_j}{\alpha(s_j, w_j, r_j)}$ is monotonic for task j assuming the function $\alpha(s_j, w_j, r_j)$ is monotonic decreasing.*

Proof. In order to misreport the task private value and deadline must be less than the true value. The opposite is true for the required resources (storage, compute and result data) with the misreported value being greater than the true value. Therefore the α function will increase as the resource requirements increase as well, meaning that density will decrease. □

4.3 Decentralised Iterative Auction

VCG (Vickrey-Clark-Grove) auction [21] [5] [8] is proven to be economically efficient, budget balanced and incentive compatible. A task's price is found by the difference of the social welfare for when the task exists compared to the social welfare when the task doesn't exist. Our auction uses the same principle for pricing by finding the difference between the current server revenue and the revenue when the task is allocated (at £0).

The auction iteratively lets a task advertise its requirements to all of the servers who respond with their price for the task. This price is equal to the server's current revenue minus the solution to the the problem in section 4.3.1 plus a small value called the price change variable. Being the reverse of the VCG mechanism, such that the price is found for when the task exists rather than when it doesn't exist. The price change variable allows for the increase in the revenue of the server and is can be chosen by the server. Once all of the server have responded, the task can compare the minimum server price to its private value. If the price is less then the task will accept the servers with the minimum price offer, otherwise the task will stop looking as the price for the task to run on any server is greater than its reserve price.

venue for a server m given a new task p
ted tasks N has a similar formulation to
onal variable is considered, a task's price

ase.

$$(11)$$

$$(12)$$

$$m, \quad (13)$$

$$W_m, \quad (14)$$

$$+ (r_p^{'} + s_p^{'}) \leq R_m, \quad (15)$$

$$, \quad \forall n \in N \cup \{p\}, \quad (16)$$

$$\forall n \in N \cup \{p\} \quad (17)$$

$$\forall n \in N \cup \{p\} \quad (18)$$

$$\forall n \in N \cup \{p\} \quad (19)$$

$$\forall n \in N \quad (20)$$

is to maximize the price of all tasks (not
the price is zero). The server resource
imilar to the constraints in the standard
3.2 however with the assumption that
here is no need to consider if the task is
lline and non-negative resource speeds
are all the same equation with the new
e other tasks. The equation to check that
a single server is not included as only
k's price.
properties are considered: Incentive com-
economically efficient and individual ra-

Since the auction is run without an auc-
for the auction to be run in a decentralised
"middlemen" taking some money so all
ht to the servers from the tasks
al - As the server need to confirm with the
o pay an amount to be allocated, the task
inst its secret reserved price preventing
paying more than it is willing
ble - Misreporting can give a task as if
t the allocation of resources from server
can misreport so to be allocate to a cer-
herwise would result in the task being

cy - At the begin then task are almost
in till server become full and require
his means that allocation can fall into a
meaning that the server will sometime
mically efficient.

Algorithm 2 Decentralised Iterative Auction

Require: I is the set of servers
Require: J is the set of unallocated tasks, which initial is the set of all tasks to be allocated
Require: $P(i, k)$ is solution to the problem in section 4.3.1 using the server i and new task k. The server's current tasks is known to itself and its current revenue from tasks so not passed as arguments.
Require: $R(i, k)$ is a function returning the list of tasks not able to run if task k is allocated to server i
Require: $\leftarrow_R$ will randomly select an element from a set
  while $|J| > 0$ **do**
    $j \leftarrow_R J$
    $p, i \leftarrow argmin_{i \in I} P(i, j)$
    if $p \leq v_j$ **then**
      $p_j \leftarrow p$
      $x_{i,j} \leftarrow 1$
      for all $j^{'} \in R(i, j)$ **do**
        $x_{i,j^{'}} \leftarrow 0$
        $p_j^{'} \leftarrow 0$
        $J \leftarrow J \cup j^{'}$
      end for
    end if
    $J \leftarrow J \setminus \{j\}$
  end while

The algorithm 2 is a centralised version of the decentralised iterative auction. It works through iteratively checking a currently unallocated job to find the price if the job was currently allocated on a server. This is done through first solving the program in section 4.3.1 which calculates the new revenue if the task was forced to be allocated with a price of zero. The task price is equal to the current server revenue − new revenue with the task allocated + a price change variable to increase the revenue of the server. The minimum price returned by $P(i, k)$ is then compared to the job's maximum reserve price (that would be private in the equivalent decentralised algorithm) to confirm if the job is willing to pay at that price. If the job is willing then the job is allocated to the minimum price server and the job price set to the agreed price. However in the process of allocating a job then the currently allocated jobs on the server could be unallocated so these jobs allocation's and price's are reset then appended to the set of unallocated jobs.

4.4 Attributes of proposed algorithms

In table 3, the important attributes for the proposed algorithm

I	CVA	DIA
	Yes	No
	No	No
	Yes	No
	All	Not the reserve value
w	Low	High
	No	Yes

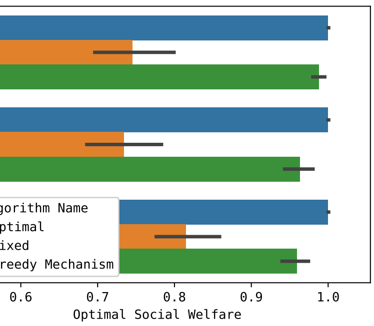
the proposed algorithms: Greedy ical Value auction(CVA) and Decen- n (DIA)

ALUATION

ented in section 4, synthetic models have st of tasks and servers.

ave been handcrafted with each attribute gaussian distribution with a mean and

algorithm to the optimal elastic allocation, implemented to solve the problem in mpare to fixed speed equivalent models, rce required to run the job is found and ls for all of the tasks, with the optimal ob with the fixed speeds is found as well. mechanism, the value density function tion was $argmin_{\forall i \in I} S_i' + W_i' + R_i'$ and the $ins_j' + w_j' + r_j'$ for job j and servers I.



of the social welfare for the greedy elaxed problem, time limited branch

greedy mechanism achieves 98% of the small models, the mechanism achieves dels. In comparison, the fixed allocation al solution and always does worse than reedy mechanism.

ocial welfare of the auction mechanisms: l vcg, critical value auction and the de- on with different price change variables.

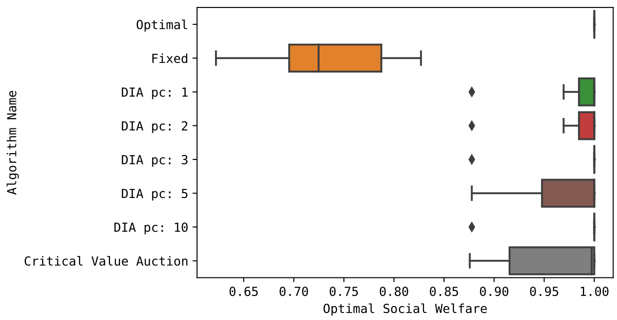


Figure 6: Comparison of the social welfare for the auction mechanisms

VCG is an economically efficient auction that requires the optimal solution to the problem in section 3.2.

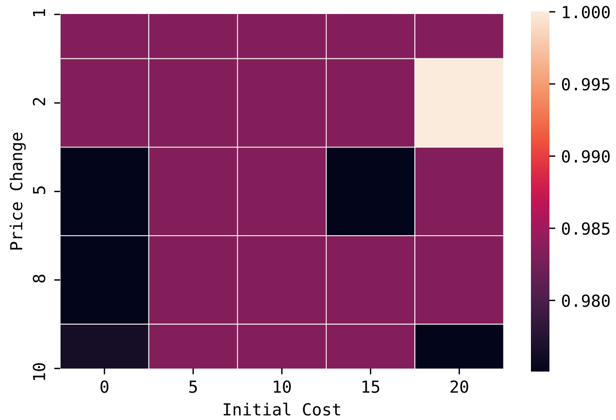
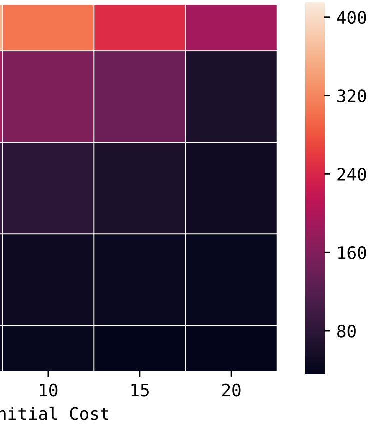


Figure 7: Average number of rounds with a price change variables and task initial cost

Within the context of edge cloud computing, the number of rounds for the decentralised iterative auction is important to making it a feasible auction as it is proportional to the time required to run. We investigated the effect of two heuristic on the number of rounds and social welfare of the auction; the price change variable and initial cost heuristic. With an auction using as minimum heuristic values for the price change and initial cost, figure 7, on average 400 rounds were required for the price to converge while an auction using a price change of 10 and initial cost of 20 means that only on average 80 rounds are required, 5x less. But by using high initial cost and price change heuristics, this can prevent tasks from being allocated, figure 8, shows that the difference in social welfare is only 2% from minimum to maximum heuristics.

6 CONCLUSIONS

In this paper, we studied a resource allocation problem in edge clouds, where resources are elastic and can be allocated to tasks at varying speeds to satisfy heterogeneous requirements and deadlines.



**l welfare with a price change vari-
st**

roposed a centralized greedy mechanism
mance bound, and a number of auction-
lso consider the elasticity of resources
r strategic manipulation. We show that
ge of resource elasticity leads to signifi-
e than current approaches that assume

to consider the dynamic scenario where
rom the system over time, and to also
ask preemption is allowed.

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