DOT^ω Candidate Rules

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$$
\begin{array}{rcr}
 & X, Y, Z... & \textbf{Type Variable} \\
 & x, y, z... & \textbf{Term Variable} \\
 & M & \textbf{Type Label} \\
 & \ell & \textbf{Term Label} \\
A, B, C ::= & X \mid \lambda(X : K).A \mid x.A \mid X\ Y & \textbf{Type} \\
(\tau, \rho, S, U) & \mid \top \mid \bot \mid (x : \tau) \to \rho \mid \tau \wedge \rho \mid \mu(x.\tau) & \text{(Proper types)} \\
 & \mid \{\textbf{val}\ x : \tau\} \mid \{\textbf{type}\ X : K\} & \\
J, K ::= & * \mid S..U \mid \Pi(X : J).K & \textbf{Kind} \\
e ::= & \cdots \mid \text{let } \textbf{type}\ X = \tau \text{ in } e & \textbf{Term} \\
\Gamma ::= & \varnothing \mid \Gamma, x : A \mid \Gamma, X : K \mid \Gamma, X = K & \textbf{Context}
\end{array}
$$

Figure 1: Syntax

$$
\frac{}{\varnothing\ \texttt{ctx}} \qquad \frac{\Gamma\ \texttt{ctx} \qquad \Gamma \vdash K\ \texttt{kd}}{\Gamma, X : K\ \texttt{ctx}} \qquad \frac{\Gamma\ \texttt{ctx} \qquad \Gamma \vdash A : *}{\Gamma, x : A\ \texttt{ctx}}
$$

$$
\frac{\Gamma\ \texttt{ctx} \qquad \Gamma \vdash A : k}{\Gamma, x = A\ \texttt{ctx}}
$$

Figure 2: Context formation

$$
\frac{\Gamma \vdash S : * \qquad \Gamma \vdash U : *}{\Gamma \vdash S..U\ \texttt{kd}}\ \textsc{Wf-Intv} \qquad \frac{\Gamma \vdash J\ \texttt{kd} \qquad \Gamma, X : J \vdash K\ \texttt{kd}}{\Gamma \vdash \Pi(X : J).K\ \texttt{kd}}\ \textsc{Wf-DArr}
$$

Figure 3: Kind formation

$$\frac{\Gamma \vdash S_2 \leq S_1 : * \qquad \Gamma \vdash U_1 \leq U_2 : *}{\Gamma \vdash S_1..S_1 \leq U_2..U_2} \text{ SK-INTV}$$

$$\frac{\Gamma \vdash \Pi(X : J_1).K_1 \ \texttt{kd} \qquad \Gamma \vdash J_2 \leq J_1 \qquad \Gamma, X : J_2 \vdash K_1 \leq K_2}{\Gamma \vdash \Pi(X : J_1).K_1 \leq \Pi(X : J_2).K_2} \text{ SK-DARR}$$

Figure 4: Subkinding

$$\frac{\Gamma, X : K \ \texttt{ctx}}{\Gamma, X : K \vdash X : K} \text{ K-VAR} \qquad \frac{\Gamma, X = A \ \texttt{ctx}}{\Gamma, X = A \vdash X : K} \text{ K-ALIAS} \qquad \frac{}{\Gamma \vdash \top : *} \text{ K-TOP}$$

$$\frac{}{\Gamma \vdash \bot : *} \text{ K-BOT} \qquad \frac{\Gamma \vdash A : S..U}{\Gamma \vdash A : A..A} \text{ K-SING} \qquad \frac{\Gamma \vdash A : * \qquad \Gamma \vdash B : *}{\Gamma \vdash A \to B : *} \text{ K-ARR}$$

$$\frac{\Gamma \vdash J \ \texttt{kd} \qquad \Gamma, X : J \vdash A : K \qquad \Gamma, X : J \vdash K \ \texttt{kd}}{\Gamma \vdash \lambda(X : J).A : \Pi(X : J).K} \text{ K-ABS}$$

$$\frac{\Gamma \vdash X : \Pi(Z : J).K \qquad \Gamma \vdash Y : J \qquad \Gamma, Z : J \vdash K \ \texttt{kd} \qquad \Gamma \vdash K[Y/Z] \ \texttt{kd}}{\Gamma \vdash X \ Y : K[Y/X]} \text{ K-APP}$$

$$\frac{\Gamma \vdash A : S_1..U_1 \qquad \Gamma \vdash B : S_2..U_2}{\Gamma \vdash A \wedge B : S_1 \vee S_2..U_1 \wedge U_2} \text{ K-INTERSECT} \qquad \frac{\Gamma \vdash A : S..U}{\Gamma \vdash \{\mathbf{val}\ \ell : A\}} \text{ K-FIELD}$$

$$\frac{\Gamma \vdash K \ \texttt{kd}}{\Gamma \vdash \{\mathbf{type}\ M : K\}} \text{ K-TYP} \qquad \frac{\Gamma \vdash x : \{\mathbf{type}\ M : K\}}{\Gamma \vdash x.M : K} \text{ K-TYP-MEM}$$

$$\frac{\Gamma, x : \tau \vdash \tau : K}{\Gamma \vdash \mu(x.\tau) : K} \text{ K-REC} \qquad \frac{\Gamma \vdash A : J \qquad \Gamma \vdash J \leq K}{\Gamma \vdash A : K} \text{ K-SUB}$$

Figure 5: Kind assignment

$$\frac{\Gamma \vdash A : K}{\Gamma \vdash A \leq A : K}\ \text{ST-}\textsc{refl} \qquad \frac{\Gamma \vdash A \leq B : K \qquad \Gamma \vdash B \leq C : K}{\Gamma \vdash A \leq C : K}\ \text{ST-}\textsc{trans}$$

$$\frac{\Gamma \vdash A : S..U}{\Gamma \vdash A \leq \top : *}\ \text{ST-}\textsc{top} \qquad \frac{\Gamma \vdash A : S..U}{\Gamma \vdash \bot \leq A : *}\ \text{ST-}\textsc{bot}$$

$$\frac{\Gamma, X = \tau\ \texttt{ctx} \qquad \Gamma \vdash \tau : k}{\Gamma, X = \tau \vdash X \leq \tau : k}\ \text{ST-}\textsc{alias}_1$$

$$\frac{\Gamma, X = \tau\ \texttt{ctx} \qquad \Gamma \vdash \tau : k}{\Gamma, X = \tau \vdash \tau \leq X : k}\ \text{ST-}\textsc{alias}_2 \qquad \frac{\Gamma \vdash A \wedge B : K}{\Gamma \vdash A \wedge B \leq B : K}\ \text{ST-}\textsc{and}\text{-}\ell_1$$

$$\frac{\Gamma \vdash A \wedge B : K}{\Gamma \vdash A \wedge B \leq B : K}\ \text{ST-}\textsc{and}\text{-}\ell_2$$

$$\frac{\Gamma \vdash S \leq A : K \qquad \Gamma \vdash S \leq B : K}{\Gamma \vdash S \leq A \wedge B : K}\ \text{ST-}\textsc{and-r}$$

$$\frac{\Gamma \vdash A \leq B : K}{\Gamma \vdash \{\mathbf{val}\ \ell : A\} \leq \{\mathbf{val}\ \ell : B\} : *}\ \text{ST-}\textsc{field}$$

$$\frac{\Gamma \vdash J \leq K}{\Gamma \vdash \{\mathbf{type}\ M : J\} \leq \{\mathbf{type}\ M : K\} : *}\ \text{ST-}\textsc{typ}$$

$$\frac{\Gamma \vdash X = \lambda(Z : J).A : \Pi(Z : J).K \qquad \Gamma \vdash Y : J}{\Gamma \vdash X\ Y \leq A[Y/Z] : K[Y/Z]}\ \text{ST-}\beta_1$$

$$\frac{\Gamma \vdash X = \lambda(Z : J).A : \Pi(Z : J).K \qquad \Gamma \vdash Y : J}{\Gamma \vdash A[Y/Z] \leq X\ Y : K[Y/Z]}\ \text{ST-}\beta_1$$

Figure 6: Subtyping

$$\frac{\Gamma \vdash A \leq B : K \qquad \Gamma \vdash B \leq A : K}{\Gamma \vdash A = B : K}\ \textsc{Eq}$$

Figure 7: Type equality

Type assignment rules are the same as Rapoport et al. [2] and Amin et al. [1], with changes and additions below.

$$\frac{\Gamma \vdash \tau : k \qquad \Gamma, X = \tau \vdash e : \rho}{\Gamma \vdash \text{let } \mathbf{type}\ X = \tau \text{ in } e : \rho}\ \textsc{Let-type}$$

$$\frac{\Gamma \vdash \tau : k}{\Gamma \vdash \{\mathbf{type}\ M = \tau\} : \{\mathbf{type}\ M : k\}}\ \textsc{Def-type}$$

Figure 8: Type assignment, adjusted rules