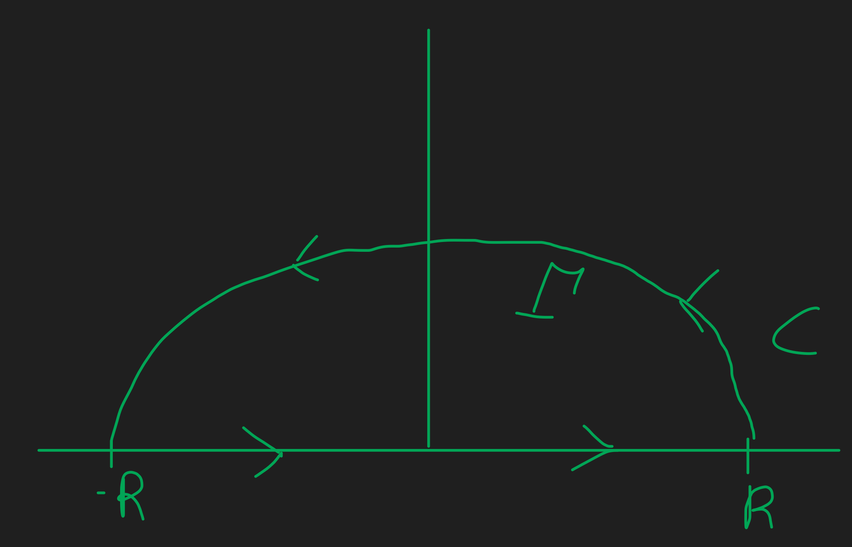
GROUPWORK

Show that $\displaystyle\int_0^\infty \frac{x\cos(x)}{x^2+9}\,dx = 0$

$$\lim_{z\to 3i} \cancel{(z-3i)} \frac{ze^{iz}}{\cancel{(z-3i)}(z+3i)} = \frac{\cancel{3i}\, e^{-3}}{\cancel{6i}} = \frac{e^{-3}}{2}$$

$$I = \int_0^\infty \frac{x\cos(x)}{x^2+9}\,dx = \frac{1}{2}\,\mathrm{Re}\int_{-\infty}^{\infty} \frac{xe^{ix}}{x^2+9}\,dx$$

$$f(z) = \frac{ze^{iz}}{z^2+9}, \quad \text{poles: } z = \pm 3i$$



$$\oint_C = \int_\Gamma + \int_{-R}^{R}$$

$$\oint_C \frac{ze^{iz}}{z^2+9}\,dz = 2\pi i \sum \mathrm{Res}(f)$$

$$\Rightarrow \oint_C \frac{ze^{iz}}{z^2+9}\,dz = \cancel{2}\pi i \cdot \frac{e^{-3}}{\cancel{2}} = \pi i e^{-3}$$

$$\pi i e^{-3} = \int_\Gamma + \int_{-R}^{R}$$

$$\left|\int_\Gamma\right| \leq \int_0^\pi \left| \frac{Re^{i\phi} e^{iRe^{i\phi}}}{R^2e^{2i\phi}+9}\, iRe^{i\phi} \right| d\phi = \int_0^\pi \left| \frac{Re^{iRe^{i\phi}}}{R^2e^{2i\phi}+9} R \right| d\phi$$

$$= \int_0^\pi \left| \frac{R^2 e^{iR\cos(\phi) - R\sin(\phi)}}{R^2e^{2i\phi}+9} \right| d\phi = R^2 \int_0^\pi \frac{e^{-R\sin(\phi)}}{|R^2e^{2i\phi}+9|}\,d\phi$$

$$\left| R^2e^{2i\phi} - (-9) \right| \geq \left| |R^2e^{2i\phi}| - |-9| \right| = |R^2-9| = R^2-9$$

$$\frac{1}{|R^2e^{2i\phi}+9|} \leq \frac{1}{R^2-9}$$

$$\leq R^2 \int_0^\pi \frac{e^{-R\sin(\phi)}}{R^2-9}\,d\phi = \frac{R^2}{R^2-9}\int_0^\pi e^{-R\sin(\phi)}\,d\phi \quad [0,\pi] \to (0,\pi)$$

$$\xrightarrow{R\to\infty} 0 \Rightarrow \left|\int_\Gamma\right| = 0 \Rightarrow \int_\Gamma = 0$$

$$\therefore \pi i e^{-3} = \int_{-R}^{R} \frac{xe^{ix}}{x^2+9}\,dx$$

$$\xrightarrow{R\to\infty} \mathrm{Re}(\pi i e^{-3}) = \mathrm{Re}\int_{-\infty}^{\infty} \frac{xe^{ix}}{x^2+9}\,dx$$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{x\cos(x)}{x^2+9}\,dx = 0$$

$$I = \frac{1}{2}\cdot 0 = 0$$

$\boxtimes$