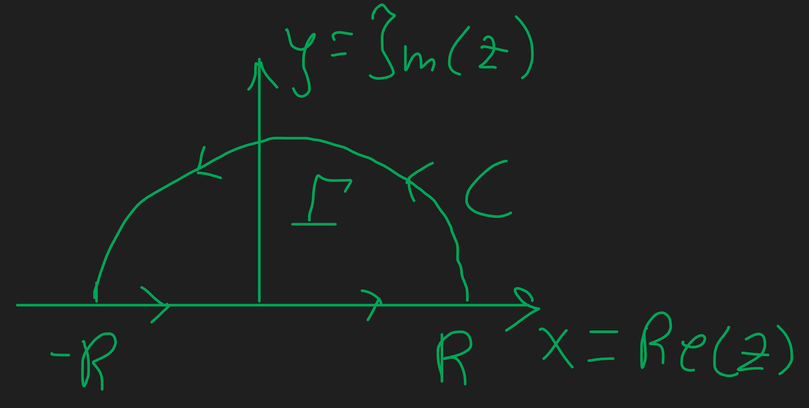
Show that $\displaystyle\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^3}\,dx = \frac{3\pi}{8}$

$$I = \int_{-\infty}^{\infty} \frac{1}{(x^2+1)^3}\,dx$$

$$f(z) = \frac{1}{(z^2+1)^3}\,, \text{ poles: } z^2+1=0 \Rightarrow z = \pm i$$

$$\oint_C = \int_\Gamma + \int_{-R}^{R}$$



$$\oint_C \frac{1}{(z^2+1)^3}\,dz = 2\pi i \cdot \frac{1}{2!} \lim_{z\to i} \left(\frac{d}{dz}\right)^2 \left[(z-i)^3 \frac{1}{(z^2+1)^3}\right]$$

$$= \pi i \lim_{z\to i} \left(\frac{d}{dz}\right)^2 \left[(z-i)^3 \frac{1}{(z+i)(z-i)(z^2+1)^2}\right]$$

$$= \pi i \lim_{z\to i} \left(\frac{d}{dz}\right)^2 \left[(z-i)^2 \frac{1}{(z+i)(z^2+1)(z^2+1)}\right]$$

$$= \pi i \lim_{z\to i} \left(\frac{d}{dz}\right)^2 \left[\frac{(z-i)^2}{(z+i)(z+i)(z-i)(z+i)(z-i)} \frac{1}{}\right]$$

$$= \pi i \lim_{z\to i} \frac{12}{(z+i)^5} = \pi i \cdot \frac{12}{32i} = \frac{3\pi}{8}$$

$$\frac{3\pi}{8} = \oint_\Gamma + \int_{-R}^{R}$$

$$\left|\oint_\Gamma\right| \le \int_0^\pi \left|\frac{iRe^{i\phi}}{(R^2e^{2i\phi}+1)^3}\right| d\phi = \int_0^\pi \frac{R}{|(R^2e^{2i\phi}+1)^3|}\,d\phi$$

$$= \int_0^\pi \frac{R}{R^3\left|R^4e^{6i\phi} + 3R^2e^{4i\phi} + 3e^{2i\phi} + \frac{1}{R^2}\right|}\,d\phi$$

$$\Rightarrow \lim_{R\to\infty} \left|\oint_\Gamma\right| = 0 \Rightarrow \oint_\Gamma = 0$$

$$\therefore \frac{3\pi}{8} = \lim_{R\to\infty} \left(\int_{-R}^{R} \frac{1}{(x^2+1)^3}\,dx\right)$$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{1}{(x^2+1)^3}\,dx = \frac{3\pi}{8} \quad \blacksquare$$