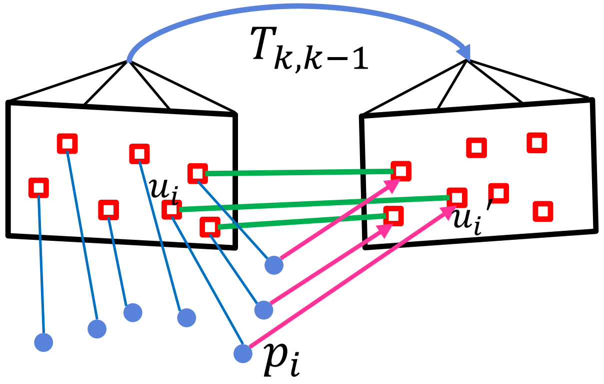
Robotic Navigation and Exploration

Week 9: SLAM - Direct Method

Min-Chun Hu anitahu@cs.nthu.edu.tw
CS, NTHU / CSIE, NCKU

Direct Method & Indirect Method

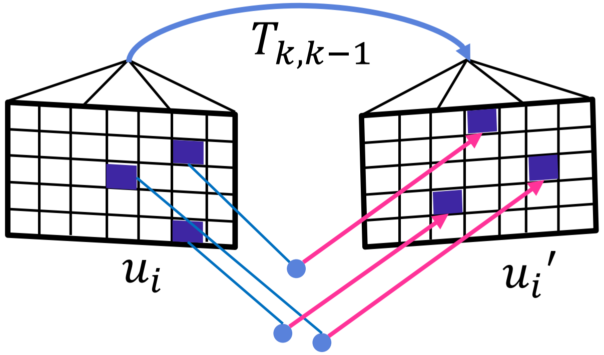
Indirect Method



$$T_{k,k-1} = \underset{T}{argmin} \sum_{i}^{N} ||u_i' - \pi p_i||^2$$

Minimize Geometric Error (Reprojection Error)

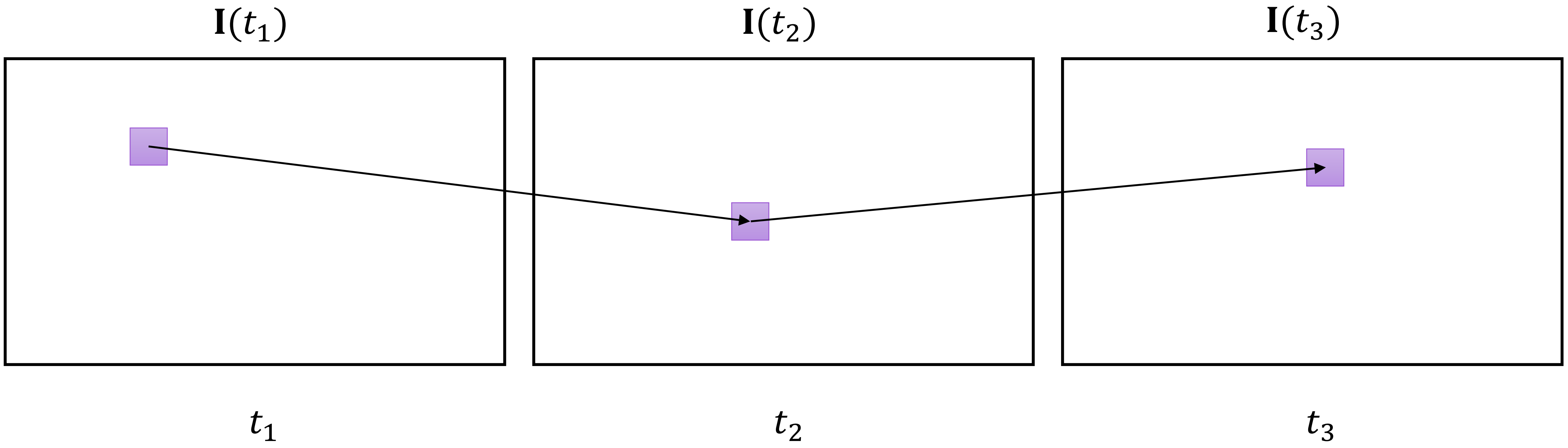
Direct Method



$$T_{k,k-1} = \underset{T}{argmin} \sum_{i}^{N} ||I_k(u_i') - I_{k-1}(u_i))||^2$$

Minimize Photometric Error (Pixel Grayscale)

Optical Flow



Assumption : the displacement of the image content between two nearby frames is small and approximately constant within a neighborhood of the point p under consideration

$$\mathbf{I}(x + dx, y + dy, t + dt) = \mathbf{I}(x, y, t)$$

Lucas –Kanade (L-K) Method

$$\mathbf{I}(x+dx, y+dy, t+dt) \approx \mathbf{I}(x,y,t) + \frac{\partial \mathbf{I}}{\partial x}dx + \frac{\partial \mathbf{I}}{\partial y}dy + \frac{\partial \mathbf{I}}{\partial t}dt$$

Intensity Invariant:

$$\frac{\partial \mathbf{I}}{\partial x}dx + \frac{\partial \mathbf{I}}{\partial y}dy + \frac{\partial \mathbf{I}}{\partial t}dt = 0$$

$$\frac{\partial \mathbf{I}}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \mathbf{I}}{\partial y}\frac{\partial y}{\partial t} = -\frac{\partial \mathbf{I}}{\partial t}$$

$$[\mathbf{I}_x \quad \mathbf{I}_y]\begin{bmatrix} u \\ v \end{bmatrix} = -\mathbf{I}_t$$

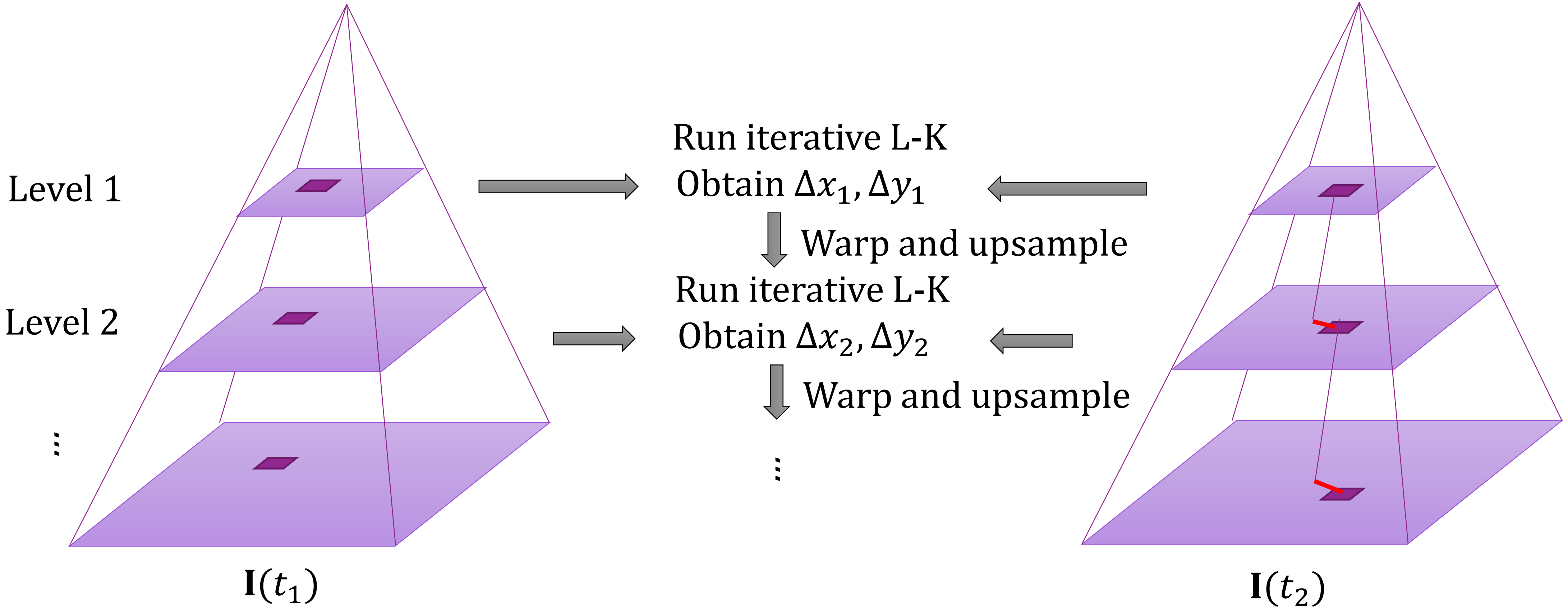
Local consistency:

$$[\mathbf{I}_x \quad \mathbf{I}_y]_k \begin{bmatrix} u \\ v \end{bmatrix} = -\mathbf{I}_{tk},\ k = 1, \dots, w^2$$

$$\mathbf{A} = \begin{bmatrix} [\mathbf{I}_x \quad \mathbf{I}_y]_1 \\ \vdots \\ [\mathbf{I}_x \quad \mathbf{I}_y]_k \end{bmatrix},\ \mathbf{b} = \begin{bmatrix} \mathbf{I}_{t1} \\ \vdots \\ \mathbf{I}_{tk} \end{bmatrix} \qquad \mathbf{A}\begin{bmatrix} u \\ v \end{bmatrix} = -\mathbf{b} \qquad \begin{bmatrix} u \\ v \end{bmatrix}^* = -\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Coarse-to-fine Optical Flow

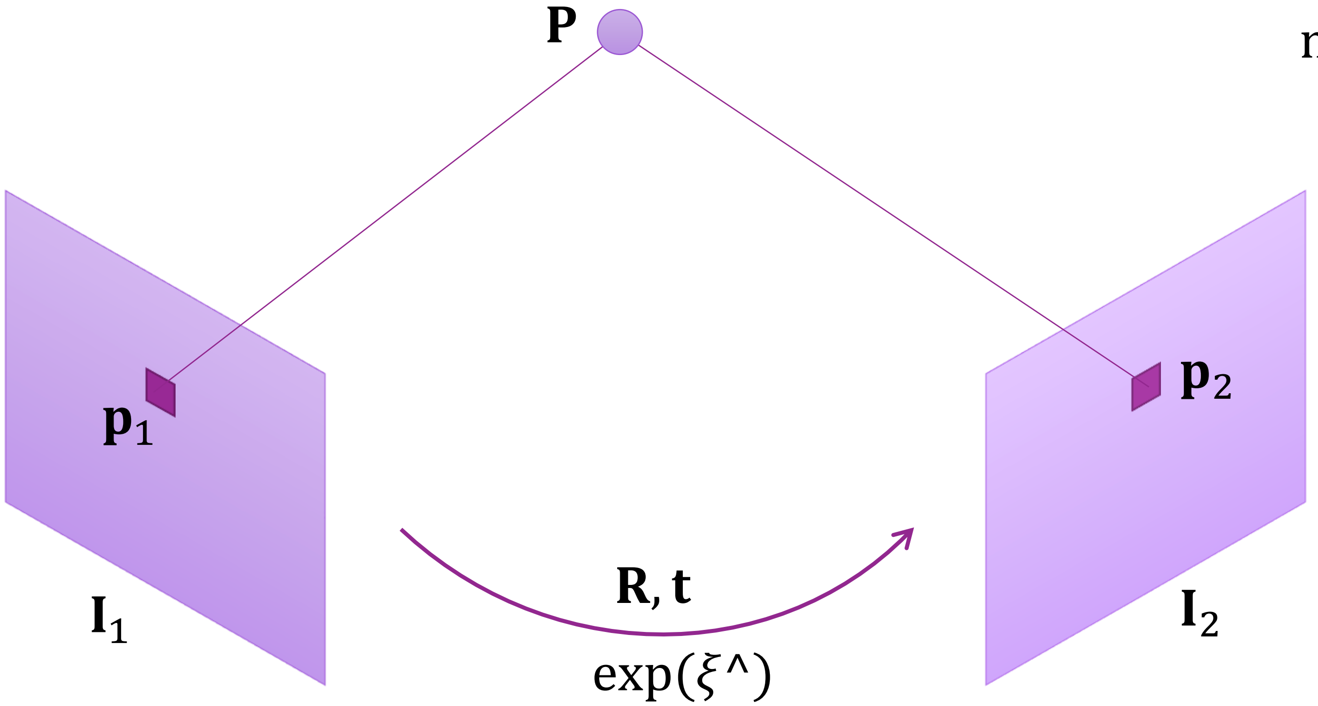
$$\min_{\Delta x, \Delta \boldsymbol{y}} \|\mathbf{I}(x, y, t) - \mathbf{I}(x + \Delta x, y + \Delta y, t + \Delta t)\|_2^2$$



Direct Method

$$e_i = \mathbf{I}_1(\mathbf{p}_{1,i}) - \mathbf{I}_2(\mathbf{p}_{2,\mathrm{i}})$$

$$\min_{\mathbf{T}} \sum_{i=1}^{N} e_i^T e_i$$



$$\mathbf{p}_1 = \begin{bmatrix} u_1 \\ v_1 \\ 1 \end{bmatrix} = \frac{1}{Z_1}\mathbf{KP}$$

$$\mathbf{p_2} = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \frac{1}{Z_2}\mathbf{K}(\mathbf{RP} + \mathbf{t}) = \frac{1}{Z_2}\mathbf{K}(\mathbf{Tp})$$

Direct Method

$$e_i = \mathbf{I}_1(\mathbf{p}_{1,i}) - \mathbf{I}_2(\mathbf{p}_{2,\mathrm{i}}) \qquad \min_{\mathbf{T}} \sum_{i=1}^{N} e_i^T e_i$$

$$\boxed{\mathbf{p}_2 = \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \frac{1}{Z_2}\mathbf{K}(\mathbf{RP}+\mathbf{t}) = \frac{1}{Z_2}\mathbf{K}(\mathbf{Tp})}$$

$$\mathbf{q} = \mathbf{Tp} \qquad \mathbf{u} = \frac{1}{Z_2}\mathbf{Kq}$$

$$e(\mathbf{T}) = \mathbf{I}_1(\mathbf{p}_1) - \mathbf{I}_2(\boldsymbol{u}) \qquad \frac{\partial e}{\partial \mathbf{T}} = \frac{\partial \mathbf{I}_2}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial \delta \boldsymbol{\xi}} \delta \boldsymbol{\xi}$$

$$\frac{\partial \mathbf{u}}{\partial \mathbf{q}} = \begin{bmatrix} \frac{\partial u}{\partial X} & \frac{\partial u}{\partial Y} & \frac{\partial u}{\partial Z} \\ \frac{\partial v}{\partial X} & \frac{\partial v}{\partial Y} & \frac{\partial v}{\partial Z} \end{bmatrix} = \begin{bmatrix} \frac{f_x}{Z} & 0 & -\frac{f_x X}{Z^2} \\ 0 & \frac{f_y}{Z} & -\frac{f_y Y}{Z^2} \end{bmatrix}$$

$$\boxed{u = \frac{f_x X}{Z},\ v = \frac{f_y X}{Z}}$$

$$\frac{\partial \mathbf{q}}{\partial \delta \boldsymbol{\xi}} = [\mathbf{I}, -\mathbf{q}^\wedge]$$

$$\begin{aligned} \frac{\partial (Rp)}{\partial \psi} &= \lim_{\psi \to 0} \frac{\exp(\psi^\wedge)\exp(\phi^\wedge)\,p - \exp(\phi^\wedge)\,p}{\psi} \\ &= \lim_{\psi \to 0} \frac{(I+\psi^\wedge)\exp(\phi^\wedge)\,p - \exp(\phi^\wedge)\,p}{\psi} \\ &= \lim_{\psi \to 0} \frac{\psi^\wedge Rp}{\psi} = \lim_{\psi \to 0} \frac{-(Rp)^\wedge \psi}{\psi} = -(Rp)^\wedge \end{aligned}$$

Direct Method

$$\frac{\partial e}{\partial \mathbf{T}} = \frac{\partial \mathbf{I}_2}{\partial \mathbf{u}} \boxed{\frac{\partial \mathbf{u}}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial \delta \boldsymbol{\xi}}} \delta \boldsymbol{\xi}$$

$$\frac{\partial \mathbf{u}}{\partial \delta \boldsymbol{\xi}} = \begin{bmatrix} \frac{f_x}{Z} & 0 & -\frac{f_x X}{Z^2} & -\frac{f_x XY}{Z^2} & f_x + \frac{f_x X^2}{Z^2} & -\frac{f_x Y}{Z} \\ 0 & \frac{f_y}{Z} & -\frac{f_y Y}{Z^2} & -f_y - \frac{f_y Y^2}{Z^2} & \frac{f_y XY}{Z^2} & \frac{f_y X}{Z} \end{bmatrix}$$

$$\boldsymbol{J} = -\frac{\partial \mathbf{I}_2}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \delta \boldsymbol{\xi}}$$

Apply Gauss Newton or Levenberg–Marquardt algorithm to solve the optimization problem.