

CHERI-MIPS capability monotonicity proof

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1 Abstract model of CHERI ISAs

1.1 Capability abstraction

Generated by Lem from *capabilities.lem*.

theory *Capabilities*

imports

Main

LEM.Lem-pervasives-extra

Sail.Sail2-values
Sail.Sail2-prompt-monad

begin

— *open import Pervasives-extra*
— *open import Sail2-values*
— *open import Sail2-prompt-monad*

record *perms* =

permit-ccall :: *bool*

permit-execute :: *bool*

permit-load :: *bool*

permit-load-capability :: *bool*

permit-seal :: *bool*

permit-store :: *bool*

permit-store-capability :: *bool*

permit-store-local-capability :: *bool*

permit-system-access :: *bool*

permit-unseal :: *bool*

record *'c Capability-class* =

is-tagged-method :: *'c* ⇒ *bool*

is-sealed-method :: *'c* ⇒ *bool*

get-mem-region-method :: *'c* ⇒ *nat set*

get-obj-type-method :: *'c* ⇒ *nat*

get-perms-method :: *'c* ⇒ *perms*

get-cursor-method :: *'c* ⇒ *nat*

get-global-method :: *'c* ⇒ *bool*

set-tag-method :: *'c* ⇒ *bool* ⇒ *'c*

```

set-seal-method :: 'c ⇒ bool ⇒ 'c

set-obj-type-method :: 'c ⇒ nat ⇒ 'c

set-perms-method :: 'c ⇒ perms ⇒ 'c

set-global-method :: 'c ⇒ bool ⇒ 'c

cap-of-mem-bytes-method :: memory-byte list ⇒ bitU ⇒ 'c option

— val seal : forall 'cap. Capability 'cap => 'cap -> nat -> 'cap
definition seal :: 'cap Capability-class ⇒ 'cap ⇒ nat ⇒ 'cap where
  seal dict-Capabilities-Capability-cap c obj-type = (
    (set-seal-method dict-Capabilities-Capability-cap) ((set-obj-type-method dict-Capabilities-Capability-cap)
c obj-type) True )
  for dict-Capabilities-Capability-cap :: 'cap Capability-class
  and c :: 'cap
  and obj-type :: nat

— val unseal : forall 'cap. Capability 'cap => 'cap -> bool -> 'cap
definition unseal :: 'cap Capability-class ⇒ 'cap ⇒ bool ⇒ 'cap where
  unseal dict-Capabilities-Capability-cap c global1 = (
    (set-seal-method dict-Capabilities-Capability-cap) ((set-obj-type-method dict-Capabilities-Capability-cap)
((set-global-method dict-Capabilities-Capability-cap) c (global1 ∧ (get-global-method
dict-Capabilities-Capability-cap) c))(( 0 :: nat))) False )
  for dict-Capabilities-Capability-cap :: 'cap Capability-class
  and c :: 'cap
  and global1 :: bool

— val leq-perms : perms -> perms -> bool
definition leq-perms :: perms ⇒ perms ⇒ bool where
  leq-perms p1 p2 = (
    ((permit-ccall p1) → (permit-ccall p2)) ∧
    (((permit-execute p1) → (permit-execute p2)) ∧
    (((permit-load p1) → (permit-load p2)) ∧
    (((permit-load-capability p1) → (permit-load-capability p2)) ∧
    (((permit-store p1) → (permit-store p2)) ∧
    (((permit-store-capability p1) → (permit-store-capability p2)) ∧
    (((permit-store-local-capability p1) → (permit-store-local-capability p2)) ∧
    (((permit-system-access p1) → (permit-system-access p2)) ∧
    ((permit-unseal p1) → (permit-unseal p2))))))))))
  for p1 :: perms
  and p2 :: perms

```

— *val leq-cap : forall 'cap. Capability 'cap, Eq 'cap => 'cap -> 'cap -> bool*

definition *leq-cap* :: 'cap Capability-class => 'cap => 'cap => bool **where**

```

    leq-cap dict-Capabilities-Capability-cap c1 c2 = (
      (c1 = c2) ∨
      ((¬ ((is-tagged-method dict-Capabilities-Capability-cap) c1)) ∨
        (((is-tagged-method dict-Capabilities-Capability-cap) c2) ∧
          (¬ ((is-sealed-method dict-Capabilities-Capability-cap) c1) ∧ ¬ ((is-sealed-method
dict-Capabilities-Capability-cap) c2)) ∧
          (((get-mem-region-method dict-Capabilities-Capability-cap) c1) ⊆ ((get-mem-region-method
dict-Capabilities-Capability-cap) c2)) ∧
          (((get-global-method dict-Capabilities-Capability-cap) c1) →
            (get-global-method dict-Capabilities-Capability-cap) c2) ∧
          (leq-perms ((get-perms-method dict-Capabilities-Capability-cap) c1) ((get-perms-method
dict-Capabilities-Capability-cap) c2))))))
    for dict-Capabilities-Capability-cap :: 'cap Capability-class
    and c1 :: 'cap
    and c2 :: 'cap

```

— *val invokable : forall 'cap. Capability 'cap => 'cap -> 'cap -> bool*

definition *invokable* :: 'cap Capability-class => 'cap => 'cap => bool **where**

```

    invokable dict-Capabilities-Capability-cap cc cd1 = (
      (let pc = ((get-perms-method dict-Capabilities-Capability-cap) cc) in
      (let pd = ((get-perms-method dict-Capabilities-Capability-cap) cd1) in (is-tagged-method
dict-Capabilities-Capability-cap) cc ∧ ((is-tagged-method dict-Capabilities-Capability-cap)
cd1 ∧
      ((is-sealed-method dict-Capabilities-Capability-cap) cc ∧ ((is-sealed-method dict-Capabilities-Capability-cap)
cd1 ∧
      ((permit-ccall pc) ∧ ((permit-ccall pd) ∧
      (((get-obj-type-method dict-Capabilities-Capability-cap) cc) = (get-obj-type-method
dict-Capabilities-Capability-cap) cd1) ∧
      ((permit-execute pc) ∧ ((
      (get-cursor-method dict-Capabilities-Capability-cap) cc ∈ (get-mem-region-method
dict-Capabilities-Capability-cap) cc) ∧
      ¬(permit-execute pd))))))))))
    for dict-Capabilities-Capability-cap :: 'cap Capability-class
    and cc :: 'cap
    and cd1 :: 'cap

```

— *Derivation of capabilities, bounded by derivation depth to guarantee termination*

— *val cap-derivable-bounded : forall 'cap. Capability 'cap, SetType 'cap, Eq 'cap
=> nat -> set 'cap -> 'cap -> bool*

fun *cap-derivable-bounded* :: 'cap Capability-class => nat => 'cap set => 'cap =>
bool **where**

```

    cap-derivable-bounded dict-Capabilities-Capability-cap 0 C c = ( (c ∈ C))
    for dict-Capabilities-Capability-cap :: 'cap Capability-class
    and C :: 'cap set

```

```

and c :: 'cap
| cap-derivable-bounded dict-Capabilities-Capability-cap ((Suc n)) C c = (
  (( $\exists$  c'. cap-derivable-bounded
    dict-Capabilities-Capability-cap n C c'  $\wedge$  leq-cap
    dict-Capabilities-Capability-cap c c'))  $\vee$ 
    (( $\exists$  c'.  $\exists$  c''.
      cap-derivable-bounded
      dict-Capabilities-Capability-cap n C c'  $\wedge$ 
      (cap-derivable-bounded
        dict-Capabilities-Capability-cap n C c''  $\wedge$ 
        ((is-tagged-method dict-Capabilities-Capability-cap) c'  $\wedge$  ((is-tagged-method
dict-Capabilities-Capability-cap) c''  $\wedge$  ( $\neg$  (
  (is-sealed-method dict-Capabilities-Capability-cap) c'')  $\wedge$ 
    (((is-sealed-method dict-Capabilities-Capability-cap) c'  $\wedge$  ((permit-unseal (
      (get-perms-method dict-Capabilities-Capability-cap) c'')  $\wedge$  (((get-obj-type-method
dict-Capabilities-Capability-cap) c' = (get-cursor-method dict-Capabilities-Capability-cap)
c'')  $\wedge$  (unseal dict-Capabilities-Capability-cap c' ((get-global-method dict-Capabilities-Capability-cap)
c'') = c))))))  $\vee$ 
      ( $\neg$  ((is-sealed-method dict-Capabilities-Capability-cap) c')  $\wedge$  ((permit-seal
(
  (get-perms-method dict-Capabilities-Capability-cap) c'')  $\wedge$  (seal dict-Capabilities-Capability-cap
c' ((get-cursor-method dict-Capabilities-Capability-cap) c'') = c))))))))))
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and n :: nat
and C :: 'cap set
and c :: 'cap

```

— *TODO: Prove an upper bound for the derivation depth. For a finite set of n capabilities, it seems a derivation depth of $n+1$ should be enough: If all but one capabilities in C are sealed, up to $n-1$ unsealing operations and possibly a restriction and a sealing operation might be necessary to derive the desired capability.*

```

definition cap-derivable :: 'a Capability-class  $\Rightarrow$  'a set  $\Rightarrow$  'a  $\Rightarrow$  bool where
  cap-derivable dict-Capabilities-Capability-a C c = ( ( $\exists$  n. cap-derivable-bounded
    dict-Capabilities-Capability-a n C c)))
for dict-Capabilities-Capability-a :: 'a Capability-class
and C :: 'a set
and c :: 'a

```

```

— val reads-from-reg : forall 'regval. event 'regval  $\rightarrow$  maybe register-name
fun reads-from-reg :: 'regval event  $\Rightarrow$  (string)option where
  reads-from-reg (E-read-reg r -) = ( Some r )
for r :: string
| reads-from-reg - = ( None )

```

— *val reads-reg-caps : forall 'regval 'cap. Capability 'cap, SetType 'cap \Rightarrow ('regval*

```

-> set 'cap) -> event 'regval -> set 'cap
fun reads-reg-caps :: 'cap Capability-class =>('regval => 'cap set)> 'regval event
=> 'cap set where
    reads-reg-caps dict-Capabilities-Capability-cap caps-of-regval1 (E-read-reg - v)
= ( set-filter
  (is-tagged-method dict-Capabilities-Capability-cap) (caps-of-regval1 v))
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and caps-of-regval1 :: 'regval => 'cap set
and v :: 'regval
| reads-reg-caps dict-Capabilities-Capability-cap caps-of-regval1 - = ( {} )
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and caps-of-regval1 :: 'regval => 'cap set

— val writes-to-reg : forall 'regval. event 'regval -> maybe register-name
fun writes-to-reg :: 'regval event =>(string)option where
    writes-to-reg (E-write-reg r -) = ( Some r )
for r :: string
| writes-to-reg - = ( None )

— val writes-reg-caps : forall 'regval 'cap. Capability 'cap, SetType 'cap => ('regval
-> set 'cap) -> event 'regval -> set 'cap
fun writes-reg-caps :: 'cap Capability-class =>('regval => 'cap set)> 'regval event
=> 'cap set where
    writes-reg-caps dict-Capabilities-Capability-cap caps-of-regval1 (E-write-reg -
v) = ( set-filter
  (is-tagged-method dict-Capabilities-Capability-cap) (caps-of-regval1 v))
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and caps-of-regval1 :: 'regval => 'cap set
and v :: 'regval
| writes-reg-caps dict-Capabilities-Capability-cap caps-of-regval1 - = ( {} )
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and caps-of-regval1 :: 'regval => 'cap set

— val reads-mem-val : forall 'regval. event 'regval -> maybe (nat * nat * list
memory-byte * bitU)
fun reads-mem-val :: 'regval event =>(nat*nat*(memory-byte)list*bitU)option
where
    reads-mem-val (E-read-memt - addr sz (v, t)) = ( Some (addr, sz, v, t))
for addr :: nat
and sz :: nat
and t :: bitU
and v :: (memory-byte)list
| reads-mem-val (E-read-mem - addr sz v) = ( Some (addr, sz, v, B0))
for addr :: nat
and sz :: nat
and v :: (memory-byte)list

```

| reads-mem-val - = (None)

— val reads-mem-cap : forall 'regval 'cap. Capability 'cap => event 'regval -> maybe (nat * nat * 'cap)

definition reads-mem-cap :: 'cap Capability-class => 'regval event =>(nat*nat*'cap)option
where

```

    reads-mem-cap dict-Capabilities-Capability-cap e = (
    Option.bind (reads-mem-val e) ( λx .
    (case x of
    (addr, sz, v, t) =>
    Option.bind
    ((cap-of-mem-bytes-method dict-Capabilities-Capability-cap) v t)
    (λ c .
    if(is-tagged-method dict-Capabilities-Capability-cap) c then
    Some (addr, sz, c) else None)
    )))
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and e :: 'regval event

```

— val writes-mem-val : forall 'regval. event 'regval -> maybe (nat * nat * list memory-byte * bitU)

fun writes-mem-val :: 'regval event =>(nat*nat*(memory-byte)list*bitU)option
where

```

    writes-mem-val (E-write-memt - addr sz v t -) = ( Some (addr, sz, v, t))
for addr :: nat
and sz :: nat
and t :: bitU
and v :: (memory-byte)list
| writes-mem-val (E-write-mem - addr sz v -) = ( Some (addr, sz, v, B0))
for addr :: nat
and sz :: nat
and v :: (memory-byte)list
| writes-mem-val - = ( None )

```

— val writes-mem-cap : forall 'regval 'cap. Capability 'cap => event 'regval -> maybe (nat * nat * 'cap)

definition writes-mem-cap :: 'cap Capability-class => 'regval event =>(nat*nat*'cap)option
where

```

    writes-mem-cap dict-Capabilities-Capability-cap e = (
    Option.bind (writes-mem-val e) ( λx .
    (case x of
    (addr, sz, v, t) =>
    Option.bind
    ((cap-of-mem-bytes-method dict-Capabilities-Capability-cap) v t)
    (λ c .
    if(is-tagged-method dict-Capabilities-Capability-cap) c then

```

```

        Some (addr, sz, c) else None)
    )))
  for dict-Capabilities-Capability-cap :: 'cap Capability-class
  and e :: 'regval event

end
theory Cheri-axioms

imports
  Main
  LEM.Lem-pervasives-extra
  Sail.Sail2-values
  Sail.Sail2-prompt-monad
  Sail.Sail2-operators
  Capabilities

begin

```

1.2 ISA abstraction

Generated by Lem from *cheri-axioms.lem*.

```

— open import Pervasives-extra
— open import Sail2-values
— open import Sail2-prompt-monad
— open import Sail2-operators
— open import Capabilities

— TODO: Maybe add a read-kind for instruction fetches, so that we can distinguish
loads from fetches in events and don't need to carry around the is-fetch parameter
below
datatype acctype = Load | Store | Fetch

record( 'cap, 'regval, 'instr, 'e) isa =

  instr-sem :: 'instr ⇒ ('regval, unit, 'e) monad

  instr-fetch :: ('regval, 'instr, 'e) monad

  tag-granule :: nat

  PCC :: register-name — trace 'regval -> set

  KCC :: register-name — trace 'regval -> set

  IDC :: register-name — trace 'regval -> set

  caps-of-regval :: 'regval ⇒ 'cap set

  invokes-caps :: 'instr ⇒ 'regval trace ⇒ bool

```


instr-raises-ex :: 'instr \Rightarrow 'regval trace \Rightarrow bool

fetch-raises-ex :: 'regval trace \Rightarrow bool

exception-targets :: 'regval set \Rightarrow 'cap set

privileged-regs :: register-name \rightarrow trace 'regval \rightarrow set

translation-tables :: 'regval trace \Rightarrow nat set

translate-address :: nat \Rightarrow acctype \Rightarrow 'regval trace \Rightarrow nat option

definition *writes-mem-val-at-idx* :: nat \Rightarrow ('a event)list \Rightarrow (nat*nat*(memory-byte)list*bitU)option
where

writes-mem-val-at-idx i t = (Option.bind (index t i) writes-mem-val)
for i :: nat
and t :: ('a event)list

definition *writes-mem-cap-at-idx* :: 'a Capability-class \Rightarrow nat \Rightarrow ('b event)list
 \Rightarrow (nat*nat*'a)option **where**

writes-mem-cap-at-idx dict-Capabilities-Capability-a i t = (Option.bind (index t i)
(writes-mem-cap dict-Capabilities-Capability-a))
for dict-Capabilities-Capability-a :: 'a Capability-class
and i :: nat
and t :: ('b event)list

definition *writes-to-reg-at-idx* :: nat \Rightarrow ('a event)list \Rightarrow (string)option **where**

writes-to-reg-at-idx i t = (Option.bind (index t i) writes-to-reg)
for i :: nat
and t :: ('a event)list

definition *writes-reg-caps-at-idx* :: 'd Capability-class \Rightarrow ('d,'c,'b,'a)isa \Rightarrow nat
 \Rightarrow ('c event)list \Rightarrow 'd set **where**

writes-reg-caps-at-idx dict-Capabilities-Capability-d ISA i t = (case-option {}
(writes-reg-caps
dict-Capabilities-Capability-d(caps-of-regval ISA)) (index t i))
for dict-Capabilities-Capability-d :: 'd Capability-class
and ISA :: ('d,'c,'b,'a)isa
and i :: nat
and t :: ('c event)list

definition *reads-mem-val-at-idx* :: nat \Rightarrow ('a event)list \Rightarrow (nat*nat*(memory-byte)list*bitU)option
where

reads-mem-val-at-idx i t = (Option.bind (index t i) reads-mem-val)
for i :: nat
and t :: ('a event)list

definition *reads-mem-cap-at-idx* :: 'a Capability-class \Rightarrow nat \Rightarrow ('b event)list \Rightarrow (nat*nat*'a)option **where**
reads-mem-cap-at-idx dict-Capabilities-Capability-a i t = (Option.bind (index t i)
 (reads-mem-cap dict-Capabilities-Capability-a))
for dict-Capabilities-Capability-a :: 'a Capability-class
and i :: nat
and t :: ('b event)list

definition *reads-from-reg-at-idx* :: nat \Rightarrow ('a event)list \Rightarrow (string)option **where**
reads-from-reg-at-idx i t = (Option.bind (index t i) reads-from-reg)
for i :: nat
and t :: ('a event)list

definition *reads-reg-caps-at-idx* :: 'd Capability-class \Rightarrow ('d,'c,'b,'a)isa \Rightarrow nat \Rightarrow ('c event)list \Rightarrow 'd set **where**
reads-reg-caps-at-idx dict-Capabilities-Capability-d ISA i t = (case-option { }
 (reads-reg-caps
 dict-Capabilities-Capability-d(caps-of-regval ISA)) (index t i))
for dict-Capabilities-Capability-d :: 'd Capability-class
and ISA :: ('d,'c,'b,'a)isa
and i :: nat
and t :: ('c event)list

— val address-range : nat \rightarrow nat \rightarrow list nat

definition *address-range* :: nat \Rightarrow nat \Rightarrow (nat)list **where**
address-range start len = (genlist (λ n . start + n) len)
for start :: nat
and len :: nat

— val address-tag-aligned : forall 'cap 'regval 'instr 'e. isa 'cap 'regval 'instr 'e
 \rightarrow nat \rightarrow bool

definition *address-tag-aligned* :: ('cap,'regval,'instr,'e)isa \Rightarrow nat \Rightarrow bool **where**
address-tag-aligned ISA addr = (((addr mod(tag-granule ISA)) = (0 ::
 nat)))
for ISA :: ('cap,'regval,'instr,'e)isa
and addr :: nat

— val cap-reg-written-before-idx : forall 'cap 'regval 'instr 'e. Capability 'cap, Eq
 'cap, SetType 'cap \Rightarrow isa 'cap 'regval 'instr 'e \rightarrow nat \rightarrow register-name \rightarrow
 trace 'regval \rightarrow bool

definition *cap-reg-written-before-idx* :: 'cap Capability-class \Rightarrow ('cap,'regval,'instr,'e)isa
 \Rightarrow nat \Rightarrow string \Rightarrow ('regval event)list \Rightarrow bool **where**

$cap_reg_written_before_idx \text{ dict-Capabilities-Capability-cap ISA } i \ r \ t = ((\exists \ j. (j < i) \wedge ((writes_to_reg_at_idx \ j \ t = Some \ r) \wedge \neg (writes_reg_caps_at_idx \ dict-Capabilities-Capability-cap \ ISA \ j \ t = \{\})))))$
for $dict-Capabilities-Capability-cap :: 'cap \ \text{Capability-class}$
and $ISA :: ('cap, 'regval, 'instr, 'e)isa$
and $i :: nat$
and $r :: string$
and $t :: ('regval \ event)list$

— $val \ system_access_permitted_before_idx : forall \ 'cap \ 'regval \ 'instr \ 'e. \ \text{Capability } 'cap, \ SetType \ 'cap, \ Eq \ 'cap \Rightarrow isa \ 'cap \ 'regval \ 'instr \ 'e \rightarrow nat \rightarrow trace \ 'regval \rightarrow bool$

definition $system_access_permitted_before_idx :: 'cap \ \text{Capability-class} \Rightarrow ('cap, 'regval, 'instr, 'e)isa \Rightarrow nat \Rightarrow ('regval \ event)list \Rightarrow bool$ **where**

$system_access_permitted_before_idx \text{ dict-Capabilities-Capability-cap ISA } i \ t = ($
 $((\exists \ j. \exists \ r. \exists \ c. (j < i) \wedge ((reads_from_reg_at_idx \ j \ t = Some \ r) \wedge (\neg (cap_reg_written_before_idx \ dict-Capabilities-Capability-cap \ ISA \ j \ r \ t) \wedge ((c \in (reads_reg_caps_at_idx \ dict-Capabilities-Capability-cap \ ISA \ j \ t)) \wedge ((r \in (PCC \ ISA)) \wedge (r \notin (privileged_regs \ ISA)) \wedge ((is_tagged_method \ dict-Capabilities-Capability-cap) \ c \wedge (\neg ((is_sealed_method \ dict-Capabilities-Capability-cap) \ c) \wedge (permit_system_access \ ((get_perms_method \ dict-Capabilities-Capability-cap) \ c))))))))))$
for $dict-Capabilities-Capability-cap :: 'cap \ \text{Capability-class}$
and $ISA :: ('cap, 'regval, 'instr, 'e)isa$
and $i :: nat$
and $t :: ('regval \ event)list$

— $val \ permits_cap_load : forall \ 'cap. \ \text{Capability } 'cap \Rightarrow 'cap \rightarrow nat \rightarrow nat \rightarrow bool$

definition $permits_cap_load :: 'cap \ \text{Capability-class} \Rightarrow 'cap \Rightarrow nat \Rightarrow nat \Rightarrow bool$ **where**

$permits_cap_load \text{ dict-Capabilities-Capability-cap } c \ vaddr \ sz = ($
 $((is_tagged_method \ dict-Capabilities-Capability-cap) \ c \wedge (\neg ((is_sealed_method \ dict-Capabilities-Capability-cap) \ c) \wedge ((List.set \ (address_range \ vaddr \ sz) \subseteq ($
 $(get_mem_region_method \ dict-Capabilities-Capability-cap) \ c)) \wedge (permit_load_capability \ ((get_perms_method \ dict-Capabilities-Capability-cap) \ c))))))$
for $dict-Capabilities-Capability-cap :: 'cap \ \text{Capability-class}$
and $c :: 'cap$
and $vaddr :: nat$
and $sz :: nat$

— *val available-caps : forall 'cap 'regval 'instr 'e. Capability 'cap, Eq 'cap, SetType 'cap => isa 'cap 'regval 'instr 'e -> nat -> trace 'regval -> set 'cap*
fun available-caps :: 'cap Capability-class =>('cap,'regval,'instr,'e)isa => nat
=>('regval event)list => 'cap set **where**
available-caps dict-Capabilities-Capability-cap ISA 0 t = ({})
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and ISA :: ('cap,'regval,'instr,'e)isa
and t :: ('regval event)list
| available-caps dict-Capabilities-Capability-cap ISA ((Suc i)) t = (
(let caps-of = (λ e .
((case reads-mem-cap dict-Capabilities-Capability-cap e of
Some (_, -, c) => {c}
| None => {}
)) ∪
((case reads-from-reg e of
Some r =>
if (¬
(cap-reg-written-before-idx
dict-Capabilities-Capability-cap ISA i
r t) ∧
(system-access-permitted-before-idx
dict-Capabilities-Capability-cap ISA i
t ∨ ¬ (r ∈ (privileged-regs ISA))))
then
reads-reg-caps dict-Capabilities-Capability-cap
(caps-of-regval ISA) e else {}
| None => {}
))) in
(let new-caps = (case-option {} caps-of (index t i)) in
(available-caps dict-Capabilities-Capability-cap ISA i t) ∪ new-caps)))
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and ISA :: ('cap,'regval,'instr,'e)isa
and i :: nat
and t :: ('regval event)list

1.3 Intra-instruction properties

— *val read-reg-axiom : forall 'cap 'regval 'instr 'e. Capability 'cap, SetType 'cap, Eq 'cap, Eq 'regval => isa 'cap 'regval 'instr 'e -> bool -> trace 'regval -> bool*

definition read-reg-axiom :: 'cap Capability-class =>('cap,'regval,'instr,'e)isa => bool =>('regval event)list => bool **where**
read-reg-axiom dict-Capabilities-Capability-cap ISA has-ex t = (
((∀ i. ∀ r. ∀ v.
((index t i = Some (E-read-reg r v)) ∧ (r ∈ (privileged-regs ISA)))
→
(system-access-permitted-before-idx
dict-Capabilities-Capability-cap ISA i t ∨

```

      (has-ex — && r IN ISA.KCC))))))
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and ISA :: ('cap,'regval,'instr,'e)isa
and has-ex :: bool
and t :: ('regval event)list

— val store-cap-mem-axiom : forall 'cap 'regval 'instr 'e. Capability 'cap, SetType
'cap, Eq 'cap => isa 'cap 'regval 'instr 'e -> trace 'regval -> bool
definition store-cap-mem-axiom :: 'cap Capability-class => ('cap,'regval,'instr,'e)isa
=> ('regval event)list => bool where
  store-cap-mem-axiom dict-Capabilities-Capability-cap ISA t = (
    (( $\forall$  i.  $\forall$  c.  $\forall$  addr.  $\forall$  sz.
      (writes-mem-cap-at-idx
        dict-Capabilities-Capability-cap i t = Some (addr, sz, c))
       $\longrightarrow$ 
      (cap-derivable dict-Capabilities-Capability-cap (available-caps dict-Capabilities-Capability-cap
        ISA i t) c))))
    for dict-Capabilities-Capability-cap :: 'cap Capability-class
    and ISA :: ('cap,'regval,'instr,'e)isa
    and t :: ('regval event)list

— val store-cap-reg-axiom : forall 'cap 'regval 'instr 'e. Capability 'cap, SetType
'cap, SetType 'regval, Eq 'cap, Eq 'regval => isa 'cap 'regval 'instr 'e -> bool ->
bool -> trace 'regval -> bool
definition store-cap-reg-axiom :: 'cap Capability-class => ('cap,'regval,'instr,'e)isa
=> bool => bool => ('regval event)list => bool where
  store-cap-reg-axiom dict-Capabilities-Capability-cap ISA has-ex invokes-caps1
  t = (
    (( $\forall$  i.  $\forall$  c.  $\forall$  r.
      ((writes-to-reg-at-idx i t = Some r)  $\wedge$  (c  $\in$  (writes-reg-caps-at-idx
        dict-Capabilities-Capability-cap ISA i t))))
       $\longrightarrow$ 
      (cap-derivable dict-Capabilities-Capability-cap (available-caps dict-Capabilities-Capability-cap
        ISA i t) c  $\vee$ 
        ((has-ex  $\wedge$ 
          (
            ( $\neg$  exists r' v' j. j < i && index t j = Just (E-read-reg r' v') && c  $\in$  (exception-targets
              ISA) {v'. ( $\exists$  r'.  $\exists$  j. (j < i)  $\wedge$  ((index t j = Some (E-read-reg r' v'))  $\wedge$  (r'
                 $\in$  (KCC ISA)))))})  $\wedge$ 
              (
                ( $\neg$  reads-from-reg-at-idx j t = Just r' && c' IN (reads-reg-caps-at-idx ISA
                  j t) && leq-cap c c' && r' IN (ISA.KCC (take j t)) && r  $\in$  ((PCC ISA) — take
                    i t))))  $\vee$ 
                (( $\exists$  cc.  $\exists$  cd0.
                  invokes-caps1  $\wedge$ 
                  (cap-derivable
                    dict-Capabilities-Capability-cap (available-caps dict-Capabilities-Capability-cap ISA

```

```

i t) cc ∧
  (cap-derivable
  dict-Capabilities-Capability-cap (available-caps dict-Capabilities-Capability-cap ISA
i t) cd0 ∧
  (invokable dict-Capabilities-Capability-cap cc cd0 ∧
  ((leq-cap dict-Capabilities-Capability-cap c (unseal dict-Capabilities-Capability-cap
cc True) ∧ (r ∈ (PCC ISA))) ∨
  (leq-cap dict-Capabilities-Capability-cap c (unseal dict-Capabilities-Capability-cap
cd0 True) ∧ (r ∈ (IDC ISA)))))))))
for dict-Capabilities-Capability-cap :: 'cap Capability-class
and ISA :: ('cap, 'regval, 'instr, 'e) isa
and has-ex :: bool
and invokes-caps1 :: bool
and t :: ('regval event) list

```

— *val load-mem-axiom : forall 'cap 'regval 'instr 'e. Capability 'cap, SetType 'cap, Eq 'cap => isa 'cap 'regval 'instr 'e -> bool -> trace 'regval -> bool*

definition load-mem-axiom :: 'cap Capability-class => ('cap, 'regval, 'instr, 'e) isa => bool => ('regval event) list => bool **where**

```

  load-mem-axiom dict-Capabilities-Capability-cap ISA is-fetch t = (
  ((∀ i. ∀ paddr. ∀ sz. ∀ v. ∀ tag.
  ((reads-mem-val-at-idx i t = Some (paddr, sz, v, tag)) ∧
  ¬ (paddr ∈ ((translation-tables ISA) (List.take i t))))
  →
  ((∃ c'. ∃ vaddr.
  cap-derivable
  dict-Capabilities-Capability-cap (available-caps dict-Capabilities-Capability-cap ISA
i t) c' ∧ (
  (is-tagged-method dict-Capabilities-Capability-cap) c' ∧ (¬ ((is-sealed-method
dict-Capabilities-Capability-cap) c') ∧
  (((translate-address ISA) vaddr (if is-fetch then Fetch else Load) (List.take
i t) = Some paddr) ∧
  ((List.set (address-range vaddr sz) ⊆ (
  (get-mem-region-method dict-Capabilities-Capability-cap) c')) ∧
  ((if is-fetch then (permit-execute (
  (get-perms-method dict-Capabilities-Capability-cap) c')) else (permit-load (
  (get-perms-method dict-Capabilities-Capability-cap) c')))) ∧
  ((is-fetch → (tag = B0)) ∧
  ((tag ≠ B0) → ((permit-load-capability (
  (get-perms-method dict-Capabilities-Capability-cap) c')) ∧ ((sz = (tag-granule
ISA)) ∧ address-tag-aligned ISA paddr))))))))))
  for dict-Capabilities-Capability-cap :: 'cap Capability-class
  and ISA :: ('cap, 'regval, 'instr, 'e) isa
  and is-fetch :: bool
  and t :: ('regval event) list

```

— *val mem-val-is-cap : forall 'cap 'regval 'instr 'e. Capability 'cap, SetType 'cap,*

$Eq \text{ 'cap} \Rightarrow isa \text{ 'cap 'regval 'instr 'e} \rightarrow list \text{ memory-byte} \rightarrow bitU \rightarrow bool$
definition $mem\text{-}val\text{-}is\text{-}cap :: 'cap \text{ Capability-class} \Rightarrow ('cap, 'regval, 'instr, 'e) isa \Rightarrow (memory\text{-}byte)list \Rightarrow bitU \Rightarrow bool$ **where**
 $mem\text{-}val\text{-}is\text{-}cap \text{ dict-Capabilities-Capability-cap} - v \ t = ((\exists \ c. (cap\text{-}of\text{-}mem\text{-}bytes\text{-}method \text{ dict-Capabilities-Capability-cap}) \ v \ t = Some \ (c :: 'cap))))$
for $dict\text{-}Capabilities\text{-}Capability\text{-}cap :: 'cap \text{ Capability-class}$
and $v :: (memory\text{-}byte)list$
and $t :: bitU$

— $val \text{ mem-val-is-local-cap} : forall \text{ 'cap 'regval 'instr 'e. Capability 'cap, SetType 'cap, Eq 'cap} \Rightarrow isa \text{ 'cap 'regval 'instr 'e} \rightarrow list \text{ memory-byte} \rightarrow bitU \rightarrow bool$
definition $mem\text{-}val\text{-}is\text{-}local\text{-}cap :: 'cap \text{ Capability-class} \Rightarrow ('cap, 'regval, 'instr, 'e) isa \Rightarrow (memory\text{-}byte)list \Rightarrow bitU \Rightarrow bool$ **where**
 $mem\text{-}val\text{-}is\text{-}local\text{-}cap \text{ dict-Capabilities-Capability-cap} - v \ t = ((\exists \ c. ((cap\text{-}of\text{-}mem\text{-}bytes\text{-}method \text{ dict-Capabilities-Capability-cap}) \ v \ t = Some \ (c :: 'cap)) \wedge \neg ((get\text{-}global\text{-}method \text{ dict-Capabilities-Capability-cap}) \ c))))$
for $dict\text{-}Capabilities\text{-}Capability\text{-}cap :: 'cap \text{ Capability-class}$
and $v :: (memory\text{-}byte)list$
and $t :: bitU$

— $val \text{ store-tag-axiom} : forall \text{ 'cap 'regval 'instr 'e. Capability 'cap, SetType 'cap, Eq 'cap} \Rightarrow isa \text{ 'cap 'regval 'instr 'e} \rightarrow trace \text{ 'regval} \rightarrow bool$
definition $store\text{-}tag\text{-}axiom :: 'cap \text{ Capability-class} \Rightarrow ('cap, 'regval, 'instr, 'e) isa \Rightarrow ('regval \text{ event})list \Rightarrow bool$ **where**
 $store\text{-}tag\text{-}axiom \text{ dict-Capabilities-Capability-cap} \text{ ISA} \ t = ($
 $((\forall \ i. \forall \ paddr. \forall \ sz. \forall \ v. \forall \ tag.$
 $(writes\text{-}mem\text{-}val\text{-}at\text{-}idx \ i \ t = Some \ (paddr, sz, v, tag))$
 \longrightarrow
 $((List.length \ v = sz) \wedge$
 $((tag = B0) \vee (tag = B1)) \wedge$
 $((tag = B1) \longrightarrow (address\text{-}tag\text{-}aligned \text{ ISA} \ paddr \wedge (sz = (tag\text{-}granule \text{ ISA}))))))$
for $dict\text{-}Capabilities\text{-}Capability\text{-}cap :: 'cap \text{ Capability-class}$
and $\text{ISA} :: ('cap, 'regval, 'instr, 'e) isa$
and $t :: ('regval \text{ event})list$

— $val \text{ store-mem-axiom} : forall \text{ 'cap 'regval 'instr 'e. Capability 'cap, SetType 'cap, Eq 'cap} \Rightarrow isa \text{ 'cap 'regval 'instr 'e} \rightarrow trace \text{ 'regval} \rightarrow bool$
definition $store\text{-}mem\text{-}axiom :: 'cap \text{ Capability-class} \Rightarrow ('cap, 'regval, 'instr, 'e) isa \Rightarrow ('regval \text{ event})list \Rightarrow bool$ **where**
 $store\text{-}mem\text{-}axiom \text{ dict-Capabilities-Capability-cap} \text{ ISA} \ t = ($
 $((\forall \ i. \forall \ paddr. \forall \ sz. \forall \ v. \forall \ tag.$
 $((writes\text{-}mem\text{-}val\text{-}at\text{-}idx \ i \ t = Some \ (paddr, sz, v, tag)) \wedge$
 $\neg (paddr \in ((translation\text{-}tables \text{ ISA}) (List.take \ i \ t))))$

```

      →
      ((∃ c'. ∃ vaddr.
        cap-derivable
        dict-Capabilities-Capability-cap (available-caps dict-Capabilities-Capability-cap ISA
        i t) c' ∧ (
          (is-tagged-method dict-Capabilities-Capability-cap) c' ∧ (¬ ((is-sealed-method
        dict-Capabilities-Capability-cap) c') ∧
          (((translate-address ISA) vaddr Store (List.take i t) = Some paddr) ∧
          ((List.set (address-range vaddr sz) ⊆ (
        (get-mem-region-method dict-Capabilities-Capability-cap) c')) ∧
          ((permit-store (
        (get-perms-method dict-Capabilities-Capability-cap) c')) ∧
          (((mem-val-is-cap
        dict-Capabilities-Capability-cap ISA v tag ∧ (tag = B1)) → (permit-store-capability
        (
          (get-perms-method dict-Capabilities-Capability-cap) c')))) ∧
          ((mem-val-is-local-cap
        dict-Capabilities-Capability-cap ISA v tag ∧ (tag = B1)) → (permit-store-local-capability
        (
          (get-perms-method dict-Capabilities-Capability-cap) c'))))))))))))
      for dict-Capabilities-Capability-cap :: 'cap Capability-class
      and ISA :: ('cap, 'regval, 'instr, 'e) isa
      and t :: ('regval event) list

```

— *val cheri-axioms* : forall 'cap 'regval 'instr 'e. Capability 'cap, SetType 'cap, SetType 'regval, Eq 'cap, Eq 'regval => isa 'cap 'regval 'instr 'e -> bool -> bool -> bool -> trace 'regval -> bool

definition *cheri-axioms* :: 'cap Capability-class => ('cap, 'regval, 'instr, 'e) isa => bool => bool => bool => ('regval event) list => bool **where**

```

  cheri-axioms dict-Capabilities-Capability-cap ISA is-fetch has-ex invokes-caps1
  t = (
    store-cap-mem-axiom
    dict-Capabilities-Capability-cap ISA t ∧
    (store-cap-reg-axiom
    dict-Capabilities-Capability-cap ISA has-ex invokes-caps1 t ∧
    (read-reg-axiom dict-Capabilities-Capability-cap ISA has-ex t ∧
    (load-mem-axiom dict-Capabilities-Capability-cap ISA is-fetch t ∧
    (store-tag-axiom dict-Capabilities-Capability-cap ISA t ∧
    store-mem-axiom dict-Capabilities-Capability-cap ISA t))))))
  for dict-Capabilities-Capability-cap :: 'cap Capability-class
  and ISA :: ('cap, 'regval, 'instr, 'e) isa
  and is-fetch :: bool
  and has-ex :: bool
  and invokes-caps1 :: bool
  and t :: ('regval event) list

```

end

theory *Capabilities-lemmas*


```

imports Capabilities
begin

```

1.4 Helper definitions and lemmas

```

locale Capabilities =
  fixes CC :: 'cap Capability-class
  assumes is-tagged-set-tag[simp]:  $\bigwedge c \text{ tag. is-tagged-method } CC \text{ (set-tag-method } CC \text{ } c \text{ tag)} = \text{tag}$ 
  and is-tagged-set-seal[simp]:  $\bigwedge c \text{ s. is-tagged-method } CC \text{ (set-seal-method } CC \text{ } c \text{ s)} = \text{is-tagged-method } CC \text{ } c$ 
  and is-tagged-set-obj-type[simp]:  $\bigwedge c \text{ t. is-tagged-method } CC \text{ (set-obj-type-method } CC \text{ } c \text{ t)} = \text{is-tagged-method } CC \text{ } c$ 
  and is-tagged-set-perms[simp]:  $\bigwedge c \text{ p. is-tagged-method } CC \text{ (set-perms-method } CC \text{ } c \text{ p)} = \text{is-tagged-method } CC \text{ } c$ 
  and is-tagged-cap-of-mem-bytes[simp]:  $\bigwedge c \text{ bytes tag. cap-of-mem-bytes-method } CC \text{ bytes tag} = \text{Some } c \implies \text{is-tagged-method } CC \text{ } c \longleftrightarrow \text{tag} = B1$ 
begin

inductive-set derivable :: 'cap set  $\Rightarrow$  'cap set for C :: 'cap set where
  Copy:  $c \in C \implies c \in \text{derivable } C$ 
| Restrict:  $c' \in \text{derivable } C \implies \text{leq-cap } CC \text{ } c \text{ } c' \implies c \in \text{derivable } C$ 
| Unseal:
   $\llbracket c' \in \text{derivable } C; c'' \in \text{derivable } C; \text{is-tagged-method } CC \text{ } c'; \text{is-tagged-method } CC \text{ } c'';$ 
   $\neg \text{is-sealed-method } CC \text{ } c''; \text{is-sealed-method } CC \text{ } c'; \text{permit-unseal (get-perms-method } CC \text{ } c'') \rrbracket$ 
   $\implies$ 
   $\text{get-obj-type-method } CC \text{ } c' = \text{get-cursor-method } CC \text{ } c' \rrbracket \implies$ 
   $\text{unseal } CC \text{ } c' \text{ (get-global-method } CC \text{ } c'') \in \text{derivable } C$ 
| Seal:
   $\llbracket c' \in \text{derivable } C; c'' \in \text{derivable } C; \text{is-tagged-method } CC \text{ } c'; \text{is-tagged-method } CC \text{ } c'';$ 
   $\neg \text{is-sealed-method } CC \text{ } c''; \neg \text{is-sealed-method } CC \text{ } c'; \text{permit-seal (get-perms-method } CC \text{ } c'') \rrbracket \implies$ 
   $\text{seal } CC \text{ } c' \text{ (get-cursor-method } CC \text{ } c'') \in \text{derivable } C$ 

lemma leq-cap-refl[simp, intro]:
  leq-cap CC c c
  by (simp add: leq-cap-def)

lemma leq-cap-tag-imp[intro]:
  assumes leq-cap CC c c'
  and is-tagged-method CC c
  shows is-tagged-method CC c'
  using assms
  by (auto simp: leq-cap-def)

lemma derivable-mono:
  assumes C  $\subseteq$  C'

```

shows *derivable* $C \subseteq \text{derivable } C'$
proof
 fix c
 assume $c \in \text{derivable } C$
 then show $c \in \text{derivable } C'$ **using** *assms* **by** *induction* (*auto intro: derivable.intros*)
qed

lemma *cap-derivable-bounded-gteq*:
 assumes $c: \text{cap-derivable-bounded } CC\ n\ C\ c$
 and $m: m \geq n$
 shows *cap-derivable-bounded* $CC\ m\ C\ c$
proof –
 from m obtain k where $m = n + k$ **using** *less-iff-Suc-add*[*of* $n\ m$] **by** *auto*
 also have *cap-derivable-bounded* $CC\ (n + k)\ C\ c$ **using** c **by** (*induction* k) (*auto*)
 finally show ?thesis .
qed

lemma *derivable-refl*: $C \subseteq \text{derivable } C$ **by** (*auto intro: derivable.intros*)

lemma *derivable-union-subseteq-absorb*:
 assumes $C' \subseteq \text{derivable } C$
 shows *derivable* $(C \cup C') = \text{derivable } C$
proof
 show *derivable* $(C \cup C') \subseteq \text{derivable } C$
proof
 fix c
 assume $c \in \text{derivable } (C \cup C')$
 then show $c \in \text{derivable } C$ **using** *assms* **by** *induction* (*auto intro: derivable.intros*)
qed
 show *derivable* $C \subseteq \text{derivable } (C \cup C')$ **by** (*intro derivable-mono*) *auto*
qed

lemma *derivable-minus-subseteq*: *derivable* $(C - C') \subseteq \text{derivable } C$
proof
 fix c
 assume $c \in \text{derivable } (C - C')$
 then show $c \in \text{derivable } C$ **by** *induction* (*auto intro: derivable.intros*)
qed

lemma *cap-derivable-iff-derivable*: *cap-derivable* $CC\ C\ c \longleftrightarrow c \in \text{derivable } C$
proof
 assume *cap-derivable* $CC\ C\ c$
 then obtain n where $c: \text{cap-derivable-bounded } CC\ n\ C\ c$ **by** (*auto simp: cap-derivable-def*)
 then show $c \in \text{derivable } C$
 by (*induction* $CC \equiv CC\ n\ C\ c$ *rule: cap-derivable-bounded.induct*)

```

      (auto intro: derivable.intros)
next
  assume c: c ∈ derivable C
  then have ∃ n. cap-derivable-bounded CC n C c
  proof (induction rule: derivable.induct)
    case (Copy c)
    then have cap-derivable-bounded CC 0 C c by auto
    then show ?case by blast
  next
    case (Restrict c' c)
    then obtain n where cap-derivable-bounded CC n C c' by auto
    then have cap-derivable-bounded CC (Suc n) C c using Restrict.hyps by auto
    then show ?case by blast
  next
    case (Unseal c' c'')
    then obtain n' n''
      where cap-derivable-bounded CC n' C c' and cap-derivable-bounded CC n''
C c''
      by blast
    then have cap-derivable-bounded CC (max n' n'') C c'
      and cap-derivable-bounded CC (max n' n'') C c''
      by (auto intro: cap-derivable-bounded-gteq)
    then have cap-derivable-bounded CC (Suc (max n' n'')) C (unseal CC c'
(get-global-method CC c''))
      using Unseal.hyps
      by auto
    then show ?case by blast
  next
    case (Seal c' c'')
    then obtain n' n''
      where cap-derivable-bounded CC n' C c' and cap-derivable-bounded CC n''
C c''
      by blast
    then have cap-derivable-bounded CC (max n' n'') C c'
      and cap-derivable-bounded CC (max n' n'') C c''
      by (auto intro: cap-derivable-bounded-gteq)
    then have cap-derivable-bounded CC (Suc (max n' n'')) C (seal CC c' (get-cursor-method
CC c''))
      using Seal.hyps
      by auto
    then show ?case by blast
  qed
  then show cap-derivable CC C c by (simp add: cap-derivable-def)
qed
end

end
theory Cheri-axioms-lemmas

```

```

imports Capabilities-lemmas Cheri-axioms
begin

locale Capability-ISA = Capabilities CC
  for CC :: 'cap Capability-class +
  fixes ISA :: ('cap, 'regval, 'instr, 'e) isa

lemma reads-from-reg-at-idx-Some-iff[simp]:
  reads-from-reg-at-idx i t = Some r  $\longleftrightarrow$  reads-from-reg (t ! i) = Some r  $\wedge$  i <
length t
  by (auto simp: reads-from-reg-at-idx-def bind-eq-Some-conv)

lemma reads-from-reg-SomeE[elim!]:
  assumes reads-from-reg e = Some r
  obtains v where e = E-read-reg r v
  using assms
  by (cases e) auto

lemma reads-from-reg-Some-iff:
  reads-from-reg e = Some r  $\longleftrightarrow$  ( $\exists$  v. e = E-read-reg r v)
  by (cases e) auto

lemma member-reads-reg-caps-at-idx-iff[simp]:
  c  $\in$  reads-reg-caps-at-idx CC ISA i t  $\longleftrightarrow$ 
  c  $\in$  reads-reg-caps CC (caps-of-regval ISA) (t ! i)  $\wedge$  i < length t
  by (auto simp: reads-reg-caps-at-idx-def split: option.splits)

lemma member-reads-reg-caps-iff:
  c  $\in$  reads-reg-caps CC c-of-r e  $\longleftrightarrow$ 
  ( $\exists$  r v. e = E-read-reg r v  $\wedge$  c  $\in$  c-of-r v  $\wedge$  is-tagged-method CC c)
  by (cases e) auto

lemma member-reads-reg-capsE[elim!]:
  assumes c  $\in$  reads-reg-caps CC c-of-r e
  obtains r v where e = E-read-reg r v and c  $\in$  c-of-r v and is-tagged-method
CC c
  using assms
  by (auto simp: member-reads-reg-caps-iff)

lemma reads-reg-caps-Some-reads-mem-cap-None[simp]:
  assumes c  $\in$  reads-reg-caps CC cor e
  shows reads-mem-cap CC e = None
  using assms by (cases e) (auto simp: reads-mem-cap-def)

lemma writes-to-reg-at-idx-Some-iff[simp]:
  writes-to-reg-at-idx i t = Some r  $\longleftrightarrow$  writes-to-reg (t ! i) = Some r  $\wedge$  i < length
t
  by (auto simp: writes-to-reg-at-idx-def bind-eq-Some-conv)

```

lemma *writes-to-reg-SomeE*[*elim!*]:
assumes *writes-to-reg e = Some r*
obtains *v* **where** *e = E-write-reg r v*
using *assms*
by (*cases e*) *auto*

lemma *writes-to-reg-Some-iff*:
writes-to-reg e = Some r \longleftrightarrow $(\exists v. e = E\text{-write-reg } r \ v)$
by (*cases e*) *auto*

lemma *member-writes-reg-caps-at-idx-iff*[*simp*]:
 $c \in \text{writes-reg-caps-at-idx } CC \ ISA \ i \ t \longleftrightarrow$
 $c \in \text{writes-reg-caps } CC \ (\text{caps-of-regval } ISA) \ (t \ ! \ i) \wedge i < \text{length } t$
by (*auto simp: writes-reg-caps-at-idx-def split: option.splits*)

lemma *member-writes-reg-capsE*[*elim!*]:
assumes $c \in \text{writes-reg-caps } CC \ c\text{-of-}r \ e$
obtains *r v* **where** $e = E\text{-write-reg } r \ v$ **and** $c \in c\text{-of-}r \ v$ **and** *is-tagged-method*
 $CC \ c$
using *assms*
by (*cases e*) *auto*

lemma *writes-mem-cap-at-idx-Some-iff*[*simp*]:
 $\text{writes-mem-cap-at-idx } CC \ i \ t = \text{Some } (addr, sz, c) \longleftrightarrow$
 $\text{writes-mem-cap } CC \ (t \ ! \ i) = \text{Some } (addr, sz, c) \wedge i < \text{length } t$
by (*auto simp: writes-mem-cap-at-idx-def bind-eq-Some-conv*)

lemma *reads-mem-cap-at-idx-Some-iff*[*simp*]:
 $\text{reads-mem-cap-at-idx } CC \ i \ t = \text{Some } (addr, sz, c) \longleftrightarrow$
 $\text{reads-mem-cap } CC \ (t \ ! \ i) = \text{Some } (addr, sz, c) \wedge i < \text{length } t$
by (*auto simp: reads-mem-cap-at-idx-def bind-eq-Some-conv*)

lemma *nth-append-left*:
assumes $i < \text{length } xs$
shows $(xs \ @ \ ys) \ ! \ i = xs \ ! \ i$
using *assms* **by** (*auto simp: nth-append*)

context *Capability-ISA*
begin

lemma *writes-mem-cap-SomeE*[*elim!*]:
assumes $\text{writes-mem-cap } CC \ e = \text{Some } (addr, sz, c)$
obtains *wk bytes r* **where** $e = E\text{-write-memt } wk \ addr \ sz \ \text{bytes } B1 \ r$ **and**
 $\text{cap-of-mem-bytes-method } CC \ \text{bytes } B1 = \text{Some } c$ **and** *is-tagged-method* $CC \ c$
using *assms*
by (*cases e*) (*auto simp: writes-mem-cap-def bind-eq-Some-conv split: if-splits*)

lemma *writes-mem-cap-Some-iff*:
 $\text{writes-mem-cap } CC \ e = \text{Some } (addr, sz, c) \longleftrightarrow$

(\exists *wk* bytes *r*. *e* = *E-write-memt* *wk* *addr* *sz* bytes *B1* *r* \wedge *cap-of-mem-bytes-method* *CC* bytes *B1* = *Some* *c* \wedge *is-tagged-method* *CC* *c*)

by (*cases* *e*) (*auto simp: writes-mem-cap-def bind-eq-Some-conv*)

lemma *reads-mem-cap-SomeE*[*elim!*]:

assumes *reads-mem-cap* *CC* *e* = *Some* (*addr*, *sz*, *c*)

obtains *wk* bytes *r* **where** *e* = *E-read-memt* *wk* *addr* *sz* (bytes, *B1*) **and**

cap-of-mem-bytes-method *CC* bytes *B1* = *Some* *c* **and** *is-tagged-method* *CC* *c*

using *assms*

by (*cases* *e*) (*auto simp: reads-mem-cap-def bind-eq-Some-conv split: if-splits*)

lemma *reads-mem-cap-Some-iff*:

reads-mem-cap *CC* *e* = *Some* (*addr*, *sz*, *c*) \longleftrightarrow

(\exists *wk* bytes. *e* = *E-read-memt* *wk* *addr* *sz* (bytes, *B1*) \wedge *cap-of-mem-bytes-method* *CC* bytes *B1* = *Some* *c* \wedge *is-tagged-method* *CC* *c*)

by (*cases* *e*; *fastforce simp: reads-mem-cap-def bind-eq-Some-conv*)

lemma *available-caps-cases*:

assumes *c* \in *available-caps* *CC* *ISA* *i* *t*

obtains (*Reg*) *r* *v* *j* **where** *t* ! *j* = *E-read-reg* *r* *v*

and *c* \in *caps-of-regval* *ISA* *v*

and \neg *cap-reg-written-before-idx* *CC* *ISA* *j* *r* *t*

and *r* \in *privileged-regs* *ISA* \longrightarrow *system-access-permitted-before-idx* *CC* *ISA* *j* *t*

and *j* < *i* **and** *j* < *length* *t* **and** *is-tagged-method* *CC* *c*

| (*Mem*) *wk* *paddr* bytes *j* *sz* **where** *t* ! *j* = *E-read-memt* *wk* *paddr* *sz* (bytes, *B1*)

and *cap-of-mem-bytes-method* *CC* bytes *B1* = *Some* *c*

and *j* < *i* **and** *j* < *length* *t* **and** *is-tagged-method* *CC* *c*

using *assms*

by (*induction* *i*) (*auto split: option.splits if-splits*)

lemma *cap-reg-written-before-idx-0-False*[*simp*]:

cap-reg-written-before-idx *CC* *ISA* 0 *r* *t* \longleftrightarrow *False*

by (*auto simp: cap-reg-written-before-idx-def*)

lemma *cap-reg-written-before-idx-Suc-iff*[*simp*]:

cap-reg-written-before-idx *CC* *ISA* (*Suc* *i*) *r* *t* \longleftrightarrow

(*cap-reg-written-before-idx* *CC* *ISA* *i* *r* *t* \vee

(\exists *v* *c*. *i* < *length* *t* \wedge *t* ! *i* = *E-write-reg* *r* *v* \wedge *c* \in *caps-of-regval* *ISA* *v* \wedge *is-tagged-method* *CC* *c*))

by (*fastforce simp: cap-reg-written-before-idx-def less-Suc-eq*)

definition *accessible-regs-at-idx* :: *nat* \Rightarrow '*regval* *trace* \Rightarrow *register-name* *set* **where**

accessible-regs-at-idx *i* *t* =

{*r*. \neg *cap-reg-written-before-idx* *CC* *ISA* *i* *r* *t* \wedge

(*r* \in *privileged-regs* *ISA* \longrightarrow *system-access-permitted-before-idx* *CC* *ISA* *i*

t)}}

fun *accessed-reg-caps* :: *register-name* *set* \Rightarrow '*regval* *event* \Rightarrow '*cap* *set* **where**

accessed-reg-caps *regs* (*E-read-reg* *r* *v*) =

$\{c. r \in \text{regs} \wedge c \in \text{caps-of-regval ISA } v \wedge \text{is-tagged-method CC } c\}$
 $| \text{accessed-reg-caps regs} - = \{\}$

lemma *member-accessed-reg-capsE*[*elim!*]:
assumes $c \in \text{accessed-reg-caps regs } e$
obtains $r \ v$ **where** $e = E\text{-read-reg } r \ v$ **and** $r \in \text{regs}$
and $c \in \text{caps-of-regval ISA } v$ **and** $\text{is-tagged-method CC } c$
using *assms*
by (*cases e*) *auto*

fun *accessed-mem-caps* :: *'regval event* \Rightarrow *'cap set* **where**
accessed-mem-caps ($E\text{-read-memt rk } a \ sz \ val$) =
 (*case cap-of-mem-bytes-method CC* (*fst val*) (*snd val*) of
Some c \Rightarrow *if is-tagged-method CC c then* $\{c\}$ *else* $\{\}$
| None $\Rightarrow \{\}$)
 $| \text{accessed-mem-caps} - = \{\}$

lemma *member-accessed-mem-capsE*[*elim!*]:
assumes $c \in \text{accessed-mem-caps } e$
obtains $rk \ a \ sz \ \text{bytes} \ \text{tag}$ **where** $e = E\text{-read-memt rk } a \ sz \ (\text{bytes}, \text{tag})$
and *cap-of-mem-bytes-method CC bytes tag* = *Some c* **and** $\text{is-tagged-method CC } c$
using *assms*
by (*cases e*) (*auto split: option.splits if-splits*)

fun *allows-system-reg-access* :: *register-name set* \Rightarrow *'regval event* \Rightarrow *bool* **where**
allows-system-reg-access accessible-regs ($E\text{-read-reg } r \ v$) =
 ($\exists c \in \text{caps-of-regval ISA } v.$
 $\text{is-tagged-method CC } c \wedge \neg \text{is-sealed-method CC } c \wedge$
 $\text{permit-system-access (get-perms-method CC } c) \wedge$
 $r \in \text{PCC ISA} \cap \text{accessible-regs}$)
 $| \text{allows-system-reg-access accessible-regs} - = \text{False}$

lemma *system-access-permitted-before-idx-0*[*simp*]:
 $\text{system-access-permitted-before-idx CC ISA } 0 \ t = \text{False}$
by (*auto simp: system-access-permitted-before-idx-def*)

lemma *system-access-permitted-before-idx-Suc*[*simp*]:
 $\text{system-access-permitted-before-idx CC ISA (Suc } i) \ t \longleftrightarrow$
 $(\text{system-access-permitted-before-idx CC ISA } i \ t \vee$
 $(i < \text{length } t \wedge \text{allows-system-reg-access (accessible-regs-at-idx } i \ t) (t ! i)))$
by (*fastforce simp: system-access-permitted-before-idx-def accessible-regs-at-idx-def*
less-Suc-eq
 $\text{elim!: allows-system-reg-access.elims}$)

lemma *accessible-regs-at-idx-0*[*simp*]:
 $\text{accessible-regs-at-idx } 0 \ t = (\neg \text{privileged-regs ISA})$
by (*auto simp: accessible-regs-at-idx-def*)

```

lemma accessible-regs-at-idx-Suc:
  accessible-regs-at-idx (Suc i) t =
    (accessible-regs-at-idx i t  $\cup$ 
      (if i < length t  $\wedge$  allows-system-reg-access (accessible-regs-at-idx i t) (t ! i)
        then {r  $\in$  privileged-regs ISA.  $\neg$ cap-reg-written-before-idx CC ISA i r t} else
          {})) -
      {r.  $\exists c v. i < \text{length } t \wedge t ! i = E\text{-write-reg } r v \wedge c \in \text{caps-of-regval } ISA v \wedge$ 
        is-tagged-method CC c}
  by (auto simp: accessible-regs-at-idx-def)

declare available-caps.simps[simp del]

lemma reads-from-reg-None-reads-reg-caps-empty[simp]:
  reads-from-reg e = None  $\implies$  reads-reg-caps CC cor e = {}
  by (cases e) auto

lemma available-caps-0[simp]: available-caps CC ISA 0 t = {}
  by (auto simp: available-caps.simps)

lemma available-caps-Suc:
  available-caps CC ISA (Suc i) t =
    available-caps CC ISA i t  $\cup$ 
    (if i < length t
      then accessed-mem-caps (t ! i)  $\cup$ 
        accessed-reg-caps (accessible-regs-at-idx i t) (t ! i)
      else {})
  by (cases t ! i)
    (auto simp: available-caps.simps accessible-regs-at-idx-def reads-mem-cap-def
      bind-eq-Some-conv split: option.splits)

abbreviation instr-sem-ISA ( $\llbracket - \rrbracket$ ) where  $\llbracket instr \rrbracket \equiv instr\text{-sem } ISA \text{ instr}$ 

end

end
theory Properties
imports Cheri-axioms-lemmas Sail.Sail2-state-lemmas
begin

locale CHERI-ISA = Capability-ISA CC ISA
  for CC :: 'cap Capability-class and ISA :: ('cap, 'regval, 'instr, 'e) isa +
  fixes fetch-assms :: 'regval trace  $\Rightarrow$  bool and instr-assms :: 'regval trace  $\Rightarrow$  bool
  assumes instr-cheri-axioms:  $\bigwedge t \text{ instr}. hasTrace \text{ } t \llbracket instr \rrbracket \implies instr\text{-assms } t \implies$ 
cheri-axioms CC ISA False (instr-raises-ex ISA instr t) (invokes-caps ISA instr t)
t
  and fetch-cheri-axioms:  $\bigwedge t. hasTrace \text{ } t (instr\text{-fetch } ISA) \implies fetch\text{-assms } t \implies$ 

```


cheri-axioms $CC\ ISA\ True\ (fetch\text{-}raises\text{-}ex\ ISA\ t)\ False\ t$
and *instr-assms-appendE*: $\bigwedge t\ t'\ instr.\ instr\text{-}assms\ (t\ @\ t') \implies Run\ \llbracket instr \rrbracket\ t$
 $() \implies instr\text{-}assms\ t \wedge fetch\text{-}assms\ t'$
and *fetch-assms-appendE*: $\bigwedge t\ t'\ instr.\ fetch\text{-}assms\ (t\ @\ t') \implies Run\ (instr\text{-}fetch\ ISA)\ t\ instr \implies fetch\text{-}assms\ t \wedge instr\text{-}assms\ t'$

locale *Register-Accessors* =
fixes *read-regval* :: *register-name* \Rightarrow *'regs* \Rightarrow *'regval option*
and *write-regval* :: *register-name* \Rightarrow *'regval* \Rightarrow *'regs* \Rightarrow *'regs option*
begin

abbreviation *s-emit-event* $e\ s \equiv emitEventS\ (read\text{-}regval,\ write\text{-}regval)\ e\ s$
abbreviation *s-run-trace* $t\ s \equiv runTraceS\ (read\text{-}regval,\ write\text{-}regval)\ t\ s$
abbreviation *s-allows-trace* $t\ s \equiv \exists s'.\ s\text{-run-trace}\ t\ s = Some\ s'$

end

locale *CHERI-ISA-State* = *CHERI-ISA* $CC\ ISA + Register\text{-}Accessors\ read\text{-}regval\ write\text{-}regval$
for *ISA* :: (*'cap*, *'regval*, *'instr*, *'e*) *isa*
and *CC* :: *'cap* *Capability-class*
and *read-regval* :: *register-name* \Rightarrow *'regs* \Rightarrow *'regval option*
and *write-regval* :: *register-name* \Rightarrow *'regval* \Rightarrow *'regs* \Rightarrow *'regs option* +

fixes *s-translation-tables* :: *'regs sequential-state* \Rightarrow *nat set*
and *s-translate-address* :: *nat* \Rightarrow *acctype* \Rightarrow *'regs sequential-state* \Rightarrow *nat option*
assumes *read-absorb-write*: $\bigwedge r\ v\ s\ s'.\ write\text{-}regval\ r\ v\ s = Some\ s' \implies read\text{-}regval\ r\ s' = Some\ v$
and *read-ignore-write*: $\bigwedge r\ r'\ v\ s\ s'.\ write\text{-}regval\ r\ v\ s = Some\ s' \implies r' \neq r \implies read\text{-}regval\ r'\ s' = read\text{-}regval\ r'\ s$
and *translation-tables-sound*: $\bigwedge t\ s.\ s\text{-allows-trace}\ t\ s \implies translation\text{-}tables\ ISA\ t \subseteq s\text{-translation-tables}\ s$
and *translate-address-sound*: $\bigwedge t\ s\ vaddr\ paddr\ load.\ s\text{-allows-trace}\ t\ s \implies$
 $translate\text{-}address\ ISA\ vaddr\ load\ t = Some\ paddr \implies$
 $s\text{-translate-address}\ vaddr\ load\ s = Some\ paddr$
and *translate-address-tag-aligned-iff*: $\bigwedge s\ vaddr\ paddr\ load.\ s\text{-translate-address}\ vaddr\ load\ s = Some\ paddr \implies$
 $address\text{-}tag\text{-aligned}\ ISA\ paddr \longleftrightarrow address\text{-}tag\text{-aligned}\ ISA\ vaddr$
begin

1.5 Reachable capabilities

fun *get-reg-val* :: *register-name* \Rightarrow *'regs sequential-state* \Rightarrow *'regval option* **where**
get-reg-val $r\ s = read\text{-}regval\ r\ (regstate\ s)$

fun *put-reg-val* :: *register-name* \Rightarrow *'regval* \Rightarrow *'regs sequential-state* \Rightarrow *'regs sequential-state option* **where**
put-reg-val $r\ v\ s = map\text{-}option\ (\lambda rs'.\ s\llbracket regstate := rs' \rrbracket) (write\text{-}regval\ r\ v\ (regstate\ s))$

s))

fun *get-reg-caps* :: *register-name* \Rightarrow '*regs sequential-state* \Rightarrow '*cap set* **where**
get-reg-caps *r s* = (case *read-regval* *r* (*regstate s*) of *Some v* \Rightarrow {*c* \in *caps-of-regval* *ISA v*. *is-tagged-method* *CC c*} | *None* \Rightarrow {})

fun *get-mem-cap* :: *nat* \Rightarrow *nat* \Rightarrow '*regs sequential-state* \Rightarrow '*cap option* **where**
get-mem-cap *addr sz s* =
Option.bind (*get-mem-bytes* *addr sz s*) (λ (*bytes, tag*).
Option.bind (*cap-of-mem-bytes-method* *CC bytes tag*) (λ *c*.
if is-tagged-method *CC c* then *Some c* else *None*))

fun *get-aligned-mem-cap* :: *nat* \Rightarrow *nat* \Rightarrow '*regs sequential-state* \Rightarrow '*cap option* **where**
get-aligned-mem-cap *vaddr sz s* =
(*if address-tag-aligned* *ISA vaddr* \wedge *sz* = *tag-granule* *ISA* then *get-mem-cap* *vaddr sz s* else *None*)

inductive-set *reachable-caps* :: '*regs sequential-state* \Rightarrow '*cap set* **for** *s* :: '*regs sequential-state* **where**

Reg: [*c* \in *get-reg-caps* *r s*; *r* \notin *privileged-regs* *ISA*; *is-tagged-method* *CC c*] \Longrightarrow *c* \in *reachable-caps s*

| *SysReg*:

[*c* \in *get-reg-caps* *r s*; *r* \in *privileged-regs* *ISA*; *c'* \in *reachable-caps s*;
permit-system-access (*get-perms-method* *CC c'*); \neg *is-sealed-method* *CC c'*;
is-tagged-method *CC c*]
 \Longrightarrow *c* \in *reachable-caps s*

| *Mem*:

[*get-aligned-mem-cap* *addr* (*tag-granule* *ISA*) *s* = *Some c*;
s-translate-address *vaddr* *Load s* = *Some addr*;
c' \in *reachable-caps s*; *is-tagged-method* *CC c'*; \neg *is-sealed-method* *CC c'*;
set (*address-range* *vaddr* (*tag-granule* *ISA*)) \subseteq *get-mem-region-method* *CC c'*;
permit-load-capability (*get-perms-method* *CC c'*);
is-tagged-method *CC c*]
 \Longrightarrow *c* \in *reachable-caps s*

| *Restrict*: [*c* \in *reachable-caps s*; *leq-cap* *CC c' c*] \Longrightarrow *c'* \in *reachable-caps s*

| *Seal*:

[*c'* \in *reachable-caps s*; *c''* \in *reachable-caps s*; *is-tagged-method* *CC c'*; *is-tagged-method* *CC c''*;
 \neg *is-sealed-method* *CC c''*; \neg *is-sealed-method* *CC c'*; *permit-seal* (*get-perms-method* *CC c''*)]
 \Longrightarrow
seal *CC c'* (*get-cursor-method* *CC c''*) \in *reachable-caps s*

| *Unseal*:

[*c'* \in *reachable-caps s*; *c''* \in *reachable-caps s*; *is-tagged-method* *CC c'*; *is-tagged-method* *CC c''*;
 \neg *is-sealed-method* *CC c''*; *is-sealed-method* *CC c'*; *permit-unseal* (*get-perms-method* *CC c''*)];
get-obj-type-method *CC c'* = *get-cursor-method* *CC c''*] \Longrightarrow
unseal *CC c'* (*get-global* *CC c''*) \in *reachable-caps s*

lemma *derivable-subseteq-reachableI*:
 assumes $C \subseteq \text{reachable-caps } s$
 shows *derivable* $C \subseteq \text{reachable-caps } s$
proof
 fix c
 assume $c \in \text{derivable } C$
 then show $c \in \text{reachable-caps } s$ **using** *assms*
 by *induction* (*auto intro: reachable-caps.intros*)
qed

lemma *derivable-subseteq-reachableE*:
 assumes *derivable* $C \subseteq \text{reachable-caps } s$
 shows $C \subseteq \text{reachable-caps } s$
using *assms* **by** (*auto intro: derivable.intros*)

lemma *derivable-reachable-caps-idem[simp]*: *derivable* ($\text{reachable-caps } s$) = $\text{reachable-caps } s$
using *derivable-subseteq-reachableI* [*of reachable-caps s s*] *derivable-refl*
by *auto*

lemma *runTraceS-rev-induct* [*consumes 1, case-names Init Step*]:
 assumes $s\text{-run-trace } t \ s = \text{Some } s'$
 and *Init*: $P \ [] \ s$
 and *Step*: $\bigwedge t \ e \ s'' \ s'. \ s\text{-run-trace } t \ s = \text{Some } s'' \implies s\text{-emit-event } e \ s'' = \text{Some } s' \implies P \ t \ s'' \implies P \ (t \ @ \ [e]) \ s'$
 shows $P \ t \ s'$
using *assms*
by (*induction t arbitrary: s' rule: rev-induct*)
 (*auto elim: runTraceS-appendE runTraceS-ConsE simp: bind-eq-Some-conv*)

lemma *get-reg-val-s-run-trace-cases*:
 assumes $v: \text{get-reg-val } r \ s' = \text{Some } v$ **and** $c: c \in \text{caps-of-regval ISA } v$
 and $s': s\text{-run-trace } t \ s = \text{Some } s'$
 obtains (*Init*) $\text{get-reg-val } r \ s = \text{Some } v$
 | (*Update*) $j \ v' \text{ where } t ! j = E\text{-write-reg } r \ v' \text{ and } c \in \text{caps-of-regval ISA } v' \text{ and } j < \text{length } t$
proof (*use s' v c in induction rule: runTraceS-rev-induct*)
 case (*Step* $t \ e \ s'' \ s'$)
 note *Init* = *Step*(4)
 note *Update* = *Step*(5)
 note $c = \langle c \in \text{caps-of-regval ISA } v \rangle$
 show ?*case*
proof *cases*
 assume $v\text{-s}'': \text{get-reg-val } r \ s'' = \text{Some } v$
 show ?*thesis*
proof (*rule Step.IH[OF - - v-s'' c]*)
 assume $\text{get-reg-val } r \ s = \text{Some } v$
 then show *thesis* **by** (*intro Init*)

```

next
  fix j v'
  assume t ! j = E-write-reg r v' and c ∈ caps-of-regval ISA v' and j < length
t
  then show thesis by (intro Update[of j v']) (auto simp: nth-append-left)
qed
next
  assume v-s'': get-reg-val r s'' ≠ Some v
  note e = ⟨s-emit-event e s'' = Some s'⟩
  note v-s' = ⟨get-reg-val r s' = Some v⟩
  from e v-s' v-s'' have e = E-write-reg r v
  proof (cases rule: emitEventS-update-cases)
    case (Write-reg r' v' rs')
    then show ?thesis
    using v-s' v-s''
    by (cases r' = r) (auto simp: read-ignore-write read-absorb-write)
  qed (auto simp: put-mem-bytes-def Let-def)
  then show thesis using c by (auto intro: Update[of length t v])
qed
qed auto

lemma reads-reg-cap-at-idx-provenance[consumes 5]:
  assumes r: t ! i = E-read-reg r v and c: c ∈ caps-of-regval ISA v and tag:
is-tagged-method CC c
  and s': s-run-trace t s = Some s' and i: i < length t
  obtains (Initial) c ∈ get-reg-caps r s
  | (Update) j where c ∈ writes-reg-caps CC (caps-of-regval ISA) (t ! j)
  and writes-to-reg (t ! j) = Some r and j < i
proof -
  from s' i obtain s1 s2
  where s1: s-run-trace (take i t) s = Some s1
  and s2: s-emit-event (t ! i) s1 = Some s2
  by (blast elim: runTraceS-nth-split)
  from s2 c r tag have c ∈ get-reg-caps r s1
  by (auto simp: bind-eq-Some-conv split: option.splits if-splits)
  with s1 Update show thesis using i
  proof (induction take i t s1 arbitrary: i t rule: runTraceS-rev-induct)
    case Init
    then show ?case by (intro Initial)
  next
    case (Step t' e s'' s' i t)
    then obtain j where j: i = Suc j by (cases i) auto
    then have t': t' = take j t and e: e = t ! j
    using Step by (auto simp: take-hd-drop[symmetric] hd-drop-conv-nth)
    note IH = Step(3)[of j t]
    note Update = Step(5)
    note i = ⟨i < length t⟩
    show ?case
  proof (use ⟨s-emit-event e s'' = Some s'⟩ in ⟨cases rule: emitEventS-update-cases⟩)

```

```

    case (Write-mem wk addr sz v tag res)
    then have c: c ∈ get-reg-caps r s''
      using Step
      by (auto simp: put-mem-bytes-def bind-eq-Some-conv Let-def)
    show ?thesis
      by (rule IH) (use c t' i j Update in ⟨auto⟩)
  next
    case (Write-reg r' v rs')
    show ?thesis
    proof cases
      assume r' = r
      then show ?thesis
        using Write-reg e j ⟨c ∈ get-reg-caps r s'⟩
        by (intro Update[of j]) (auto simp: read-absorb-write)
    next
      assume r' ≠ r
      show ?thesis
        by (rule IH)
          (use ⟨r' ≠ r⟩ Write-reg e t' i j ⟨c ∈ get-reg-caps r s'⟩ Update in
            ⟨auto simp: read-ignore-write⟩)
    qed
  next
    case Read
    show ?thesis
      by (rule IH) (use Read Step.premis t' i j Update in ⟨auto⟩)
    qed
  qed
qed

```

lemma *reads-reg-cap-at-idx-from-initial*:

assumes $r: t \vdash i = E\text{-read-reg } r \ v$ and $c: c \in \text{caps-of-regval } ISA \ v$ and tag :

is-tagged-method $CC \ c$

and $s': s\text{-run-trace } t \ s = \text{Some } s'$ and $i: i < \text{length } t$

and $\neg \text{cap-reg-written-before-idx } CC \ ISA \ i \ r \ t$

shows $c \in \text{get-reg-caps } r \ s$

using *assms*

by (*elim reads-reg-cap-at-idx-provenance*)

(*auto simp: cap-reg-written-before-idx-def writes-reg-caps-at-idx-def*)

1.6 Capability monotonicity

lemma *available-caps-mono*:

assumes $j: j < i$

shows $\text{available-caps } CC \ ISA \ j \ t \subseteq \text{available-caps } CC \ ISA \ i \ t$

proof —

have $\text{available-caps } CC \ ISA \ j \ t \subseteq \text{available-caps } CC \ ISA \ (\text{Suc } (j + k)) \ t$ for k

by (*induction k*) (*auto simp: available-caps-Suc image-iff subset-iff*)

then show ?thesis using *assms less-iff-Suc-add*[of $j \ i$] by *blast*

qed

lemma *reads-reg-cap-non-privileged-accessible*[intro]:
assumes $c \in \text{caps-of-regval } ISA \ v$ **and** $t ! j = E\text{-read-reg } r \ v$
and $\neg \text{cap-reg-written-before-idx } CC \ ISA \ j \ r \ t$
and $r \notin \text{privileged-regs } ISA$
and $\text{is-tagged-method } CC \ c$
and $j < i$
and $j < \text{length } t$
shows $c \in \text{available-caps } CC \ ISA \ i \ t$
proof –
from *assms* **have** $c : c \in \text{available-caps } CC \ ISA \ (Suc \ j) \ t$
by (*auto simp: bind-eq-Some-conv image-iff available-caps.simps*)
consider $i = Suc \ j \mid Suc \ j < i$ **using** ($j < i$)
by (*cases i = Suc j*) *auto*
then show $c \in \text{available-caps } CC \ ISA \ i \ t$
using $c \in \text{available-caps-mono}[of \ Suc \ j \ i \ t]$
by cases auto
qed

lemma *system-access-permitted-at-idx-available-caps*:
assumes $\text{system-access-permitted-before-idx } CC \ ISA \ i \ t$
obtains c **where** $c \in \text{available-caps } CC \ ISA \ i \ t$ **and** $\text{is-tagged-method } CC \ c$
and $\neg \text{is-sealed-method } CC \ c$ **and** $\text{permit-system-access } (\text{get-perms-method } CC \ c)$
using *assms*
by (*auto simp: system-access-permitted-before-idx-def; blast*)

lemma *writes-reg-cap-nth-provenance*[consumes 4]:
assumes $t ! i = E\text{-write-reg } r \ v$ **and** $c \in \text{caps-of-regval } ISA \ v$
and $\text{cheri-axioms } CC \ ISA \ \text{is-fetch has-ex inv-caps } t$
and $i < \text{length } t$
and $\text{tagged: is-tagged-method } CC \ c$
obtains $(\text{Accessible}) \ c \in \text{derivable } (\text{available-caps } CC \ ISA \ i \ t)$
 $\mid (\text{Exception}) \ v' \ r' \ j$ **where** $c \in \text{exception-targets } ISA \ \{v. \exists r \ j. j < i \wedge j < \text{length } t \wedge t ! j = E\text{-read-reg } r \ v \wedge r \in KCC \ ISA\}$

and $r \in PCC \ ISA$ **and** has-ex
 $\mid (CCall) \ cc \ cd \ c'$ **where** inv-caps **and** $\text{invokable } CC \ cc \ cd$
and $cc \in \text{derivable } (\text{available-caps } CC \ ISA \ i \ t)$
and $cd \in \text{derivable } (\text{available-caps } CC \ ISA \ i \ t)$
and $(r \in PCC \ ISA \wedge \text{leq-cap } CC \ c \ (\text{unseal } CC \ cc \ \text{True})) \vee$
 $(r \in IDC \ ISA \wedge \text{leq-cap } CC \ c \ (\text{unseal } CC \ cd \ \text{True}))$

using *assms*

unfolding *cheri-axioms-def store-cap-reg-axiom-def writes-reg-caps-at-idx-def cap-derivable-iff-derivable*
by (*elim impE conjE allE[where x = i] allE[where x = c]*)
(auto simp: eq-commute[where b = t ! j for t j])

lemma *get-mem-cap-run-trace-cases*:
assumes $c : \text{get-mem-cap addr } (\text{tag-granule } ISA) \ s' = \text{Some } c$

```

    and s': s-run-trace t s = Some s'
    and tagged: is-tagged-method CC c
    and aligned: address-tag-aligned ISA addr
    and axiom: store-tag-axiom CC ISA t
  obtains (Initial) get-mem-cap addr (tag-granule ISA) s = Some c
  | (Update) k wk bytes r where k < length t
    and t ! k = E-write-memt wk addr (tag-granule ISA) bytes B1 r
    and cap-of-mem-bytes-method CC bytes B1 = Some c
proof (use s' c axiom in (induction rule: runTraceS-rev-induct))
  case (Step t e s'' s')
  note Update = Step.premis(2)
  have axiom: store-tag-axiom CC ISA t
    using (store-tag-axiom CC ISA (t @ [e]))
  by (auto simp: store-tag-axiom-def writes-mem-val-at-idx-def nth-append bind-eq-Some-conv
split: if-splits; metis less-SucI)
  have IH: thesis if get-mem-cap addr (tag-granule ISA) s'' = Some c
  proof (rule Step.IH[OF - - that axiom])
    assume get-mem-cap addr (tag-granule ISA) s = Some c
    then show thesis by (rule Initial)
  next
  fix k wk bytes r
  assume k < length t and t ! k = E-write-memt wk addr (tag-granule ISA)
bytes B1 r
    and cap-of-mem-bytes-method CC bytes B1 = Some c
  then show thesis
    by (intro Update[of k wk bytes r]) (auto simp: nth-append)
qed
obtain v tag
  where v: get-mem-bytes addr (tag-granule ISA) s' = Some (v, tag)
  and cv: cap-of-mem-bytes-method CC v tag = Some c
  using (get-mem-cap addr (tag-granule ISA) s' = Some c)
  by (auto simp: bind-eq-Some-conv bool-of-bitU-def split: if-splits)
then have tag: tag = B1 using tagged by auto
from (s-emit-event e s'' = Some s') show thesis
proof (cases rule: emitEventS-update-cases)
  case (Write-mem wk addr' sz' v' tag' r)
  have sz': tag' = B1 → (address-tag-aligned ISA addr' ∧ sz' = tag-granule
ISA)
    and len-v': length v' = sz'
    using (store-tag-axiom CC ISA (t @ [e])) Write-mem
  by (auto simp: store-tag-axiom-def writes-mem-val-at-idx-def bind-eq-Some-conv
nth-append split: if-splits)
  show ?thesis
  proof cases
    assume addr-disj: {addr'..<tag-granule ISA + addr'} ∩ {addr'..<sz' + addr'}
= {}
    then have get-mem-bytes addr (tag-granule ISA) s'' = get-mem-bytes addr
(tag-granule ISA) s'
      using Write-mem len-v'

```

```

    by (intro get-mem-bytes-cong) (auto simp: memstate-put-mem-bytes tagstate-put-mem-bytes)
  then show thesis
    using v cv tag
    by (intro IH) auto
next
  assume addr-overlap: {addr.. $\text{tag-granule ISA} + \text{addr}$ }  $\cap$  {addr'.. $\text{sz}' + \text{addr}'$ }
  then have tag': tag' = B1
  proof -
    obtain addr''
      where addr-orig: addr''  $\in$  {addr.. $\text{tag-granule ISA} + \text{addr}$ }
      and addr-prime: addr''  $\in$  {addr'.. $\text{sz}' + \text{addr}'$ }
      using addr-overlap
      by blast
    have tagstate s' addr'' = Some B1
      using addr-orig get-mem-bytes-tagged-tagstate[OF v[unfolded tag]]
      by auto
    then show tag' = B1
      using addr-prime Write-mem len-v'
      by (auto simp: tagstate-put-mem-bytes)
  qed
  with addr-overlap aligned sz' have addr': addr' = addr
    by (auto simp: address-tag-aligned-def dvd-def mult-Suc-right[symmetric]
      simp del: mult-Suc-right)
  then have v': v' = v
    using v tag tag' sz' len-v' Write-mem
    by (auto simp: get-mem-bytes-put-mem-bytes-same-addr)
  then show thesis
    using Write-mem cv tag' addr' sz' tag
    by (intro Step.prem(2)[of length t]) (auto simp: writes-mem-cap-def)
  qed
next
  case (Write-reg r v rs')
  with (get-mem-cap addr (tag-granule ISA) s' = Some c) show thesis
    by (auto intro: IH simp: get-mem-bytes-def)
next
  case Read
  with (get-mem-cap addr (tag-granule ISA) s' = Some c) show thesis
    by (auto intro: IH)
  qed
qed auto

lemma reads-mem-cap-at-idx-provenance:
  assumes read:  $t ! i = E\text{-read-mem}t \text{ rk addr (tag-granule ISA) (bytes, B1)}$ 
  and c: cap-of-mem-bytes-method CC bytes B1 = Some c
  and s': s-run-trace t s = Some s'
  and axioms: cheri-axioms CC ISA is-fetch has-ex inv-caps t
  and i:  $i < \text{length } t$ 
  and tagged: is-tagged-method CC c

```


and *aligned*: *address-tag-aligned ISA addr*
obtains (*Initial*) *get-mem-cap addr (tag-granule ISA) s = Some c*
| (*Update*) *k wk bytes' r* **where** *k < i*
and *t ! k = E-write-memt wk addr (tag-granule ISA) bytes' B1 r*
and *cap-of-mem-bytes-method CC bytes' B1 = Some c*
proof –
obtain *s''*
where *s''*: *s-run-trace (take i t) s = Some s''*
and *c'*: *get-mem-cap addr (tag-granule ISA) s'' = Some c*
using *s' i read c tagged*
by (*cases rule: runTraceS-nth-split; cases t ! i*)
(*auto simp: bind-eq-Some-conv reads-mem-cap-def split: if-splits*)
have *store-tag-axiom CC ISA (take i t)*
using *axioms*
by (*fastforce simp: cheri-axioms-def store-tag-axiom-def writes-mem-val-at-idx-def*
bind-eq-Some-conv)
with *c' s'' tagged aligned* **show** *thesis*
by (*cases rule: get-mem-cap-run-trace-cases*) (*auto intro: that*)
qed

fun *s-invariant* :: (*'regs sequential-state* \Rightarrow *'a*) \Rightarrow *'regval trace* \Rightarrow *'regs sequential-state*
 \Rightarrow *bool* **where**
s-invariant f [] s = True
| *s-invariant f (e # t) s = (case s-emit-event e s of Some s' \Rightarrow f s' = f s \wedge*
s-invariant f t s' | None \Rightarrow False)

abbreviation *s-invariant-holds* :: (*'regs sequential-state* \Rightarrow *bool*) \Rightarrow *'regval trace*
 \Rightarrow *'regs sequential-state* \Rightarrow *bool* **where**
s-invariant-holds P t s \equiv P s \wedge s-invariant P t s

lemma *s-invariant-append*:
s-invariant f (β @ α) s \longleftrightarrow
($\exists s'$. *s-invariant f β s \wedge s-run-trace β s = Some s' \wedge s-invariant f α s')*
by (*induction β arbitrary: s*) (*auto split: option.splits simp: runTraceS-Cons-tl*)

lemma *s-invariant-takeI*:
assumes *s-invariant f t s*
shows *s-invariant f (take n t) s*
proof –
from *assms* **have** *s-invariant f (take n t @ drop n t) s* **by** *auto*
then show *?thesis* **unfolding** *s-invariant-append* **by** *auto*
qed

lemma *s-invariant-run-trace-eq*:
assumes *s-invariant f t s* **and** *s-run-trace t s = Some s'*
shows *f s' = f s*
using *assms*
by (*induction f t s rule: s-invariant.induct*)
(*auto split: option.splits elim: runTraceS-ConsE*)

definition *no-caps-in-translation-tables* :: 'regs sequential-state \Rightarrow bool **where**
no-caps-in-translation-tables $s \equiv$
 \forall addr sz c. *get-mem-cap* addr sz $s = \text{Some } c \wedge \text{is-tagged-method } CC \ c \longrightarrow$
 $\text{addr} \notin \text{s-translation-tables } s$

lemma *derivable-available-caps-subseteq-reachable-caps*:
assumes *axioms*: *cheri-axioms* $CC \ ISA$ *is-fetch* *has-ex* *inv-caps* t
and t : *s-run-trace* $t \ s = \text{Some } s'$
and *translation-table-addrs-invariant*: *s-invariant* *s-translation-tables* $t \ s$
and *no-caps-in-translation-tables*: *s-invariant-holds* *no-caps-in-translation-tables*
 $t \ s$
shows *derivable* (*available-caps* $CC \ ISA \ i \ t$) \subseteq *reachable-caps* s
proof (*induction* i *rule*: *less-induct*)
case (*less* i)
show ?*case* **proof**
fix c
assume $c \in \text{derivable} (\text{available-caps } CC \ ISA \ i \ t)$
then show $c \in \text{reachable-caps } s$
proof *induction*
fix c
assume $c \in \text{available-caps } CC \ ISA \ i \ t$
then show $c \in \text{reachable-caps } s$
proof (*cases* *rule*: *available-caps-cases*)
case (*Reg* $r \ v \ j$)
with t **have** *initial*: $c \in \text{get-reg-caps } r \ s$
by (*blast* *intro*: *reads-reg-cap-at-idx-from-initial*)
show ?*thesis*
proof *cases*
assume $r \in \text{privileged-regs } ISA$
then obtain c' **where** c' : $c' \in \text{reachable-caps } s$ **and** *is-tagged-method* $CC \ c'$
and $\neg \text{is-sealed-method } CC \ c'$ **and** p : *permit-system-access* (*get-perms-method* $CC \ c'$)
using *Reg* *less*.*IH*[*OF* $\langle j < i \rangle$] *derivable-refl*[*of* *available-caps* $CC \ ISA \ j \ t$]
by (*auto* *elim*!: *system-access-permitted-at-idx-available-caps*)
then show ?*thesis*
using *Reg*
by (*auto* *intro*: *reachable-caps.SysReg*[*OF* *initial* $r \ c'$])
next
assume $r \notin \text{privileged-regs } ISA$
then show ?*thesis* **using** *initial* *Reg* **by** (*auto* *intro*: *reachable-caps.Reg*)
qed
next
case (*Mem* $wk \ paddr \ \text{bytes } j \ sz$)
note $\text{read} = \langle t \ ! \ j = E\text{-read-memt } wk \ paddr \ sz \ (\text{bytes}, B1) \rangle$
note $\text{bytes} = \langle \text{cap-of-mem-bytes-method } CC \ \text{bytes } B1 = \text{Some } c \rangle$
have $\text{addr} \notin \text{translation-tables } ISA \ (\text{take } j \ t)$
proof

```

assume paddr-j: paddr ∈ translation-tables ISA (take j t)
then have paddr ∈ s-translation-tables s
  using translation-tables-sound[of take j t s] t (j < length t)
  by (auto elim: runTraceS-nth-split)
moreover have paddr ∉ s-translation-tables s
proof –
  obtain s''
    where s'': s-run-trace (take j t) s = Some s''
      and c-s'': get-mem-cap paddr sz s'' = Some c
    using t (j < length t) read bytes (is-tagged-method CC c)
    by (cases rule: runTraceS-nth-split; cases t ! j)
      (auto simp: bind-eq-Some-conv reads-mem-cap-def split: if-splits)
    moreover have no-caps-in-translation-tables s''
      using no-caps-in-translation-tables s''
      using s-invariant-takeI[of no-caps-in-translation-tables t s j]
      using s-invariant-run-trace-eq[of no-caps-in-translation-tables take j t
s s'']
      by auto
    moreover have s-translation-tables s'' = s-translation-tables s
      using translation-table-addr-invariant s''
      by (intro s-invariant-run-trace-eq) (auto intro: s-invariant-takeI)
    ultimately show ?thesis
      using (is-tagged-method CC c)
      by (fastforce simp: no-caps-in-translation-tables-def bind-eq-Some-conv)
  qed
ultimately show False by blast
qed
then obtain vaddr c'
  where vaddr: translate-address ISA vaddr Load (take j t) = Some paddr
    and c': c' ∈ derivable (available-caps CC ISA j t)
      is-tagged-method CC c' ¬is-sealed-method CC c'
      set (address-range vaddr sz) ⊆ get-mem-region-method CC c'
      permit-load (get-perms-method CC c')
      permit-load-capability (get-perms-method CC c')
    and sz: sz = tag-granule ISA
    and aligned: address-tag-aligned ISA paddr
  using read t axioms (j < length t) (is-tagged-method CC c)
  unfolding cheri-axioms-def load-mem-axiom-def reads-mem-cap-def
by (fastforce simp: reads-mem-val-at-idx-def bind-eq-Some-conv cap-derivable-iff-derivable
split: if-splits)
  have s-vaddr: s-translate-address vaddr Load s = Some paddr
    using vaddr t (j < length t)
  by (blast intro: translate-address-sound[of take j t] elim: runTraceS-nth-split)
  from read[unfolded sz] bytes t axioms (j < length t) (is-tagged-method CC
c) aligned
  show ?thesis
proof (cases rule: reads-mem-cap-at-idx-provenance)
  case Initial
  then show ?thesis

```

```

    using Mem s-vaddr less.IH[of j] c' aligned sz
    by (intro reachable-caps.Mem[of paddr s c vaddr c'])
      (auto simp: bind-eq-Some-conv translate-address-tag-aligned-iff
permits-cap-load-def)
  next
    case (Update k wk bytes' r)
    then show ?thesis
      using axioms ⟨is-tagged-method CC c⟩ ⟨j < length t⟩ ⟨j < i⟩ less.IH[of k]
      unfolding cheri-axioms-def store-cap-mem-axiom-def
      by (auto simp: writes-mem-cap-at-idx-def writes-mem-cap-Some-iff
bind-eq-Some-conv cap-derivable-iff-derivable)
    qed
  qed
qed (auto intro: reachable-caps.intros)
qed
qed

```

lemma *put-regval-get-mem-cap*:

```

  assumes s': put-reg-val r v s = Some s'
  and s-translate-address addr acctype s' = s-translate-address addr acctype s
  shows get-mem-cap addr sz s' = get-mem-cap addr sz s
  using assms by (auto cong: bind-option-cong simp: get-mem-bytes-def)

```

definition *system-access-reachable* :: 'regs sequential-state \Rightarrow bool **where**

```

  system-access-reachable s  $\equiv \exists c \in \text{reachable-caps } s.$ 
  permit-system-access (get-perms-method CC c)  $\wedge \neg \text{is-sealed-method CC c}$ 

```

lemma *get-reg-cap-intra-domain-trace-reachable*:

```

  assumes r: c  $\in$  get-reg-caps r s'
  and s': s-run-trace t s = Some s'
  and axioms: cheri-axioms CC ISA is-fetch False False t

```

```

  and translation-table-addr-invariant: s-invariant s-translation-tables t s

```

```

  and no-caps-in-translation-tables: s-invariant-holds no-caps-in-translation-tables

```

t s

```

  and tag: is-tagged-method CC c

```

```

  and priv: r  $\in$  privileged-regs ISA  $\longrightarrow$  system-access-reachable s

```

```

  shows c  $\in$  reachable-caps s

```

proof –

```

  from r obtain v where v: get-reg-val r s' = Some v and c: c  $\in$  caps-of-regval
ISA v

```

```

  by (auto simp: bind-eq-Some-conv split: option.splits)

```

```

  from v c s' show c  $\in$  reachable-caps s

```

```

proof (cases rule: get-reg-val-s-run-trace-cases)

```

```

  case Init

```

```

  show ?thesis

```

```

proof cases

```

```

  assume r: r  $\in$  privileged-regs ISA

```

```

  with priv obtain c' where c': c'  $\in$  reachable-caps s

```

```

    and permit-system-access (get-perms-method CC c') and  $\neg$ is-sealed-method
CC c'
    by (auto simp: system-access-reachable-def)
    then show ?thesis using Init c tag by (intro reachable-caps.SysReg[OF - r
c']) auto
  next
    assume r  $\notin$  privileged-regs ISA
    then show ?thesis using Init c tag by (intro reachable-caps.Reg) auto
  qed
next
  case (Update j v')
  then have *: c  $\in$  writes-reg-caps CC (caps-of-regval ISA) (t ! j)
    and writes-to-reg (t ! j) = Some r
    using c tag by auto
  then have c  $\in$  derivable (available-caps CC ISA j t)
    using axioms tag (j < length t)
    unfolding cheri-axioms-def store-cap-reg-axiom-def
    by (fastforce simp: cap-derivable-iff-derivable)
  moreover have derivable (available-caps CC ISA j t)  $\subseteq$  reachable-caps s
    using axioms s' translation-table-addr-invariant no-caps-in-translation-tables
    by (intro derivable-available-caps-subseteq-reachable-caps)
  ultimately show ?thesis by auto
qed
qed

```

lemma *reachable-caps-trace-intradomain-monotonicity:*

```

assumes axioms: cheri-axioms CC ISA is-fetch False False t
    and s': s-run-trace t s = Some s'
    and addr-trans-inv: s-invariant ( $\lambda s'$  addr load. s-translate-address addr load s')
t s
    and translation-table-addr-invariant: s-invariant s-translation-tables t s
    and no-caps-in-translation-tables: s-invariant-holds no-caps-in-translation-tables
t s
shows reachable-caps s'  $\subseteq$  reachable-caps s
proof
  fix c
  assume c  $\in$  reachable-caps s'
  then show c  $\in$  reachable-caps s
  proof induction
    case (Reg r c)
    then show ?case
      using axioms s' translation-table-addr-invariant no-caps-in-translation-tables
      by (intro get-reg-cap-intra-domain-trace-reachable) auto
  next
    case (SysReg r c c')
    then show ?case
      using axioms s' translation-table-addr-invariant no-caps-in-translation-tables
      by (intro get-reg-cap-intra-domain-trace-reachable) (auto simp: system-access-reachable-def)
  next

```

```

case (Mem addr c vaddr c')
then have c: get-mem-cap addr (tag-granule ISA) s' = Some c
  and aligned: address-tag-aligned ISA addr
  by (auto split: if-splits)
have axiom: store-tag-axiom CC ISA t
  using axioms
  by (auto simp: cheri-axioms-def)
from c s' (is-tagged-method CC c) aligned axiom show ?case
proof (cases rule: get-mem-cap-run-trace-cases)
case Initial
have s-translate-address vaddr Load s' = s-translate-address vaddr Load s
  using s-invariant-run-trace-eq[OF addr-trans-inv s']
  by meson
then show ?thesis
  using Initial Mem
  by (intro reachable-caps.Mem[of addr s c vaddr c']) (auto split: if-splits)
next
case (Update k wk bytes r)
have derivable (available-caps CC ISA k t)  $\subseteq$  reachable-caps s
  using assms axioms
  by (intro derivable-available-caps-subseteq-reachable-caps)
then show ?thesis
  using Update (is-tagged-method CC c) axioms
  unfolding cheri-axioms-def store-cap-mem-axiom-def cap-derivable-iff-derivable
  by (auto simp: writes-mem-cap-at-idx-def writes-mem-cap-Some-iff)
qed
qed (auto intro: reachable-caps.intros)
qed

lemma reachable-caps-instr-trace-intradomain-monotonicity:
  assumes t: hasTrace t (instr-sem ISA instr)
  and ta: instr-assms t
  and s': s-run-trace t s = Some s'
  and no-exception:  $\neg$ instr-raises-ex ISA instr t
  and no-ccall:  $\neg$ invokes-caps ISA instr t
  and addr-trans-inv: s-invariant ( $\lambda s'$  addr load. s-translate-address addr load s')
  t s
  and translation-table-addr-invariant: s-invariant s-translation-tables t s
  and no-caps-in-translation-tables: s-invariant-holds no-caps-in-translation-tables
  t s
  shows reachable-caps s'  $\subseteq$  reachable-caps s
  using assms instr-cheri-axioms[OF t ta]
  by (intro reachable-caps-trace-intradomain-monotonicity) auto

lemma reachable-caps-fetch-trace-intradomain-monotonicity:
  assumes t: hasTrace t (instr-fetch ISA)
  and ta: fetch-assms t
  and s': s-run-trace t s = Some s'
  and no-exception:  $\neg$ fetch-raises-ex ISA t

```

```

    and addr-trans-inv: s-invariant ( $\lambda s'$  addr load. s-translate-address addr load s')
  t s
    and translation-table-addr-invariant: s-invariant s-translation-tables t s
    and no-caps-in-translation-tables: s-invariant-holds no-caps-in-translation-tables
  t s
    shows reachable-caps s'  $\subseteq$  reachable-caps s
    using assms fetch-cheri-axioms[OF t ta]
    by (intro reachable-caps-trace-intradomain-monotonicity) auto
end

```

Multi-instruction sequences

```

fun fetch-execute-loop :: ('cap, 'regval, 'instr, 'e) isa  $\Rightarrow$  nat  $\Rightarrow$  ('regval, unit, 'e)
monad where
  fetch-execute-loop ISA (Suc bound) = (instr-fetch ISA  $\gg$  instr-sem ISA)  $\gg$ 
  fetch-execute-loop ISA bound
| fetch-execute-loop ISA 0 = return ()

```

```

fun instrs-raise-ex :: ('cap, 'regval, 'instr, 'e) isa  $\Rightarrow$  nat  $\Rightarrow$  'regval trace  $\Rightarrow$  bool
where
  instrs-raise-ex ISA (Suc bound) t =
    ( $\exists$  tf t'. t = tf @ t'  $\wedge$  hasTrace tf (instr-fetch ISA)  $\wedge$ 
      (fetch-raises-ex ISA tf  $\vee$ 
        ( $\exists$  instr ti t''. t' = ti @ t''  $\wedge$ 
          runTrace tf (instr-fetch ISA) = Some (Done instr)  $\wedge$ 
          hasTrace ti (instr-sem ISA instr)  $\wedge$ 
          (instr-raises-ex ISA instr ti  $\vee$ 
            instrs-raise-ex ISA bound t')))))
| instrs-raise-ex ISA 0 t = False

```

```

fun instrs-invoke-caps :: ('cap, 'regval, 'instr, 'e) isa  $\Rightarrow$  nat  $\Rightarrow$  'regval trace  $\Rightarrow$ 
bool where
  instrs-invoke-caps ISA (Suc bound) t =
    ( $\exists$  tf t'. t = tf @ t'  $\wedge$  hasTrace tf (instr-fetch ISA)  $\wedge$ 
      ( $\exists$  instr ti t''. t' = ti @ t''  $\wedge$ 
        runTrace tf (instr-fetch ISA) = Some (Done instr)  $\wedge$ 
        hasTrace ti (instr-sem ISA instr)  $\wedge$ 
        (invokes-caps ISA instr ti  $\vee$ 
          instrs-invoke-caps ISA bound t')))))
| instrs-invoke-caps ISA 0 t = False

```

```

context CHERI-ISA-State
begin

```

```

lemma reachable-caps-instrs-trace-intradomain-monotonicity:
  assumes t: hasTrace t (fetch-execute-loop ISA n)
  and ta: fetch-assms t
  and s': s-run-trace t s = Some s'
  and no-exception:  $\neg$ instrs-raise-ex ISA n t

```

```

    and no-ccall:  $\neg \text{instrs-invoke-caps } \text{ISA } n \ t$ 
    and addr-trans-inv:  $s\text{-invariant } (\lambda s' \text{ addr load. } s\text{-translate-address addr load } s')$ 
  t s
    and translation-table-addr-invariant:  $s\text{-invariant } s\text{-translation-tables } t \ s$ 
    and no-caps-in-translation-tables:  $s\text{-invariant-holds no-caps-in-translation-tables}$ 
  t s
    shows  $\text{reachable-caps } s' \subseteq \text{reachable-caps } s$ 
  proof (use assms in (induction n arbitrary: s t))
    case 0
    then show ?case by (auto simp: return-def hasTrace-iff-Traces-final)
  next
    case (Suc n)
    then obtain m'
      where (instr-fetch ISA  $\gg (\lambda \text{instr. } \llbracket \text{instr} \rrbracket \gg \text{fetch-execute-loop ISA } n), t, m'$ )
    ∈ Traces
      and m': final m'
    by (auto simp: hasTrace-iff-Traces-final)
    then show ?case
  proof (cases rule: bind-Traces-cases)
    case (Left m'')
    then have hasTrace t (instr-fetch ISA)
      using m'
    by (auto elim!: final-bind-cases) (auto simp: hasTrace-iff-Traces-final final-def)
    then show ?thesis
      using Suc.prem
    by (intro reachable-caps-fetch-trace-intradomain-monotonicity) auto
  next
    case (Bind tf instr t')
    obtain s'' where s'':  $s\text{-run-trace tf } s = \text{Some } s''$  and t':  $s\text{-run-trace } t' \ s'' = \text{Some } s'$ 
    using Bind Suc
    by (auto elim: runTraceS-appendE)
    have tf: hasTrace tf (instr-fetch ISA)
      using Bind
    by (auto simp: hasTrace-iff-Traces-final final-def)
    have invs':
      s-invariant  $(\lambda s' \text{ addr load. } s\text{-translate-address addr load } s') \ t' \ s''$ 
      s-invariant s-translation-tables t' s''
      s-invariant-holds no-caps-in-translation-tables t' s''
    using tf s'' Bind Suc.prem
    using s-invariant-run-trace-eq[of no-caps-in-translation-tables tf s s'']
    by (auto simp: s-invariant-append)
    have ta': fetch-assms tf instr-assms t'
      using Bind Suc.prem fetch-assms-appendE
    by auto
    from  $(\llbracket \text{instr} \rrbracket \gg \text{fetch-execute-loop ISA } n, t', m') \in \text{Traces}$ 
    have  $\text{reachable-caps } s' \subseteq \text{reachable-caps } s''$ 
  proof (cases rule: bind-Traces-cases)
    case (Left m'')

```



```

then have hasTrace t' [instr]
  using m'
by (auto elim!: final-bind-cases) (auto simp: hasTrace-iff-Traces-final final-def)
then show ?thesis
  using tf t' s'' Bind Suc.premis invs' ta'
  by (intro reachable-caps-instr-trace-intradomain-monotonicity)
    (auto simp: runTrace-iff-Traces)
next
case (Bind ti am t'')
obtain s''' where s''': s-run-trace ti s'' = Some s''' and t'': s-run-trace t''
s''' = Some s'
  using Bind t'
  by (auto elim: runTraceS-appendE)
have ti: hasTrace ti [instr]
  using Bind
  by (auto simp: hasTrace-iff-Traces-final final-def)
have invs'':
  s-invariant (λs' addr load. s-translate-address addr load s') t'' s'''
  s-invariant s-translation-tables t'' s'''
  s-invariant-holds no-caps-in-translation-tables t'' s'''
  using invs' s''' Bind
  using s-invariant-run-trace-eq[of no-caps-in-translation-tables ti s'' s''']
  by (auto simp: s-invariant-append)
have ta'': instr-assms ti fetch-assms t''
  using Bind ta' instr-assms-appendE
  by auto
have no-exception': ¬fetch-raises-ex ISA tf ¬instr-raises-ex ISA instr ti
  and no-ccall': ¬invokes-caps ISA instr ti
  and no-exception'': ¬instrs-raise-ex ISA n t''
  and no-ccall'': ¬instrs-invoke-caps ISA n t''
  using ti tf Suc.premis Bind ⟨t = tf @ t'⟩
  using ⟨Run (instr-fetch ISA) tf instr⟩
  by (auto simp: runTrace-iff-Traces)
then have reachable-caps s' ⊆ reachable-caps s'''
  using Bind m' t'' invs'' ta''
  by (intro Suc.IH) (auto simp: hasTrace-iff-Traces-final final-def)
also have reachable-caps s''' ⊆ reachable-caps s''
  using ti s''' no-exception' no-ccall' invs' ⟨t' = ti @ t'⟩ ta''
  by (intro reachable-caps-instr-trace-intradomain-monotonicity)
    (auto simp: s-invariant-append)
finally show ?thesis .
qed
also have reachable-caps s'' ⊆ reachable-caps s
  using tf s'' Bind Suc.premis ta'
  by (intro reachable-caps-fetch-trace-intradomain-monotonicity)
    (auto simp: s-invariant-append)
finally show ?thesis .
qed
qed

```

end

end

theory *Trace-Assumptions*

imports *Sail.Sail2-state-lemmas HOL-Eisbach.Eisbach-Tools*

begin

2 Verification infrastructure

lemma *return-Traces-iff[simp]*:

$(\text{return } x, t, m') \in \text{Traces} \longleftrightarrow t = [] \wedge m' = \text{Done } x$

by (*auto simp: return-def*)

lemma *Run-read-regE*:

assumes *Run (read-reg r) t v*

obtains *(Read) rv where t = [E-read-reg (name r) rv] and of-regval r rv = Some v*

using *assms*

by (*auto simp: read-reg-def elim!: Read-reg-TracesE split: option.splits*)

lemmas *Run-elim = Run-bindE Run-or-boolM-E Run-returnE Run-letE Run-and-boolM-E Run-ifE*

lemma *Run-assert-exp-iff[simp]*: *Run (assert-exp c m) t a \longleftrightarrow c \wedge t = [] \wedge a = ()*

by (*auto simp: assert-exp-def*)

lemma *Run-liftR-assert-exp-iff[simp]*:

Run (liftR (assert-exp c msg :: ('r, unit, 'ex) monad)) t a \longleftrightarrow Run (assert-exp c msg :: ('r, unit, 'ex) monad) t a

by (*auto simp: assert-exp-def liftR-def*)

lemma *Run-foreachM-appendE*:

assumes *Run (foreachM (xs @ ys) vars body) t vars'*

obtains *txs tys vars''*

where *t = txs @ tys*

and *Run (foreachM xs vars body) txs vars''*

and *Run (foreachM ys vars'' body) tys vars'*

proof –

have $\exists txs tys vars''.$

$t = txs @ tys \wedge$

$Run (foreachM xs vars body) txs vars'' \wedge$

$Run (foreachM ys vars'' body) tys vars'$

proof (*use assms in (induction xs arbitrary: vars t)*)

case (*Cons x xs*)

then obtain *vars'' tx t'*

where *tx: Run (body x vars) tx vars''*

and *t': Run (foreachM (xs @ ys) vars'' body) t' vars'*

```

    and  $t: t = tx @ t'$ 
  by (auto elim: Run-bindE)
from Cons.IH[OF  $t'$ ] obtain  $vars''' txs tys$ 
where  $t' = txs @ tys$ 
  and  $Run (foreachM xs vars'' body) txs vars'''$ 
  and  $tys: Run (foreachM ys vars''' body) tys vars'$ 
by blast
then have  $Run (foreachM (x \# xs) vars body) (tx @ txs) vars'''$ 
  using  $tx$ 
  by (auto intro: Traces-bindI)
then show ?case
  using  $tys$ 
  unfolding  $\langle t = tx @ t' \rangle$  and  $\langle t' = txs @ tys \rangle$  and  $append-assoc[symmetric]$ 
  by blast
qed auto
then show thesis
  using that
  by blast
qed

lemma Run-foreachM-elim:
  assumes  $Run (foreachM xs vars body) t vars'$ 
  and  $\bigwedge^n tl tn tr vars' vars''.$ 
     $\llbracket t = tl @ tn @ tr;$ 
     $P tl vars';$ 
     $Run (body (xs ! n) vars') tn vars'';$ 
     $n < length xs$ 
   $\implies P (tl @ tn) vars''$ 
  and  $P [] vars$ 
shows  $P t vars'$ 
  using assms
proof (use assms in (induction xs arbitrary:  $t vars'$  rule: rev-induct))
  case (snoc  $x xs$ )
  then obtain  $txs tx vars''$ 
  where  $t: t = txs @ tx$ 
    and  $txs: Run (foreachM xs vars body) txs vars''$ 
    and  $tx: Run (body x vars'') tx vars'$ 
  by (elim Run-foreachM-appendE) auto
  then have  $P txs vars''$ 
  using  $\langle P [] vars \rangle$ 
  by (intro snoc.IH[OF  $txs$ ]) (auto simp: nth-append intro!: snoc.prem1(2))
  then show ?case
  using  $t txs tx$ 
  using snoc.prem1(2)[where  $tl = txs$  and  $tn = tx$  and  $tr = []$  and  $n = length$ 
 $xs$ ]
  by auto
qed auto

```

lemma Run-try-catchE:

```

assumes Run (try-catch m h) t a
obtains (Run) Run m t a
| (Catch) tm e th where (m, tm, Exception e) ∈ Traces and Run (h e) th a and
t = tm @ th
proof (use assms in (cases rule: try-catch-Traces-cases))
  case (NoEx m')
  then show ?thesis
    by (cases (m', h) rule: try-catch.cases) (auto elim!: Run Catch)
next
  case (Ex tm ex th)
  show ?thesis using Catch[OF Ex] .
qed

```

```

lemma throw-Traces-iff[simp]:
  (throw e, t, m') ∈ Traces  $\longleftrightarrow$  t = [] ∧ m' = Exception e
by (auto simp: throw-def)

```

```

lemma early-return-Traces-iff[simp]:
  (early-return a, t, m') ∈ Traces  $\longleftrightarrow$  t = [] ∧ m' = Exception (Inl a)
by (auto simp: early-return-def)

```

```

lemma Run-catch-early-returnE:
assumes Run (catch-early-return m) t a
obtains (Run) Run m t a
| (Early) (m, t, Exception (Inl a)) ∈ Traces
using assms
unfolding catch-early-return-def
by (elim Run-try-catchE) (auto split: sum.splits)

```

2.1 Assumptions about register reads and writes

```

definition no-reg-writes-to Rs m  $\equiv$  ( $\forall t m' r v. (m, t, m') \in \text{Traces} \wedge r \in Rs \longrightarrow$ 
E-write-reg r v  $\notin$  set t)

```

```

definition runs-no-reg-writes-to Rs m  $\equiv$  ( $\forall t a r v. \text{Run } m \ t \ a \wedge r \in Rs \longrightarrow$ 
E-write-reg r v  $\notin$  set t)

```

```

locale Register-State =
  fixes get-regval :: string  $\Rightarrow$  'regstate  $\Rightarrow$  'regval option
    and set-regval :: string  $\Rightarrow$  'regval  $\Rightarrow$  'regstate  $\Rightarrow$  'regstate option
begin

```

```

fun updates-regs :: string set  $\Rightarrow$  'regval trace  $\Rightarrow$  'regstate  $\Rightarrow$  'regstate option where
  updates-regs R [] s = Some s
| updates-regs R (E-write-reg r v # t) s =
  (if r ∈ R
    then Option.bind (set-regval r v s) (updates-regs R t)
    else updates-regs R t s)
| updates-regs R (- # t) s = updates-regs R t s

```

```

fun reads-regs-from :: string set  $\Rightarrow$  'regval trace  $\Rightarrow$  'regstate  $\Rightarrow$  bool where
  reads-regs-from R [] s = True
| reads-regs-from R (E-read-reg r v # t) s =
  (if r  $\in$  R
   then get-regval r s = Some v  $\wedge$  reads-regs-from R t s
   else reads-regs-from R t s)
| reads-regs-from R (E-write-reg r v # t) s =
  (if r  $\in$  R
   then (case set-regval r v s of Some s'  $\Rightarrow$  reads-regs-from R t s' | None  $\Rightarrow$ 
False)
   else reads-regs-from R t s)
| reads-regs-from R (- # t) s = reads-regs-from R t s

lemma reads-regs-from-updates-regs-Some:
  assumes reads-regs-from R t s
  obtains s' where updates-regs R t s = Some s'
  using assms
  by (induction R t s rule: reads-regs-from.induct) (auto split: if-splits option.splits)

named-theorems regstate-simp

lemma updates-regs-append-iff[regstate-simp]:
  updates-regs R (t @ t') s = Option.bind (updates-regs R t s) (updates-regs R t')
  by (induction R t s rule: updates-regs.induct) (auto split: bind-splits)

lemma reads-regs-from-append-iff[regstate-simp]:
  reads-regs-from R (t @ t') s  $\longleftrightarrow$  (reads-regs-from R t s  $\wedge$  reads-regs-from R t'
(the (updates-regs R t s)))
  by (induction R t s rule: reads-regs-from.induct) (auto split: option.splits)

lemma reads-regs-from-appendE-simp:
  assumes reads-regs-from Rs t regs and t = t1 @ t2
  and the (updates-regs Rs t1 regs) = regs'
  obtains reads-regs-from Rs t1 regs and reads-regs-from Rs t2 regs'
  using assms
  by (auto simp: reads-regs-from-append-iff)

lemma no-reg-writes-to-updates-regs-inv[simp]:
  assumes (m, t, m')  $\in$  Traces
  and no-reg-writes-to Rs m
  shows updates-regs Rs t s = Some s
  using assms
proof -
  have  $\forall r \in Rs. \forall v. E\text{-write-reg } r \ v \notin \text{set } t$ 
  using assms
  by (auto simp: no-reg-writes-to-def)
  then show updates-regs Rs t s = Some s
  by (induction Rs t s rule: updates-regs.induct) auto
qed

```

lemma *no-reg-writes-to-updates-regsE*:

assumes $(m, t, m') \in \text{Traces}$

and *no-reg-writes-to* $Rs\ m$

obtains *updates-regs* $Rs\ t\ s = \text{Some } s$

using *assms*

by *auto*

named-theorems *no-reg-writes-toI*

named-theorems *runs-no-reg-writes-toI*

lemma *no-reg-writes-runs-no-reg-writes*:

no-reg-writes-to $Rs\ m \implies \text{runs-no-reg-writes-to } Rs\ m$

by (*auto simp: no-reg-writes-to-def runs-no-reg-writes-to-def*)

lemma *no-reg-writes-to-bindI*[*intro, simp, no-reg-writes-toI*]:

assumes *no-reg-writes-to* $Rs\ m$ **and** $\bigwedge t\ a. \text{Run } m\ t\ a \implies \text{no-reg-writes-to } Rs\ (f\ a)$

shows *no-reg-writes-to* $Rs\ (m \ggg f)$

using *assms*

by (*auto simp: no-reg-writes-to-def elim: bind-Traces-cases*)

lemma *runs-no-reg-writes-to-bindI*[*intro, simp, runs-no-reg-writes-toI*]:

assumes *runs-no-reg-writes-to* $Rs\ m$ **and** $\bigwedge t\ a. \text{Run } m\ t\ a \implies \text{runs-no-reg-writes-to } Rs\ (f\ a)$

shows *runs-no-reg-writes-to* $Rs\ (m \ggg f)$

using *assms*

by (*auto simp: runs-no-reg-writes-to-def elim: Run-bindE*)

lemma *no-reg-writes-to-return*[*simp, no-reg-writes-toI*]:

no-reg-writes-to $Rs\ (\text{return } a)$

by (*auto simp: no-reg-writes-to-def*)

lemma *no-reg-writes-to-throw*[*simp, no-reg-writes-toI*]:

no-reg-writes-to $Rs\ (\text{throw } e)$

by (*auto simp: no-reg-writes-to-def*)

lemma *no-reg-writes-to-Fail*[*simp, no-reg-writes-toI*]:

no-reg-writes-to $Rs\ (\text{Fail } \text{msg})$

by (*auto simp: no-reg-writes-to-def*)

lemma *no-reg-writes-to-try-catchI*[*intro, simp, no-reg-writes-toI*]:

assumes *no-reg-writes-to* $Rs\ m$ **and** $\bigwedge e. \text{no-reg-writes-to } Rs\ (h\ e)$

shows *no-reg-writes-to* $Rs\ (\text{try-catch } m\ h)$

using *assms*

by (*auto simp: no-reg-writes-to-def elim!: try-catch-Traces-cases*)

lemma *no-reg-writes-to-catch-early-returnI*[*intro, simp, no-reg-writes-toI*]:

assumes *no-reg-writes-to* $Rs\ m$

shows *no-reg-writes-to* *Rs* (*catch-early-return m*)
using *assms*
by (*auto simp: catch-early-return-def split: sum.splits*)

lemma *no-reg-writes-to-early-return*[*intro, simp, no-reg-writes-toI*]:
shows *no-reg-writes-to* *Rs* (*early-return a*)
by (*auto simp: early-return-def*)

lemma *no-reg-writes-to-liftR-I*[*intro, simp, no-reg-writes-toI*]:
assumes *no-reg-writes-to* *Rs m*
shows *no-reg-writes-to* *Rs* (*liftR m*)
using *assms*
by (*auto simp: liftR-def*)

lemma *no-reg-writes-to-let*[*simp, no-reg-writes-toI*]:
no-reg-writes-to *Rs* (*f x*) \implies *no-reg-writes-to* *Rs* (*let a = x in f a*)
by *auto*

lemma *no-reg-writes-to-if*[*simp, no-reg-writes-toI*]:
assumes *c* \implies *no-reg-writes-to* *Rs m1* **and** $\neg c \implies$ *no-reg-writes-to* *Rs m2*
shows *no-reg-writes-to* *Rs* (*if c then m1 else m2*)
using *assms*
by *auto*

lemma *runs-no-reg-writes-to-if*[*simp, runs-no-reg-writes-toI*]:
assumes *c* \implies *runs-no-reg-writes-to* *Rs m1* **and** $\neg c \implies$ *runs-no-reg-writes-to*
Rs m2
shows *runs-no-reg-writes-to* *Rs* (*if c then m1 else m2*)
using *assms*
by *auto*

lemma *no-reg-writes-to-case-prod*[*intro, simp, no-reg-writes-toI*]:
assumes $\bigwedge x y. \text{no-reg-writes-to } Rs (f x y)$
shows *no-reg-writes-to* *Rs* (*case z of (x, y) \Rightarrow f x y*)
using *assms*
by (*cases z*) *auto*

lemma *runs-no-reg-writes-to-case-prod*[*intro, simp, runs-no-reg-writes-toI*]:
assumes $\bigwedge x y. \text{runs-no-reg-writes-to } Rs (f x y)$
shows *runs-no-reg-writes-to* *Rs* (*case z of (x, y) \Rightarrow f x y*)
using *assms*
by (*cases z*) *auto*

lemma *no-reg-writes-to-choose-bool*[*simp, no-reg-writes-toI*]:
no-reg-writes-to *Rs* (*choose-bool desc*)
by (*auto simp: choose-bool-def no-reg-writes-to-def elim: Traces-cases*)

lemma *no-reg-writes-to-undefined-bool*[*simp, no-reg-writes-toI*]:
no-reg-writes-to *Rs* (*undefined-bool ()*)

by (auto simp: undefined-bool-def)

lemma no-reg-writes-to-foreachM-inv[*simp*, no-reg-writes-toI]:
 assumes $\bigwedge x \text{ vars. no-reg-writes-to } Rs \text{ (body } x \text{ vars)}$
 shows no-reg-writes-to *Rs* (foreachM *xs vars body*)
 using *assms*
 by (induction *xs vars body* rule: foreachM.induct) auto

lemma no-reg-writes-to-bool-of-bitU-nondet[*simp*, no-reg-writes-toI]:
 no-reg-writes-to *Rs* (bool-of-bitU-nondet *b*)
 by (cases *b*) (auto simp: bool-of-bitU-nondet-def)

lemma no-reg-writes-to-and-boolM[*intro*, *simp*, no-reg-writes-toI]:
 assumes no-reg-writes-to *Rs m1* and no-reg-writes-to *Rs m2*
 shows no-reg-writes-to *Rs* (and-boolM *m1 m2*)
 using *assms*
 by (auto simp: and-boolM-def)

lemma no-reg-writes-to-or-boolM[*intro*, *simp*, no-reg-writes-toI]:
 assumes no-reg-writes-to *Rs m1* and no-reg-writes-to *Rs m2*
 shows no-reg-writes-to *Rs* (or-boolM *m1 m2*)
 using *assms*
 by (auto simp: or-boolM-def)

lemma no-reg-writes-to-assert-exp[*simp*, no-reg-writes-toI]:
 no-reg-writes-to *Rs* (assert-exp *c m*)
 by (auto simp: assert-exp-def no-reg-writes-to-def)

lemma no-reg-writes-to-exit[*simp*, no-reg-writes-toI]:
 no-reg-writes-to *Rs* (exit0 ())
 by (auto simp: exit0-def no-reg-writes-to-def)

lemma no-reg-writes-to-read-reg[*simp*, no-reg-writes-toI]:
 no-reg-writes-to *Rs* (read-reg *r*)
 by (auto simp: no-reg-writes-to-def read-reg-def elim: Read-reg-TracesE split: option.splits)

lemma no-reg-writes-to-write-reg[*simp*, no-reg-writes-toI]:
 assumes name *r* $\notin Rs$
 shows no-reg-writes-to *Rs* (write-reg *r v*)
 using *assms*
 by (auto simp: no-reg-writes-to-def write-reg-def elim!: Write-reg-TracesE)

lemma no-reg-writes-to-read-mem-bytes[*simp*, no-reg-writes-toI]:
 no-reg-writes-to *Rs* (read-mem-bytes *BC BC' rk addr bytes*)
 by (auto simp: read-mem-bytes-def no-reg-writes-to-def maybe-fail-def
 elim: Traces-cases split: option.splits)

lemma no-reg-writes-to-read-mem[*simp*, no-reg-writes-toI]:

no-reg-writes-to Rs (read-mem BC BC' rk addr-length addr bytes)
by (*auto simp: read-mem-def split: option.splits*)

lemma *no-reg-writes-to-write-mem-ea*[*simp, no-reg-writes-toI*]:
no-reg-writes-to Rs (write-mem-ea BC wk addr-length addr bytes)
by (*auto simp: write-mem-ea-def no-reg-writes-to-def maybe-fail-def split: option.splits elim: Traces-cases*)

lemma *no-reg-writes-to-write-mem*[*simp, no-reg-writes-toI*]:
no-reg-writes-to Rs (write-mem BC BC' wk addr-length addr bytes value)
by (*auto simp: write-mem-def no-reg-writes-to-def split: option.splits elim: Traces-cases*)

lemma *no-reg-writes-to-genlistM*[*simp, no-reg-writes-toI*]:
assumes $\bigwedge i. \text{no-reg-writes-to } Rs \text{ (f } i)$
shows *no-reg-writes-to Rs (genlistM f n)*
using *assms*
by (*auto simp: genlistM-def*)

lemma *no-reg-writes-to-choose-bools*[*simp, no-reg-writes-toI*]:
shows *no-reg-writes-to Rs (choose-bools desc n)*
by (*auto simp: choose-bools-def*)

lemma *no-reg-writes-to-chooseM*[*simp, no-reg-writes-toI*]:
shows *no-reg-writes-to Rs (chooseM desc xs)*
by (*auto simp: chooseM-def split: option.splits*)

lemma *no-reg-writes-to-internal-pick*[*simp, no-reg-writes-toI*]:
shows *no-reg-writes-to Rs (internal-pick xs)*
by (*auto simp: internal-pick-def*)

lemma *no-reg-writes-to-bools-of-bits-nondet*[*simp, no-reg-writes-toI*]:
shows *no-reg-writes-to Rs (bools-of-bits-nondet bits)*
by (*auto simp: bools-of-bits-nondet-def*)

lemma *no-reg-writes-to-of-bits-nondet*[*simp, no-reg-writes-toI*]:
shows *no-reg-writes-to Rs (of-bits-nondet BC bits)*
by (*auto simp: of-bits-nondet-def*)

lemmas *no-reg-write-builtins* =
no-reg-writes-to-return no-reg-writes-to-throw no-reg-writes-to-Fail
no-reg-writes-to-early-return no-reg-writes-to-assert-exp
no-reg-writes-to-read-reg no-reg-writes-to-chooseM no-reg-writes-to-internal-pick
no-reg-writes-to-choose-bool no-reg-writes-to-undefined-bool
no-reg-writes-to-bool-of-bitU-nondet no-reg-writes-to-bools-of-bits-nondet
no-reg-writes-to-of-bits-nondet no-reg-writes-to-choose-bools no-reg-writes-to-exit
no-reg-writes-to-read-mem-bytes no-reg-writes-to-read-mem
no-reg-writes-to-write-mem-ea no-reg-writes-to-write-mem

method *no-reg-writes-toI* **uses** *simp* =

(intro runs-no-reg-writes-toI no-reg-writes-runs-no-reg-writes no-reg-writes-toI conjI
impI;

auto simp: simp split del: if-split split: option.splits)

lemma *Run-choose-bool-updates-regs*[regstate-simp]:
 assumes *Run* (choose-bool desc) *t b*
 shows *updates-regs* *Rs t* *regs* = *Some regs*
 using *assms*
 by (auto simp: choose-bool-def elim!: *Traces-cases*[where *t = t*])

lemma *Run-choose-bools-updates-regs*[regstate-simp]:
 assumes *Run* (choose-bools desc *n*) *t b*
 shows *updates-regs* *Rs t* *regs* = *Some regs*
 using *assms*
 by (auto simp: choose-bools-def genlistM-def regstate-simp elim!: *Run-foreachM-elim*
Run-bindE)

lemma *Run-undefined-bool-updates-regs*[regstate-simp]:
 assumes *Run* (undefined-bool *u*) *t b*
 shows *updates-regs* *Rs t* *regs* = *Some regs*
 using *assms*
 unfolding *undefined-bool-def*
 by (elim *Run-choose-bool-updates-regs*)

lemma *Run-internal-pick-updates-regs*[regstate-simp]:
 assumes *Run* (*internal-pick xs*) *t a*
 shows *updates-regs* *Rs t* *regs* = *Some regs*
 using *assms*
 by (auto simp: *internal-pick-def* chooseM-def regstate-simp elim!: *Run-elim*s split:
option.splits)

named-theorems *RunE*

method *RunE* **uses** *elim* =
 (match **premises** in *R*[thin]: ⟨*Run m t a*⟩ **and** *regs*[thin]: *reads-regs-from* *Rs t*
regs **for** *m t a Rs regs* ⇒
 ⟨match *elim RunE* in *E*: ⟨*R'* ⇒ *regs'* ⇒ -⟩ **for** *R' regs'* ⇒
 ⟨match (⟨*Run m t a*⟩) in *R'* ⇒
 ⟨match (⟨*reads-regs-from* *Rs t regs*⟩) in *regs'* ⇒
 ⟨rule *E*[OF *R regs*]; (*RunE elim*: *elim*)?⟩⟩⟩)

end

2.2 State invariants

locale *State-Invariant* = *Register-State* *get-regval set-regval*
for *get-regval* :: *string* ⇒ '*regstate* ⇒ '*regval* *option*
and *set-regval* :: *string* ⇒ '*regval* ⇒ '*regstate* ⇒ '*regstate* *option*
 + **fixes** *invariant* :: '*regstate* ⇒ *bool* **and** *inv-regs* :: *register-name* *set*

begin

definition

$Run\text{-}inv\ m\ t\ a\ regs \equiv Run\ m\ t\ a \wedge reads\text{-}regs\text{-}from\ inv\text{-}regs\ t\ regs \wedge invariant\ regs$

definition $trace\text{-}preserves\text{-}invariant :: 'regval\ trace \Rightarrow bool$ **where**

$trace\text{-}preserves\text{-}invariant\ t \equiv$
 $(\forall s. invariant\ s \wedge reads\text{-}regs\text{-}from\ inv\text{-}regs\ t\ s \longrightarrow invariant\ (the\ (updates\text{-}regs\ inv\text{-}regs\ t\ s)))$

lemma $trace\text{-}preserves\text{-}invariantE$:

assumes $trace\text{-}preserves\text{-}invariant\ t$ **and** $reads\text{-}regs\text{-}from\ inv\text{-}regs\ t\ s$ **and** $invariant\ s$

obtains s' **where** $updates\text{-}regs\ inv\text{-}regs\ t\ s = Some\ s'$ **and** $invariant\ s'$

using $assms$

by $(fastforce\ simp: trace\text{-}preserves\text{-}invariant\text{-}def\ elim: reads\text{-}regs\text{-}from\ updates\text{-}regs\text{-}Some)$

lemma $trace\text{-}preserves\text{-}invariant\text{-}appendI$:

assumes $t1: trace\text{-}preserves\text{-}invariant\ t1$ **and** $t2: trace\text{-}preserves\text{-}invariant\ t2$

shows $trace\text{-}preserves\text{-}invariant\ (t1\ @\ t2)$

using $t2$

by $(auto\ simp: trace\text{-}preserves\text{-}invariant\text{-}def\ regstate\text{-}simp\ elim: trace\text{-}preserves\text{-}invariantE[OF\ t1])$

definition $traces\text{-}preserve\text{-}invariant :: ('regval, 'a, 'e)\ monad \Rightarrow bool$ **where**

$traces\text{-}preserve\text{-}invariant\ m \equiv (\forall t\ m'. (m, t, m') \in Traces \longrightarrow trace\text{-}preserves\text{-}invariant\ t)$

definition $runs\text{-}preserve\text{-}invariant :: ('regval, 'a, 'e)\ monad \Rightarrow bool$ **where**

$runs\text{-}preserve\text{-}invariant\ m \equiv (\forall t\ a. Run\ m\ t\ a \longrightarrow trace\text{-}preserves\text{-}invariant\ t)$

definition $exceptions\text{-}preserve\text{-}invariant :: ('regval, 'a, 'e)\ monad \Rightarrow bool$ **where**

$exceptions\text{-}preserve\text{-}invariant\ m \equiv (\forall t\ e. (m, t, Exception\ e) \in Traces \longrightarrow trace\text{-}preserves\text{-}invariant\ t)$

lemma $traces\text{-}runs\text{-}preserve\text{-}invariantI$:

assumes $traces\text{-}preserve\text{-}invariant\ m$

shows $runs\text{-}preserve\text{-}invariant\ m$

using $assms$

by $(auto\ simp: traces\text{-}preserve\text{-}invariant\text{-}def\ runs\text{-}preserve\text{-}invariant\text{-}def)$

lemma $traces\text{-}exceptions\text{-}preserve\text{-}invariantI$:

assumes $traces\text{-}preserve\text{-}invariant\ m$

shows $exceptions\text{-}preserve\text{-}invariant\ m$

using $assms$

by $(auto\ simp: traces\text{-}preserve\text{-}invariant\text{-}def\ exceptions\text{-}preserve\text{-}invariant\text{-}def)$

lemma $traces\text{-}preserve\text{-}invariantE$:

assumes *traces-preserve-invariant m*
and $(m, t, m') \in \text{Traces}$ **and** *invariant s* **and** *reads-regs-from inv-regs t s*
obtains s' **where** *updates-regs inv-regs t s = Some s'* **and** *invariant s'*
using *assms*
by (*auto simp: traces-preserve-invariant-def elim: trace-preserves-invariantE*)

lemma *runs-preserve-invariantE*:
assumes *runs-preserve-invariant m*
and *Run m t a* **and** *invariant s* **and** *reads-regs-from inv-regs t s*
obtains s' **where** *updates-regs inv-regs t s = Some s'* **and** *invariant s'*
using *assms*
by (*auto simp: runs-preserve-invariant-def elim: trace-preserves-invariantE*)

lemma *Run-inv-bindE*:
assumes *Run-inv (m \gg f) t a regs* **and** *runs-preserve-invariant m*
obtains *tm am tf* **where** $t = tm @ tf$ **and** *Run-inv m tm am regs*
and *Run-inv (f am) tf a (the (updates-regs inv-regs tm regs))*
proof –
from *assms*
obtain *tm am tf*
where $t: t = tm @ tf$ **and** $tm: \text{Run } m \text{ } tm \text{ } am$ **and** $tf: \text{Run } (f \text{ } am) \text{ } tf \text{ } a$
and *regs: reads-regs-from inv-regs tm regs*
and $regs': \text{reads-regs-from inv-regs } tf \text{ } (the (updates-regs inv-regs tm regs))$
and *inv: invariant regs*
using *assms unfolding Run-inv-def*
by (*auto simp: regstate-simp elim!: Run-bindE*)
moreover obtain $regs'$ **where** $regs': \text{updates-regs inv-regs } tm \text{ } regs = \text{Some } regs'$
and $inv': \text{invariant } regs'$
using *assms tm inv regs*
by (*elim runs-preserve-invariantE*)
ultimately show *thesis*
using *that*
unfolding *Run-inv-def*
by *auto*
qed

named-theorems *preserves-invariantI*
named-theorems *trace-preserves-invariantI*

lemma *no-reg-writes-trace-preserves-invariantI*:
assumes *no-reg-writes-to inv-regs m*
and $(m, t, m') \in \text{Traces}$
shows *trace-preserves-invariant t*
using *assms*
by (*auto simp: trace-preserves-invariant-def*)

lemma *no-reg-writes-traces-preserve-invariantI*:
assumes *no-reg-writes-to inv-regs m*
shows *traces-preserve-invariant m*

```

using assms
by (auto simp: traces-preserve-invariant-def intro: no-reg-writes-trace-preserves-invariantI)

```

```

method preserves-invariantI uses intro simp elim =
  (intro intro preserves-invariantI conjI allI impI traces-runs-preserve-invariantI
traces-exceptions-preserve-invariantI;
  auto simp: simp elim!: elim)

```

```

lemma traces-preserve-invariant-bindI[preserves-invariantI]:
  assumes m: traces-preserve-invariant m
  and f:  $\bigwedge s\ t\ a.$  Run-inv m t a s  $\implies$  traces-preserve-invariant (f a)
  shows traces-preserve-invariant (m  $\ggg$  f)
proof -
  { fix s t m'
    assume (m  $\ggg$  f, t, m')  $\in$  Traces and s: invariant s and regs: reads-regs-from
inv-regs t s
    then have invariant (the (updates-regs inv-regs t s))
    proof (cases rule: bind-Traces-cases)
      case (Left m'')
        with m s regs show ?thesis
        by (auto simp: traces-preserve-invariant-def trace-preserves-invariant-def)
      next
        case (Bind tm am tf)
          then obtain s'
            where regs': reads-regs-from inv-regs tm s
              and s': updates-regs inv-regs tm s = Some s'
            using regs
            by (auto simp: regstate-simp elim: reads-regs-from-updates-regs-Some)
          then have invariant s' and Run-inv m tm am s
            using s m (Run m tm am)
            by (fastforce simp: traces-preserve-invariant-def trace-preserves-invariant-def
Run-inv-def)+
          then show ?thesis
            using Bind s' regs f[OF (Run-inv m tm am s)]
            by (auto simp: traces-preserve-invariant-def trace-preserves-invariant-def
regstate-simp)
          qed
        }
    then show ?thesis
      by (simp add: traces-preserve-invariant-def trace-preserves-invariant-def)
    qed
  }

```

```

lemma runs-preserve-invariant-bindI[preserves-invariantI]:
  assumes runs-preserve-invariant m and  $\bigwedge t\ a.$  Run m t a  $\implies$  runs-preserve-invariant
(f a)
  shows runs-preserve-invariant (m  $\ggg$  f)
  using assms

```

by (fastforce simp: runs-preserve-invariant-def elim!: Run-bindE intro: trace-preserves-invariant-appendI)

lemma runs-preserve-invariant-try-catchI[preserves-invariantI]:
 assumes runs-preserve-invariant m
 and exceptions-preserve-invariant m
 and $\bigwedge t e. (m, t, \text{Exception } e) \in \text{Traces} \implies \text{runs-preserve-invariant } (h \ e)$
 shows runs-preserve-invariant (try-catch m h)
 using assms
 by (fastforce simp: runs-preserve-invariant-def exceptions-preserve-invariant-def
 elim!: Run-try-catchE intro: trace-preserves-invariant-appendI)

lemma preserves-invariant-case-sum[preserves-invariantI]:
 assumes $\bigwedge a. \text{traces-preserve-invariant } (f \ a)$ and $\bigwedge b. \text{traces-preserve-invariant } (g \ b)$
 shows traces-preserve-invariant (case x of Inl a \Rightarrow f a | Inr b \Rightarrow g b)
 using assms
 by (auto split: sum.splits)

lemma preserves-invariant-case-option[preserves-invariantI]:
 assumes $\bigwedge a. \text{traces-preserve-invariant } (f \ a)$ and traces-preserve-invariant g
 shows traces-preserve-invariant (case x of Some a \Rightarrow f a | None \Rightarrow g)
 using assms
 by (auto split: option.splits)

lemma preserves-invariant-case-prod[preserves-invariantI]:
 assumes $\bigwedge x y. \text{traces-preserve-invariant } (f \ x \ y)$
 shows traces-preserve-invariant (case z of (x, y) \Rightarrow f x y)
 using assms
 by auto

lemmas no-reg-write-builtins-preserve-invariant[preserves-invariantI] =
 no-reg-write-builtins[THEN no-reg-writes-traces-preserve-invariantI]

lemma preserves-invariant-if[preserves-invariantI]:
 assumes $c \implies \text{traces-preserve-invariant } m1$ and $\neg c \implies \text{traces-preserve-invariant } m2$
 shows traces-preserve-invariant (if c then m1 else m2)
 using assms
 by auto

lemma preserves-invariant-try-catch[preserves-invariantI]:
 assumes traces-preserve-invariant m
 and $\bigwedge t e. (m, t, \text{Exception } e) \in \text{Traces} \implies \text{traces-preserve-invariant } (h \ e)$
 shows traces-preserve-invariant (try-catch m h)
 using assms
 by (fastforce simp: traces-preserve-invariant-def elim!: try-catch-Traces-cases
 intro: trace-preserves-invariant-appendI)

lemma preserves-invariant-catch-early-return[preserves-invariantI]:

```

assumes traces-preserve-invariant m
shows traces-preserve-invariant (catch-early-return m)
using assms
by (auto simp: catch-early-return-def intro: preserves-invariantI)

lemma runs-preserve-invariant-catch-early-returnI[preserves-invariantI]:
  assumes runs-preserve-invariant m
    and exceptions-preserve-invariant m
  shows runs-preserve-invariant (catch-early-return m)
  using assms
  unfolding catch-early-return-def
  by (auto intro!: preserves-invariantI no-reg-write-builtins-preserve-invariant [THEN
traces-runs-preserve-invariantI] split: sum.splits)

lemma preserves-invariant-liftR[preserves-invariantI]:
  assumes traces-preserve-invariant m
  shows traces-preserve-invariant (liftR m)
  using assms
  by (auto simp: liftR-def intro: preserves-invariantI)

lemma Nil-preserves-invariant[intro, simp]:
  trace-preserves-invariant []
  by (auto simp: trace-preserves-invariant-def)

lemma preserves-invariant-and-boolM[preserves-invariantI]:
  assumes traces-preserve-invariant m1 and traces-preserve-invariant m2
  shows traces-preserve-invariant (and-boolM m1 m2)
  using assms
  by (auto simp: and-boolM-def intro: preserves-invariantI)

lemma preserves-invariant-or-boolM[preserves-invariantI]:
  assumes traces-preserve-invariant m1 and traces-preserve-invariant m2
  shows traces-preserve-invariant (or-boolM m1 m2)
  using assms
  by (auto simp: or-boolM-def intro: preserves-invariantI)

lemma preserves-invariant-let[preserves-invariantI]:
  assumes traces-preserve-invariant (f y)
  shows traces-preserve-invariant (let x = y in f x)
  using assms
  by auto

lemma runs-preserve-invariant-foreachM[preserves-invariantI]:
  assumes  $\bigwedge x \text{ vars. runs-preserve-invariant (body x vars)}$ 
  shows runs-preserve-invariant (foreachM xs vars body)
  using assms
  by (induction xs arbitrary: vars) (auto intro: preserves-invariantI traces-runs-preserve-invariantI)

lemma preserves-invariant-foreachM[preserves-invariantI]:

```

assumes $\bigwedge x \text{ vars. } \text{traces-preserve-invariant } (\text{body } x \text{ vars})$
shows $\text{traces-preserve-invariant } (\text{foreachM } xs \text{ vars } \text{body})$
using *assms*
by (*induction xs arbitrary: vars*) (*auto intro: preserves-invariantI*)

end

2.3 Deterministic expressions

context *State-Invariant*
begin

definition $\text{Traces-inv regs} \equiv \{(m, t, m') \mid m \text{ t } m'. (m, t, m') \in \text{Traces} \wedge \text{reads-regs-from inv-regs } t \text{ regs} \wedge \text{invariant regs} \wedge \text{final } m'\}$

definition $\text{determ-traces } m \equiv (\forall t1 \ m1' \ \text{regs1 } t2 \ m2' \ \text{regs2}. (m, t1, m1') \in \text{Traces-inv regs1} \wedge (m, t2, m2') \in \text{Traces-inv regs2} \longrightarrow m1' = m2')$

definition $\text{determ-runs } m \equiv (\forall t1 \ a1 \ \text{regs1 } t2 \ a2 \ \text{regs2}. \text{Run-inv } m \ t1 \ a1 \ \text{regs1} \wedge \text{Run-inv } m \ t2 \ a2 \ \text{regs2} \longrightarrow a1 = a2)$

definition $\text{the-outcome } m \equiv (\text{THE } m'. \exists t \ \text{regs}. (m, t, m') \in \text{Traces-inv regs})$

definition $\text{the-result } m \equiv (\text{THE } a. \exists t \ \text{regs}. \text{Run-inv } m \ t \ a \ \text{regs})$

lemma *in-Traces-inv-iff*:

$(m, t, m') \in \text{Traces-inv regs} \longleftrightarrow (m, t, m') \in \text{Traces} \wedge \text{reads-regs-from inv-regs } t \text{ regs} \wedge \text{invariant regs} \wedge \text{final } m'$
by (*auto simp: Traces-inv-def*)

lemma *Run-inv-iff-Traces-inv*:

$\text{Run-inv } m \ t \ a \ \text{regs} \longleftrightarrow (m, t, \text{Done } a) \in \text{Traces-inv regs}$

unfolding *Run-inv-def in-Traces-inv-iff*

by (*auto simp: final-def*)

lemma *determ-tracesI*:

assumes $\bigwedge t \ m'' \ \text{regs}. (m, t, m'') \in \text{Traces-inv regs} \implies m'' = m'$

shows $\text{determ-traces } m$

using *assms*

unfolding *determ-traces-def*

by *blast*

lemma *determ-runsI*:

assumes $\bigwedge t \ a \ \text{regs}. \text{Run-inv } m \ t \ a \ \text{regs} \implies a = c$

shows $\text{determ-runs } m$

using *assms*

unfolding *determ-runs-def*

by *blast*

named-theorems *determ*

lemma *determ-the-outcome-eq*:
 assumes *determ-traces* *m* and $(m, t, m') \in \text{Traces-inv regs}$
 shows *the-outcome* $m = m'$
 using *assms*
 unfolding *the-outcome-def* *determ-traces-def* *in-Traces-inv-iff*
 by *blast*

lemma *determ-the-result-eq*:
 assumes *determ-runs* *m* and *Run-inv* *m* *t* *a* *regs*
 shows *the-result* $m = a$
 using *assms*
 unfolding *the-result-def* *determ-runs-def*
 by *blast*

lemma *determ-traces-runs*:
 assumes *determ-traces* *m*
 shows *determ-runs* *m*
 using *assms*
 unfolding *determ-traces-def* *determ-runs-def* *Run-inv-iff* *Traces-inv*
 by *blast*

lemma *determ-runs-return*[*determ*]: *determ-runs* (*return* *a*)
 by (*auto simp: determ-runs-def Run-inv-def*)

lemma *determ-traces-return*[*determ*]: *determ-traces* (*return* *a*)
 by (*auto simp: determ-traces-def in-Traces-inv-iff*)

lemma *determ-traces-throw*[*determ*]: *determ-traces* (*throw* *e*)
 by (*auto simp: determ-traces-def in-Traces-inv-iff*)

lemma *determ-runs-bindI*:
 assumes *determ-runs* *m* and *determ-runs* (*f* (*the-result* *m*)) and *runs-preserve-invariant* *m*
 shows *determ-runs* ($m \gg f$)
 using *assms*
 by (*intro* *determ-runsI*[**where** $c = \text{the-result } (f (\text{the-result } m))$])
 (*auto elim!: Run-inv-bindE simp: determ-the-result-eq*)

lemma *final-simps*[*intro*, *simp*]:
final (*Exception* *e*)
final (*Fail* *msg*)
 by (*auto simp: final-def*)

lemma *runs-preserve-invariant-Run-invariant*[*simp*]:
 assumes *runs-preserve-invariant* *m*
 and *Run* *m* *t* *a* and *invariant* *s* and *reads-regs-from* *inv-regs* *t* *s*
 shows *invariant* (*the* (*updates-regs* *inv-regs* *t* *s*))
 using *assms*
 by (*auto elim!: runs-preserve-invariantE*)

lemma *traces-preserve-invariant-Traces-invariant[simp]*:

assumes *traces-preserve-invariant m*

and $(m, t, m') \in \text{Traces}$ **and** *invariant s* **and** *reads-regs-from inv-regs t s*

shows *invariant (the (updates-regs inv-regs t s))*

using *assms*

by (*auto elim!: traces-preserve-invariantE*)

lemma *bind-Traces-inv-cases*:

assumes $(m \gg f, t, m') \in \text{Traces-inv regs}$ **and** *runs-preserve-invariant m*

obtains $(Ex) e$ **where** $(m, t, \text{Exception } e) \in \text{Traces-inv regs}$ **and** $m' = \text{Exception } e$

| $(\text{Fail}) \text{ msg}$ **where** $(m, t, \text{Fail msg}) \in \text{Traces-inv regs}$ **and** $m' = \text{Fail msg}$

| $(\text{Bind}) \text{ tm am tf}$ **where** $t = \text{tm} @ \text{tf}$ **and** *Run-inv m tm am regs*

and $(f \text{ am}, \text{tf}, m') \in \text{Traces-inv (the (updates-regs inv-regs tm regs))}$

using *assms Bind[of t []]*

unfolding *in-Traces-inv-iff*

by (*auto elim!: bind-Traces-cases final-bind-cases simp: Run-inv-def regstate-simp elim: final-cases*)

lemma *determ-traces-bindI*:

assumes *determ-traces m* **and** *runs-preserve-invariant m*

and $\bigwedge t \text{ a regs. } \text{Run-inv m t a regs} \implies \text{determ-traces (f a)}$

shows *determ-traces (m \gg f)*

unfolding *determ-traces-def*

using *assms*

by (*auto simp: Run-inv-iff-Traces-inv elim!: bind-Traces-inv-cases final-bind-cases dest!: determ-the-outcome-eq[OF assms(1), rotated] determ-the-outcome-eq[OF assms(3), rotated]*)

lemma *try-catch-eq-iff*:

$(\text{try-catch } m \text{ h} = \text{Done } a) \longleftrightarrow (m = \text{Done } a \vee (\exists e. m = \text{Exception } e \wedge h \text{ e} = \text{Done } a))$

$(\text{try-catch } m \text{ h} = \text{Exception } e) \longleftrightarrow (\exists e'. m = \text{Exception } e' \wedge h \text{ e}' = \text{Exception } e)$

$(\text{try-catch } m \text{ h} = \text{Fail msg}) \longleftrightarrow (m = \text{Fail msg} \vee (\exists e. m = \text{Exception } e \wedge h \text{ e} = \text{Fail msg}))$

by (*cases m; auto*)⁺

lemma *try-catch-Traces-inv-cases*:

assumes $(\text{try-catch } m \text{ h}, t, \text{mtc}) \in \text{Traces-inv regs}$ **and** *traces-preserve-invariant m*

obtains $(\text{Done}) a$ **where** *Run-inv m t a regs* **and** $\text{mtc} = \text{Done } a$

| $(\text{Fail}) \text{ msg}$ **where** $(m, t, \text{Fail msg}) \in \text{Traces-inv regs}$ **and** $\text{mtc} = \text{Fail msg}$

| $(Ex) \text{ tm ex th}$ **where** $(m, \text{tm}, \text{Exception ex}) \in \text{Traces-inv regs}$

and $(h \text{ ex}, \text{th}, \text{mtc}) \in \text{Traces-inv (the (updates-regs inv-regs tm regs))}$ **and** $t = \text{tm} @ \text{th}$

using *assms*

unfolding *in-Traces-inv-iff Run-inv-def*

by (*auto elim!:* *try-catch-Traces-cases final-cases simp: regstate-simp try-catch-eq-iff;*
fastforce)

lemma *determ-traces-try-catchI:*

assumes *determ-traces m and traces-preserve-invariant m*
and $\bigwedge e. \text{determ-traces } (h \ e)$
shows *determ-traces (try-catch m h)*
unfolding *determ-traces-def*
using *assms determ-the-outcome-eq[OF assms(3)] determ-the-outcome-eq[OF*
assms(1)]
by (*fastforce simp: Run-inv-iff-Traces-inv elim!:* *try-catch-Traces-inv-cases*
dest!: *determ-the-outcome-eq[OF assms(1), rotated] determ-the-outcome-eq[OF*
assms(3), rotated])

lemma *determ-traces-liftR[determ]:*

assumes *determ-traces m and traces-preserve-invariant m*
shows *determ-traces (liftR m)*
using *assms*
unfolding *liftR-def*
by (*auto intro!:* *determ-traces-try-catchI determ*)

lemma *determ-traces-catch-early-return[determ]:*

assumes *determ-traces m and traces-preserve-invariant m*
shows *determ-traces (catch-early-return m)*
using *assms*
unfolding *catch-early-return-def*
by (*auto intro!:* *determ-traces-try-catchI determ split: sum.splits*)

lemma *determ-traces-early-return[determ]:*

determ-traces (early-return a)
by (*auto simp: early-return-def intro: determ*)

lemma *determ-traces-foreachM:*

assumes $\bigwedge x \text{ vars}. x \in \text{set } xs \implies \text{determ-traces } (\text{body } x \text{ vars})$
and $\bigwedge x \text{ vars}. x \in \text{set } xs \implies \text{runs-preserve-invariant } (\text{body } x \text{ vars})$
shows *determ-traces (foreachM xs vars body)*
using *assms*
by (*induction xs arbitrary: vars*) (*auto intro: determ determ-traces-bindI*)

lemma *determ-runs-if:*

determ-runs (if c then m1 else m2) if c \implies determ-runs m1 and $\neg c \implies$
determ-runs m2
using *that*
by *auto*

lemma *determ-traces-if:*

determ-traces (if c then m1 else m2) if c \implies determ-traces m1 and $\neg c \implies$
determ-traces m2
using *that*

by auto

lemma *determ-traces-read-inv-reg*:

assumes $\text{name } r \in \text{inv-regs}$
 and $\forall \text{regs. invariant regs} \longrightarrow \text{get-regval } (\text{name } r) \text{ regs} = \text{Some } v \wedge \text{of-regval } r$
 $v = \text{Some } (\text{read-from } r \text{ regs})$
 shows *determ-traces* (*read-reg* r)
 using *assms*
 by (intro *determ-tracesI* [where $m' = \text{Done } (\text{the } (\text{of-regval } r \ v)))$)
 (auto simp: *Traces-inv-def read-reg-def elim!*: *Read-reg-TracesE final-cases split: option.splits*)

lemma *determ-runs-read-inv-reg*:

determ-runs (*read-reg* r) if $\text{name } r \in \text{inv-regs}$ and $\bigwedge \text{regs. invariant regs} \implies$
 $\text{get-regval } (\text{name } r) \text{ regs} = \text{Some } v$
 using *that*
 by (intro *determ-runsI* [where $c = \text{the } (\text{of-regval } r \ v)]$)
 (auto simp: *determ-runs-def Run-inv-def elim!*: *Run-read-regE*)

lemma *determ-runs-or-boolM*[*determ*]:

determ-runs (*or-boolM* $m1 \ m2$) if *determ-runs* $m1$ and *determ-runs* $m2$ and
runs-preserve-invariant $m1$
 using *that*
 unfolding *or-boolM-def*
 by (auto intro!: *determ-runs-bindI determ-runs-return*)

lemma *determ-runs-assert-exp*[*determ*]: *determ-runs* (*assert-exp* $e \ \text{msg}$)

by (intro *determ-runsI*) auto

lemma *determ-runs-case-prod*[*determ*]:

determ-runs (*case* x of $(y, z) \Rightarrow f \ y \ z$) if $\bigwedge y \ z. x = (y, z) \implies \text{determ-runs } (f \ y \ z)$
 using *that*
 by auto

lemma *determ-runs-case-option*[*determ*]:

determ-runs (*case* x of $\text{Some } y \Rightarrow f \ y \mid \text{None} \Rightarrow g$) if $\bigwedge y. x = \text{Some } y \implies$
determ-runs ($f \ y$) and *determ-runs* g
 using *that*
 by (*cases* x) auto

lemma *determ-traces-exit*[*determ*]: *determ-traces* (*exit0* u)

by (intro *determ-tracesI*) (auto simp: *exit0-def in-Traces-inv-iff*)

lemmas *determ-runs-exit0* = *determ-traces-exit*[*THEN determ-traces-runs, determ*]

end

(Conditionally) deterministic monadic expressions

definition *deterministic-if* $P\ m\ c \equiv (\forall\ t\ a.\ \text{Run}\ m\ t\ a \wedge P\ t \longrightarrow a = c)$

definition *prefix-closed* $P \equiv (\forall\ t1\ t2.\ P\ (t1\ @\ t2) \longrightarrow P\ t1)$

lemma *Run-bind-deterministic-ifE*:

assumes *prefix-closed* P

and *deterministic-if* $P\ m\ c$

and $\text{Run}\ (m\ \gg\ f)\ t\ a$

and $P\ t$

obtains $tm\ tf$ **where** $\text{Run}\ m\ tm\ c$ and $\text{Run}\ (f\ c)\ tf\ a$ and $t = tm\ @\ tf$

using *assms*

by (*elim Run-bindE*) (*auto simp: deterministic-if-def prefix-closed-def*)

abbreviation *deterministic-exp* $\equiv \text{deterministic-if}\ (\lambda\ -. \ \text{True})$

lemma *Run-bind-deterministic-expE*:

assumes *deterministic-exp* $m\ c$

and $\text{Run}\ (m\ \gg\ f)\ t\ a$

obtains $tm\ tf$ **where** $\text{Run}\ m\ tm\ c$ and $\text{Run}\ (f\ c)\ tf\ a$ and $t = tm\ @\ tf$

using *assms*

by (*elim Run-bindE*) (*auto simp: deterministic-if-def*)

end

theory *Recognising-Automata*

imports *Cheri-axioms-lemmas Sail.Sail2-state-lemmas Trace-Assumptions*

begin

2.4 Verification tools for CHERI properties

For proving that a concrete ISA satisfies the CHERI axioms, we define an automaton for each axiom that only accepts traces satisfying the axiom. The state of the automaton keeps track of relevant information, e.g. the capabilities read so far.

This makes it easy to decompose proofs about complete instruction traces into proofs about parts of a trace, e.g. corresponding to calls to auxiliary functions.

locale *Deterministic-Automaton* =

fixes *enabled* :: $'s \Rightarrow 'rv\ event \Rightarrow bool$

and *step* :: $'s \Rightarrow 'rv\ event \Rightarrow 's$

and *initial* :: $'s$

and *final* :: $'s \Rightarrow bool$

begin

fun *trace-enabled* :: $'s \Rightarrow 'rv\ trace \Rightarrow bool$ **where**

trace-enabled $s\ [] = \text{True}$

| *trace-enabled* $s\ (e\ \# \ t) = (\text{enabled}\ s\ e \wedge \text{trace-enabled}\ (\text{step}\ s\ e)\ t)$

abbreviation *run* :: $'s \Rightarrow 'rv\ trace \Rightarrow 's$ **where** $\text{run}\ s\ t \equiv \text{foldl}\ \text{step}\ s\ t$

definition *accepts-from* :: 's \Rightarrow 'rv trace \Rightarrow bool **where**
accepts-from s t \equiv trace-enabled s t \wedge final (run s t)

abbreviation *accepts* \equiv *accepts-from initial*

lemma *trace-enabled-append-iff*: trace-enabled s (t @ t') \longleftrightarrow trace-enabled s t \wedge trace-enabled (run s t) t'
by (induction t arbitrary: s) auto

lemma *accepts-from-append-iff*: *accepts-from* s (t @ t') \longleftrightarrow trace-enabled s t \wedge *accepts-from* (run s t) t'
by (auto simp: *accepts-from-def* trace-enabled-append-iff)

lemma *accepts-from-Cons[simp]*: *accepts-from* s (e # t) \longleftrightarrow enabled s e \wedge *accepts-from* (step s e) t
by (auto simp: *accepts-from-def*)

lemma *accepts-from-id-take-nth-drop*:
assumes i < length t
shows *accepts-from* s t = *accepts-from* s (take i t @ t ! i # drop (Suc i) t)
using *assms*
by (auto simp: *id-take-nth-drop[symmetric]*)

lemma *accepts-from-trace-enabledI*:
assumes *accepts-from* s t
shows trace-enabled s t
using *assms*
by (auto simp: *accepts-from-def*)

lemma *accepts-from-trace-enabled-takeI*:
assumes *accepts-from* s t
shows trace-enabled s (take i t)
using *assms*
by (cases i < length t)
(auto simp: *accepts-from-id-take-nth-drop* *accepts-from-append-iff* intro: *accepts-from-trace-enabledI*)

lemma *accepts-from-nth-enabledI*:
assumes *accepts-from* s t
and i < length t
shows enabled (run s (take i t)) (t ! i)
using *assms*
by (auto simp: *accepts-from-id-take-nth-drop* *accepts-from-append-iff*)

lemma *accepts-from-iff-all-enabled-final*:
accepts-from s t \longleftrightarrow (\forall i < length t. enabled (run s (take i t)) (t ! i)) \wedge final (run s t)
by (induction t arbitrary: s)
(auto simp: *accepts-from-def* *nth-Cons* split: nat.splits)

lemma *trace-enabled-acceptI*:
assumes *trace-enabled s t* **and** *final (run s t)*
shows *accepts-from s t*
using *assms*
by (*auto simp: accepts-from-def*)

named-theorems *trace-simp*
named-theorems *trace-elim*

lemma *Nil-trace-enabled[trace-elim]*:
assumes $t = []$
shows *trace-enabled s t*
using *assms*
by *auto*

lemma *bind-TracesE*:
assumes $(m \gg f, t, m') \in \text{Traces}$
and $\bigwedge tm\ tf\ m''. (m, tm, m'') \in \text{Traces} \implies t = tm @ tf \implies P\ tm$
and $\bigwedge tm\ am\ tf. (f\ am, tf, m') \in \text{Traces} \implies \text{Run } m\ tm\ am \implies t = tm @ tf$
 $\implies P\ tm \implies P\ (tm @ tf)$
shows $P\ t$
proof (*use assms in (cases rule: bind-Traces-cases)*)
case (*Left m''*)
then show *?thesis* **using** *assms(2)* [**where** $tm = t$ **and** $tf = []$] **by** *auto*
next
case (*Bind tm am tf*)
then show *?thesis* **using** *assms(2)* *assms(3)* **by** *auto*
qed

lemma *Run-bind-trace-enabled[trace-elim]*:
assumes $\text{Run } (m \gg f) t\ a$
and $\bigwedge tm\ tf\ am. t = tm @ tf \implies \text{Run } m\ tm\ am \implies \text{trace-enabled } s\ tm$
and $\bigwedge tm\ tf\ am. t = tm @ tf \implies \text{Run } m\ tm\ am \implies \text{Run } (f\ am) tf\ a \implies$
 $\text{trace-enabled } (\text{run } s\ tm) tf$
shows *trace-enabled s t*
using *assms*
by (*elim Run-bindE*) (*auto simp: trace-enabled-append-iff*)

lemma *Exception-bind-trace-enabled*:
assumes $(m \gg f, t, \text{Exception } e) \in \text{Traces}$
and $(m, t, \text{Exception } e) \in \text{Traces} \implies \text{trace-enabled } s\ t$
and $\bigwedge tm\ tf\ am. t = tm @ tf \implies \text{Run } m\ tm\ am \implies \text{trace-enabled } s\ tm$
and $\bigwedge tm\ tf\ am. t = tm @ tf \implies \text{Run } m\ tm\ am \implies (f\ am, tf, \text{Exception } e)$
 $\in \text{Traces} \implies \text{trace-enabled } (\text{run } s\ tm) tf$
shows *trace-enabled s t*
proof (*use assms in (cases rule: bind-Traces-cases)*)
case (*Left m''*)
then consider $(Ex)\ m'' = \text{Exception } e \mid (Done)\ a$ **where** $m'' = Done\ a$ **and** f
 $a = \text{Exception } e$

```

    by (cases m'') auto
  then show ?thesis
    using  $\langle (m, t, m'') \in \text{Traces} \rangle$  assms
    by cases auto
next
  case (Bind tm am tf)
  then show ?thesis
    using assms
    by (auto simp: trace-enabled-append-iff)
qed

```

lemma *bind-Traces-trace-enabled*[*trace-elim*]:

```

  assumes  $(m \gg f, t, m') \in \text{Traces}$ 
    and  $\bigwedge tm\ tf\ m''. (m, tm, m'') \in \text{Traces} \implies t = tm @ tf \implies \text{trace-enabled } s\ tm$ 
    and  $\bigwedge tm\ am\ tf. (f\ am, tf, m') \in \text{Traces} \implies \text{Run } m\ tm\ am \implies t = tm @ tf$ 
   $\implies \text{trace-enabled } (\text{run } s\ tm)\ tf$ 
  shows trace-enabled  $s\ t$ 
  using assms
  by (elim bind-TracesE) (auto simp: trace-enabled-append-iff)

```

lemma *try-catch-trace-enabled*[*trace-elim*]:

```

  assumes  $(\text{try-catch } m\ h, t, m') \in \text{Traces}$ 
    and  $\bigwedge n\ m''. (m, \text{take } n\ t, m'') \in \text{Traces} \implies \text{trace-enabled } s\ (\text{take } n\ t)$ 
    and  $\bigwedge tm\ ex\ th. (h\ ex, th, m') \in \text{Traces} \implies (m, tm, \text{Exception } ex) \in \text{Traces}$ 
   $\implies t = tm @ th \implies \text{trace-enabled } (\text{run } s\ tm)\ th$ 
  shows trace-enabled  $s\ t$ 
proof (use assms in  $\langle \text{cases rule: try-catch-Traces-cases} \rangle$ )
  case (NoEx m'')
  then show ?thesis using assms(2)[of length  $t\ m''$ ] by auto
next
  case (Ex tm ex th)
  then show ?thesis using assms(2)[of length  $tm$ ] assms(3) by (auto simp:
    trace-enabled-append-iff)
qed

```

lemma *if-Traces-trace-enabled*[*trace-elim*]:

```

  assumes  $(\text{if } b \text{ then } m1 \text{ else } m2, t, m') \in \text{Traces}$ 
    and  $b \implies (m1, t, m') \in \text{Traces} \implies \text{trace-enabled } s\ t$ 
    and  $\neg b \implies (m2, t, m') \in \text{Traces} \implies \text{trace-enabled } s\ t$ 
  shows trace-enabled  $s\ t$ 
  using assms by (cases b) auto

```

lemma *let-Traces-trace-enabled*[*trace-elim*]:

```

  assumes  $(\text{let } x = y \text{ in } f\ x, t, m') \in \text{Traces}$ 
    and  $(f\ y, t, m') \in \text{Traces} \implies \text{trace-enabled } s\ t$ 
  shows trace-enabled  $s\ t$ 
  using assms by auto

```


lemma *case-prod-Traces-trace-enabled*[*trace-elim*]:
assumes (*case p of (a, b) \Rightarrow f a b, t, m'*) \in *Traces*
and $\bigwedge x y. p = (x, y) \Longrightarrow (f\ x\ y, t, m') \in \textit{Traces} \Longrightarrow \textit{trace-enabled}\ s\ t$
shows *trace-enabled s t*
using *assms by (cases p) auto*

lemma *case-option-Traces-trace-enabled*[*trace-elim*]:
assumes (*case x of Some y \Rightarrow f y | None \Rightarrow m, t, m'*) \in *Traces*
and $\bigwedge y. (f\ y, t, m') \in \textit{Traces} \Longrightarrow x = \textit{Some}\ y \Longrightarrow \textit{trace-enabled}\ s\ t$
and $(m, t, m') \in \textit{Traces} \Longrightarrow x = \textit{None} \Longrightarrow \textit{trace-enabled}\ s\ t$
shows *trace-enabled s t*
using *assms by (cases x) auto*

lemma *return-trace-enabled*[*trace-elim*]:
assumes (*return a, t, m'*) \in *Traces*
shows *trace-enabled s t*
using *assms*
by (*auto simp: return-def*)

lemma *throw-trace-enabled*[*trace-elim*]:
assumes (*throw e, t, m'*) \in *Traces*
shows *trace-enabled s t*
using *assms*
by (*auto simp: throw-def*)

lemma *early-return-trace-enabled*[*trace-elim*]:
assumes (*early-return a, t, m'*) \in *Traces*
shows *trace-enabled s t*
using *assms*
by (*auto simp: early-return-def elim!: trace-elim*)

lemma *catch-early-return-trace-enabled*[*trace-elim*]:
assumes (*catch-early-return m, t, m'*) \in *Traces*
and $\bigwedge n m''. (m, \textit{take}\ n\ t, m'') \in \textit{Traces} \Longrightarrow \textit{trace-enabled}\ s\ (\textit{take}\ n\ t)$
shows *trace-enabled s t*
using *assms*
by (*auto simp: catch-early-return-def elim!: trace-elim split: sum.splits*)

lemma *liftR-trace-enabled*[*trace-elim*]:
assumes (*liftR m, t, m'*) \in *Traces*
and $\bigwedge n m''. (m, \textit{take}\ n\ t, m'') \in \textit{Traces} \Longrightarrow \textit{trace-enabled}\ s\ (\textit{take}\ n\ t)$
shows *trace-enabled s t*
using *assms*
by (*auto simp: liftR-def elim!: trace-elim*)

lemma *foreachM-inv-trace-enabled*:
assumes (*foreachM xs vars body, t, m'*) \in *Traces*
and $\bigwedge s\ x\ vars\ t\ m'. (body\ x\ vars, t, m') \in \textit{Traces} \Longrightarrow P\ s \Longrightarrow x \in \textit{set}\ xs \Longrightarrow \textit{trace-enabled}\ s\ t$

and $\bigwedge s \ x \ \text{vars} \ t \ \text{vars}'. \text{Run} \ (\text{body} \ x \ \text{vars}) \ t \ \text{vars}' \implies P \ s \implies x \in \text{set} \ xs \implies$
 $P \ (\text{run} \ s \ t)$
and $P \ s$
shows $\text{trace-enabled} \ s \ t$
using assms
by $(\text{induction} \ xs \ \text{arbitrary}: s \ t \ \text{vars}) \ (\text{auto} \ \text{simp}: \text{trace-enabled-append-iff} \ \text{elim}!:$
 $\text{trace-elim})$

lemma $\text{foreachM-const-trace-enabled}[\text{trace-elim}]$:
assumes $(\text{foreachM} \ xs \ \text{vars} \ \text{body}, t, m') \in \text{Traces}$
and $\bigwedge x \ \text{vars} \ t \ m'. (\text{body} \ x \ \text{vars}, t, m') \in \text{Traces} \implies x \in \text{set} \ xs \implies \text{trace-enabled}$
 $s \ t$
and $\bigwedge x \ \text{vars} \ t \ \text{vars}'. \text{Run} \ (\text{body} \ x \ \text{vars}) \ t \ \text{vars}' \implies x \in \text{set} \ xs \implies \text{run} \ s \ t = s$
shows $\text{trace-enabled} \ s \ t$
using assms
by $(\text{elim} \ \text{foreachM-inv-trace-enabled}[\text{where} \ P = \lambda s'. s' = s]) \ \text{auto}$

lemma $\text{Run-and-boolM-trace-enabled}[\text{trace-elim}]$:
assumes $\text{Run} \ (\text{and-boolM} \ l \ r) \ t \ a$
and $\bigwedge tl \ tr \ al. t = tl \ @ \ tr \implies \text{Run} \ l \ tl \ al \implies \text{trace-enabled} \ s \ tl$
and $\bigwedge tl \ tr. t = tl \ @ \ tr \implies \text{Run} \ l \ tl \ \text{True} \implies \text{Run} \ r \ tr \ a \implies \text{trace-enabled}$
 $(\text{run} \ s \ tl) \ tr$
shows $\text{trace-enabled} \ s \ t$
using assms
unfolding and-boolM-def
by $(\text{elim} \ \text{Run-bind-trace-enabled}) \ (\text{auto} \ \text{simp}: \text{return-def} \ \text{split}: \text{if-splits})$

lemma $\text{and-boolM-trace-enabled}[\text{trace-elim}]$:
assumes $(\text{and-boolM} \ m1 \ m2, t, m') \in \text{Traces}$
and $\bigwedge tm \ tf \ m''. (m1, tm, m'') \in \text{Traces} \implies t = tm \ @ \ tf \implies \text{trace-enabled} \ s$
 tm
and $\bigwedge tm \ tf. (m2, tf, m') \in \text{Traces} \implies \text{Run} \ m1 \ tm \ \text{True} \implies t = tm \ @ \ tf \implies$
 $\text{trace-enabled} \ (\text{run} \ s \ tm) \ tf$
shows $\text{trace-enabled} \ s \ t$
using assms
by $(\text{auto} \ \text{simp}: \text{and-boolM-def} \ \text{elim}!:\ \text{trace-elim})$

lemma $\text{Run-or-boolM-trace-enabled}[\text{trace-elim}]$:
assumes $\text{Run} \ (\text{or-boolM} \ l \ r) \ t \ a$
and $\bigwedge tl \ tr \ al. t = tl \ @ \ tr \implies \text{Run} \ l \ tl \ al \implies \text{trace-enabled} \ s \ tl$
and $\bigwedge tl \ tr. t = tl \ @ \ tr \implies \text{Run} \ l \ tl \ \text{False} \implies \text{Run} \ r \ tr \ a \implies \text{trace-enabled}$
 $(\text{run} \ s \ tl) \ tr$
shows $\text{trace-enabled} \ s \ t$
using assms
unfolding or-boolM-def
by $(\text{elim} \ \text{Run-bind-trace-enabled}) \ (\text{auto} \ \text{simp}: \text{return-def} \ \text{split}: \text{if-splits})$

lemma $\text{or-boolM-trace-enabled}[\text{trace-elim}]$:
assumes $(\text{or-boolM} \ m1 \ m2, t, m') \in \text{Traces}$

```

and  $\bigwedge tm\ tf\ m''. (m1, tm, m'') \in Traces \implies t = tm @ tf \implies trace-enabled\ s$ 
 $tm$ 
and  $\bigwedge tm\ tf. (m2, tf, m') \in Traces \implies Run\ m1\ tm\ False \implies t = tm @ tf$ 
 $\implies trace-enabled\ (run\ s\ tm)\ tf$ 
shows  $trace-enabled\ s\ t$ 
using assms
by (auto simp: or-boolM-def elim!: trace-elim)

end

```

An automaton for the axiom that capabilities stored to memory must be derivable from accessible capabilities

```

record ('cap, 'regval) axiom-state =
  accessed-caps :: 'cap set
  system-reg-access :: bool
  read-from-KCC :: 'regval set
  written-regs :: string set

```

```

locale Cap-Axiom-Automaton = Capability-ISA CC ISA
  for CC :: 'cap Capability-class and ISA :: ('cap, 'regval, 'instr, 'e) isa +
  fixes enabled :: ('cap, 'regval) axiom-state  $\Rightarrow$  'regval event  $\Rightarrow$  bool
begin

```

```

definition accessible-regs :: ('cap, 'regval) axiom-state  $\Rightarrow$  register-name set where
  accessible-regs s = {r. r  $\notin$  written-regs s  $\wedge$  (r  $\in$  privileged-regs ISA  $\longrightarrow$  system-reg-access s)}
```

```

definition axiom-step :: ('cap, 'regval) axiom-state  $\Rightarrow$  'regval event  $\Rightarrow$  ('cap, 'regval)
axiom-state where
  axiom-step s e = ( $\bigwedge$  accessed-caps = accessed-caps s  $\cup$  accessed-mem-caps e  $\cup$ 
accessed-reg-caps (accessible-regs s) e,
    system-reg-access = system-reg-access s  $\vee$  allows-system-reg-access
(accessible-regs s) e,
    read-from-KCC = read-from-KCC s  $\cup$  {v.  $\exists r \in KCC\ ISA. e =$ 
E-read-reg r v}},
    written-regs = written-regs s  $\cup$  {r.  $\exists v\ c. e = E-write-reg\ r\ v \wedge c$ 
 $\in caps-of-regval\ ISA\ v \wedge is-tagged-method\ CC\ c$ })
```

```

lemma step-selectors[simp]:
  accessed-caps (axiom-step s e) = accessed-caps s  $\cup$  accessed-mem-caps e  $\cup$  accessed-reg-caps
(accessible-regs s) e
  system-reg-access (axiom-step s e)  $\longleftrightarrow$  system-reg-access s  $\vee$  allows-system-reg-access
(accessible-regs s) e
  read-from-KCC (axiom-step s e) = read-from-KCC s  $\cup$  {v.  $\exists r \in KCC\ ISA. e =$ 
E-read-reg r v}
  written-regs (axiom-step s e) = written-regs s  $\cup$  {r.  $\exists v\ c. e = E-write-reg\ r\ v \wedge$ 
 $c \in caps-of-regval\ ISA\ v \wedge is-tagged-method\ CC\ c$ }
  by (auto simp: axiom-step-def)

```

abbreviation $initial \equiv \langle accessed-caps = \{\}, system-reg-access = False, read-from-KCC = \{\}, written-regs = \{\} \rangle$

lemma *accessible-regs-initial-iff*[simp]:
 $r \in accessible-regs\ initial \longleftrightarrow r \notin privileged-regs\ ISA$
by (auto simp: accessible-regs-def)

sublocale *Deterministic-Automaton enabled axiom-step initial* $\lambda\cdot$. *True* .

lemma *cap-reg-written-before-idx-written-regs*:
 $cap-reg-written-before-idx\ CC\ ISA\ i\ r\ t \longleftrightarrow r \in written-regs\ (run\ initial\ (take\ i\ t))$
proof (induction i)
 case (Suc i)
 then show ?case
 by (cases i < length t) (auto simp: take-Suc-conv-app-nth)
qed auto

lemma *accessible-regs-axiom-step*:
 $accessible-regs\ (axiom-step\ s\ e) =$
 $accessible-regs\ s \cup$
 $(if\ allows-system-reg-access\ (accessible-regs\ s)\ e\ then\ privileged-regs\ ISA\ else\ \{\}) -$
 $written-regs\ (axiom-step\ s\ e)$
by (auto simp: accessible-regs-def)

lemma *system-reg-access-run-take-eq*[simp]:
 $system-access-permitted-before-idx\ CC\ ISA\ i\ t \longleftrightarrow system-reg-access\ (run\ initial\ (take\ i\ t))$
 (is ?sys-reg-access i)
 $accessible-regs-at-idx\ i\ t = accessible-regs\ (run\ initial\ (take\ i\ t))$
 (is ?accessible-regs i)
proof (induction i)
 case (Suc i)
 show ?accessible-regs (Suc i)
 by (cases i < length t)
 (auto simp: Suc.IH accessible-regs-def accessible-regs-at-idx-def
 cap-reg-written-before-idx-written-regs take-Suc-conv-app-nth)
 show ?sys-reg-access (Suc i)
 by (cases i < length t) (auto simp: Suc.IH take-Suc-conv-app-nth)
qed (auto simp: accessible-regs-def)

lemma *accessed-caps-run-take-eq*[simp]:
 $available-caps\ CC\ ISA\ i\ t = accessed-caps\ (run\ initial\ (take\ i\ t))$
proof (induction i)
 case (Suc i)
 then show ?case
 by (cases i < length t) (auto simp add: available-caps-Suc take-Suc-conv-app-nth)
qed auto

lemma *read-from-KCC-run-take-eq*:

read-from-KCC (*run initial* (*take i t*)) = {*v*. $\exists r j. j < i \wedge j < \text{length } t \wedge t ! j$
= *E-read-reg* *r v* $\wedge r \in \text{KCC ISA}$ }

proof (*induction i*)

case (*Suc i*)

then show ?case

using *system-reg-access-run-take-eq*(1)[*of i t*]

by (cases *i < length t*) (auto simp: *take-Suc-conv-app-nth less-Suc-eq*)

qed auto

lemma *write-only-regs-run-take-eq*:

written-regs (*run initial* (*take i t*)) = {*r*. $\exists v c j. t ! j = \text{E-write-reg } r v \wedge j < i$
 $\wedge j < \text{length } t \wedge c \in \text{caps-of-regval ISA } v \wedge \text{is-tagged-method } CC c$ }

proof (*induction i*)

case (*Suc i*)

then show ?case

by (cases *i < length t*) (auto simp: *take-Suc-conv-app-nth less-Suc-eq*)

qed auto

lemmas *step-defs* = *axiom-step-def reads-mem-cap-def*

abbreviation *special-reg-names* $\equiv PCC \text{ ISA} \cup IDC \text{ ISA} \cup KCC \text{ ISA} \cup \text{privileged-regs ISA}$

definition *non-cap-reg* :: ('regstate, 'regval, 'a) register-ref \Rightarrow bool **where**

non-cap-reg *r* \equiv

name *r* $\notin PCC \text{ ISA} \cup IDC \text{ ISA} \cup KCC \text{ ISA} \cup \text{privileged-regs ISA} \wedge$

$(\forall rv v. \text{of-regval } r rv = \text{Some } v \longrightarrow \text{caps-of-regval ISA } rv = \{\}) \wedge$

$(\forall v. \text{caps-of-regval ISA } (\text{regval-of } r v) = \{\})$

fun *non-cap-event* :: 'regval event \Rightarrow bool **where**

non-cap-event (*E-read-reg* *r v*) = (*r* $\notin PCC \text{ ISA} \cup IDC \text{ ISA} \cup KCC \text{ ISA} \cup$
privileged-regs ISA $\wedge \text{caps-of-regval ISA } v = \{\}$)

| *non-cap-event* (*E-write-reg* *r v*) = (*r* $\notin PCC \text{ ISA} \cup IDC \text{ ISA} \cup KCC \text{ ISA} \cup$
privileged-regs ISA $\wedge \text{caps-of-regval ISA } v = \{\}$)

| *non-cap-event* (*E-read-memt* - - -) = False

| *non-cap-event* (*E-read-mem* - - -) = False

| *non-cap-event* (*E-write-memt* - - - -) = False

| *non-cap-event* (*E-write-mem* - - - -) = False

| *non-cap-event* - = True

fun *non-mem-event* :: 'regval event \Rightarrow bool **where**

non-mem-event (*E-read-memt* - - -) = False

| *non-mem-event* (*E-read-mem* - - -) = False

| *non-mem-event* (*E-write-memt* - - - -) = False

| *non-mem-event* (*E-write-mem* - - - -) = False

| *non-mem-event* - = True

definition *non-cap-trace* :: 'regval trace \Rightarrow bool **where**
non-cap-trace $t \equiv (\forall e \in \text{set } t. \text{non-cap-event } e)$

definition *non-mem-trace* :: 'regval trace \Rightarrow bool **where**
non-mem-trace $t \equiv (\forall e \in \text{set } t. \text{non-mem-event } e)$

lemma *non-cap-trace-Nil*[intro, simp]:
non-cap-trace []
by (auto simp: *non-cap-trace-def*)

lemma *non-cap-trace-Cons*[iff]:
non-cap-trace ($e \# t$) \longleftrightarrow *non-cap-event* $e \wedge$ *non-cap-trace* t
by (auto simp: *non-cap-trace-def*)

lemma *non-cap-trace-append*[iff]:
non-cap-trace ($t1 @ t2$) \longleftrightarrow *non-cap-trace* $t1 \wedge$ *non-cap-trace* $t2$
by (induction $t1$) auto

lemma *non-mem-trace-Nil*[intro, simp]:
non-mem-trace []
by (auto simp: *non-mem-trace-def*)

lemma *non-mem-trace-Cons*[iff]:
non-mem-trace ($e \# t$) \longleftrightarrow *non-mem-event* $e \wedge$ *non-mem-trace* t
by (auto simp: *non-mem-trace-def*)

lemma *non-mem-trace-append*[iff]:
non-mem-trace ($t1 @ t2$) \longleftrightarrow *non-mem-trace* $t1 \wedge$ *non-mem-trace* $t2$
by (induction $t1$) auto

lemma *non-cap-event-non-mem-event*:
non-mem-event e **if** *non-cap-event* e
using *that*
by (cases e) auto

lemma *non-cap-trace-non-mem-trace*:
non-mem-trace t **if** *non-cap-trace* t
using *that*
by (auto simp: *non-mem-trace-def* *non-cap-trace-def* intro: *non-cap-event-non-mem-event*)

lemma *non-cap-event-axiom-step-inv*:
assumes *non-cap-event* e
shows *axiom-step* $s \ e = s$
using *assms*
by (elim *non-cap-event.elims*) (auto simp: *step-defs* *bind-eq-Some-conv* *split*: *option.splits*)

lemma *non-cap-trace-run-inv*:
assumes *non-cap-trace* t

shows $\text{run } s \ t = s$
using *assms*
by (*induction t*) (*auto simp: non-cap-event-axiom-step-inv*)

definition *non-cap-exp* :: ('regval, 'a, 'exception) monad \Rightarrow bool **where**
 $\text{non-cap-exp } m = (\forall t \ m'. (m, t, m') \in \text{Traces} \longrightarrow (\text{non-cap-trace } t \vee (\exists t' \ r \ v \text{ msg. } t = t' @ [E\text{-read-reg } r \ v] \wedge r \notin \text{special-reg-names} \wedge \text{non-cap-trace } t' \wedge m' = \text{Fail msg})))$

definition *non-mem-exp* :: ('regval, 'a, 'exception) monad \Rightarrow bool **where**
 $\text{non-mem-exp } m = (\forall t \ m'. (m, t, m') \in \text{Traces} \longrightarrow \text{non-mem-trace } t)$

lemma *non-cap-exp-Traces-cases*:
assumes *non-cap-exp m*
and $(m, t, m') \in \text{Traces}$
obtains (*Non-cap*) *non-cap-trace t*
 $|$ (*Fail*) $t' \ r \ v \text{ msg}$ **where** $t = t' @ [E\text{-read-reg } r \ v]$ **and** $r \notin \text{special-reg-names}$
and $m' = \text{Fail msg}$ **and** *non-cap-trace t'*
using *assms*
unfolding *non-cap-exp-def*
by *blast*

lemma *non-cap-exp-non-mem-exp*:
 $\text{non-mem-exp } m$ **if** *non-cap-exp m*
by (*auto simp: non-mem-exp-def elim!: non-cap-exp-Traces-cases[OF that] intro: non-cap-trace-non-mem-trace*)

lemma *non-cap-exp-Run-non-cap-trace*:
assumes $m: \text{non-cap-exp } m$
and $t: \text{Run } m \ t \ a$
shows *non-cap-trace t*
using *t*
by (*elim non-cap-exp-Traces-cases[OF m] auto*)

lemmas *non-cap-exp-Run-run-invI* = *non-cap-exp-Run-non-cap-trace[THEN non-cap-trace-run-inv]*

named-theorems *non-cap-expI*
named-theorems *non-mem-expI*

lemma *non-cap-exp-return[non-cap-expI]*:
 $\text{non-cap-exp } (\text{return } a)$
by (*auto simp: non-cap-exp-def return-def*)

lemma *non-cap-exp-bindI[intro!]*:
assumes $m: \text{non-cap-exp } m$
and $f: \bigwedge t \ a. \text{Run } m \ t \ a \Longrightarrow \text{non-cap-exp } (f \ a)$
shows $\text{non-cap-exp } (m \gg f)$
proof (*unfold non-cap-exp-def, intro allI impI*)
fix $t \ m'$

```

assume  $(m \gg= f, t, m') \in \text{Traces}$ 
then show  $\text{non-cap-trace } t \vee (\exists t' r v \text{ msg. } t = t' @ [E\text{-read-reg } r \ v] \wedge r \notin$ 
 $\text{special-reg-names} \wedge \text{non-cap-trace } t' \wedge m' = \text{Fail msg})$ 
proof (cases rule: bind-Traces-cases)
  case (Left  $m''$ )
    then show ?thesis
    by (elim non-cap-exp-Traces-cases[OF  $m$ ]) auto
  next
    case (Bind  $tm \ am \ tf$ )
    then show ?thesis
    using non-cap-exp-Run-non-cap-trace[OF  $m \ \langle \text{Run } m \ tm \ am \rangle$ ]
    by (elim f[OF  $\langle \text{Run } m \ tm \ am \rangle$ , THEN non-cap-exp-Traces-cases]) auto
qed
qed

```

```

lemma non-mem-exp-bindI[intro!]:
  assumes non-mem-exp  $m$ 
  and  $\bigwedge t a. \text{Run } m \ t \ a \implies \text{non-mem-exp } (f \ a)$ 
  shows non-mem-exp  $(m \gg= f)$ 
  using assms
  by (fastforce simp: non-mem-exp-def elim!: bind-Traces-cases)

```

```

lemma non-cap-exp-try-catch[intro!]:
  assumes  $m: \text{non-cap-exp } m$ 
  and  $h: \bigwedge t ex. (m, t, \text{Exception } ex) \in \text{Traces} \implies \text{non-cap-exp } (h \ ex)$ 
  shows non-cap-exp (try-catch  $m \ h$ )
proof (unfold non-cap-exp-def, intro allI impI)
  fix  $t m'$ 
  assume  $(\text{try-catch } m \ h, t, m') \in \text{Traces}$ 
  then show  $\text{non-cap-trace } t \vee (\exists t' r v \text{ msg. } t = t' @ [E\text{-read-reg } r \ v] \wedge r \notin$ 
 $\text{special-reg-names} \wedge \text{non-cap-trace } t' \wedge m' = \text{Fail msg})$ 
  proof (cases rule: try-catch-Traces-cases)
    case (NoEx  $m''$ )
    then show ?thesis
    by (elim non-cap-exp-Traces-cases[OF  $m$ ]) auto
  next
    case (Ex  $tm \ ex \ th$ )
    then show ?thesis
    by (elim non-cap-exp-Traces-cases[OF  $m$ ]
      h[OF  $\langle (m, tm, \text{Exception } ex) \in \text{Traces} \rangle$ , THEN non-cap-exp-Traces-cases])
      auto
qed
qed

```

```

lemma non-mem-exp-try-catch:
  assumes non-mem-exp  $m$ 
  and  $\bigwedge t ex. (m, t, \text{Exception } ex) \in \text{Traces} \implies \text{non-mem-exp } (h \ ex)$ 
  shows non-mem-exp (try-catch  $m \ h$ )
  using assms

```


by (*fastforce simp: non-mem-exp-def elim!: try-catch-Traces-cases*)

lemma *non-cap-exp-throw*[*non-cap-expI*]:
non-cap-exp (*throw e*)
by (*auto simp: non-cap-exp-def*)

lemma *non-cap-exp-early-return*[*non-cap-expI*]:
non-cap-exp (*early-return a*)
by (*auto simp: early-return-def intro!: non-cap-expI*)

lemma *non-cap-exp-catch-early-return*[*intro!*]:
non-cap-exp (*catch-early-return m*) **if** *non-cap-exp m*
by (*auto simp: catch-early-return-def intro!: that non-cap-expI split: sum.splits*)

lemma *non-mem-exp-catch-early-return*:
non-mem-exp (*catch-early-return m*) **if** *non-mem-exp m*
by (*auto simp: catch-early-return-def intro!: that non-mem-exp-try-catch non-cap-expI [THEN non-cap-exp-non-mem-exp] split: sum.splits*)

lemma *non-cap-exp-liftR*[*intro!*]:
non-cap-exp (*liftR m*) **if** *non-cap-exp m*
by (*auto simp: liftR-def intro!: that non-cap-expI*)

lemma *non-mem-exp-liftR*:
non-mem-exp (*liftR m*) **if** *non-mem-exp m*
by (*auto simp: liftR-def intro!: that non-mem-exp-try-catch non-cap-expI [THEN non-cap-exp-non-mem-exp]*)

lemma *non-cap-exp-assert-exp*[*non-cap-expI*]:
non-cap-exp (*assert-exp c msg*)
by (*auto simp: assert-exp-def non-cap-exp-def*)

lemma *non-cap-exp-foreachM*[*intro!*]:
assumes $\bigwedge x \text{ vars. } x \in \text{set } xs \implies \text{non-cap-exp } (\text{body } x \text{ vars})$
shows *non-cap-exp* (*foreachM xs vars body*)
using *assms*
by (*induction xs vars body rule: foreachM.induct*) (*auto intro: non-cap-expI*)

lemma *non-mem-exp-foreachM*:
assumes $\bigwedge x \text{ vars. } x \in \text{set } xs \implies \text{non-mem-exp } (\text{body } x \text{ vars})$
shows *non-mem-exp* (*foreachM xs vars body*)
using *assms*
by (*induction xs vars body rule: foreachM.induct*) (*auto intro: non-cap-expI [THEN non-cap-exp-non-mem-exp]*)

lemma *non-cap-exp-choose-bool*[*non-cap-expI*]:
non-cap-exp (*choose-bool desc*)
by (*auto simp: choose-bool-def non-cap-exp-def elim: Traces-cases*)

lemma *non-cap-exp-undefined-bool*[*non-cap-expI*]:
non-cap-exp (*undefined-bool* ())
by (*auto simp: undefined-bool-def intro: non-cap-expI*)

lemma *non-cap-exp-bool-of-bitU-nondet*[*non-cap-expI*]:
non-cap-exp (*bool-of-bitU-nondet* *b*)
unfolding *bool-of-bitU-nondet-def*
by (*cases b*) (*auto intro: non-cap-expI*)

lemma *non-cap-exp-genlistM*:
assumes $\bigwedge n. \text{non-cap-exp } (f\ n)$
shows *non-cap-exp* (*genlistM* *f* *n*)
using *assms*
by (*auto simp: genlistM-def intro!: non-cap-expI*)

lemma *non-cap-exp-choose-bools*[*non-cap-expI*]:
non-cap-exp (*choose-bools* *desc* *n*)
by (*auto simp: choose-bools-def intro: non-cap-expI non-cap-exp-genlistM*)

lemma *non-cap-exp-Fail*[*non-cap-expI*]:
non-cap-exp (*Fail* *msg*)
by (*auto simp: non-cap-exp-def*)

lemma *non-cap-exp-exit*[*non-cap-expI*]:
non-cap-exp (*exit0* ())
unfolding *exit0-def*
by (*rule non-cap-exp-Fail*)

lemma *non-cap-exp-chooseM*[*non-cap-expI*]:
non-cap-exp (*chooseM* *desc* *xs*)
by (*auto simp: chooseM-def intro!: non-cap-expI split: option.splits*)

lemma *non-cap-exp-internal-pick*[*non-cap-expI*]:
non-cap-exp (*internal-pick* *xs*)
by (*auto simp: internal-pick-def intro!: non-cap-expI*)

lemma *non-cap-exp-and-boolM*[*intro!*]:
non-cap-exp (*and-boolM* *m1* *m2*) **if** *non-cap-exp* *m1* **and** *non-cap-exp* *m2*
by (*auto simp: and-boolM-def intro!: that non-cap-expI*)

lemma *non-mem-exp-and-boolM*:
non-mem-exp (*and-boolM* *m1* *m2*) **if** *non-mem-exp* *m1* **and** *non-mem-exp* *m2*
by (*auto simp: and-boolM-def intro!: that non-cap-expI [THEN non-cap-exp-non-mem-exp]*)

lemma *non-cap-exp-or-boolM*[*intro!*]:
non-cap-exp (*or-boolM* *m1* *m2*) **if** *non-cap-exp* *m1* **and** *non-cap-exp* *m2*
by (*auto simp: or-boolM-def intro!: that non-cap-expI*)

lemma *non-mem-exp-or-boolM*:

non-mem-exp (*or-boolM* *m1 m2*) **if** *non-mem-exp* *m1* **and** *non-mem-exp* *m2*
by (*auto simp: or-boolM-def intro!: that non-cap-expI [THEN non-cap-exp-non-mem-exp]*)

lemma *non-cap-exp-let[intro!]*:
non-cap-exp (*let* *x = a in m x*) **if** *non-cap-exp* (*m a*)
by (*auto intro: that*)

lemma *non-mem-exp-let*:
non-mem-exp (*let* *x = a in m x*) **if** *non-mem-exp* (*m a*)
by (*auto intro: that*)

lemma *non-cap-exp-if*:
assumes *c* \implies *non-cap-exp* *m1* **and** $\neg c \implies$ *non-cap-exp* *m2*
shows *non-cap-exp* (*if* *c* **then** *m1* **else** *m2*)
using *assms*
by *auto*

lemma *non-mem-exp-if*:
assumes *c* \implies *non-mem-exp* *m1* **and** $\neg c \implies$ *non-mem-exp* *m2*
shows *non-mem-exp* (*if* *c* **then** *m1* **else** *m2*)
using *assms*
by *auto*

lemma *non-cap-exp-read-non-cap-reg*:
assumes *non-cap-reg* *r*
shows *non-cap-exp* (*read-reg* *r* :: ('regval, 'r, 'exception) monad)
proof –
have *non-cap-trace* *t* $\vee (\exists v \text{ msg. } t = [E\text{-read-reg } (\text{name } r) v] \wedge \text{name } r \notin$
special-reg-names $\wedge m' = \text{Fail msg})$
if (*read-reg* *r*, *t*, *m'* :: ('regval, 'r, 'exception) monad) $\in \text{Traces}$ **for** *t m'*
using *that assms*
by (*auto simp: read-reg-def non-cap-exp-def non-cap-reg-def elim!: Read-reg-TracesE*
split: option.splits)
then show ?thesis
unfolding *non-cap-exp-def*
by *blast*
qed

lemma
non-mem-exp-read-reg[non-mem-expI]: *non-mem-exp* (*read-reg* *r*) **and**
non-mem-exp-write-reg[non-mem-expI]: *non-mem-exp* (*write-reg* *r v*)
unfolding *non-mem-exp-def read-reg-def write-reg-def*
by (*auto elim!: Read-reg-TracesE Write-reg-TracesE split: option.splits*)

lemma *non-cap-exp-write-non-cap-reg*:
assumes *non-cap-reg* *r*
shows *non-cap-exp* (*write-reg* *r v*)
using *assms*
unfolding *write-reg-def*

by (*auto simp: non-cap-exp-def non-cap-reg-def elim!: Write-reg-TracesE*)

method *non-cap-expI* **uses** *simp* =
 (*auto simp: simp intro!: non-cap-expI non-cap-exp-if non-cap-exp-read-non-cap-reg*
non-cap-exp-write-non-cap-reg
split del: if-split split: option.split sum.split prod.split)

lemmas *non-mem-exp-combinators* =
non-mem-exp-bindI non-mem-exp-if non-mem-exp-let non-mem-exp-and-boolM
non-mem-exp-or-boolM
non-mem-exp-foreachM non-mem-exp-try-catch non-mem-exp-catch-early-return
non-mem-exp-liftR

method *non-mem-expI* **uses** *simp* =
 (*auto simp: simp intro!: non-mem-expI non-mem-exp-combinators non-cap-expI [THEN*
non-cap-exp-non-mem-exp]
split del: if-split split: option.split sum.split prod.split)

lemma *Run-write-reg-no-cap[trace-simp]*:
assumes *Run (write-reg r v) t a*
and *non-cap-reg r*
shows *run s t = s*
using *assms*
by (*cases s*) (*auto simp: write-reg-def step-defs non-cap-reg-def elim!: Write-reg-TracesE*)

lemma *Run-write-reg-run-gen*:
assumes *Run (write-reg r v) t a*
shows *run s t =*
s(|written-regs := written-regs s ∪
(if (∃ c ∈ caps-of-regval ISA (regval-of r v). is-tagged-method
CC c)
then {name r} else {}))|)
using *assms*
by (*cases s*) (*auto simp: write-reg-def step-defs elim!: Write-reg-TracesE*)

lemma *Run-read-non-cap-reg-run[trace-simp]*:
assumes *Run (read-reg r) t v*
and *non-cap-reg r*
shows *run s t = s*
using *assms*
by (*auto simp: step-defs non-cap-reg-def elim!: Run-read-regE*)

lemma *no-reg-writes-to-written-regs-run-inv[trace-simp]*:
assumes *Run m t a*
and *no-reg-writes-to UNIV m*
shows *written-regs (run s t) = written-regs s*
proof –
have *E-write-reg r v ∉ set t* **for** *r v*
using *assms*

```

    by (auto simp: no-reg-writes-to-def)
  then show ?thesis
    by (induction t rule: rev-induct) auto
qed

```

```

method trace-enabledI uses simp elim =
  (auto simp: simp trace-simp elim!: elim trace-elim)

```

```

end

```

```

locale Write-Cap-Automaton = Capability-ISA CC ISA
  for CC :: 'cap Capability-class and ISA :: ('cap, 'regval, 'instr, 'e) isa +
  fixes ex-traces :: bool and invocation-traces :: bool
begin

fun enabled :: ('cap, 'regval) axiom-state  $\Rightarrow$  'regval event  $\Rightarrow$  bool where
  enabled s (E-write-reg r v) =
    ( $\forall$  c. (c  $\in$  caps-of-regval ISA v  $\wedge$  is-tagged-method CC c)
       $\longrightarrow$ 
      (c  $\in$  derivable (accessed-caps s)  $\vee$ 
       (c  $\in$  exception-targets ISA (read-from-KCC s)  $\wedge$  ex-traces  $\wedge$  r  $\in$  PCC ISA)
       $\vee$ 
      ( $\exists$  cc cd. invocation-traces  $\wedge$  cc  $\in$  derivable (accessed-caps s)  $\wedge$  cd  $\in$ 
       derivable (accessed-caps s)  $\wedge$ 
       invokable CC cc cd  $\wedge$  (r  $\in$  PCC ISA  $\wedge$  leq-cap CC c (unseal CC cc
        True)  $\vee$  r  $\in$  IDC ISA  $\wedge$  leq-cap CC c (unseal CC cd True))))))
| enabled s (E-read-reg r v) = (r  $\in$  privileged-regs ISA  $\longrightarrow$  (system-reg-access s  $\vee$ 
  ex-traces))
| enabled s (E-write-memt - addr sz bytes tag -) =
  ( $\forall$  c. cap-of-mem-bytes-method CC bytes tag = Some c  $\wedge$  is-tagged-method CC
  c  $\longrightarrow$  c  $\in$  derivable (accessed-caps s))
| enabled s - = True

```

```

lemma enabled-E-write-reg-cases:
  assumes enabled s (E-write-reg r v)
    and c  $\in$  caps-of-regval ISA v
    and is-tagged-method CC c
  obtains (Derivable) c  $\in$  derivable (accessed-caps s)
| (KCC) c  $\in$  exception-targets ISA (read-from-KCC s) and ex-traces and
  r  $\in$  PCC ISA and c  $\notin$  derivable (accessed-caps s)
| (CCall) cc cd where invocation-traces and invokable CC cc cd and
  cc  $\in$  derivable (accessed-caps s) and cd  $\in$  derivable (accessed-caps s) and
  r  $\in$  PCC ISA  $\wedge$  leq-cap CC c (unseal CC cc True)  $\vee$  r  $\in$  IDC ISA  $\wedge$  leq-cap
  CC c (unseal CC cd True) and
  c  $\notin$  derivable (accessed-caps s)
  using assms by (cases c  $\in$  derivable (accessed-caps s)) auto

```

sublocale *Cap-Axiom-Automaton CC ISA enabled ..*

lemma *non-cap-event-enabledI:*

assumes *non-cap-event e*
shows *enabled s e*
using *assms*
by (*elim non-cap-event.elims*) *auto*

lemma *non-cap-trace-enabledI:*

assumes *non-cap-trace t*
shows *trace-enabled s t*
using *assms*
by (*induction t*) (*auto simp: non-cap-event-enabledI non-cap-event-axiom-step-inv*)

lemma *non-cap-exp-trace-enabledI:*

assumes *non-cap-exp m*
and $(m, t, m') \in \text{Traces}$
shows *trace-enabled s t*
by (*cases rule: non-cap-exp-Traces-cases[OF assms]*)
(auto intro: non-cap-trace-enabledI simp: trace-enabled-append-iff)

lemma *index-eq-some': (index l n = Some x) = (n < length l ∧ l ! n = x)*

by *auto*

lemma *recognises-store-cap-reg-read-reg-axioms:*

assumes *t: accepts t*
shows *store-cap-reg-axiom CC ISA ex-traces invocation-traces t*
and *store-cap-mem-axiom CC ISA t*
and *read-reg-axiom CC ISA ex-traces t*
proof –
show *read-reg-axiom CC ISA ex-traces t*
using *assms*
unfolding *accepts-from-iff-all-enabled-final read-reg-axiom-def*
by (*auto elim!: enabled.elims*)
show *store-cap-reg-axiom CC ISA ex-traces invocation-traces t*
proof (*unfold store-cap-reg-axiom-def, intro allI impI, goal-cases Idx*)
case (*Idx i c r*)
then show *?case*
proof *cases*
assume *i: i < length t*
then obtain *v where e: index t i = Some (E-write-reg r v)*
and *c: c ∈ caps-of-regval ISA v*
and *tag: is-tagged-method CC c*
using *Idx*
by (*cases t ! i*) *auto*
then have *enabled (run initial (take i t)) (E-write-reg r v)*
using *accepts-from-nth-enabledI[OF t i]*

```

    by auto
  from this c tag
  show ?thesis
  proof (cases rule: enabled-E-write-reg-cases)
    case Derivable
    then show ?thesis
      by (auto simp: cap-derivable-iff-derivable)
  next
    case KCC

    show ?thesis
      using KCC
      unfolding index-eq-some'
      by (auto simp: cap-derivable-iff-derivable read-from-KCC-run-take-eq)
  next
    case (CCall cc cd)
    then show ?thesis
      by (auto simp: cap-derivable-iff-derivable)
  qed
qed auto
qed
show store-cap-mem-axiom CC ISA t
  using assms
  unfolding accepts-from-iff-all-enabled-final store-cap-mem-axiom-def
  by (auto simp: cap-derivable-iff-derivable writes-mem-cap-Some-iff)
qed

end

locale Cap-Axiom-Inv-Automaton = Cap-Axiom-Automaton CC ISA enabled +
  State-Invariant get-regval set-regval invariant inv-regs
for CC :: 'cap Capability-class and ISA :: ('cap, 'regval, 'instr, 'e) isa
  and enabled :: ('cap, 'regval) axiom-state  $\Rightarrow$  'regval event  $\Rightarrow$  bool
  and get-regval :: string  $\Rightarrow$  'regstate  $\Rightarrow$  'regval option
  and set-regval :: string  $\Rightarrow$  'regval  $\Rightarrow$  'regstate  $\Rightarrow$  'regstate option
  and invariant :: 'regstate  $\Rightarrow$  bool and inv-regs :: register-name set +
fixes ex-traces :: bool
assumes non-cap-event-enabled:  $\bigwedge e. \text{non-cap-event } e \implies \text{enabled } s \ e$ 
  and read-non-special-regs-enabled:  $\bigwedge r \ v. r \notin \text{PCC ISA} \cup \text{IDC ISA} \cup \text{KCC ISA}$ 
 $\cup \text{privileged-regs ISA} \implies \text{enabled } s \ (E\text{-read-reg } r \ v)$ 
begin

definition isException m  $\equiv ((\exists e. m = \text{Exception } e) \vee (\exists \text{msg}. m = \text{Fail } \text{msg})) \wedge$ 
  ex-traces

definition finished :: ('regval, 'a, 'ex) monad  $\Rightarrow$  bool where
  finished m =  $((\exists a. m = \text{Done } a) \vee \text{isException } m)$ 

lemma finishedE:

```

```

assumes finished m
obtains (Done) a where m = Done a
| (Ex) isException m
using assms
by (auto simp: finished-def)

lemma finished-cases:
  assumes finished m
  obtains (Done) a where m = Done a | (Fail) msg where m = Fail msg | (Ex)
e where m = Exception e
  using assms
  by (auto simp: finished-def isException-def)

lemma finished-Done[intro, simp]:
  finished (Done a)
by (auto simp: finished-def)

lemma finished-Fail[intro, simp]:
  finished (Fail msg)  $\implies$  finished (Fail msg')
unfolding finished-def isException-def
by auto

lemma finished-Exception[intro, simp]:
  finished (Exception e)  $\implies$  finished (Exception e')
unfolding finished-def isException-def
by auto

lemma finished-isException[intro, simp]:
  isException m  $\implies$  finished m
by (auto simp: finished-def)

lemma finished-bind-left:
  assumes finished (m  $\gg$  f)
  shows finished m
  using assms
  unfolding finished-def isException-def
  by (cases m) auto

definition
  traces-enabled m s regs  $\equiv$ 
   $\forall t m'. (m, t, m') \in \text{Traces} \wedge \text{finished } m' \wedge \text{reads-regs-from } \text{inv-regs } t \text{ regs} \wedge$ 
invariant regs  $\longrightarrow$  trace-enabled s t

lemma traces-enabled-accepts-fromI:
  assumes hasTrace t m and traces-enabled m s regs and hasException t m  $\vee$ 
hasFailure t m  $\longrightarrow$  ex-traces
  and reads-regs-from inv-regs t regs and invariant regs
  shows accepts-from s t
  using assms

```


unfolding *traces-enabled-def finished-def isException-def*
unfolding *hasTrace-iff-Traces-final hasException-iff-Traces-Exception hasFailure-iff-Traces-Fail*
unfolding *runTrace-iff-Traces[symmetric]*
by (*intro trace-enabled-acceptI*) (*auto elim!:* *final-cases*)

named-theorems *traces-enabledI*

lemma *traces-enabled-bind[traces-enabledI]:*
assumes *runs-preserve-invariant m and traces-enabled m s regs*
and $\bigwedge t a. \text{Run-inv } m \ t \ a \ \text{regs} \implies \text{traces-enabled } (f \ a) \ (\text{run } s \ t) \ (\text{the } (\text{updates-reg} \text{ inv-reg} \text{ } t \ \text{regs}))$
shows *traces-enabled (m \gg f) s regs*
using *assms*
by (*auto simp: traces-enabled-def Run-inv-def regstate-simp trace-enabled-append-iff*
dest!: *finished-bind-left elim!:* *bind-Traces-cases elim!:* *runs-preserve-invariantE;*
fastforce)

lemma *non-cap-trace-enabledI:*
assumes *non-cap-trace t*
shows *trace-enabled s t*
using *assms*
by (*induction t*) (*auto simp: non-cap-event-enabled non-cap-event-axiom-step-inv*)

lemma *non-cap-exp-trace-enabledI:*
assumes *m: non-cap-exp m*
and *t: (m, t, m') \in Traces*
shows *trace-enabled s t*
by (*cases rule: non-cap-exp-Traces-cases[OF m t]*)
(auto intro: non-cap-trace-enabledI read-non-special-reg-enabled simp: trace-enabled-append-iff)

lemma *non-cap-exp-traces-enabledI:*
assumes *non-cap-exp m*
shows *traces-enabled m s regs*
using *assms*
by (*auto simp: traces-enabled-def intro: non-cap-exp-trace-enabledI*)

lemma *Run-inv-RunI[simp]: Run-inv m t a regs \implies Run m t a*
by (*simp add: Run-inv-def*)

lemma *traces-enabled-let[traces-enabledI]:*
assumes *traces-enabled (f y) s regs*
shows *traces-enabled (let x = y in f x) s regs*
using *assms*
by *auto*

lemma *traces-enabled-case-prod[traces-enabledI]:*
assumes $\bigwedge x y. z = (x, y) \implies \text{traces-enabled } (f \ x \ y) \ s \ \text{regs}$
shows *traces-enabled (case z of (x, y) \Rightarrow f x y) s regs*
using *assms*

by auto

lemma *traces-enabled-if*[*traces-enabledI*]:

assumes $c \implies \text{traces-enabled } m1 \ s \ \text{regs}$ **and** $\neg c \implies \text{traces-enabled } m2 \ s \ \text{regs}$
shows *traces-enabled* (if c then $m1$ else $m2$) $s \ \text{regs}$
using *assms*
by auto

lemma *traces-enabled-if-ignore-cond*:

assumes *traces-enabled* $m1 \ s \ \text{regs}$ **and** *traces-enabled* $m2 \ s \ \text{regs}$
shows *traces-enabled* (if c then $m1$ else $m2$) $s \ \text{regs}$
using *assms*
by auto

lemma *traces-enabled-and-boolM*[*traces-enabledI*]:

assumes *runs-preserve-invariant* $m1$ **and** *traces-enabled* $m1 \ s \ \text{regs}$
and $\bigwedge t. \text{Run-inv } m1 \ t \ \text{True } \text{regs} \implies \text{traces-enabled } m2 \ (\text{run } s \ t) \ (\text{the } (\text{updates-regs } \text{inv-regs } t \ \text{regs}))$
shows *traces-enabled* (*and-boolM* $m1 \ m2$) $s \ \text{regs}$
using *assms*
by (auto simp: *and-boolM-def* intro!: *traces-enabledI* intro: *non-cap-exp-traces-enabledI* *non-cap-expI*)

lemma *traces-enabled-or-boolM*[*traces-enabledI*]:

assumes *runs-preserve-invariant* $m1$ **and** *traces-enabled* $m1 \ s \ \text{regs}$
and $\bigwedge t. \text{Run-inv } m1 \ t \ \text{False } \text{regs} \implies \text{traces-enabled } m2 \ (\text{run } s \ t) \ (\text{the } (\text{updates-regs } \text{inv-regs } t \ \text{regs}))$
shows *traces-enabled* (*or-boolM* $m1 \ m2$) $s \ \text{regs}$
using *assms*
by (auto simp: *or-boolM-def* intro!: *traces-enabledI* intro: *non-cap-exp-traces-enabledI* *non-cap-expI*)

lemma *traces-enabled-foreachM-inv*:

assumes $\bigwedge x \ \text{vars} \ s \ \text{regs}. P \ \text{vars} \ s \ \text{regs} \implies x \in \text{set } xs \implies \text{traces-enabled } (\text{body } x \ \text{vars}) \ s \ \text{regs}$
and $\bigwedge x \ \text{vars}. x \in \text{set } xs \implies \text{runs-preserve-invariant } (\text{body } x \ \text{vars})$
and $\bigwedge x \ \text{vars} \ s \ \text{regs} \ t \ \text{vars}'. P \ \text{vars} \ s \ \text{regs} \implies x \in \text{set } xs \implies \text{Run-inv } (\text{body } x \ \text{vars}) \ t \ \text{vars}' \ \text{regs} \implies P \ \text{vars}' \ (\text{run } s \ t) \ (\text{the } (\text{updates-regs } \text{inv-regs } t \ \text{regs}))$
and $P \ \text{vars} \ s \ \text{regs}$
shows *traces-enabled* (*foreachM* $xs \ \text{vars} \ \text{body}$) $s \ \text{regs}$
by (use *assms* **in** $\langle \text{induction } xs \ \text{arbitrary: vars } s \ \text{regs} \rangle$;
fastforce intro!: *traces-enabledI* intro: *non-cap-exp-traces-enabledI* *non-cap-expI*)

lemma *traces-enabled-try-catch*:

assumes *traces-enabled* $m \ s \ \text{regs}$
and $\bigwedge tm \ e \ th \ m'. (m, tm, \text{Exception } e) \in \text{Traces} \implies (h \ e, th, m') \in \text{Traces} \implies \text{finished } m' \implies$
reads-regs-from $\text{inv-regs } (tm \ @ \ th) \ \text{regs} \implies \text{invariant } \text{regs} \implies$

$\text{trace-enabled } s \ (tm \ @ \ th)$
shows $\text{traces-enabled } (\text{try-catch } m \ h) \ s \ \text{regs}$
proof –

have *: $\text{finished } (\text{try-catch } m \ h) \longleftrightarrow (\exists a. m = \text{Done } a) \vee (\exists msg. m = \text{Fail } msg \wedge \text{finished } m) \vee (\exists e. m = \text{Exception } e \wedge (h \ e, [], h \ e) \in \text{Traces} \wedge \text{finished } (h \ e))$
for m
by $(\text{cases } m) \ (\text{auto simp: finished-def isException-def})$
show ?thesis
using assms
by $(\text{fastforce simp: traces-enabled-def regstate-simp trace-enabled-append-iff Run-inv-def } *)$
 $\text{elim! : try-catch-Traces-cases elim: traces-preserve-invariantE}$
qed

lemma $\text{traces-enabled-liftR}[\text{traces-enabledI}]$:
assumes $\text{traces-enabled } m \ s \ \text{regs}$
shows $\text{traces-enabled } (\text{liftR } m) \ s \ \text{regs}$
using assms
unfolding liftR-def
by $(\text{intro traces-enabled-try-catch}) \ (\text{auto simp: traces-enabled-def Run-inv-def})$

definition

$\text{early-returns-enabled } m \ s \ \text{regs} \equiv$
 $\text{traces-enabled } m \ s \ \text{regs} \wedge$
 $(\forall t \ a. (m, t, \text{Exception } (\text{Inl } a)) \in \text{Traces} \wedge \text{reads-regs-from inv-regs } t \ \text{regs} \wedge$
 $\text{invariant } \text{regs} \longrightarrow \text{trace-enabled } s \ t)$

lemma $\text{traces-enabled-catch-early-return}[\text{traces-enabledI}]$:
assumes $\text{early-returns-enabled } m \ s \ \text{regs}$
shows $\text{traces-enabled } (\text{catch-early-return } m) \ s \ \text{regs}$
using assms
unfolding $\text{catch-early-return-def}$
by $(\text{intro traces-enabled-try-catch})$
 $(\text{auto simp: traces-enabled-def early-returns-enabled-def Run-inv-def split: sum.splits})$

lemma $\text{liftR-no-early-return}[\text{simp}]$:
shows $(\text{liftR } m, t, \text{Exception } (\text{Inl } e)) \in \text{Traces} \longleftrightarrow \text{False}$
by $(\text{induction } m \ \text{arbitrary: } t) \ (\text{auto simp: liftR-def elim: Traces-cases})$

lemma $\text{early-returns-enabled-liftR}[\text{traces-enabledI}]$:
assumes $\text{traces-enabled } m \ s \ \text{regs}$
shows $\text{early-returns-enabled } (\text{liftR } m) \ s \ \text{regs}$
using assms
by $(\text{auto simp: early-returns-enabled-def intro: traces-enabled-liftR})$

lemma $\text{early-returns-enabled-return}[\text{traces-enabledI}]$:
 $\text{early-returns-enabled } (\text{return } a) \ s \ \text{regs}$

by (auto simp: early-returns-enabled-def traces-enabled-def)

lemma *early-returns-enabled-bind*[traces-enabledI]:
 assumes *inv*: traces-preserve-invariant *m*
 and *m*: early-returns-enabled *m* *s* *regs*
 and *f*: $\bigwedge t a. \text{Run-inv } m \ t \ a \ \text{regs} \implies \text{early-returns-enabled } (f \ a) \ (\text{run } s \ t) \ (\text{the } (\text{updates-regs } \text{inv-regs } t \ \text{regs}))$
 shows early-returns-enabled (*m* $\gg=$ *f*) *s* *regs*
proof –
 { fix *t a*
 assume (*m* $\gg=$ *f*, *t*, *Exception* (*Inl a*)) \in *Traces* and *t*: reads-regs-from *inv-regs* *t* *regs* and *regs*: invariant *regs*
 then have trace-enabled *s* *t*
proof (cases rule: bind-*Traces*-cases)
 case (Left *m''*)
 then consider *m''* = *Exception* (*Inl a*) | *a'* where *m''* = *Done a'* and *f a'*
 = *Exception* (*Inl a*)
 by (cases *m''*) auto
 then show ?thesis
 using Left *m* *t* *regs*
 by cases (auto simp: early-returns-enabled-def traces-enabled-def)
 next
 case (Bind *tm am tf*)
 then obtain *regs'*
 where updates-regs *inv-regs* *tm* *regs* = *Some* *regs'* and invariant *regs'*
 and reads-regs-from *inv-regs* *tm* *regs* and reads-regs-from *inv-regs* *tf* *regs'*
 using *t* *regs*
 by (elim traces-preserve-invariantE[OF *inv*]) (auto simp: regstate-simp)
 then show ?thesis
 using Bind *m* *f*[of *tm am*] *regs*
 by (auto simp: trace-enabled-append-iff early-returns-enabled-def traces-enabled-def Run-inv-def)
 qed
 }
 then show ?thesis
 using *assms*
 by (auto intro: traces-enabled-bind traces-runs-preserve-invariantI simp: early-returns-enabled-def)
 qed

lemma *early-returns-enabled-early-return*[traces-enabledI]:
 early-returns-enabled (early-return *a*) *s* *regs*
 by (auto simp: early-returns-enabled-def early-return-def throw-def traces-enabled-def)

lemma *early-returns-enabled-let*[traces-enabledI]:
 assumes early-returns-enabled (*f* *y*) *s* *regs*
 shows early-returns-enabled (let *x* = *y* in *f* *x*) *s* *regs*
 using *assms*
 by auto

lemma *early-returns-enabled-case-prod*[*traces-enabledI*]:
assumes $\bigwedge x y. z = (x, y) \implies \text{early-returns-enabled } (f x y) s \text{ regs}$
shows $\text{early-returns-enabled } (\text{case } z \text{ of } (x, y) \Rightarrow f x y) s \text{ regs}$
using *assms*
by *auto*

lemma *early-returns-enabled-if*[*traces-enabledI*]:
assumes $c \implies \text{early-returns-enabled } m1 s \text{ regs}$ **and** $\neg c \implies \text{early-returns-enabled } m2 s \text{ regs}$
shows $\text{early-returns-enabled } (\text{if } c \text{ then } m1 \text{ else } m2) s \text{ regs}$
using *assms*
by *auto*

lemma *early-returns-enabled-if-ignore-cond*:
assumes $\text{early-returns-enabled } m1 s \text{ regs}$ **and** $\text{early-returns-enabled } m2 s \text{ regs}$
shows $\text{early-returns-enabled } (\text{if } c \text{ then } m1 \text{ else } m2) s \text{ regs}$
using *assms*
by *auto*

lemma *early-returns-enabled-and-boolM*[*traces-enabledI*]:
assumes $\text{traces-preserve-invariant } m1$ **and** $\text{early-returns-enabled } m1 s \text{ regs}$
and $\bigwedge t. \text{Run-inv } m1 t \text{ True } \text{regs} \implies \text{early-returns-enabled } m2 (\text{run } s t) \text{ (the (updates-regs inv-regs } t \text{ regs))}$
shows $\text{early-returns-enabled } (\text{and-boolM } m1 m2) s \text{ regs}$
using *assms*
by (*auto simp: and-boolM-def intro!: traces-enabledI intro: non-cap-exp-traces-enabledI non-cap-expI*)

lemma *early-returns-enabled-or-boolM*[*traces-enabledI*]:
assumes $\text{traces-preserve-invariant } m1$ **and** $\text{early-returns-enabled } m1 s \text{ regs}$
and $\bigwedge t. \text{Run-inv } m1 t \text{ False } \text{regs} \implies \text{early-returns-enabled } m2 (\text{run } s t) \text{ (the (updates-regs inv-regs } t \text{ regs))}$
shows $\text{early-returns-enabled } (\text{or-boolM } m1 m2) s \text{ regs}$
using *assms*
by (*auto simp: or-boolM-def intro!: traces-enabledI intro: non-cap-exp-traces-enabledI non-cap-expI*)

lemma *early-returns-enabled-foreachM-inv*:
assumes $\bigwedge x \text{ vars } s \text{ regs}. P \text{ vars } s \text{ regs} \implies x \in \text{set } xs \implies \text{early-returns-enabled } (\text{body } x \text{ vars}) s \text{ regs}$
and $\bigwedge x \text{ vars}. x \in \text{set } xs \implies \text{traces-preserve-invariant } (\text{body } x \text{ vars})$
and $\bigwedge x \text{ vars } s \text{ regs } t \text{ vars}'. P \text{ vars } s \text{ regs} \implies x \in \text{set } xs \implies \text{Run-inv } (\text{body } x \text{ vars}) t \text{ vars}' \text{ regs} \implies P \text{ vars}' (\text{run } s t) \text{ (the (updates-regs inv-regs } t \text{ regs))}$
and $P \text{ vars } s \text{ regs}$
shows $\text{early-returns-enabled } (\text{foreachM } xs \text{ vars body}) s \text{ regs}$
by (*use assms in (induction xs arbitrary: vars s regs); fastforce intro!: traces-enabledI intro: non-cap-exp-traces-enabledI non-cap-expI*)

lemma *non-cap-exp-Run-inv-traces-enabled-runE*:

assumes *Run-inv m1 t a regs* **and** *non-cap-exp m1* **and** *traces-enabled m2 s regs'*
shows *traces-enabled m2 (run s t) regs'*
using *assms*
by (*auto simp: Run-inv-def non-cap-exp-Run-run-invI*)

lemma *no-reg-writes-Run-inv-traces-enabled-updates-regsE*:
assumes *Run-inv m1 t a regs* **and** *no-reg-writes-to inv-regs m1* **and** *traces-enabled m2 s regs*
shows *traces-enabled m2 s (the (updates-regs inv-regs t) regs)*
using *assms*
by (*auto simp: Run-inv-def*)

lemma *non-cap-exp-Run-inv-early-returns-enabled-runE*:
assumes *Run-inv m1 t a regs* **and** *non-cap-exp m1* **and** *early-returns-enabled m2 s regs'*
shows *early-returns-enabled m2 (run s t) regs'*
using *assms*
by (*auto simp: Run-inv-def non-cap-exp-Run-run-invI*)

lemma *no-reg-writes-Run-inv-early-returns-enabled-updates-regsE*:
assumes *Run-inv m1 t a regs* **and** *no-reg-writes-to inv-regs m1* **and** *early-returns-enabled m2 s regs*
shows *early-returns-enabled m2 s (the (updates-regs inv-regs t) regs)*
using *assms*
by (*auto simp: Run-inv-def*)

lemma *accessible-regs-no-writes-run*:
assumes *t: Run m t a*
and *m: runs-no-reg-writes-to {r} m*
and *s: r ∈ accessible-regs s*
shows *r ∈ accessible-regs (run s t)*
proof –
have *no-write: ∀ v. E-write-reg r v ∉ set t*
using *m t*
by (*auto simp: runs-no-reg-writes-to-def Run-inv-def*)
show *?thesis*
proof (*use s no-write in (induction t arbitrary: s)*)
case (*Cons e t*)
then have *r ∈ accessible-regs (axiom-step s e)* **and** *∀ v. E-write-reg r v ∉ set t*
by (*auto simp: accessible-regs-def*)
from *Cons.IH[OF this]* **show** *?case* **by** *auto*
qed *auto*
qed

lemma *no-reg-writes-to-mono*:
assumes *runs-no-reg-writes-to Rs m*
and *Rs' ⊆ Rs*
shows *runs-no-reg-writes-to Rs' m*
using *assms*

```

by (auto simp: runs-no-reg-writes-to-def)

lemma accessible-regs-no-writes-run-subset:
  assumes  $t$ :  $\text{Run } m \ t \ a$  and  $m$ :  $\text{runs-no-reg-writes-to } Rs \ m$ 
  and  $Rs$ :  $R_s \subseteq \text{accessible-regs } s$ 
  shows  $R_s \subseteq \text{accessible-regs } (\text{run } s \ t)$ 
  using  $t \ Rs$  no-reg-writes-to-mono[OF  $m$ ]
  by (auto intro: accessible-regs-no-writes-run)

lemma accessible-regs-no-writes-run-inv-subset:
  assumes  $t$ :  $\text{Run-inv } m \ t \ a \ \text{regs}$  and  $m$ :  $\text{runs-no-reg-writes-to } Rs \ m$ 
  and  $Rs$ :  $R_s \subseteq \text{accessible-regs } s$ 
  shows  $R_s \subseteq \text{accessible-regs } (\text{run } s \ t)$ 
  using  $assms$ 
  by (intro accessible-regs-no-writes-run-subset) (auto simp: Run-inv-def)

named-theorems accessible-regsE
named-theorems accessible-regsI

method accessible-regs-step uses simp  $assms$  =
  ((erule accessible-regsE eqTrueE)
   | (rule accessible-regsI preserves-invariantI TrueI)
   | (erule accessible-regs-no-writes-run-inv-subset accessible-regs-no-writes-run-subset,
      solves (use  $assms$  in (no-reg-writes-toI simp: simp))))

method accessible-regsI-with methods solve uses simp  $assms$  =
  ((erule accessible-regsE eqTrueE; accessible-regsI-with solve simp: simp  $assms$ :
 $assms$ )
   | (rule accessible-regsI preserves-invariantI TrueI; accessible-regsI-with solve
simp: simp  $assms$ :  $assms$ )
   | (erule accessible-regs-no-writes-run-inv-subset accessible-regs-no-writes-run-subset,
      solves (use  $assms$  in (no-reg-writes-toI simp: simp)),
      accessible-regsI-with solve simp: simp  $assms$ :  $assms$ )
   | solve)

method accessible-regsI uses simp  $assms$  =
  (accessible-regsI-with
   (use  $assms$  in (no-reg-writes-toI simp: simp))
   | (use  $assms$  in (auto simp: simp))
   simp: simp  $assms$ :  $assms$ )

definition derivable-caps  $s \equiv \{c. \text{is-tagged-method } CC \ c \longrightarrow c \in \text{derivable } (\text{accessed-caps } s)\}$ 

named-theorems derivable-capsI
named-theorems derivable-capsE

```

```

lemma accessed-caps-run-mono:
  accessed-caps s  $\subseteq$  accessed-caps (run s t)
  by (rule subsetI) (induction t arbitrary: s; auto)

lemma derivable-caps-run-mono:
  derivable-caps s  $\subseteq$  derivable-caps (run s t)
  using derivable-mono[OF accessed-caps-run-mono]
  by (auto simp: derivable-caps-def)

lemma derivable-caps-run-imp:
   $c \in \text{derivable-caps } s \implies c \in \text{derivable-caps } (\text{run } s \ t)$ 
  using derivable-caps-run-mono
  by auto

method derivable-caps-step =
  (rule derivable-capsI preserves-invariantI TrueI
   | erule derivable-capsE eqTrueE
   | rule derivable-caps-run-imp)

method derivable-capsI-with methods solve uses simp assms =
  ((rule derivable-capsI preserves-invariantI TrueI
   | erule derivable-capsE eqTrueE
   | rule derivable-caps-run-imp
   | solve );
   derivable-capsI-with solve simp: simp assms: assms)

method derivable-capsI uses simp assms =
  (derivable-capsI-with  $\langle \text{solves } \langle \text{accessible-regsI } \text{simp: simp assms: assms} \rangle \rangle$  simp:
   simp assms: assms)

method try-simp-traces-enabled =
  ((match conclusion in  $\langle \text{traces-enabled } m2 \ (\text{run } s \ t) \ (\text{the } (\text{updates-regs inv-regs } t \ \text{regs})) \rangle$ 
   for  $m2 \ s \ t \ \text{regs} \Rightarrow$ 
     $\langle \text{match premises in } m1: \langle \text{Run-inv } m1 \ t \ a \ \text{regs} \rangle \text{ for } m1 \ a \Rightarrow$ 
       $\langle \langle \text{rule non-cap-exp-Run-inv-traces-enabled-runE}[\text{OF } m1], \text{ solves } \langle \text{non-cap-expI} \rangle \rangle?,$ 
         $(\text{rule no-reg-writes-Run-inv-traces-enabled-updates-regsE}[\text{OF } m1], \text{ solves}$ 
         $\langle \text{no-reg-writes-toI} \rangle \rangle?) \rangle$ 
      |  $\langle \text{early-returns-enabled } m2 \ (\text{run } s \ t) \ (\text{the } (\text{updates-regs inv-regs } t \ \text{regs})) \rangle$ 
      for  $m2 \ s \ t \ \text{regs} \Rightarrow$ 
         $\langle \text{match premises in } m1: \langle \text{Run-inv } m1 \ t \ a \ \text{regs} \rangle \text{ for } m1 \ a \Rightarrow$ 
           $\langle \langle \text{rule non-cap-exp-Run-inv-early-returns-enabled-runE}[\text{OF } m1], \text{ solves } \langle \text{non-cap-expI} \rangle \rangle?,$ 
             $(\text{rule no-reg-writes-Run-inv-early-returns-enabled-updates-regsE}[\text{OF } m1],$ 
             $\text{solves } \langle \text{no-reg-writes-toI} \rangle \rangle?) \rangle$ 
        )
    )

named-theorems traces-enabled-combinatorI

lemmas traces-enabled-builtin-combinatorsI =
  traces-enabled-bind traces-enabled-and-boolM traces-enabled-or-boolM
  early-returns-enabled-bind early-returns-enabled-and-boolM early-returns-enabled-or-boolM

```



```

named-theorems traces-enabled-split
declare option.split[where  $P = \lambda m. \text{traces-enabled } m \text{ } s \text{ regs}$  for  $s \text{ regs}$ , traces-enabled-split]
declare prod.split[where  $P = \lambda m. \text{traces-enabled } m \text{ } s \text{ regs}$  for  $s \text{ regs}$ , traces-enabled-split]

method traces-enabled-step uses intro elim =
  ((rule intro TrueI)
   | (erule elim eqTrueE)
   | ((rule traces-enabled-combinatorI traces-enabled-builtin-combinatorsI[rotated
2], try-simp-traces-enabled))
   | (rule traces-enabledI preserves-invariantI)
   | (rule traces-enabled-split[THEN iffD2]; intro conjI impI))

method traces-enabledI-with methods solve uses intro elim =
  ((rule intro TrueI; traces-enabledI-with solve intro: intro elim: elim)
   | (erule elim eqTrueE; traces-enabledI-with solve intro: intro elim: elim)
   | ((rule traces-enabled-combinatorI traces-enabled-builtin-combinatorsI[rotated
2], try-simp-traces-enabled); traces-enabledI-with solve intro: intro elim: elim)
   | (rule traces-enabledI; traces-enabledI-with solve intro: intro elim: elim)
   | (preserves-invariantI intro: intro elim: elim; traces-enabledI-with solve intro:
intro elim: elim)
   | (rule traces-enabled-split[THEN iffD2]; intro conjI impI; traces-enabledI-with
solve intro: intro elim: elim)
   | solve)

method traces-enabledI uses simp intro elim assms =
  ((traces-enabled-step intro: intro elim: elim; traces-enabledI simp: simp intro:
intro elim: elim assms: assms)
   | (accessible-regs-step simp: simp assms: assms; solves (traces-enabledI simp:
simp intro: intro elim: elim assms: assms))
   | (derivable-caps-step; solves (traces-enabledI simp: simp intro: intro elim: elim
assms: assms))
   | (solves (no-reg-writes-toI simp: simp))
   | (solves (preserves-invariantI simp: simp))
   | (use assms in (auto intro!: intro elim!: elim simp: simp)?))

lemma if-derivable-capsI[derivable-capsI]:
  assumes  $\text{cond} \implies c1 \in \text{derivable-caps } s$  and  $\neg \text{cond} \implies c2 \in \text{derivable-caps } s$ 
shows (if cond then c1 else c2)  $\in \text{derivable-caps } s$ 
using assms
by auto

end

locale Write-Cap-Inv-Automaton =

```

```

Write-Cap-Automaton CC ISA ex-traces invocation-traces +
State-Invariant get-regval set-regval invariant inv-regs
for CC :: 'cap Capability-class and ISA :: ('cap, 'regval, 'instr, 'e) isa
  and ex-traces :: bool and invocation-traces :: bool
  and get-regval :: string  $\Rightarrow$  'regstate  $\Rightarrow$  'regval option
  and set-regval :: string  $\Rightarrow$  'regval  $\Rightarrow$  'regstate  $\Rightarrow$  'regstate option
  and invariant :: 'regstate  $\Rightarrow$  bool and inv-regs :: register-name set
begin

sublocale Cap-Axiom-Inv-Automaton where enabled = enabled
proof
  fix s e
  assume non-cap-event e
  then show enabled s e
    by (cases e) auto
next
  fix s r v
  assume r  $\notin$  special-reg-names
  then show enabled s (E-read-reg r v)
    by auto
qed

lemma read-reg-trace-enabled:
  assumes t: (read-reg r, t, m')  $\in$  Traces
    and r: name r  $\in$  privileged-regs ISA  $\longrightarrow$  system-reg-access s  $\vee$  ex-traces
  shows trace-enabled s t
  by (use t in (auto simp: read-reg-def elim!: Read-reg-TracesE split: option.splits))
    (use r in (auto))

lemma traces-enabled-read-reg:
  assumes name r  $\in$  privileged-regs ISA  $\longrightarrow$  (system-reg-access s  $\vee$  ex-traces)
  shows traces-enabled (read-reg r) s regs
  using assms
  unfolding traces-enabled-def
  by (blast intro: read-reg-trace-enabled)

lemma write-reg-trace-enabled:
  assumes (write-reg r v, t, m')  $\in$  Traces
    and enabled s (E-write-reg (name r) (regval-of r v))
  shows trace-enabled s t
  using assms
  by (auto simp add: write-reg-def simp del: enabled.simps elim!: Write-reg-TracesE)

lemma traces-enabled-write-reg:
  assumes enabled s (E-write-reg (name r) (regval-of r v))
  shows traces-enabled (write-reg r v) s regs
  using assms
  unfolding traces-enabled-def
  by (blast intro: write-reg-trace-enabled)

```

```

lemma traces-enabled-reg-axioms:
  assumes traces-enabled m initial regs and hasTrace t m
    and reads-regs-from inv-regs t regs and invariant regs
    and hasException t m  $\vee$  hasFailure t m  $\longrightarrow$  ex-traces
  shows store-cap-reg-axiom CC ISA ex-traces invocation-traces t
    and store-cap-mem-axiom CC ISA t
    and read-reg-axiom CC ISA ex-traces t
  using assms
  by (intro recognises-store-cap-reg-read-reg-axioms;
    elim traces-enabled-accepts-fromI[where regs = regs];
    auto)+

end

locale Capability-ISA-Fixed-Translation = Capability-ISA CC ISA
  for CC :: 'cap Capability-class and ISA :: ('cap, 'regval, 'instr, 'e) isa +
  fixes translation-assm :: 'regval trace  $\Rightarrow$  bool
  assumes fixed-translation-tables:  $\bigwedge i t.$  translation-assm t  $\Longrightarrow$  translation-tables
ISA (take i t) = translation-tables ISA []
  and fixed-translation:  $\bigwedge i t$  addr load. translation-assm t  $\Longrightarrow$  translate-address
ISA addr load (take i t) = translate-address ISA addr load []

fun non-store-event :: 'regval event  $\Rightarrow$  bool where
  non-store-event (E-write-mem - paddr sz v -) = False
  | non-store-event (E-write-memt - paddr sz v tag -) = False
  | non-store-event - = True

abbreviation non-store-trace :: 'regval trace  $\Rightarrow$  bool where
  non-store-trace t  $\equiv$  ( $\forall e \in \text{set } t.$  non-store-event e)

lemma (in Cap-Axiom-Automaton) non-mem-trace-mem-axiomsI:
  assumes non-mem-trace t
  shows store-mem-axiom CC ISA t and store-tag-axiom CC ISA t and load-mem-axiom
CC ISA is-fetch t
proof –
  have i: non-mem-event (t ! i) if i < length t for i
    using assms that
    by (auto simp: non-mem-trace-def)
  show store-mem-axiom CC ISA t
    using i
    by (fastforce simp: store-mem-axiom-def writes-mem-val-at-idx-def bind-eq-Some-conv
elim!: writes-mem-val.elims)
  show store-tag-axiom CC ISA t
    using i
    by (fastforce simp: store-tag-axiom-def writes-mem-val-at-idx-def bind-eq-Some-conv
elim!: writes-mem-val.elims)
  show load-mem-axiom CC ISA is-fetch t
    using i

```

by (fastforce simp: load-mem-axiom-def reads-mem-val-at-idx-def bind-eq-Some-conv
elim!: reads-mem-val.elims)

qed

locale *Mem-Automaton* = *Capability-ISA-Fixed-Translation* **where** $CC = CC$
and $ISA = ISA$
for $CC :: 'cap$ *Capability-class* **and** $ISA :: ('cap, 'regval, 'instr, 'e)$ *isa* +
fixes *is-fetch* :: *bool*
begin

definition *paddr-in-mem-region* :: $'cap \Rightarrow acctype \Rightarrow nat \Rightarrow nat \Rightarrow bool$ **where**
paddr-in-mem-region c $acctype$ $paddr$ sz =
 $(\exists vaddr. set (address-range vaddr sz) \subseteq get-mem-region-method\ CC\ c \wedge$
 $translate-address\ ISA\ vaddr\ acctype\ [] = Some\ paddr)$

definition *has-access-permission* :: $perms \Rightarrow acctype \Rightarrow bool \Rightarrow bool \Rightarrow bool$ **where**
has-access-permission $perms$ $acctype$ $is-cap$ $is-local-cap$ =
 $(case\ acctype\ of$
 $\quad Fetch \Rightarrow permit-execute\ perms$
 $\quad | Load \Rightarrow permit-load\ perms \wedge (is-cap \longrightarrow permit-load-capability\ perms)$
 $\quad | Store \Rightarrow permit-store\ perms \wedge (is-cap \longrightarrow permit-store-capability\ perms) \wedge$
 $(is-local-cap \longrightarrow permit-store-local-capability\ perms))$

definition *authorises-access* :: $'cap \Rightarrow acctype \Rightarrow bool \Rightarrow bool \Rightarrow nat \Rightarrow nat \Rightarrow$
 $bool$ **where**
authorises-access c $acctype$ $is-cap$ $is-local-cap$ $paddr$ sz =
 $(is-tagged-method\ CC\ c \wedge \neg is-sealed-method\ CC\ c \wedge paddr-in-mem-region\ c$
 $acctype\ paddr\ sz \wedge$
 $has-access-permission\ (get-perms-method\ CC\ c)\ acctype\ is-cap\ is-local-cap)$

definition *access-enabled* :: $('cap, 'regval)$ *axiom-state* $\Rightarrow acctype \Rightarrow nat \Rightarrow nat$
 $\Rightarrow memory-byte\ list \Rightarrow bitU \Rightarrow bool$ **where**
access-enabled s $acctype$ $paddr$ sz v tag =
 $((tag \neq B0 \longrightarrow address-tag-aligned\ ISA\ paddr \wedge sz = tag-granule\ ISA) \wedge$
 $(case\ acctype\ of\ Load \Rightarrow True$
 $\quad | Store \Rightarrow (tag = B0 \vee tag = B1) \wedge length\ v = sz$
 $\quad | Fetch \Rightarrow tag = B0) \wedge$
 $(paddr \in translation-tables\ ISA\ [] \vee$
 $(\exists c' \in derivable\ (accessed-caps\ s).$
 $\quad let\ is-cap = tag \neq B0\ in$
 $\quad let\ is-local-cap = mem-val-is-local-cap\ CC\ ISA\ v\ tag \wedge tag = B1\ in$
 $\quad authorises-access\ c'\ acctype\ is-cap\ is-local-cap\ paddr\ sz)))$

lemmas *access-enabled-defs* = *access-enabled-def* *authorises-access-def* *paddr-in-mem-region-def*
has-access-permission-def

fun *enabled* :: $('cap, 'regval)$ *axiom-state* $\Rightarrow 'regval\ event \Rightarrow bool$ **where**
 $enabled\ s\ (E-write-mem - paddr\ sz\ v -) = access-enabled\ s\ Store\ paddr\ sz\ v\ B0$
 $| enabled\ s\ (E-write-memt - paddr\ sz\ v\ tag -) = access-enabled\ s\ Store\ paddr\ sz\ v$

```

tag
| enabled s (E-read-mem - paddr sz v) = access-enabled s (if is-fetch then Fetch
else Load) paddr sz v B0
| enabled s (E-read-memt - paddr sz v-tag) = access-enabled s (if is-fetch then Fetch
else Load) paddr sz (fst v-tag) (snd v-tag)
| enabled s - = True

```

sublocale *Cap-Axiom-Automaton* **where** *enabled* = *enabled* ..

lemma *accepts-store-mem-axiom*:
assumes *: *translation-assm t* **and** **: *accepts t*
shows *store-mem-axiom CC ISA t*
using *accepts-from-nth-enabledI*[*OF* **]
unfolding *store-mem-axiom-def*
unfolding *writes-mem-val-at-idx-def cap-derivable-iff-derivable*
unfolding *fixed-translation-tables*[*OF* *] *fixed-translation*[*OF* *]
by (*fastforce simp: access-enabled-defs bind-eq-Some-conv elim!: writes-mem-val.elims*)

lemma *accepts-store-tag-axiom*:
assumes *accepts t*
shows *store-tag-axiom CC ISA t*
using *accepts-from-nth-enabledI*[*OF* *assms*]
unfolding *store-tag-axiom-def writes-mem-val-at-idx-def*
by (*fastforce simp: access-enabled-defs bind-eq-Some-conv elim!: writes-mem-val.elims*)

lemma *accepts-load-mem-axiom*:
assumes *: *translation-assm t* **and** **: *accepts t*
shows *load-mem-axiom CC ISA is-fetch t*
unfolding *load-mem-axiom-def*
unfolding *reads-mem-val-at-idx-def cap-derivable-iff-derivable*
unfolding *fixed-translation-tables*[*OF* *] *fixed-translation*[*OF* *]
by (*auto simp: bind-eq-Some-conv elim!: reads-mem-val.elims dest!: accepts-from-nth-enabledI*[*OF* **] *split del: if-split*;
cases is-fetch; fastforce simp: access-enabled-defs)

lemma *non-mem-event-enabledI*:
enabled s e **if** *non-mem-event e*
using *that*
by (*auto elim: non-mem-event.elims*)

lemma *non-mem-trace-enabledI*:
trace-enabled s t **if** *non-mem-trace t*
using *that*
by (*induction t arbitrary: s*) (*auto intro: non-mem-event-enabledI*)

end

locale *Mem-Inv-Automaton* =
Mem-Automaton translation-assm CC ISA is-fetch +

```

    State-Invariant get-regval set-regval invariant inv-regs
  for CC :: 'cap Capability-class and ISA :: ('cap, 'regval, 'instr, 'e) isa
    and translation-asm :: 'regval event list  $\Rightarrow$  bool
    and is-fetch :: bool and ex-traces :: bool
    and get-regval :: string  $\Rightarrow$  'regstate  $\Rightarrow$  'regval option
    and set-regval :: string  $\Rightarrow$  'regval  $\Rightarrow$  'regstate  $\Rightarrow$  'regstate option
    and invariant :: 'regstate  $\Rightarrow$  bool and inv-regs :: register-name set
  begin

  sublocale Cap-Axiom-Inv-Automaton where enabled = enabled and ex-traces =
    ex-traces
  proof
    fix s e
    assume non-cap-event e
    then show enabled s e
      by (cases e) auto
  next
    fix s r v
    assume r  $\notin$  special-reg-names
    then show enabled s (E-read-reg r v)
      by auto
  qed

  lemma non-mem-exp-trace-enabledI:
    trace-enabled s t if non-mem-exp m and (m, t, m')  $\in$  Traces
    using that
    by (auto simp: non-mem-exp-def intro: non-mem-trace-enabledI)

  lemma non-mem-exp-traces-enabledI:
    traces-enabled m s regs if non-mem-exp m
    using that
    by (auto simp: traces-enabled-def intro: non-mem-exp-trace-enabledI)

  lemma traces-enabled-mem-axioms:
    assumes traces-enabled m initial regs and hasTrace t m
      and reads-regs-from inv-regs t regs and invariant regs
      and hasException t m  $\vee$  hasFailure t m  $\longrightarrow$  ex-traces
      and translation-asm t
    shows store-mem-axiom CC ISA t
      and store-tag-axiom CC ISA t
      and load-mem-axiom CC ISA is-fetch t
    using assms
    by (intro accepts-store-mem-axiom accepts-store-tag-axiom accepts-load-mem-axiom
      traces-enabled-accepts-fromI[where m = m and regs = regs];
      auto)+

  end

```

```

end
theory Cheri-reg-lemmas
imports Sail-CHERI-MIPS.Cheri-lemmas
begin

termination execute by size-change

definition
  register-names  $\equiv$ 
    {"InstCount", "CID", "ErrorEPCC", "KDC", "KR2C", "KR1C", "CPLR",
     "CULR", "C31", "C30", "C29", "C28", "C27", "C26", "C25",
     "C24", "C23", "C22", "C21", "C20", "C19", "C18", "C17",
     "C16", "C15", "C14", "C13", "C12", "C11", "C10", "C09",
     "C08", "C07", "C06", "C05", "C04", "C03", "C02", "C01",
     "DDC", "CapCause", "NextPCC", "DelayedPCC", "PCC", "KCC", "EPCC",
     "UART-RVALID", "UART-RDATA", "UART-WRITTEN", "UART-WDATA",
     "GPR",
     "LO", "HI", "DelayedPC", "BranchPending", "InBranchDelay",
     "NextInBranchDelay", "CP0Status", "CP0ConfigK0", "CP0UserLocal",
     "CP0HWREna", "CP0Count", "CP0BadInstrP", "CP0BadInstr",
     "LastInstrBits", "CurrentInstrBits", "CP0BadVAddr", "CP0LLAddr",
     "CP0LLBit", "CP0Cause", "CP0Compare", "TLBEntry63", "TLBEntry62",
     "TLBEntry61", "TLBEntry60", "TLBEntry59", "TLBEntry58", "TLBEntry57",
     "TLBEntry56", "TLBEntry55", "TLBEntry54", "TLBEntry53", "TLBEntry52",
     "TLBEntry51", "TLBEntry50", "TLBEntry49", "TLBEntry48", "TLBEntry47",
     "TLBEntry46", "TLBEntry45", "TLBEntry44", "TLBEntry43", "TLBEntry42",
     "TLBEntry41", "TLBEntry40", "TLBEntry39", "TLBEntry38", "TLBEntry37",
     "TLBEntry36", "TLBEntry35", "TLBEntry34", "TLBEntry33", "TLBEntry32",
     "TLBEntry31", "TLBEntry30", "TLBEntry29", "TLBEntry28", "TLBEntry27",
     "TLBEntry26", "TLBEntry25", "TLBEntry24", "TLBEntry23", "TLBEntry22",
     "TLBEntry21", "TLBEntry20", "TLBEntry19", "TLBEntry18", "TLBEntry17",
     "TLBEntry16", "TLBEntry15", "TLBEntry14", "TLBEntry13", "TLBEntry12",
     "TLBEntry11", "TLBEntry10", "TLBEntry09", "TLBEntry08", "TLBEntry07",
     "TLBEntry06", "TLBEntry05", "TLBEntry04", "TLBEntry03", "TLBEntry02",
     "TLBEntry01", "TLBEntry00", "TLBXContext", "TLBEntryHi", "TLBWired",
     "TLBPageMask", "TLBContext", "TLBEntryLo1", "TLBEntryLo0",
     "TLBRandom", "TLBIndex", "TLBProbe", "NextPC", "PC"}

lemma register-name-cases:
  obtains r = "InstCount"
  | r = "CID"
  | r = "ErrorEPCC"
  | r = "KDC"
  | r = "KR2C"
  | r = "KR1C"
  | r = "CPLR"
  | r = "CULR"

```

```

| r = "C31"
| r = "C30"
| r = "C29"
| r = "C28"
| r = "C27"
| r = "C26"
| r = "C25"
| r = "C24"
| r = "C23"
| r = "C22"
| r = "C21"
| r = "C20"
| r = "C19"
| r = "C18"
| r = "C17"
| r = "C16"
| r = "C15"
| r = "C14"
| r = "C13"
| r = "C12"
| r = "C11"
| r = "C10"
| r = "C09"
| r = "C08"
| r = "C07"
| r = "C06"
| r = "C05"
| r = "C04"
| r = "C03"
| r = "C02"
| r = "C01"
| r = "DDC"
| r = "CapCause"
| r = "NextPCC"
| r = "DelayedPCC"
| r = "PCC"
| r = "KCC"
| r = "EPCC"
| r = "UART-RVALID"
| r = "UART-RDATA"
| r = "UART-WRITTEN"
| r = "UART-WDATA"
| r = "GPR"
| r = "LO"
| r = "HI"
| r = "DelayedPC"
| r = "BranchPending"
| r = "InBranchDelay"
| r = "NextInBranchDelay"

```



```

| r = "CP0Status"
| r = "CP0ConfigK0"
| r = "CP0UserLocal"
| r = "CP0HWREna"
| r = "CP0Count"
| r = "CP0BadInstrP"
| r = "CP0BadInstr"
| r = "LastInstrBits"
| r = "CurrentInstrBits"
| r = "CP0BadVAddr"
| r = "CP0LLAddr"
| r = "CP0LLBit"
| r = "CP0Cause"
| r = "CP0Compare"
| r = "TLBEntry63"
| r = "TLBEntry62"
| r = "TLBEntry61"
| r = "TLBEntry60"
| r = "TLBEntry59"
| r = "TLBEntry58"
| r = "TLBEntry57"
| r = "TLBEntry56"
| r = "TLBEntry55"
| r = "TLBEntry54"
| r = "TLBEntry53"
| r = "TLBEntry52"
| r = "TLBEntry51"
| r = "TLBEntry50"
| r = "TLBEntry49"
| r = "TLBEntry48"
| r = "TLBEntry47"
| r = "TLBEntry46"
| r = "TLBEntry45"
| r = "TLBEntry44"
| r = "TLBEntry43"
| r = "TLBEntry42"
| r = "TLBEntry41"
| r = "TLBEntry40"
| r = "TLBEntry39"
| r = "TLBEntry38"
| r = "TLBEntry37"
| r = "TLBEntry36"
| r = "TLBEntry35"
| r = "TLBEntry34"
| r = "TLBEntry33"
| r = "TLBEntry32"
| r = "TLBEntry31"
| r = "TLBEntry30"
| r = "TLBEntry29"

```

```

| r = "TLBEntry28"
| r = "TLBEntry27"
| r = "TLBEntry26"
| r = "TLBEntry25"
| r = "TLBEntry24"
| r = "TLBEntry23"
| r = "TLBEntry22"
| r = "TLBEntry21"
| r = "TLBEntry20"
| r = "TLBEntry19"
| r = "TLBEntry18"
| r = "TLBEntry17"
| r = "TLBEntry16"
| r = "TLBEntry15"
| r = "TLBEntry14"
| r = "TLBEntry13"
| r = "TLBEntry12"
| r = "TLBEntry11"
| r = "TLBEntry10"
| r = "TLBEntry09"
| r = "TLBEntry08"
| r = "TLBEntry07"
| r = "TLBEntry06"
| r = "TLBEntry05"
| r = "TLBEntry04"
| r = "TLBEntry03"
| r = "TLBEntry02"
| r = "TLBEntry01"
| r = "TLBEntry00"
| r = "TLBXContext"
| r = "TLBEntryHi"
| r = "TLBWired"
| r = "TLBPageMask"
| r = "TLBContext"
| r = "TLBEntryLo1"
| r = "TLBEntryLo0"
| r = "TLBRandom"
| r = "TLBIndex"
| r = "TLBProbe"
| r = "NextPC"
| r = "PC"
| r ∉ register-names
proof cases
  assume r ∈ register-names
  then show ?thesis
    unfolding register-names-def
    by (elim insertE) (auto elim: that)
qed

```

lemma *set-regval-non-register-name*[simp]:
 $r \notin \text{register-names} \implies \text{set-regval } r \ v \ s = \text{None}$
by (auto simp: register-names-def set-regval-def)

lemma *get-regval-non-register-name*[simp]:
 $r \notin \text{register-names} \implies \text{get-regval } r \ s = \text{None}$
by (auto simp: register-names-def get-regval-def)

lemma *set-regval-cases*:
assumes *set-regval* $r \ v \ s = \text{Some } s'$
obtains v' **where** $r = \text{"InstCount"}$ **and** *int-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{InstCount} := v')$
 $| \ v'$ **where** $r = \text{"CID"}$ **and** *bitvector-64-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CID} := v')$
 $| \ v'$ **where** $r = \text{"ErrorEPCC"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{ErrorEPCC} := v')$
 $| \ v'$ **where** $r = \text{"KDC"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{KDC} := v')$
 $| \ v'$ **where** $r = \text{"KR2C"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{KR2C} := v')$
 $| \ v'$ **where** $r = \text{"KR1C"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{KR1C} := v')$
 $| \ v'$ **where** $r = \text{"CPLR"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CPLR} := v')$
 $| \ v'$ **where** $r = \text{"CULR"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CULR} := v')$
 $| \ v'$ **where** $r = \text{"C31"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C31} := v')$
 $| \ v'$ **where** $r = \text{"C30"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C30} := v')$
 $| \ v'$ **where** $r = \text{"C29"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C29} := v')$
 $| \ v'$ **where** $r = \text{"C28"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C28} := v')$
 $| \ v'$ **where** $r = \text{"C27"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C27} := v')$
 $| \ v'$ **where** $r = \text{"C26"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C26} := v')$
 $| \ v'$ **where** $r = \text{"C25"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C25} := v')$
 $| \ v'$ **where** $r = \text{"C24"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C24} := v')$
 $| \ v'$ **where** $r = \text{"C23"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C23} := v')$
 $| \ v'$ **where** $r = \text{"C22"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C22} := v')$
 $| \ v'$ **where** $r = \text{"C21"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C21} := v')$
 $| \ v'$ **where** $r = \text{"C20"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{C20} := v')$

```

:= v')
| v' where r = "C19" and Capability-of-regval v = Some v' and s' = s(C19
:= v')
| v' where r = "C18" and Capability-of-regval v = Some v' and s' = s(C18
:= v')
| v' where r = "C17" and Capability-of-regval v = Some v' and s' = s(C17
:= v')
| v' where r = "C16" and Capability-of-regval v = Some v' and s' = s(C16
:= v')
| v' where r = "C15" and Capability-of-regval v = Some v' and s' = s(C15
:= v')
| v' where r = "C14" and Capability-of-regval v = Some v' and s' = s(C14
:= v')
| v' where r = "C13" and Capability-of-regval v = Some v' and s' = s(C13
:= v')
| v' where r = "C12" and Capability-of-regval v = Some v' and s' = s(C12
:= v')
| v' where r = "C11" and Capability-of-regval v = Some v' and s' = s(C11
:= v')
| v' where r = "C10" and Capability-of-regval v = Some v' and s' = s(C10
:= v')
| v' where r = "C09" and Capability-of-regval v = Some v' and s' = s(C09
:= v')
| v' where r = "C08" and Capability-of-regval v = Some v' and s' = s(C08
:= v')
| v' where r = "C07" and Capability-of-regval v = Some v' and s' = s(C07
:= v')
| v' where r = "C06" and Capability-of-regval v = Some v' and s' = s(C06
:= v')
| v' where r = "C05" and Capability-of-regval v = Some v' and s' = s(C05
:= v')
| v' where r = "C04" and Capability-of-regval v = Some v' and s' = s(C04
:= v')
| v' where r = "C03" and Capability-of-regval v = Some v' and s' = s(C03
:= v')
| v' where r = "C02" and Capability-of-regval v = Some v' and s' = s(C02
:= v')
| v' where r = "C01" and Capability-of-regval v = Some v' and s' = s(C01
:= v')
| v' where r = "DDC" and Capability-of-regval v = Some v' and s' = s(DDC
:= v')
| v' where r = "CapCause" and CapCauseReg-of-regval v = Some v' and s' =
s(CapCause := v')
| v' where r = "NextPCC" and Capability-of-regval v = Some v' and s' =
s(NextPCC := v')
| v' where r = "DelayedPCC" and Capability-of-regval v = Some v' and s' =
s(DelayedPCC := v')
| v' where r = "PCC" and Capability-of-regval v = Some v' and s' = s(regstate.PCC
:= v')

```

| v' **where** $r = \text{"KCC"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{regstate.KCC} := v')$
 | v' **where** $r = \text{"EPCC"}$ **and** *Capability-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{EPCC} := v')$
 | v' **where** $r = \text{"UART-RVALID"}$ **and** *bitvector-1-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{UART-RVALID} := v')$
 | v' **where** $r = \text{"UART-RDATA"}$ **and** *bitvector-8-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{UART-RDATA} := v')$
 | v' **where** $r = \text{"UART-WRITTEN"}$ **and** *bitvector-1-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{UART-WRITTEN} := v')$
 | v' **where** $r = \text{"UART-WDATA"}$ **and** *bitvector-8-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{UART-WDATA} := v')$
 | v' **where** $r = \text{"GPR"}$ **and** *vector-of-regval* *bitvector-64-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{GPR} := v')$
 | v' **where** $r = \text{"LO"}$ **and** *bitvector-64-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{LO} := v')$
 | v' **where** $r = \text{"HI"}$ **and** *bitvector-64-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{HI} := v')$
 | v' **where** $r = \text{"DelayedPC"}$ **and** *bitvector-64-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{DelayedPC} := v')$
 | v' **where** $r = \text{"BranchPending"}$ **and** *bitvector-1-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{BranchPending} := v')$
 | v' **where** $r = \text{"InBranchDelay"}$ **and** *bitvector-1-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{InBranchDelay} := v')$
 | v' **where** $r = \text{"NextInBranchDelay"}$ **and** *bitvector-1-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{NextInBranchDelay} := v')$
 | v' **where** $r = \text{"CP0Status"}$ **and** *StatusReg-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CP0Status} := v')$
 | v' **where** $r = \text{"CP0ConfigK0"}$ **and** *bitvector-3-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CP0ConfigK0} := v')$
 | v' **where** $r = \text{"CP0UserLocal"}$ **and** *bitvector-64-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CP0UserLocal} := v')$
 | v' **where** $r = \text{"CP0HWREna"}$ **and** *bitvector-32-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CP0HWREna} := v')$
 | v' **where** $r = \text{"CP0Count"}$ **and** *bitvector-32-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CP0Count} := v')$
 | v' **where** $r = \text{"CP0BadInstrP"}$ **and** *bitvector-32-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CP0BadInstrP} := v')$
 | v' **where** $r = \text{"CP0BadInstr"}$ **and** *bitvector-32-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CP0BadInstr} := v')$
 | v' **where** $r = \text{"LastInstrBits"}$ **and** *bitvector-32-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{LastInstrBits} := v')$
 | v' **where** $r = \text{"CurrentInstrBits"}$ **and** *bitvector-32-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CurrentInstrBits} := v')$
 | v' **where** $r = \text{"CP0BadVAddr"}$ **and** *bitvector-64-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CP0BadVAddr} := v')$
 | v' **where** $r = \text{"CP0LLAddr"}$ **and** *bitvector-64-dec-of-regval* $v = \text{Some } v'$ **and** $s' = s(\text{CP0LLAddr} := v')$
 | v' **where** $r = \text{"CP0LLBit"}$ **and** *bitvector-1-dec-of-regval* $v = \text{Some } v'$ **and** s'

```

= s( $\langle CPOLLBit := v' \rangle$ )
|  $v'$  where  $r = "CP0Cause"$  and CauseReg-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle CP0Cause := v' \rangle)$   

|  $v'$  where  $r = "CP0Compare"$  and bitvector-32-dec-of-regval  $v = Some\ v'$  and  

 $s' = s(\langle CP0Compare := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry63"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry63 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry62"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry62 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry61"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry61 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry60"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry60 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry59"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry59 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry58"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry58 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry57"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry57 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry56"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry56 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry55"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry55 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry54"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry54 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry53"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry53 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry52"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry52 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry51"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry51 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry50"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry50 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry49"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry49 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry48"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry48 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry47"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry47 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry46"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry46 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry45"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry45 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry44"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry44 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry43"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry43 := v' \rangle)$   

|  $v'$  where  $r = "TLBEntry42"$  and TLBEntry-of-regval  $v = Some\ v'$  and  $s' =$   

 $s(\langle TLBEntry42 := v' \rangle)$ 
```

| v' where r = "TLBEntry41" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry41 := v')
| v' where r = "TLBEntry40" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry40 := v')
| v' where r = "TLBEntry39" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry39 := v')
| v' where r = "TLBEntry38" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry38 := v')
| v' where r = "TLBEntry37" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry37 := v')
| v' where r = "TLBEntry36" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry36 := v')
| v' where r = "TLBEntry35" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry35 := v')
| v' where r = "TLBEntry34" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry34 := v')
| v' where r = "TLBEntry33" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry33 := v')
| v' where r = "TLBEntry32" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry32 := v')
| v' where r = "TLBEntry31" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry31 := v')
| v' where r = "TLBEntry30" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry30 := v')
| v' where r = "TLBEntry29" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry29 := v')
| v' where r = "TLBEntry28" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry28 := v')
| v' where r = "TLBEntry27" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry27 := v')
| v' where r = "TLBEntry26" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry26 := v')
| v' where r = "TLBEntry25" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry25 := v')
| v' where r = "TLBEntry24" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry24 := v')
| v' where r = "TLBEntry23" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry23 := v')
| v' where r = "TLBEntry22" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry22 := v')
| v' where r = "TLBEntry21" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry21 := v')
| v' where r = "TLBEntry20" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry20 := v')
| v' where r = "TLBEntry19" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry19 := v')
| v' where r = "TLBEntry18" and TLBEntry-of-regval v = Some v' and s' =
s(TLBEntry18 := v')
| v' where r = "TLBEntry17" and TLBEntry-of-regval v = Some v' and s' =

[illegible]

| v' **where** $r = \text{"TLBRandom"}$ **and** $\text{bitvector-6-dec-of-regval } v = \text{Some } v'$ **and**
 $s' = s(\text{TLBRandom} := v')$
 | v' **where** $r = \text{"TLBIndex"}$ **and** $\text{bitvector-6-dec-of-regval } v = \text{Some } v'$ **and** $s' = s(\text{TLBIndex} := v')$
 | v' **where** $r = \text{"TLBProbe"}$ **and** $\text{bitvector-1-dec-of-regval } v = \text{Some } v'$ **and** $s' = s(\text{TLBProbe} := v')$
 | v' **where** $r = \text{"NextPC"}$ **and** $\text{bitvector-64-dec-of-regval } v = \text{Some } v'$ **and** $s' = s(\text{NextPC} := v')$
 | v' **where** $r = \text{"PC"}$ **and** $\text{bitvector-64-dec-of-regval } v = \text{Some } v'$ **and** $s' = s(\text{PC} := v')$
proof –
 from *assms* **have** $r \in \text{register-names}$
 by (cases $r \in \text{register-names}$) *auto*
 with *assms* **show** *thesis*
 by (cases r rule: *register-name-cases*) (*auto simp: register-defs elim: that*)
qed

lemma *get-regval-simps*:

$\text{get-regval "InstCount" } s = \text{Some } (\text{regval-of-int } (\text{InstCount } s))$
 $\text{get-regval "CID" } s = \text{Some } (\text{regval-of-bitvector-64-dec } (\text{CID } s))$
 $\text{get-regval "ErrorEPCC" } s = \text{Some } (\text{regval-of-Capability } (\text{ErrorEPCC } s))$
 $\text{get-regval "KDC" } s = \text{Some } (\text{regval-of-Capability } (\text{KDC } s))$
 $\text{get-regval "KR2C" } s = \text{Some } (\text{regval-of-Capability } (\text{KR2C } s))$
 $\text{get-regval "KR1C" } s = \text{Some } (\text{regval-of-Capability } (\text{KR1C } s))$
 $\text{get-regval "CPLR" } s = \text{Some } (\text{regval-of-Capability } (\text{CPLR } s))$
 $\text{get-regval "CULR" } s = \text{Some } (\text{regval-of-Capability } (\text{CULR } s))$
 $\text{get-regval "C31" } s = \text{Some } (\text{regval-of-Capability } (\text{C31 } s))$
 $\text{get-regval "C30" } s = \text{Some } (\text{regval-of-Capability } (\text{C30 } s))$
 $\text{get-regval "C29" } s = \text{Some } (\text{regval-of-Capability } (\text{C29 } s))$
 $\text{get-regval "C28" } s = \text{Some } (\text{regval-of-Capability } (\text{C28 } s))$
 $\text{get-regval "C27" } s = \text{Some } (\text{regval-of-Capability } (\text{C27 } s))$
 $\text{get-regval "C26" } s = \text{Some } (\text{regval-of-Capability } (\text{C26 } s))$
 $\text{get-regval "C25" } s = \text{Some } (\text{regval-of-Capability } (\text{C25 } s))$
 $\text{get-regval "C24" } s = \text{Some } (\text{regval-of-Capability } (\text{C24 } s))$
 $\text{get-regval "C23" } s = \text{Some } (\text{regval-of-Capability } (\text{C23 } s))$
 $\text{get-regval "C22" } s = \text{Some } (\text{regval-of-Capability } (\text{C22 } s))$
 $\text{get-regval "C21" } s = \text{Some } (\text{regval-of-Capability } (\text{C21 } s))$
 $\text{get-regval "C20" } s = \text{Some } (\text{regval-of-Capability } (\text{C20 } s))$
 $\text{get-regval "C19" } s = \text{Some } (\text{regval-of-Capability } (\text{C19 } s))$
 $\text{get-regval "C18" } s = \text{Some } (\text{regval-of-Capability } (\text{C18 } s))$
 $\text{get-regval "C17" } s = \text{Some } (\text{regval-of-Capability } (\text{C17 } s))$
 $\text{get-regval "C16" } s = \text{Some } (\text{regval-of-Capability } (\text{C16 } s))$
 $\text{get-regval "C15" } s = \text{Some } (\text{regval-of-Capability } (\text{C15 } s))$
 $\text{get-regval "C14" } s = \text{Some } (\text{regval-of-Capability } (\text{C14 } s))$
 $\text{get-regval "C13" } s = \text{Some } (\text{regval-of-Capability } (\text{C13 } s))$
 $\text{get-regval "C12" } s = \text{Some } (\text{regval-of-Capability } (\text{C12 } s))$
 $\text{get-regval "C11" } s = \text{Some } (\text{regval-of-Capability } (\text{C11 } s))$
 $\text{get-regval "C10" } s = \text{Some } (\text{regval-of-Capability } (\text{C10 } s))$
 $\text{get-regval "C09" } s = \text{Some } (\text{regval-of-Capability } (\text{C09 } s))$

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get-regval "C08" s = Some (regval-of-Capability (C08 s))
get-regval "C07" s = Some (regval-of-Capability (C07 s))
get-regval "C06" s = Some (regval-of-Capability (C06 s))
get-regval "C05" s = Some (regval-of-Capability (C05 s))
get-regval "C04" s = Some (regval-of-Capability (C04 s))
get-regval "C03" s = Some (regval-of-Capability (C03 s))
get-regval "C02" s = Some (regval-of-Capability (C02 s))
get-regval "C01" s = Some (regval-of-Capability (C01 s))
get-regval "DDC" s = Some (regval-of-Capability (DDC s))
get-regval "CapCause" s = Some (regval-of-CapCauseReg (CapCause s))
get-regval "NextPCC" s = Some (regval-of-Capability (NextPCC s))
get-regval "DelayedPCC" s = Some (regval-of-Capability (DelayedPCC s))
get-regval "PCC" s = Some (regval-of-Capability (regstate.PCC s))
get-regval "KCC" s = Some (regval-of-Capability (regstate.KCC s))
get-regval "EPCC" s = Some (regval-of-Capability (EPCC s))
get-regval "UART-RVALID" s = Some (regval-of-bitvector-1-dec (UART-RVALID
s))
get-regval "UART-RDATA" s = Some (regval-of-bitvector-8-dec (UART-RDATA
s))
get-regval "UART-WRITTEN" s = Some (regval-of-bitvector-1-dec (UART-WRITTEN
s))
get-regval "UART-WDATA" s = Some (regval-of-bitvector-8-dec (UART-WDATA
s))
get-regval "GPR" s = Some (regval-of-vector regval-of-bitvector-64-dec (GPR s))
get-regval "LO" s = Some (regval-of-bitvector-64-dec (LO s))
get-regval "HI" s = Some (regval-of-bitvector-64-dec (HI s))
get-regval "DelayedPC" s = Some (regval-of-bitvector-64-dec (DelayedPC s))
get-regval "BranchPending" s = Some (regval-of-bitvector-1-dec (BranchPending
s))
get-regval "InBranchDelay" s = Some (regval-of-bitvector-1-dec (InBranchDelay
s))
get-regval "NextInBranchDelay" s = Some (regval-of-bitvector-1-dec (NextInBranchDelay
s))
get-regval "CP0Status" s = Some (regval-of-StatusReg (CP0Status s))
get-regval "CP0ConfigK0" s = Some (regval-of-bitvector-3-dec (CP0ConfigK0
s))
get-regval "CP0UserLocal" s = Some (regval-of-bitvector-64-dec (CP0UserLocal
s))
get-regval "CP0HWREna" s = Some (regval-of-bitvector-32-dec (CP0HWREna
s))
get-regval "CP0Count" s = Some (regval-of-bitvector-32-dec (CP0Count s))
get-regval "CP0BadInstrP" s = Some (regval-of-bitvector-32-dec (CP0BadInstrP
s))
get-regval "CP0BadInstr" s = Some (regval-of-bitvector-32-dec (CP0BadInstr
s))
get-regval "LastInstrBits" s = Some (regval-of-bitvector-32-dec (LastInstrBits s))
get-regval "CurrentInstrBits" s = Some (regval-of-bitvector-32-dec (CurrentInstrBits
s))
get-regval "CP0BadVAddr" s = Some (regval-of-bitvector-64-dec (CP0BadVAddr
s))

```

```

s))
get-regval "CP0LLAddr" s = Some (regval-of-bitvector-64-dec (CP0LLAddr s))
get-regval "CP0LLBit" s = Some (regval-of-bitvector-1-dec (CP0LLBit s))
get-regval "CP0Cause" s = Some (regval-of-CauseReg (CP0Cause s))
get-regval "CP0Compare" s = Some (regval-of-bitvector-32-dec (CP0Compare
s))
get-regval "TLBEntry63" s = Some (regval-of-TLBEntry (TLBEntry63 s))
get-regval "TLBEntry62" s = Some (regval-of-TLBEntry (TLBEntry62 s))
get-regval "TLBEntry61" s = Some (regval-of-TLBEntry (TLBEntry61 s))
get-regval "TLBEntry60" s = Some (regval-of-TLBEntry (TLBEntry60 s))
get-regval "TLBEntry59" s = Some (regval-of-TLBEntry (TLBEntry59 s))
get-regval "TLBEntry58" s = Some (regval-of-TLBEntry (TLBEntry58 s))
get-regval "TLBEntry57" s = Some (regval-of-TLBEntry (TLBEntry57 s))
get-regval "TLBEntry56" s = Some (regval-of-TLBEntry (TLBEntry56 s))
get-regval "TLBEntry55" s = Some (regval-of-TLBEntry (TLBEntry55 s))
get-regval "TLBEntry54" s = Some (regval-of-TLBEntry (TLBEntry54 s))
get-regval "TLBEntry53" s = Some (regval-of-TLBEntry (TLBEntry53 s))
get-regval "TLBEntry52" s = Some (regval-of-TLBEntry (TLBEntry52 s))
get-regval "TLBEntry51" s = Some (regval-of-TLBEntry (TLBEntry51 s))
get-regval "TLBEntry50" s = Some (regval-of-TLBEntry (TLBEntry50 s))
get-regval "TLBEntry49" s = Some (regval-of-TLBEntry (TLBEntry49 s))
get-regval "TLBEntry48" s = Some (regval-of-TLBEntry (TLBEntry48 s))
get-regval "TLBEntry47" s = Some (regval-of-TLBEntry (TLBEntry47 s))
get-regval "TLBEntry46" s = Some (regval-of-TLBEntry (TLBEntry46 s))
get-regval "TLBEntry45" s = Some (regval-of-TLBEntry (TLBEntry45 s))
get-regval "TLBEntry44" s = Some (regval-of-TLBEntry (TLBEntry44 s))
get-regval "TLBEntry43" s = Some (regval-of-TLBEntry (TLBEntry43 s))
get-regval "TLBEntry42" s = Some (regval-of-TLBEntry (TLBEntry42 s))
get-regval "TLBEntry41" s = Some (regval-of-TLBEntry (TLBEntry41 s))
get-regval "TLBEntry40" s = Some (regval-of-TLBEntry (TLBEntry40 s))
get-regval "TLBEntry39" s = Some (regval-of-TLBEntry (TLBEntry39 s))
get-regval "TLBEntry38" s = Some (regval-of-TLBEntry (TLBEntry38 s))
get-regval "TLBEntry37" s = Some (regval-of-TLBEntry (TLBEntry37 s))
get-regval "TLBEntry36" s = Some (regval-of-TLBEntry (TLBEntry36 s))
get-regval "TLBEntry35" s = Some (regval-of-TLBEntry (TLBEntry35 s))
get-regval "TLBEntry34" s = Some (regval-of-TLBEntry (TLBEntry34 s))
get-regval "TLBEntry33" s = Some (regval-of-TLBEntry (TLBEntry33 s))
get-regval "TLBEntry32" s = Some (regval-of-TLBEntry (TLBEntry32 s))
get-regval "TLBEntry31" s = Some (regval-of-TLBEntry (TLBEntry31 s))
get-regval "TLBEntry30" s = Some (regval-of-TLBEntry (TLBEntry30 s))
get-regval "TLBEntry29" s = Some (regval-of-TLBEntry (TLBEntry29 s))
get-regval "TLBEntry28" s = Some (regval-of-TLBEntry (TLBEntry28 s))
get-regval "TLBEntry27" s = Some (regval-of-TLBEntry (TLBEntry27 s))
get-regval "TLBEntry26" s = Some (regval-of-TLBEntry (TLBEntry26 s))
get-regval "TLBEntry25" s = Some (regval-of-TLBEntry (TLBEntry25 s))
get-regval "TLBEntry24" s = Some (regval-of-TLBEntry (TLBEntry24 s))
get-regval "TLBEntry23" s = Some (regval-of-TLBEntry (TLBEntry23 s))
get-regval "TLBEntry22" s = Some (regval-of-TLBEntry (TLBEntry22 s))
get-regval "TLBEntry21" s = Some (regval-of-TLBEntry (TLBEntry21 s))

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get-regval "TLBEntry20" s = Some (regval-of-TLBEntry (TLBEntry20 s))
get-regval "TLBEntry19" s = Some (regval-of-TLBEntry (TLBEntry19 s))
get-regval "TLBEntry18" s = Some (regval-of-TLBEntry (TLBEntry18 s))
get-regval "TLBEntry17" s = Some (regval-of-TLBEntry (TLBEntry17 s))
get-regval "TLBEntry16" s = Some (regval-of-TLBEntry (TLBEntry16 s))
get-regval "TLBEntry15" s = Some (regval-of-TLBEntry (TLBEntry15 s))
get-regval "TLBEntry14" s = Some (regval-of-TLBEntry (TLBEntry14 s))
get-regval "TLBEntry13" s = Some (regval-of-TLBEntry (TLBEntry13 s))
get-regval "TLBEntry12" s = Some (regval-of-TLBEntry (TLBEntry12 s))
get-regval "TLBEntry11" s = Some (regval-of-TLBEntry (TLBEntry11 s))
get-regval "TLBEntry10" s = Some (regval-of-TLBEntry (TLBEntry10 s))
get-regval "TLBEntry09" s = Some (regval-of-TLBEntry (TLBEntry09 s))
get-regval "TLBEntry08" s = Some (regval-of-TLBEntry (TLBEntry08 s))
get-regval "TLBEntry07" s = Some (regval-of-TLBEntry (TLBEntry07 s))
get-regval "TLBEntry06" s = Some (regval-of-TLBEntry (TLBEntry06 s))
get-regval "TLBEntry05" s = Some (regval-of-TLBEntry (TLBEntry05 s))
get-regval "TLBEntry04" s = Some (regval-of-TLBEntry (TLBEntry04 s))
get-regval "TLBEntry03" s = Some (regval-of-TLBEntry (TLBEntry03 s))
get-regval "TLBEntry02" s = Some (regval-of-TLBEntry (TLBEntry02 s))
get-regval "TLBEntry01" s = Some (regval-of-TLBEntry (TLBEntry01 s))
get-regval "TLBEntry00" s = Some (regval-of-TLBEntry (TLBEntry00 s))
get-regval "TLBXContext" s = Some (regval-of-XContextReg (TLBXContext s))
get-regval "TLBEntryHi" s = Some (regval-of-TLBEntryHiReg (TLBEntryHi
s))
get-regval "TLBWired" s = Some (regval-of-bitvector-6-dec (TLBWired s))
get-regval "TLBPageMask" s = Some (regval-of-bitvector-16-dec (TLBPageMask
s))
get-regval "TLBContext" s = Some (regval-of-ContextReg (TLBContext s))
get-regval "TLBEntryLo1" s = Some (regval-of-TLBEntryLoReg (TLBEntryLo1
s))
get-regval "TLBEntryLo0" s = Some (regval-of-TLBEntryLoReg (TLBEntryLo0
s))
get-regval "TLBRandom" s = Some (regval-of-bitvector-6-dec (TLBRandom s))
get-regval "TLBIndex" s = Some (regval-of-bitvector-6-dec (TLBIndex s))
get-regval "TLBProbe" s = Some (regval-of-bitvector-1-dec (TLBProbe s))
get-regval "NextPC" s = Some (regval-of-bitvector-64-dec (NextPC s))
get-regval "PC" s = Some (regval-of-bitvector-64-dec (PC s))
by (auto simp: register-defs)

```

lemma *get-ignore-set-regval*:

assumes s' : *set-regval* r v $s = \text{Some } s'$ **and** r' : $r' \neq r$

shows *get-regval* r' $s' = \text{get-regval } r' s$

using *assms*

apply (*elim set-regval-cases*)

subgoal by (*cases* r' *rule*: *register-name-cases*; *simp add*: *get-regval-simps*)

subgoal by (*cases* r' *rule*: *register-name-cases*; *simp add*: *get-regval-simps*)

subgoal by (*cases* r' *rule*: *register-name-cases*; *simp add*: *get-regval-simps*)

subgoal by (*cases* r' *rule*: *register-name-cases*; *simp add*: *get-regval-simps*)

subgoal by (*cases* r' *rule*: *register-name-cases*; *simp add*: *get-regval-simps*)

[illegible]

[illegible]

bitvector-1-dec-of-regval $v = \text{Some } v'$
bitvector-3-dec-of-regval $v = \text{Some } v'$
bitvector-6-dec-of-regval $v = \text{Some } v'$
bitvector-8-dec-of-regval $v = \text{Some } v'$
bitvector-16-dec-of-regval $v = \text{Some } v'$
bitvector-32-dec-of-regval $v = \text{Some } v'$
bitvector-64-dec-of-regval $v = \text{Some } v'$
Capability-of-regval $v = \text{Some } v'$
CapCauseReg-of-regval $v = \text{Some } v'$
CauseReg-of-regval $v = \text{Some } v'$
ContextReg-of-regval $v = \text{Some } v'$
XContextReg-of-regval $v = \text{Some } v'$
StatusReg-of-regval $v = \text{Some } v'$
TLBEntry-of-regval $v = \text{Some } v'$
TLBEntryHiReg-of-regval $v = \text{Some } v'$
TLBEntryLoReg-of-regval $v = \text{Some } v'$

lemmas *regval-of-defs* =

regval-of-int-def regval-of-bitvector-64-dec-def regval-of-Capability-def regval-of-CapCauseReg-def
regval-of-CauseReg-def regval-of-ContextReg-def regval-of-XContextReg-def regval-of-StatusReg-def
regval-of-TLBEntry-def regval-of-TLBEntryHiReg-def regval-of-TLBEntryLoReg-def
regval-of-bit-def regval-of-bitvector-1-dec-def regval-of-bitvector-3-dec-def
regval-of-bitvector-6-dec-def regval-of-bitvector-8-dec-def regval-of-bitvector-16-dec-def
regval-of-bitvector-32-dec-def regval-of-vector-def

lemma *vector-of-regval-SomeE*:

assumes *: *vector-of-regval of-rv* $v = \text{Some } xs$ **and** **: $\bigwedge v v'. \text{of-rv } v = \text{Some } v' \implies \text{rv-of } v' = v$

obtains $v = \text{Regval-vector } (\text{map } \text{rv-of } xs)$

proof –

from * **obtain** vs **where** $v = \text{Regval-vector } vs$ **and** ***: $\text{map } \text{of-rv } vs = \text{map } \text{Some } xs$

by (*auto simp: vector-of-regval-def split: register-value.splits*)

moreover have $\text{map } \text{rv-of } xs = vs$

using ** ***

by (*induction xs arbitrary: vs*) *auto*

ultimately

show *?thesis*

using *that*

by *blast*

qed

lemma *get-absorb-set-regval*:

assumes *set-regval* $r \ v \ s = \text{Some } s'$

shows *get-regval* $r \ s' = \text{Some } v$

using *assms*

by (*elim set-regval-cases*)

(*auto simp: get-regval-simps regval-of-defs elim: of-regval-SomeE vector-of-regval-SomeE*)


```

end
theory CHERI-MIPS-Instantiation
imports Sail-CHERI-MIPS.Cheri-lemmas Cheri-reg-lemmas Recognising-Automata
Sail.Sail2-operators-mwords-lemmas Word-Extra
begin

```

3 Capability monotonicity in CHERI-MIPS

```

lemma more-and-or-boolM-simps[simp]:
  and-boolM (return True) m = m
  and-boolM (return False) m = return False
  or-boolM (return True) m = return True
  or-boolM (return False) m = m
  by (auto simp: and-boolM-def or-boolM-def)

lemma final-Done[intro, simp]: final (Done a)
  by (auto simp: final-def)

lemma bitU-of-bool-simps[simp]: bitU-of-bool True = B1 bitU-of-bool False = B0
  by (auto simp: bitU-of-bool-def)

lemma nat-of-mword-unat[simp]: nat-of-bv BC-mword w = Some (unat w)
  by (auto simp: nat-of-bv-def unat-def)

lemma pow2-simp[simp]: pow2 n = 2 ^ nat n
  by (auto simp: pow2-def pow-def)

lemma to-bits-mult[simp]:
  n = int (LENGTH('a))  $\implies$  to-bits n (a * b) = (to-bits n a * to-bits n b :: 'a::len
word)
  by (auto simp: to-bits-def of-bl-bin-word-of-int wi-hom-syms)

lemma to-bits-64-32[simp]: to-bits 64 32 = (32 :: 64 word)
  by eval

lemma mult-32-shiffl-5[simp]: 32 * (w :: 'a::len word) = w << 5
  by (auto simp: shiffl-t2n)

lemma shiffl-AND-mask-0[simp]: (w << n) AND mask n = 0
  by (intro word-eqI) (auto simp: word-ao-nth nth-shiffl)

lemma unat-to-bits[simp]:
  len = int (LENGTH('a))  $\implies$  unat (to-bits len i :: 'a::len word) = nat (i mod 2
^ LENGTH('a))
  by (auto simp: to-bits-def of-bl-bin-word-of-int unat-def uint-word-of-int)

lemma uint-to-bits[simp]:
  len = int (LENGTH('a))  $\implies$  uint (to-bits len i :: 'a::len word) = i mod 2 ^
LENGTH('a)

```

by (auto simp: to-bits-def of-bl-bin-word-of-int uint-word-of-int)

lemma length-take-chunks[simp]:
 $n \text{ dvd } \text{length } xs \implies \text{length } (\text{take-chunks } n \text{ } xs) = \text{length } xs \text{ div } n$
 by (induction n xs rule: take-chunks.induct) (auto simp: le-div-geq[symmetric] dvd-imp-le)

lemma length-mem-bytes-of-word[simp]:
 fixes $w :: 'a::\text{len word}$
 assumes $8 \text{ dvd } \text{LENGTH } ('a)$
 shows $\text{length } (\text{mem-bytes-of-word } w) = \text{LENGTH } ('a) \text{ div } 8$
 using assms
 by (auto simp add: mem-bytes-of-word-def simp del: take-chunks.simps)

lemma (in State-Invariant) Run-inv-assert-exp-iff[iff]:
 $\text{Run-inv } (\text{assert-exp } c \text{ } msg) \text{ } t \text{ } a \text{ } regs \longleftrightarrow c \wedge t = [] \wedge \text{invariant } regs$
 unfolding Run-inv-def
 by auto

lemma (in Cap-Axiom-Automaton) Run-runs-no-reg-writes-written-regs-eq:
 assumes $\text{Run } m \text{ } t \text{ } a$ and $\text{runs-no-reg-writes-to } \{r\} \text{ } m$
 shows $r \in \text{written-regs } (\text{run } s \text{ } t) \longleftrightarrow r \in \text{written-regs } s$
 proof –
 from assms have $E\text{-write-reg } r \text{ } v \notin \text{set } t$ for v
 unfolding runs-no-reg-writes-to-def
 by auto
 then show ?thesis
 by (induction t arbitrary: s) auto
 qed

3.1 Instantiation of the abstract model for CHERI-MIPS

definition get-cap-perms :: $\text{Capability} \Rightarrow \text{perms}$ where

$\text{get-cap-perms } c =$	
$\text{(\text{permit-ccall}}$	$= \text{Capability-permit-ccall } c,$
permit-execute	$= \text{Capability-permit-execute } c,$
permit-load	$= \text{Capability-permit-load } c,$
$\text{permit-load-capability}$	$= \text{Capability-permit-load-cap } c,$
permit-seal	$= \text{Capability-permit-seal } c,$
permit-store	$= \text{Capability-permit-store } c,$
$\text{permit-store-capability}$	$= \text{Capability-permit-store-cap } c,$
$\text{permit-store-local-capability}$	$= \text{Capability-permit-store-local-cap } c,$
$\text{permit-system-access}$	$= \text{Capability-access-system-regs } c,$
permit-unseal	$= \text{Capability-permit-unseal } c)$

definition set-cap-perms :: $\text{Capability} \Rightarrow \text{perms} \Rightarrow \text{Capability}$ where

$\text{set-cap-perms } c \text{ } p =$	
$c \text{(\text{Capability-permit-ccall}}$	$:= \text{permit-ccall } p,$
$\text{Capability-permit-execute}$	$:= \text{permit-execute } p,$

Capability-permit-load $:=$ *permit-load* *p*,
Capability-permit-load-cap $:=$ *permit-load-capability* *p*,
Capability-permit-seal $:=$ *permit-seal* *p*,
Capability-permit-store $:=$ *permit-store* *p*,
Capability-permit-store-cap $:=$ *permit-store-capability* *p*,
Capability-permit-store-local-cap $:=$ *permit-store-local-capability* *p*,
Capability-access-system-regs $:=$ *permit-system-access* *p*,
Capability-permit-unseal $:=$ *permit-unseal* *p*

fun *cap-of-mem-bytes* :: *memory-byte list* \Rightarrow *bitU* \Rightarrow *Capability option* **where**
cap-of-mem-bytes *bs* *t* =
Option.*bind* (*bool-of-bitU* *t*) (λt .
map-option (λbs . *memBitsToCapability* *t* *bs*) (*of-bits-method* *BC-mword* (*bits-of-mem-bytes*
bs)))

abbreviation

CC \equiv
(*is-tagged-method* = (λc . *Capability-tag* *c*),
is-sealed-method = (λc . *Capability-sealed* *c*),
get-mem-region-method = (λc . {*nat* (*getCapBase* *c*) $..<$ *nat* (*getCapTop* *c*)}),
get-obj-type-method = (λc . *unat* (*Capability-otype* *c*)),
get-perms-method = *get-cap-perms*,
get-cursor-method = (λc . *nat* (*getCapCursor* *c*)),
get-global-method = (λc . *Capability-global* *c*),
set-tag-method = (λc *t*. *c* (*Capability-tag* := *t*)),
set-seal-method = (λc *s*. *c* (*Capability-sealed* := *s*)),
set-obj-type-method = (λc *t*. *c* (*Capability-otype* := *of-nat* *t*)),
set-perms-method = *set-cap-perms*,
set-global-method = (λc *g*. *c* (*Capability-global* := *g*)),
cap-of-mem-bytes-method = *cap-of-mem-bytes*)

interpretation *Capabilities CC*

by *unfold-locales*
(*auto simp*: *bool-of-bitU-def* *memBitsToCapability-def* *capBitsToCapability-def*
get-cap-perms-def *set-cap-perms-def* *split*: *bitU.splits*)

abbreviation *privileged-CHERI-regs* \equiv {"EPCC", "ErrorEPCC", "KDC", "KCC",
"KR1C", "KR2C", "CapCause", "CPLR"}

definition *TLBEntries-names* \equiv *name* ' (*set* *TLBEntries*)

locale *CHERI-MIPS-ISA* =

fixes *translate-address* :: *nat* \Rightarrow *acctype* \Rightarrow *register-value trace* \Rightarrow *nat option*
begin

abbreviation *fetch-and-decode* \equiv (*fetch* ()) \gg (λres . *case* *res* *of* *Some* *ast* \Rightarrow
return *ast* | *None* \Rightarrow *Fail* "decode")

definition

```

ISA ≡
  (|instr-sem = execute,
   instr-fetch = fetch-and-decode,
   tag-granule = 32,
   PCC = {"PCC", "NextPCC", "DelayedPCC"},
   KCC = {"KCC"},
   IDC = {"C26"},
   caps-of-regval = (λrv. case rv of Regval-Capability c ⇒ {c} | - ⇒ {}),
   invokes-caps = (λinstr t. case instr of CCall - ⇒ True | - ⇒ False),
   instr-raises-ex = (λinstr t. hasException t (execute instr) ∨ hasFailure t
    (execute instr)),
   fetch-raises-ex = (λt. hasException t (fetch-and-decode) ∨ hasFailure t (fetch-and-decode)),
   exception-targets = (λrvs. ⋃ rv ∈ rvs. case rv of Regval-Capability c ⇒ {c} |
    - ⇒ {}),
   privileged-regs = privileged-CHERI-regs,
   translation-tables = (λt. {}),
   translate-address = translate-address)

```

interpretation *Capability-ISA CC ISA by unfold-locales*

sublocale *Register-State get-regval set-regval .*

lemma *ISA-simps[simp]:*
PCC ISA = {"PCC", "NextPCC", "DelayedPCC"}
KCC ISA = {"KCC"}
IDC ISA = {"C26"}
privileged-regs ISA = privileged-CHERI-regs
instr-sem ISA = execute
instr-fetch ISA = (fetch () ≫ (λres. case res of Some ast ⇒ return ast | None
⇒ Fail "decode"))
by (*auto simp: ISA-def*)

lemma *invokes-caps-iff-CCall[simp]:*
invokes-caps ISA instr t ⟷ (∃ cs cb sel. instr = CCall (cs, cb, sel))
by (*cases instr*) (*auto simp: ISA-def*)

lemma *instr-raises-ex-iff[simp]:*
instr-raises-ex ISA instr t ⟷ hasException t (execute instr) ∨ hasFailure t
(execute instr)
by (*auto simp: ISA-def*)

lemma *fetch-raises-ex-iff[simp]:*
fetch-raises-ex ISA t ⟷ hasException t (fetch-and-decode) ∨ hasFailure t (fetch-and-decode)
by (*auto simp: ISA-def*)

lemma *TLBEntries-no-cap:*
assumes *r ∈ set TLBEntries*
shows $\bigwedge c. \text{of-regval } r \text{ (Regval-Capability } c) = \text{None}$ **and** *name r ≠ "KCC"*
using *assms*

unfolding *TLBEntries-def register-defs*
by *auto*

lemma [*simp*]: *length TLBEntries = 64*
by (*auto simp: TLBEntries-def*)

lemma *vector-of-regval-Regval-Capability-None*[*simp*]:
vector-of-regval or (Regval-Capability c) = None
by (*auto simp: vector-of-regval-def*)

definition *is-cap-reg* :: ('s, register-value, Capability) register-ref \Rightarrow bool **where**
is-cap-reg r = ($\forall v c. \text{of-regval } r \ v = \text{Some } c \longleftrightarrow v = \text{Regval-Capability } c$)

lemma *Capability-of-regval-Some-iff-Regval-Capability*[*simp*]:
Capability-of-regval v = Some c \longleftrightarrow v = Regval-Capability c
by (*cases v*) *auto*

lemma *caps-of-regval-of-Capability*[*simp*]:
caps-of-regval ISA (regval-of-Capability c) = {c}
by (*auto simp: regval-of-Capability-def ISA-def*)

lemma *CapRegs-is-cap-reg*: $r \in \text{set CapRegs} \implies \text{is-cap-reg } r$
unfolding *register-defs CapRegs-def*
by (*auto simp: is-cap-reg-def*)

lemma [*simp*]: *length CapRegs = 32*
by (*auto simp: CapRegs-def*)

definition *CapRegs-names* $\equiv \text{name } '(\text{set CapRegs})$

lemma *CapRegs-names-unfold*[*simp*]:
CapRegs-names =
 $\{ "C31", "C30", "C29", "C28", "C27", "C26", "C25", "C24", "C23",$
 $"C22", "C21",$
 $"C20", "C19", "C18", "C17", "C16", "C15", "C14", "C13", "C12",$
 $"C11", "C10",$
 $"C09", "C08", "C07", "C06", "C05", "C04", "C03", "C02", "C01",$
 $"DDC" \}$
unfolding *CapRegs-names-def CapRegs-def register-defs*
by *auto*

lemma *name-CapRegs-CapRegs-names*: $r \in \text{set CapRegs} \implies \text{name } r \in \text{CapRegs-names}$
unfolding *CapRegs-names-def*
by *auto*

lemma *name-CapRegs-not-privileged*[*simp*]:
assumes $r \in \text{set CapRegs}$
shows $\text{name } r \neq "PCC"$
 $\text{name } r \neq "EPCC"$

```

    name r ≠ "ErrorEPCC"
    name r ≠ "KDC"
    name r ≠ "KCC"
    name r ≠ "KR1C"
    name r ≠ "KR2C"
    name r ≠ "CapCause"
    name r ≠ "CPLR"
  using assms
  by (auto dest: name-CapRegs-CapRegs-names)

lemma TLBEntries-names-unfold[simp]:
  TLBEntries-names =
    {"TLBEntry63", "TLBEntry62", "TLBEntry61", "TLBEntry60", "TLBEntry59",
     "TLBEntry58", "TLBEntry57", "TLBEntry56", "TLBEntry55", "TLBEntry54",
     "TLBEntry53", "TLBEntry52", "TLBEntry51", "TLBEntry50", "TLBEntry49",
     "TLBEntry48", "TLBEntry47", "TLBEntry46", "TLBEntry45", "TLBEntry44",
     "TLBEntry43", "TLBEntry42", "TLBEntry41", "TLBEntry40", "TLBEntry39",
     "TLBEntry38", "TLBEntry37", "TLBEntry36", "TLBEntry35", "TLBEntry34",
     "TLBEntry33", "TLBEntry32", "TLBEntry31", "TLBEntry30", "TLBEntry29",
     "TLBEntry28", "TLBEntry27", "TLBEntry26", "TLBEntry25", "TLBEntry24",
     "TLBEntry23", "TLBEntry22", "TLBEntry21", "TLBEntry20", "TLBEntry19",
     "TLBEntry18", "TLBEntry17", "TLBEntry16", "TLBEntry15", "TLBEntry14",
     "TLBEntry13", "TLBEntry12", "TLBEntry11", "TLBEntry10", "TLBEntry09",
     "TLBEntry08", "TLBEntry07", "TLBEntry06", "TLBEntry05", "TLBEntry04",
     "TLBEntry03", "TLBEntry02", "TLBEntry01", "TLBEntry00"}
  unfolding TLBEntries-def register-defs TLBEntries-names-def
  by auto

lemma ref-name-not-PCC[simp]:
  name CapCause-ref ≠ "PCC"
  name CP0Cause-ref ≠ "PCC"
  name CP0Status-ref ≠ "PCC"
  name TLBEntryHi-ref ≠ "PCC"
  name TLBEntryLo0-ref ≠ "PCC"
  name TLBEntryLo1-ref ≠ "PCC"
  name TLBContext-ref ≠ "PCC"
  name TLBXContext-ref ≠ "PCC"
  by (auto simp: register-defs)

lemma uint6-upper-bound[simp]: uint (idx :: 6 word) ≤ 63
  using uint-bounded[of idx]
  by auto

lemma upto-63-unfold:
  {0..63} = {0 :: int, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
    18, 19,
    20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
    37, 38, 39,
    40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56,

```

```

57, 58, 59,
    60, 61, 62, 63}
  by eval

lemma TLBEntry-name-not-PCC[simp]:
  assumes  $idx \in \{0..63\}$ 
  shows  $name (TLBEntries ! (64 - nat (idx + 1))) \neq "PCC"$ 
  using assms
  unfolding upto-63-unfold
  by (auto simp: TLBEntries-def register-defs)

lemma upto-31-unfold:  $\{0..31\} = \{0 :: int, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31\}$ 
  by eval

lemma [simp]:  $uint (idx :: 5 \text{ word}) \leq 31$ 
  using uint-bounded[of idx]
  by auto

lemma [simp]:  $caps\text{-of-regval } ISA (Regval\text{-Capability } c) = \{c\}$ 
  by (auto simp: ISA-def)

lemma [simp]:  $bits\text{-of-mem-bytes } (mem\text{-bytes-of-word } (capToMemBits c)) = map\ bitU\text{-of-bool } (to\text{-bl } (capToMemBits c))$ 
  unfolding mem-bytes-of-word-def bits-of-mem-bytes-def bits-of-bytes-def
  by (auto simp: append-assoc[symmetric] take-add[symmetric] simp del: append-assoc)

lemma [simp]:  $of\text{-bits-method } BC\text{-mword } (bits\text{-of-mem-bytes } (mem\text{-bytes-of-word } (capToMemBits c))) = Some (capToMemBits c)$ 
  by auto

lemma Capability-tag-memBitsToCapability[simp]:
   $Capability\text{-tag } (memBitsToCapability tag c) = tag$ 
  by (auto simp: memBitsToCapability-def capBitsToCapability-def)

lemma Run-throw-False[simp]:  $Run (throw e) t a \longleftrightarrow False$ 
  by (auto simp: throw-def)

lemma Run-SignalException-False[simp]:
   $Run (SignalException e) t a \longleftrightarrow False$ 
  by (auto simp: SignalException-def elim!: Run-bindE)

lemma Run-SignalException-wrappers-False[simp]:
   $Run (SignalExceptionTLB ex badAddr) t a \longleftrightarrow False$ 
   $Run (SignalExceptionBadAddr ex badAddr) t a \longleftrightarrow False$ 
  by (auto simp: SignalExceptionTLB-def SignalExceptionBadAddr-def elim!: Run-bindE)

lemma Run-raise-c2-exception-False[simp]:
   $Run (raise\text{-c2-exception8 } capEx reg8) t a \longleftrightarrow False$ 

```

$\text{Run } (\text{raise-c2-exception capEx reg5}) \ t \ a \longleftrightarrow \text{False}$
 $\text{Run } (\text{raise-c2-exception-noreg capEx}) \ t \ a \longleftrightarrow \text{False}$
by (*auto simp: raise-c2-exception8-def raise-c2-exception-def raise-c2-exception-noreg-def elim!: Run-bindE*)

lemma *Done-eq-bind-iff*:

$\text{Done } a = (m \ggg f) \longleftrightarrow (\exists a'. m = \text{Done } a' \wedge f \ a' = \text{Done } a)$
 $(m \ggg f) = \text{Done } a \longleftrightarrow (\exists a'. m = \text{Done } a' \wedge f \ a' = \text{Done } a)$
by (*cases m; auto*)+

lemma *Exception-eq-bind-iff*:

$\text{Exception } e = (m \ggg f) \longleftrightarrow (m = \text{Exception } e \vee (\exists a. m = \text{Done } a \wedge f \ a = \text{Exception } e))$
 $(m \ggg f) = \text{Exception } e \longleftrightarrow (m = \text{Exception } e \vee (\exists a. m = \text{Done } a \wedge f \ a = \text{Exception } e))$
by (*cases m; auto*)+

lemma *read-reg-no-ex*: $(\text{read-reg } r, \ t, \ \text{Exception } e) \in \text{Traces} \longleftrightarrow \text{False}$

by (*auto simp: read-reg-def elim: Read-reg-TracesE split: option.splits*)

lemma [*simp*]: $\text{bit-to-bool } (\text{bitU-of-bool } b) = b$

by (*auto simp: bitU-of-bool-def*)

lemma *to-bl-bool-to-bits*: $\text{to-bl } (\text{bool-to-bits } b) = [b]$

by (*auto simp: bool-to-bits-def eval*)

lemma *memBitsToCapability-capToMemBits*[*simp*]:

$\text{memBitsToCapability } \text{tag } (\text{capToMemBits } c) = c \langle \text{Capability-tag} := \text{tag} \rangle$

unfolding *memBitsToCapability-def capToMemBits-def capToBits-def capBitsToCapability-def*

by (*auto simp: word-bw-assocs subrange-vec-dec-subrange-list-dec slice-take word-cat-bl of-bl-append-same getCapPerms-def getCapHardPerms-def test-bit-of-bl*

nth-append append-assoc[symmetric]

simp del: append-assoc)

(*auto simp: to-bl-bool-to-bits*)

lemma [*simp*]: $\text{Capability-tag } c \implies c \langle \text{Capability-tag} := \text{True} \rangle = c$

by (*cases c*) *auto*

end

locale *CHERI-MIPS-Axiom-Automaton* = *CHERI-MIPS-ISA* +

fixes *enabled* :: $(\text{Capability}, \text{register-value}) \text{ axiom-state} \Rightarrow \text{register-value event} \Rightarrow \text{bool}$

begin

sublocale *Cap-Axiom-Automaton CC ISA enabled ..*

lemma *non-cap-exp-undefs*[*non-cap-expI*]:
non-cap-exp (*undefined-unit* *u*)
non-cap-exp (*undefined-string* *u*)
non-cap-exp (*undefined-int* *u*)
non-cap-exp (*undefined-range* *x y*)
non-cap-exp (*undefined-bitvector* *n*)
unfolding *undefined-unit-def* *undefined-string-def* *undefined-int-def* *undefined-bitvector-def*
undefined-range-def
by *non-cap-expI*

lemma *non-cap-exp-barrier*[*non-cap-expI*]:
non-cap-exp (*barrier* *b*)
unfolding *barrier-def* *non-cap-exp-def*
by (*auto elim: Traces-cases*)

lemma *non-cap-exp-skip*[*non-cap-expI*]:
non-cap-exp (*skip* *u*)
unfolding *skip-def*
by *non-cap-expI*

lemma *non-cap-exp-maybe-fail*[*non-cap-expI*]:
non-cap-exp (*maybe-fail* *msg x*)
unfolding *maybe-fail-def* *non-cap-exp-def*
by (*auto split: option.splits*)

lemma *non-cap-exp-shift-bits*[*non-cap-expI*]:
non-cap-exp (*shift-bits-left* *BCa BCb BCd v n*)
non-cap-exp (*shift-bits-right* *BCa BCb BCd v n*)
non-cap-exp (*shift-bits-right-arith* *BCa BCb BCd v n*)
unfolding *shift-bits-left-def* *shift-bits-right-def* *shift-bits-right-arith-def*
by *non-cap-expI*

lemma *no-cap-regvals*[*simp*]:
 $\bigwedge v. \text{bitvector-1-dec-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{bitvector-3-dec-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{bitvector-6-dec-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{bitvector-8-dec-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{bitvector-16-dec-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{bitvector-32-dec-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{bitvector-64-dec-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{CauseReg-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{StatusReg-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{ContextReg-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{XContextReg-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{int-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{TLBEntry-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{TLBEntryHiReg-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge v. \text{TLBEntryLoReg-of-regval } rv = \text{Some } v \implies \text{caps-of-regval ISA } rv = \{\}$
 $\bigwedge xs. \text{vector-of-regval of-rv } rv = \text{Some } xs \implies \text{caps-of-regval ISA } rv = \{\}$

$\bigwedge xs. \text{caps-of-regval } ISA \text{ (regval-of-vector } rv\text{-of } xs) = \{\}$
by (cases *rv*; auto simp: *ISA-def* *vector-of-regval-def* *regval-of-vector-def*) +

lemma *non-cap-reg-nth-TLBEntries*[*intro*, *simp*]:
 assumes $idx \in \{0..63\}$
 shows *non-cap-reg* (*TLBEntries* ! (64 - nat (*idx* + 1)))
 using *assms*
 unfolding *upto-63-unfold*
 by (elim *insertE*) (auto simp: *TLBEntries-def* *register-defs* *non-cap-reg-def*)

lemma *non-cap-exp-read-reg-access-TLBEntries*[*non-cap-expI*]:
 assumes $idx \in \{0..63\}$
 shows *non-cap-exp* (*read-reg* (*access-list-dec* *TLBEntries* *idx*))
 using *assms*
 by *non-cap-expI*

lemma *no-reg-writes-to-case-option*[*no-reg-writes-toI*]:
 assumes $\bigwedge a. \text{no-reg-writes-to } Rs \text{ (f } a)$
 and *no-reg-writes-to* *Rs* *m*
 shows *no-reg-writes-to* *Rs* (case *x* of *Some* *a* \Rightarrow *f* *a* | *None* \Rightarrow *m*)
 using *assms*
 by (cases *x*) auto

lemma *no-reg-writes-to-undefines*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*undefined-unit* *u*)
no-reg-writes-to *Rs* (*undefined-string* *u*)
no-reg-writes-to *Rs* (*undefined-int* *u*)
no-reg-writes-to *Rs* (*undefined-range* *x* *y*)
no-reg-writes-to *Rs* (*undefined-bitvector* *n*)
 unfolding *undefined-unit-def* *undefined-string-def* *undefined-int-def* *undefined-range-def*
undefined-bitvector-def
 by (*no-reg-writes-toI*) +

lemma *no-reg-writes-to-barrier*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*barrier* *b*)
 unfolding *barrier-def* *no-reg-writes-to-def*
 by (auto elim: *Traces-cases*)

lemma *no-reg-writes-to-skip*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*skip* *u*)
 unfolding *skip-def*
 by *no-reg-writes-toI*

lemma *no-reg-writes-to-maybe-fail*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*maybe-fail* *msg* *x*)
 unfolding *maybe-fail-def* *non-cap-exp-def*
 by (auto split: *option.splits*)

lemma *no-reg-writes-to-shift-bits*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (shift-bits-left BCa BCb BCd v n)
no-reg-writes-to Rs (shift-bits-right BCa BCb BCd v n)
no-reg-writes-to Rs (shift-bits-right-arith BCa BCb BCd v n)
unfolding *shift-bits-left-def shift-bits-right-def shift-bits-right-arith-def*
by *no-reg-writes-toI* +

lemma *no-reg-writes-to-write-ram*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (write-ram arg0 arg1 arg2 arg3 arg4)
unfolding *write-ram-def MEMea-def MEMval-def*
by *no-reg-writes-toI*

lemma *no-reg-writes-to-read-ram*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (read-ram arg0 arg1 arg2 arg3)
unfolding *read-ram-def MEMr-def*
by *no-reg-writes-toI*

lemma *no-reg-writes-to-read-memt-bytes*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (read-memt-bytes BCa BCb rk addr sz)
unfolding *read-memt-bytes-def maybe-fail-def*
by (*auto simp: no-reg-writes-to-def elim: bind-Traces-cases Traces-cases split: option.splits*)

lemma *no-reg-writes-to-read-memt*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (read-memt BCa BCb rk addr sz)
unfolding *read-memt-def*
by *no-reg-writes-toI*

lemma *no-reg-writes-to-write-memt*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (write-memt BCa BCb wk addr sz v t)
unfolding *write-memt-def*
by (*auto simp: no-reg-writes-to-def elim: Traces-cases split: option.splits*)

lemma *no-reg-writes-to-MEMval-tagged*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (MEMval-tagged addr sz t v)
unfolding *MEMval-tagged-def*
by *no-reg-writes-toI*

lemma *no-reg-writes-to-MEMval-tagged-conditional*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (MEMval-tagged-conditional addr sz t v)
unfolding *MEMval-tagged-conditional-def*
by *no-reg-writes-toI*

lemma *runs-no-reg-writes-to-SignalException*[*runs-no-reg-writes-toI*]:
runs-no-reg-writes-to Rs (SignalException ex)
unfolding *runs-no-reg-writes-to-def*
by *auto*

```

lemma runs-no-reg-writes-to-raise-c2-exception[runs-no-reg-writes-toI]:
  runs-no-reg-writes-to Rs (raise-c2-exception8 capEx reg8)
  runs-no-reg-writes-to Rs (raise-c2-exception capEx reg5)
  runs-no-reg-writes-to Rs (raise-c2-exception-noreg capEx)
  by (auto simp: runs-no-reg-writes-to-def)

lemma runs-no-reg-writes-to-checkCP0AccessHook[runs-no-reg-writes-toI]:
  runs-no-reg-writes-to Rs (checkCP0AccessHook u)
  unfolding checkCP0AccessHook-def pcc-access-system-regs-def
  by (no-reg-writes-toI)

lemma no-reg-writes-to-writeCapReg[no-reg-writes-toI, simp]:
  assumes CapRegs-names  $\cap$  Rs = {}
  shows no-reg-writes-to Rs (writeCapReg arg0 arg1)
  using assms name-CapRegs-CapRegs-names[of access-list-dec CapRegs (uint arg0)]
  unfolding writeCapReg-def bind-assoc capToString-def
  by (intro no-reg-writes-toI) (auto simp del: CapRegs-names-unfold)

lemma no-reg-writes-to-readCapReg[no-reg-writes-toI, simp]:
  no-reg-writes-to Rs (readCapReg arg0)
  unfolding readCapReg-def bind-assoc
  by (no-reg-writes-toI)

lemma no-reg-writes-to-readCapRegDDC[no-reg-writes-toI, simp]:
  no-reg-writes-to Rs (readCapRegDDC arg0)
  unfolding readCapRegDDC-def bind-assoc
  by (no-reg-writes-toI)

lemma non-mem-exp-rwCapReg[non-mem-expI]:
  non-mem-exp (readCapReg r)
  non-mem-exp (readCapRegDDC r)
  non-mem-exp (writeCapReg r v)
  by (non-mem-expI simp: readCapReg-def readCapRegDDC-def writeCapReg-def
  capToString-def)

declare MemAccessType.split[where  $P = \lambda m. \text{no-reg-writes-to } Rs \ m$  for  $Rs$ ,
  THEN iffD2, no-reg-writes-toI]
declare MemAccessType.split[split]
declare WordType.split[where  $P = \lambda m. \text{no-reg-writes-to } Rs \ m$  for  $Rs$ , THEN
  iffD2, no-reg-writes-toI]
declare WordType.split[split]
declare ClearRegSet.split[where  $P = \lambda m. \text{no-reg-writes-to } Rs \ m$  for  $Rs$ , THEN
  iffD2, no-reg-writes-toI]
declare ClearRegSet.split[split]

end

locale CHERI-MIPS-Axiom-Inv-Automaton = CHERI-MIPS-Axiom-Automaton
+
```

Cap-Axiom-Inv-Automaton **where** $CC = CC$ **and** $ISA = ISA$ **and** $get-regval = get-regval$ **and** $set-regval = set-regval$
begin

lemma *preserve-invariant-undefineds*[*preserves-invariantI*]:
traces-preserve-invariant (*undefined-unit* *u*)
traces-preserve-invariant (*undefined-string* *u*)
traces-preserve-invariant (*undefined-int* *u*)
traces-preserve-invariant (*undefined-range* *x y*)
traces-preserve-invariant (*undefined-bitvector* *n*)
by (*intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp*) +

lemma *preserves-invariant-barrier*[*no-reg-writes-toI, simp*]:
traces-preserve-invariant (*barrier* *b*)
by (*intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp*) +

lemma *preserves-invariant-skip*[*no-reg-writes-toI, simp*]:
traces-preserve-invariant (*skip* *u*)
by (*intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp*) +

lemma *preserves-invariant-maybe-fail*[*no-reg-writes-toI, simp*]:
traces-preserve-invariant (*maybe-fail* *msg x*)
by (*intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp*) +

lemma *preserves-invariant-shift-bits*[*no-reg-writes-toI, simp*]:
traces-preserve-invariant (*shift-bits-left* *BCa BCb BCd v n*)
traces-preserve-invariant (*shift-bits-right* *BCa BCb BCd v n*)
traces-preserve-invariant (*shift-bits-right-arith* *BCa BCb BCd v n*)
by (*intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp*) +

lemma *preserves-invariant-write-ram*[*preserves-invariantI*]:
traces-preserve-invariant (*write-ram* *arg0 arg1 arg2 arg3 arg4*)
by (*intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp*)

lemma *preserves-invariant-read-ram*[*preserves-invariantI*]:
traces-preserve-invariant (*read-ram* *arg0 arg1 arg2 arg3*)
by (*intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp*)

lemma *traces-enabled-case-option*[*traces-enabledI*]:
assumes $\bigwedge a. x = \text{Some } a \implies \text{traces-enabled } (f\ a) \ s \ \text{regs}$
and $x = \text{None} \implies \text{traces-enabled } m \ s \ \text{regs}$
shows $\text{traces-enabled } (\text{case } x \text{ of } \text{Some } a \Rightarrow f\ a \mid \text{None} \Rightarrow m) \ s \ \text{regs}$
using *assms*
by (*cases x*) *auto*

lemma *Run-inv-ifE*:
assumes $\text{Run-inv } (\text{if } c \text{ then } m1 \text{ else } m2) \ t \ a \ \text{regs}$

obtains *Run-inv m1 t a regs and c | Run-inv m2 t a regs and $\neg c$*
using *assms*
by (*auto split: if-splits*)

lemma *Run-inv-letE*:
assumes *Run-inv (let x = y in f x) t a regs*
obtains *Run-inv (f y) t a regs*
using *assms*
by *auto*

declare *Run-inv-ifE*[**where** *t = t and thesis = c \in derivable-caps (run s t) for s t c, derivable-capsE*]
declare *Run-inv-letE*[**where** *t = t and thesis = c \in derivable-caps (run s t) for s t c, derivable-capsE*]

lemma *Run-inv-return[simp]*: *Run-inv (return a) t a' regs \longleftrightarrow (a' = a \wedge t = [] \wedge invariant regs)*
unfolding *Run-inv-def*
by *auto*

lemma *null-cap-derivable[intro, simp]*: *null-cap \in derivable-caps s*
unfolding *null-cap-def derivable-caps-def*
by *auto*

lemma *read-reg-access-CapRegs-derivable-caps[derivable-capsE]*:
assumes *Run-inv (read-reg (access-list-dec CapRegs idx)) t c regs*
and *idx \in {0..31} and CapRegs-names \subseteq accessible-regs s*
shows *c \in derivable-caps (run s t)*
using *assms*
unfolding *Run-inv-def upto-31-unfold*
by (*elim insertE conjE Run-read-regE*)
(auto simp: CapRegs-def CapRegs-names-def derivable-caps-def register-defs intro!: derivable.Copy)

lemma *memt-builtins-preserve-invariant[preserves-invariantI]*:
 $\bigwedge BCa BCb rk addr sz. \text{traces-preserve-invariant (read-memt-bytes BCa BCb rk addr sz)}$
 $\bigwedge BCa BCb rk addr sz. \text{traces-preserve-invariant (read-memt BCa BCb rk addr sz)}$
 $\bigwedge BCa BCb wk addr sz v t. \text{traces-preserve-invariant (write-memt BCa BCb wk addr sz v t)}$
 $\bigwedge addr sz t v. \text{traces-preserve-invariant (MEMval-tagged addr sz t v)}$
 $\bigwedge addr sz t v. \text{traces-preserve-invariant (MEMval-tagged-conditional addr sz t v)}$
by (*intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp*)+

lemma *dvd-8-Suc-iffs[simp]*:
 $8 \text{ dvd Suc (Suc 0)} \longleftrightarrow \text{False}$
 $8 \text{ dvd Suc (Suc (Suc 0))} \longleftrightarrow \text{False}$
 $8 \text{ dvd Suc (Suc (Suc (Suc 0)))} \longleftrightarrow \text{False}$

```

8 dvd Suc (Suc (Suc (Suc (Suc 0))))  $\longleftrightarrow$  False
8 dvd Suc (Suc (Suc (Suc (Suc (Suc 0)))))  $\longleftrightarrow$  False
8 dvd Suc (Suc (Suc (Suc (Suc (Suc (Suc 0))))))  $\longleftrightarrow$  False
8 dvd Suc (Suc (Suc (Suc (Suc (Suc (Suc (Suc x)))))))  $\longleftrightarrow$  8 dvd x
by presburger+

lemma byte-chunks-eq-Some-iff[simp]:
  shows byte-chunks xs = Some ys  $\longleftrightarrow$  ys = take-chunks 8 xs  $\wedge$  8 dvd length xs
  by (induction xs arbitrary: ys rule: byte-chunks.induct) (auto simp: bind-eq-Some-conv)

lemma mem-bytes-of-bits-mword-eq-Some-iff[simp]:
  fixes w :: 'a::len word
  shows mem-bytes-of-bits BC-mword w = Some bytes  $\longleftrightarrow$  bytes = mem-bytes-of-word
  w  $\wedge$  8 dvd LENGTH('a)
  by (auto simp: mem-bytes-of-bits-def bytes-of-bits-def mem-bytes-of-word-def BC-mword-defs)

lemma concat-take-chunks[simp]:
  assumes n > 0
  shows List.concat (take-chunks n xs) = xs
  using assms
  by (induction n xs rule: take-chunks.induct) auto

lemma bits-of-mem-bytes-of-word[simp]:
  fixes w :: 'a::len word
  assumes 8 dvd LENGTH('a)
  shows bits-of-mem-bytes (mem-bytes-of-word w) = map bitU-of-bool (to-bl w)
  using assms
  by (auto simp add: bits-of-mem-bytes-def bits-of-bytes-def mem-bytes-of-word-def
  simp del: take-chunks.simps)

lemma bitU-of-bool-eq-iff[simp]:
  bitU-of-bool b = B1  $\longleftrightarrow$  b bitU-of-bool b = B0  $\longleftrightarrow$   $\neg$  b
  by (auto simp: bitU-of-bool-def)

lemma memBitsToCapability-False-derivable-caps[intro, simp, derivable-capsI]:
  shows memBitsToCapability False w  $\in$  derivable-caps s
  by (auto simp: derivable-caps-def)

lemma memBitsToCapability-ucast-256-derivable-caps[intro, simp, derivable-capsI]:
  assumes memBitsToCapability tag w  $\in$  derivable-caps s
  shows memBitsToCapability tag (ucast w)  $\in$  derivable-caps s
  using assms
  by auto

lemma memBitsToCapability-capToMemBits-derivable-caps[intro, derivable-capsI]:
  assumes c: c  $\in$  derivable-caps s and tag: tag  $\longrightarrow$  Capability-tag c
  shows memBitsToCapability tag (capToMemBits c)  $\in$  derivable-caps s
  using assms
  by (cases tag) (auto simp: derivable-caps-def)

```

lemma *read-from-KCC-run-mono*: $\text{read-from-KCC } s \subseteq \text{read-from-KCC } (\text{run } s \ t)$
proof (*induction t arbitrary: s*)
 case (*Cons e t*)
 have $\text{read-from-KCC } s \subseteq \text{read-from-KCC } (\text{axiom-step } s \ e)$
 by *auto*
 also have $\dots \subseteq \text{read-from-KCC } (\text{run } (\text{axiom-step } s \ e) \ t)$
 by (*rule Cons.IH*)
 finally show *?case*
 unfolding *foldl-Cons* .
qed *auto*

lemma *exception-targets-run-imp*:
 assumes $c \in \text{exception-targets ISA } (\text{read-from-KCC } s)$
 shows $c \in \text{exception-targets ISA } (\text{read-from-KCC } (\text{run } s \ t))$
 using *assms read-from-KCC-run-mono*
 by (*auto simp: ISA-def*)

lemma *exception-targets-insert[simp]*:
 $\text{exception-targets ISA } (\text{insert } (\text{Regval-Capability } c) \ C) = \text{insert } c \ (\text{exception-targets ISA } C)$
 by (*auto simp: ISA-def*)

lemma *read-reg-KCC-exception-targets*:
 assumes $\text{Run-inv } (\text{read-reg KCC-ref}) \ t \ c \ \text{regs}$
 shows $c \in \text{exception-targets ISA } (\text{read-from-KCC } (\text{run } s \ t))$
 using *assms*
 unfolding *Run-inv-def*
 by (*auto elim!: Run-read-regE simp: KCC-ref-def*)

lemma *leq-perms-refl[intro, simp]*: $\text{leq-perms } p \ p$
 unfolding *leq-perms-def*
 by *auto*

lemma *setCapOffset-getCapOffset-idem*:
 assumes $\text{setCapOffset } c \ \text{offset} = (\text{representable}, \ c')$
 and $\text{uint } \text{offset} = \text{getCapOffset } c$
 shows $c' = c$
 using *assms uint-bounded[of Capability-address c]*
 by (*cases c*)
 (*auto simp add: setCapOffset-def getCapOffset-def uint-word-ariths mod-add-right-eq simp flip: uint-inject*)

lemma *setCapOffset-derivable-caps[derivable-capsE]*:
 assumes $\text{setCapOffset } c \ \text{offset} = (\text{representable}, \ c')$
 and $\text{Capability-tag } c \implies \text{Capability-tag } c' \implies \text{Capability-sealed } c \wedge \text{Capability-sealed } c'$
 and $\text{uint } \text{offset} = \text{getCapOffset } c$
 and $c \in \text{derivable-caps } s$

shows $c' \in \text{derivable-caps } s$
proof –
 have $\text{leq-cap } CC \ c' \ c$
 using $\text{assms setCapOffset-getCapOffset-idem}[OF \ \text{assms}(1)]$
 by (auto simp: $\text{leq-cap-def setCapOffset-def getCapBase-def getCapTop-def}$
 get-cap-perms-def)
 then show $?thesis$
 using assms
 by (auto simp: $\text{derivable-caps-def setCapOffset-def elim: derivable.Restrict}$)
qed

lemma $\text{Run-inv-return-derivable-caps}[\text{derivable-caps}E]$:
 assumes $\text{Run-inv } (\text{return } a) \ t \ a' \ \text{regs}$ and $a \in \text{derivable-caps } s$
 shows $a' \in \text{derivable-caps } (\text{run } s \ t)$ and $a' \in \text{derivable-caps } s$
 using assms
 by auto

lemma $\text{Run-inv-bind-derivable-caps}[\text{derivable-caps}E]$:
 assumes $\text{Run-inv } (m \ggg f) \ t \ a \ \text{regs}$ and $\text{runs-preserve-invariant } m$
 and $\bigwedge tm \ am \ tf. \ t = tm \ @ \ tf \implies \text{Run-inv } m \ tm \ am \ \text{regs} \implies \text{Run-inv } (f \ am) \ tf \ a \ (\text{the } (\text{updates-regs inv-regs } tm \ \text{regs})) \implies c \in \text{derivable-caps } (\text{run } (\text{run } s \ tm) \ tf)$
 shows $c \in \text{derivable-caps } (\text{run } s \ t)$
 using assms
 by (elim Run-inv-bindE) auto

lemma $\text{int-to-cap-derivable-caps}[\text{derivable-caps}I]$:
 unrepCap $c \in \text{derivable-caps } s$
 by (auto simp: $\text{unrepCap-def derivable-caps-def}$)

lemma $\text{update-Capability-tag-derivable-caps}[\text{derivable-caps}I]$:
 assumes $t \implies c \in \text{derivable-caps } s$ and $t \implies \text{Capability-tag } c$
 shows $c(\text{Capability-tag} := t) \in \text{derivable-caps } s$
 using assms
 by (cases $\text{Capability-tag } c$) (auto simp: $\text{derivable-caps-def}$)

lemma $\text{preserves-invariant-readCapReg}[\text{preserves-invariant}I]$:
 $\bigwedge \text{arg0}. \text{traces-preserve-invariant } (\text{readCapReg } \text{arg0})$
 $\bigwedge \text{arg0}. \text{traces-preserve-invariant } (\text{readCapRegDDC } \text{arg0})$
 by (intro $\text{no-reg-writes-traces-preserve-invariantI no-reg-writes-toI; simp}$) +

lemma $\text{readCapReg-derivable}[\text{derivable-caps}E]$:
 assumes $\text{Run-inv } (\text{readCapReg } \text{arg0}) \ t \ c \ \text{regs}$ and $\text{CapRegs-names} \subseteq \text{accessible-regs}$
 s
 shows $c \in \text{derivable-caps } (\text{run } s \ t)$
 using assms
 unfolding readCapReg-def
 by (–) ($\text{derivable-capsI assms: assms}$)

lemma *readCapRegDDC-derivable*[*derivable-capsE*]:
assumes *Run-inv* (*readCapRegDDC* *arg0*) *t c regs* **and** *CapRegs-names* \subseteq *accessible-regs*
s
shows $c \in \text{derivable-caps } (\text{run } s \ t)$
using *assms*
unfolding *readCapRegDDC-def*
by $(-)$ (*derivable-capsI* *assms*: *assms*)

lemma *caps-of-CapCauseReg-empty*[*simp*]: *caps-of-regval ISA* (*regval-of-CapCauseReg* *r*) = {}
by (*auto simp: ISA-def regval-of-CapCauseReg-def*)

lemma *letI*: $P \text{ (let } x = y \text{ in } f \ x) \text{ if } P \ (f \ y)$
using *that*
by *auto*

declare *if-split*[**where** $P = \lambda m. \text{runs-preserve-invariant } m, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]
declare *option.split*[**where** $P = \lambda m. \text{runs-preserve-invariant } m, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]
declare *prod.split*[**where** $P = \lambda m. \text{runs-preserve-invariant } m, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]
declare *sum.split*[**where** $P = \lambda m. \text{runs-preserve-invariant } m, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]
declare *letI*[**where** $P = \lambda m. \text{runs-preserve-invariant } m, \text{ preserves-invariantI}$]

declare *MemAccessType.split*[**where** $P = \lambda m. \text{traces-enabled } m \ s \ \text{regs for } s \ \text{regs}, \text{ traces-enabled-split}$]
declare *MemAccessType.split*[**where** $P = \lambda m. \text{runs-preserve-invariant } m \text{ for } Rs, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]
declare *MemAccessType.split*[**where** $P = \lambda m. \text{traces-preserve-invariant } m \text{ for } Rs, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]
declare *WordType.split*[**where** $P = \lambda m. \text{traces-enabled } m \ s \ \text{regs for } s \ \text{regs}, \text{ traces-enabled-split}$]
declare *WordType.split*[**where** $P = \lambda m. \text{runs-preserve-invariant } m \text{ for } Rs, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]
declare *WordType.split*[**where** $P = \lambda m. \text{traces-preserve-invariant } m \text{ for } Rs, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]
declare *ClearRegSet.split*[**where** $P = \lambda m. \text{traces-enabled } m \ s \ \text{regs for } s \ \text{regs}, \text{ traces-enabled-split}$]
declare *ClearRegSet.split*[**where** $P = \lambda m. \text{runs-preserve-invariant } m \text{ for } Rs, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]
declare *ClearRegSet.split*[**where** $P = \lambda m. \text{traces-preserve-invariant } m \text{ for } Rs, \text{ THEN } \text{iffD2}, \text{ preserves-invariantI}$]

lemma *preserves-invariant-SignalException*[*preserves-invariantI*]:
runs-preserve-invariant (*SignalException* *ex*)
runs-preserve-invariant (*SignalExceptionBadAddr* *ex badAddr*)
runs-preserve-invariant (*SignalExceptionTLB* *ex badAddr*)
by (*auto simp: runs-preserve-invariant-def*)

lemma *Run-raise-c2-exception-False*[*simp*]:

```

Run-inv (raise-c2-exception8 capEx reg8) t a regs  $\longleftrightarrow$  False
Run-inv (raise-c2-exception capEx reg5) t a regs  $\longleftrightarrow$  False
Run-inv (raise-c2-exception-noreg capEx) t a regs  $\longleftrightarrow$  False
by (auto simp: Run-inv-def)

lemma runs-preserve-invariant-raise-c2-exception[preserves-invariantI]:
  runs-preserve-invariant (raise-c2-exception8 capEx reg8)
  runs-preserve-invariant (raise-c2-exception capEx reg5)
  runs-preserve-invariant (raise-c2-exception-noreg capEx)
  by (auto simp: runs-preserve-invariant-def)

end

locale CHERI-MIPS-Reg-Automaton = CHERI-MIPS-ISA +
  fixes ex-traces :: bool and invocation-traces :: bool
begin

abbreviation invariant where invariant regs  $\equiv$  Capability-tag (regstate.PCC regs)
 $\wedge \neg$  Capability-sealed (regstate.PCC regs)
abbreviation inv-regs :: register-name set where inv-regs  $\equiv$  {"PCC"}

sublocale Write-Cap-Inv-Automaton CC ISA ex-traces invocation-traces get-regval
set-regval invariant inv-regs ..

sublocale CHERI-MIPS-Axiom-Inv-Automaton where enabled = enabled and
invariant = invariant and inv-regs = inv-regs ..

lemma traces-enabled-read-reg-nth-CapRegs[traces-enabledI]:
  assumes idx  $\in$  {0..31}
  shows traces-enabled (read-reg (access-list-dec CapRegs idx)) s regs
  using assms
  unfolding upto-31-unfold
  by (elim insertE) (auto simp: CapRegs-def intro!: traces-enabled-read-reg)

lemma preserves-invariant-writeCapReg[preserves-invariantI]:
   $\bigwedge$  arg0 arg1. traces-preserve-invariant (writeCapReg arg0 arg1)
  by (intro no-reg-writes-traces-preserve-invariantI no-reg-writes-toI; simp)+

lemma traces-enabled-read-mem[traces-enabledI]:
  traces-enabled (read-mem BCa BCb rk addr-sz addr sz) s regs
  unfolding read-mem-def read-mem-bytes-def traces-enabled-def maybe-fail-def
  by (auto elim: bind-Traces-cases Traces-cases split: option.splits)

lemma traces-enabled-read-memt[traces-enabledI]:
  traces-enabled (read-memt BCa BCb rk addr sz) s regs
  unfolding read-memt-def read-memt-bytes-def traces-enabled-def maybe-fail-def
  by (auto elim: bind-Traces-cases Traces-cases split: option.splits)

lemma traces-enabled-write-mem-ea[traces-enabledI]:

```

traces-enabled (*write-mem-ea* *BCa wk a1 a2 a3*) *s regs*
unfolding *write-mem-ea-def traces-enabled-def maybe-fail-def*
by (*auto elim: bind-Traces-cases Traces-cases split: option.splits*)

lemma *traces-enabled-write-mem*[*traces-enabledI*]:
traces-enabled (*write-mem* *BCa BCb wk a1 a2 a3 a4*) *s regs*
unfolding *write-mem-def traces-enabled-def*
by (*auto elim: bind-Traces-cases Traces-cases split: option.splits*)

lemma *traces-enabled-write-memt*[*traces-enabledI*]:
assumes *tag = B1* \longrightarrow *memBitsToCapability True (ucast w) \in derivable-caps s*
shows *traces-enabled* (*write-memt* *BCa BC-mword wk addr sz w tag*) *s regs*
using *assms*
unfolding *write-memt-def traces-enabled-def*
by (*cases tag; auto split: option.splits simp: bind-eq-Some-conv ucast-bl derivable-caps-def elim!: Write-memt-TracesE*)

lemma *traces-enabled-write-ram*[*traces-enabledI*]:
traces-enabled (*write-ram* *a0 a1 a2 a3 a4*) *s regs*
unfolding *write-ram-def MEMval-def MEMea-def*
by (*traces-enabledI intro: non-cap-expI[THEN non-cap-exp-traces-enabledI]*)

lemma *traces-enabled-read-ram*[*traces-enabledI*]:
traces-enabled (*read-ram* *a0 a1 a2 a3*) *s regs*
unfolding *read-ram-def MEMr-def*
by (*traces-enabledI*)

lemma *traces-enabled-MEMval-tagged*[*traces-enabledI*]:
assumes *memBitsToCapability tag (ucast v) \in derivable-caps s*
shows *traces-enabled* (*MEMval-tagged* *addr sz tag v*) *s regs*
unfolding *MEMval-tagged-def*
by (*traces-enabledI intro: non-cap-expI[THEN non-cap-exp-traces-enabledI] assms: assms*)

lemma *traces-enabled-MEMval-tagged-conditional*[*traces-enabledI*]:
assumes *memBitsToCapability tag (ucast v) \in derivable-caps s*
shows *traces-enabled* (*MEMval-tagged-conditional* *addr sz tag v*) *s regs*
unfolding *MEMval-tagged-conditional-def*
by (*traces-enabledI intro: non-cap-expI[THEN non-cap-exp-traces-enabledI] assms: assms*)

lemma *traces-enabled-set-next-pcc-ex*:
assumes *arg0: arg0 \in exception-targets ISA (read-from-KCC s) and ex: ex-traces*
shows *traces-enabled* (*set-next-pcc* *arg0*) *s regs*
unfolding *set-next-pcc-def bind-assoc*
by (*traces-enabledI assms: arg0 exception-targets-run-imp*
intro: traces-enabled-write-reg ex no-reg-writes-traces-preserve-invariantI)

no-reg-writes-to-write-reg traces-runs-preserve-invariantI
simp: DelayedPCC-ref-def NextPCC-ref-def)

lemma *traces-enabled-write-reg-nth-CapRegs*[*traces-enabledI*]:
assumes $c \in \text{derivable-caps } s$ **and** $\text{idx} \in \{0..31\}$
shows *traces-enabled* (*write-reg* (*access-list-dec* *CapRegs* *idx*) *c*) *s* *regs*
using *assms*
unfolding *upto-31-unfold derivable-caps-def*
by (*elim insertE*; *auto intro!*; *traces-enabled-write-reg simp: CapRegs-def register-defs*)

lemma *traces-enabled-writeCapReg*[*traces-enabledI*]:
assumes $\text{arg1} \in \text{derivable-caps } s$
shows *traces-enabled* (*writeCapReg* *arg0* *arg1*) *s* *regs*
unfolding *writeCapReg-def bind-assoc capToString-def*
by (*traces-enabledI* *assms*; *assms intro*; *non-cap-expI*[*THEN non-cap-exp-traces-enabledI*]
no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg)

lemma *traces-enabled-readCapReg*[*traces-enabledI*]:
shows *traces-enabled* (*readCapReg* *arg0*) *s* *regs*
unfolding *readCapReg-def bind-assoc*
by (*traces-enabledI intro*; *non-cap-expI*[*THEN non-cap-exp-traces-enabledI*])

lemma *traces-enabled-readCapRegDDC*[*traces-enabledI*]:
shows *traces-enabled* (*readCapRegDDC* *arg0*) *s* *regs*
unfolding *readCapRegDDC-def bind-assoc*
by (*traces-enabledI*)

fun *trace-writes-cap-regs* :: *register-value trace* \Rightarrow *register-name set* **where**
trace-writes-cap-regs [] = {}
| *trace-writes-cap-regs* (*e* # *t*) =
{*r*. $\exists v c. e = E\text{-write-reg } r v \wedge c \in \text{caps-of-regval } ISA v \wedge \text{is-tagged-method } CC\ c$ } \cup
trace-writes-cap-regs *t*

fun *trace-allows-system-reg-access* :: *register-name set* \Rightarrow *register-value trace* \Rightarrow *bool* **where**
trace-allows-system-reg-access *Rs* [] = *False*
| *trace-allows-system-reg-access* *Rs* (*e* # *t*) = (*allows-system-reg-access* *Rs* *e* \vee *trace-allows-system-reg-access* (*Rs* - *trace-writes-cap-regs* [*e*]) *t*)

lemma *trace-allows-system-reg-access-append*:
trace-allows-system-reg-access *Rs* (*t1* @ *t2*) = (*trace-allows-system-reg-access* *Rs* *t1* \vee *trace-allows-system-reg-access* (*Rs* - *trace-writes-cap-regs* *t1*) *t2*)
by (*induction* *t1* *arbitrary*; *Rs*) (*auto simp: Diff-eq Int-assoc*)

lemma [*simp*]: *accessible-regs* *s* - *written-regs* *s* = *accessible-regs* *s*
by (*auto simp: accessible-regs-def*)

lemma *system-reg-access-run*:

system-reg-access (*run s t*) = (*system-reg-access s* \vee *trace-allows-system-reg-access* (*accessible-regs s*) *t*)
by (*induction t arbitrary: s*) (*auto simp: accessible-regs-axiom-step Diff-Un Diff-Int-distrib Diff-Int*)

lemma *pcc-access-system-regs-allows-system-reg-access*:

assumes *Run-inv* (*pcc-access-system-regs u*) *t a regs*
shows *trace-allows-system-reg-access Rs t* \longleftrightarrow *a* \wedge "*PCC*" \in *Rs*
using *assms*
unfolding *pcc-access-system-regs-def Run-inv-def*
by (*auto elim!: Run-bindE Run-read-regE simp: PCC-ref-def get-regval-def regval-of-Capability-def get-cap-perms-def*)

lemma *checkCP0Access-system-reg-access*:

assumes *Run-inv* (*checkCP0Access* ()) *t* () *regs* **and** {"*PCC*"} \subseteq *accessible-regs s*
shows *trace-allows-system-reg-access* (*accessible-regs s*) *t*
using *assms pcc-access-system-regs-allows-system-reg-access* [**where** *Rs = accessible-regs s*]
unfolding *checkCP0Access-def checkCP0AccessHook-def Run-inv-def*
by (*auto elim!: Run-bindE simp: regstate-simp system-reg-access-run pcc-access-system-regs-allows-system-reg-access trace-allows-system-reg-access-append split: if-splits*)

lemma *Run-inv-runs-no-reg-writes-written-regs-eq*:

assumes *Run-inv m t a regs* **and** *runs-no-reg-writes-to* {*r*} *m*
shows *r* \in *written-regs* (*run s t*) \longleftrightarrow *r* \in *written-regs s*
using *assms*
by (*auto simp: Run-inv-def Run-runs-no-reg-writes-written-regs-eq*)

lemmas *runs-no-reg-writes-written-regs-eq* =

Run-runs-no-reg-writes-written-regs-eq Run-inv-runs-no-reg-writes-written-regs-eq

end

abbreviation *noCP0Access s* \equiv *get-StatusReg-EXL* (*CP0Status s*) = 0 \wedge *get-StatusReg-ERL* (*CP0Status s*) = 0 \wedge *get-StatusReg-KSU* (*CP0Status s*) \neq 0 \wedge \neg (*get-StatusReg-CU* (*CP0Status s*) $!!$ 0)

locale *CHERI-MIPS-Fixed-Trans* =

fixes *trans-regstate* :: *regstate*

assumes *noCP0Access-trans-regstate: noCP0Access trans-regstate*

begin

definition *trans-regs* \equiv {"*CP0Status*", "*TLBEntryHi*", "*PCC*"} \cup *TLBEntries-names*

definition *trans-inv s* \equiv (\exists *pcc. s* \ll *regstate.PCC* := *pcc*) = *trans-regstate*)

lemma *invariant-trans-regstate*[*intro, simp*]:

trans-inv trans-regstate

```

proof –
  have trans-regstate( $\lfloor \text{regstate.PCC} := \text{regstate.PCC trans-regstate} \rfloor$ ) = trans-regstate
    by auto
  then show ?thesis
    unfolding trans-inv-def
    by blast
qed

fun MemAccessType-of-acctype :: acctype  $\Rightarrow$  MemAccessType where
  MemAccessType-of-acctype Load = LoadData
| MemAccessType-of-acctype Store = StoreData
| MemAccessType-of-acctype Fetch = Instruction

sublocale State-Invariant get-regval set-regval trans-inv trans-regs .

definition translate-addressM :: nat  $\Rightarrow$  acctype  $\Rightarrow$  nat M where
  translate-addressM vaddr acctype  $\equiv$ 
    let vaddr = word-of-int (int vaddr) in
      TLBTranslate vaddr (MemAccessType-of-acctype acctype)  $\gg$  ( $\lambda \text{paddr.}$ 
        return (unat paddr))

definition translate-address :: nat  $\Rightarrow$  acctype  $\Rightarrow$  'a  $\Rightarrow$  nat option where
  translate-address vaddr acctype - = (if ( $\exists t \ a \ \text{regs. Run-inv } (\text{translate-addressM } t \ a \ \text{regs})$ ) then Some (the-result (translate-addressM vaddr acctype))
    else None)

sublocale CHERI-MIPS-ISA where translate-address = translate-address .

end

locale CHERI-MIPS-Mem-Automaton = CHERI-MIPS-Fixed-Trans +
  fixes is-fetch :: bool and ex-traces :: bool
begin

sublocale Mem-Inv-Automaton
  where CC = CC and ISA = ISA and is-fetch = is-fetch and ex-traces =
    ex-traces
  and translation-assm =  $\lambda t. (\exists \text{regs. reads-regs-from inv-regs } t \ \text{regs} \wedge \text{trans-inv } \text{regs})$ 
  and get-regval = get-regval and set-regval = set-regval
  and invariant = trans-inv and inv-regs = trans-regs
  by unfold-locales (auto simp: ISA-def translate-address-def)

sublocale CHERI-MIPS-Axiom-Inv-Automaton
  where translate-address = translate-address and enabled = enabled
  and invariant = trans-inv and inv-regs = trans-regs and ex-traces = ex-traces
  by unfold-locales

```

lemma *preserves-invariant-tlbSearch*[*preserves-invariantI*]:
traces-preserve-invariant (*tlbSearch vAddr*)
unfolding *tlbSearch-def*
by (*preserves-invariantI*)

lemma *preserves-invariant-checkCP0AccessHook*[*preserves-invariantI*]:
runs-preserve-invariant (*checkCP0AccessHook u*)
unfolding *checkCP0AccessHook-def pcc-access-system-regs-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-getAccessLevel*[*preserves-invariantI*]:
traces-preserve-invariant (*getAccessLevel u*)
unfolding *getAccessLevel-def*
by (*preserves-invariantI*)

fun-cases *StatusReg-of-regval-SomeE*: *StatusReg-of-regval rv = Some a*

lemma *read-reg-CP0Status-inv-fields*:
assumes *Run-inv* (*read-reg CP0Status-ref*) *t a regs*
shows *get-StatusReg-EXL a = 0 and get-StatusReg-ERL a = 0 and get-StatusReg-KSU a ≠ 0*
and $\neg(\text{get-StatusReg-CU } a \text{ !! } 0)$
using *assms noCP0Access-trans-regstate*
unfolding *Run-inv-def trans-inv-def*
by (*auto elim!*: *Run-read-regE StatusReg-of-regval-SomeE simp: CP0Status-ref-def trans-regs-def get-regval-def regval-of-StatusReg-def*)

lemma *bits-to-bool-iff-one*:
bits-to-bool w \longleftrightarrow w = 1
by (*cases w rule: exhaustive-1-word*) (*auto simp: bits-to-bool-def*)

lemma *Run-inv-getAccessLevel-neq-Kernel*:
assumes *Run-inv* (*getAccessLevel u*) *t a regs*
shows *a ≠ Kernel*
using *assms*
unfolding *getAccessLevel-def Let-def or-boolM-def*
by (*auto simp: regstate-simp bits-to-bool-iff-one read-reg-CP0Status-inv-fields elim!*: *Run-inv-bindE intro!*: *preserves-invariantI traces-runs-preserve-invariantI split: if-splits*)

lemma *Run-inv-checkCP0Access-False*[*simp*]:
Run-inv (*checkCP0Access u*) *t a regs \longleftrightarrow False*
proof –
define *signal-ex* :: *unit M*
where *signal-ex* \equiv *set-CauseReg-CE CP0Cause-ref 0 \gg SignalException CpU*
have *Run-inv signal-ex t a regs \longleftrightarrow False* **for** *t a regs*
unfolding *signal-ex-def Run-inv-def*
by (*auto elim!*: *Run-bindE*)


```

then show ?thesis
unfolding checkCP0Access-def and-boolM-def bind-assoc signal-ex-def[symmetric]
by (auto elim!: Run-inv-bindE dest!: Run-inv-getAccessLevel-neq-Kernel
      intro!: preserves-invariantI traces-runs-preserve-invariantI
      simp: read-reg-CP0Status-inv-fields)
qed

lemma trans-inv-regs-eq-trans-regstate:
  assumes trans-inv regs
  shows CP0Status regs = CP0Status trans-regstate  $\wedge$  TLBEntryHi regs = TL-
    BEntryHi trans-regstate
  using assms
  by (auto simp: trans-inv-def)

lemma determ-runs-read-reg-CP0Status[determ]: determ-runs (read-reg CP0Status-ref)
  by (intro determ-runs-read-inv-reg) (auto simp: trans-regs-def register-defs dest:
    trans-inv-regs-eq-trans-regstate)

lemma determ-runs-SignalExceptionBadAddr[determ]: determ-runs (SignalExceptionBadAddr
  ex badAddr)
  by (intro determ-runsI) (auto simp: Run-inv-def)

lemma determ-runs-SignalExceptionTLB[determ]: determ-runs (SignalExceptionTLB
  ex badAddr)
  by (intro determ-runsI) (auto simp: Run-inv-def)

lemma get-regval-TLBEntries:
   $r \in \text{set TLBEntries} \implies \text{trans-inv regs} \implies \text{get-regval (name r) regs} = \text{Some}$ 
  (regval-of-TLBEntry (read-from r trans-regstate))
  by (auto simp: TLBEntries-def trans-inv-def; simp add: register-defs)

lemma read-from-TLBEntries:
  assumes  $\text{idx} \in \{0..63\}$  and trans-inv regs
  shows read-from (TLBEntries ! (64 - nat (idx + 1))) trans-regstate = read-from
  (TLBEntries ! (64 - nat (idx + 1))) regs
  using assms
  unfolding upto-63-unfold
  by (elim insertE) (auto simp: trans-inv-def TLBEntries-def register-defs)

lemma of-regval-TLBEntries-nth[simp]:
   $\text{idx} \in \{0..63\} \implies \text{of-regval (TLBEntries ! (64 - nat (idx + 1))) v} = \text{TLBEntry-of-regval}$ 
  v
  unfolding upto-63-unfold
  by (elim insertE) (auto simp: TLBEntries-def register-defs)

lemma determ-runs-read-reg-access-TLBEntries[determ]:
  determ-traces (read-reg (access-list-dec TLBEntries idx)) if  $\text{idx} \in \{0..63\}$ 
  using that
  by (intro determ-traces-read-inv-reg)

```

```

(auto simp del: TLBEntries-names-unfold
  simp add: trans-regs-def TLBEntries-names-def regval-of-TLBEntry-def
  intro!: get-regval-TLBEntries read-from-TLBEntries)

lemma determ-traces-read-reg-TLBEntryHi[determ]:
  determ-traces (read-reg TLBEntryHi-ref)
by (intro determ-traces-read-inv-reg)
  (auto simp: TLBEntryHi-ref-def trans-regs-def get-regval-def dest: trans-inv-regs-eq-trans-regstate)

lemma determ-traces-tlbSearch[determ]:
  determ-runs (tlbSearch vAddr)
unfolding tlbSearch-def Let-def
by (intro determ determ-traces-bindI determ-traces-foreachM determ-traces-if
  determ-traces-runs
    preserves-invariantI traces-runs-preserve-invariantI allI conjI impI)
  auto

lemma determ-runs-translate-addressM: determ-runs (translate-addressM vaddr
  is-load)
unfolding translate-addressM-def TLBTranslate-def TLBTranslateC-def TLBTranslate2-def
  getAccessLevel-def undefined-range-def Let-def bind-assoc
by (intro determ-runs-bindI determ-runs-if determ determ-traces-runs
  preserves-invariantI traces-runs-preserve-invariantI allI conjI impI)
  auto

lemma TLBTranslate-LoadData-translate-address-eq[simp]:
assumes Run-inv (TLBTranslate vaddr LoadData) t paddr regs
shows translate-address (unat vaddr) Load t' = Some (unat paddr)
proof -
from assms have Run-inv (translate-addressM (unat vaddr) Load) t (unat
  paddr) regs
unfolding translate-addressM-def Run-inv-def
by (auto simp flip: uint-nat intro: Traces-bindI[of - t paddr - [], simplified])
then show ?thesis
using determ-runs-translate-addressM
by (auto simp: translate-address-def determ-the-result-eq)
qed

lemma TLBTranslate-StoreData-translate-address-eq[simp]:
assumes Run-inv (TLBTranslate vaddr StoreData) t paddr regs
shows translate-address (unat vaddr) Store t' = Some (unat paddr)
proof -
from assms have Run-inv (translate-addressM (unat vaddr) Store) t (unat
  paddr) regs
unfolding translate-addressM-def Run-inv-def
by (auto simp flip: uint-nat intro: Traces-bindI[of - t paddr - [], simplified])
then show ?thesis
using determ-runs-translate-addressM
by (auto simp: translate-address-def determ-the-result-eq)

```

qed

lemma *TLBTranslate-Instruction-translate-address-eq[simp]*:
assumes *Run-inv* (*TLBTranslate vaddr Instruction*) *t paddr regs*
shows *translate-address (unat vaddr) Fetch t' = Some (unat paddr)*
proof –
from *assms* **have** *Run-inv (translate-addressM (unat vaddr) Fetch) t (unat paddr) regs*
unfolding *translate-addressM-def Run-inv-def*
by (*auto simp flip: uint-nat intro: Traces-bindI[of - t paddr - [], simplified]*)
then show *?thesis*
using *determ-runs-translate-addressM*
by (*auto simp: translate-address-def determ-the-result-eq*)
qed

end

lemma *mult-mod-plus-less*:
assumes *n dvd m and n > 0 and m > 0 and 0 ≤ i and i < n*
shows *n * q mod m + i < (m :: int)*
using *assms*
by (*auto simp: dvd-def*)
(*metis assms(2–5) mult.commute zero-less-mult-pos2 zmult2-lemma-aux4*)

lemma *dvd-nat-iff-int-dvd*:
assumes *0 ≤ i*
shows *n dvd nat i ⟷ int n dvd i*
using *assms*
by (*auto simp: dvd-def nat-mult-distrib (use nat-0-le in ‹fastforce›)*)

lemma *sail-ones-max-word[simp]*: *sail-ones n = max-word*
by (*intro word-eqI (auto simp: sail-ones-def zeros-def)*)

lemma *sail-mask-ucast[simp]*: *sail-mask n w = ucast w*
by (*auto simp: sail-mask-def vector-truncate-def zero-extend-def*)

lemma *mod2-minus-one-mask*:
 $(2^n - 1) = (\text{mask } n :: 'a::\text{len word})$
by (*auto simp: mask-def*)

lemma *slice-mask-nth*:
fixes *n i l :: int and j :: nat*
defines *w ≡ slice-mask n i l :: 'n::len word*
assumes *n = LENGTH('n)*
shows *w !! j ⟷ j < nat n ∧ nat i ≤ j ∧ j < nat i + nat l*
using *assms*
by (*auto simp: slice-mask-def nth-shiffl Let-def mod2-minus-one-mask*)

lemma *subrange-subrange-concat-ucast-right*:

```

fixes  $w1 :: 'a::len$  word and  $w2 :: 'b::len$  word
fixes  $c\ i1\ j1\ i2 :: int$ 
defines  $w \equiv \text{subrange-subrange-concat } c\ w1\ i1\ j1\ w2\ i2\ 0 :: 'c::len$  word
defines  $d \equiv \text{ucast } w2 :: 'd::len$  word
assumes  $int\ LENGTH('d) \leq i2 + 1$  and  $0 \leq i2\ LENGTH('b) \geq LENGTH('d)$ 
 $LENGTH('c) \geq LENGTH('d)$ 
shows  $\text{ucast } w = d$ 
using assms
by (intro word-eqI)
  (auto simp: subrange-subrange-concat-def nth-ucast word-ao-nth nth-shiffl nth-shiftr
nat-add-distrib slice-mask-nth)

context CHERI-MIPS-Fixed-Trans
begin

lemma [simp]: tag-granule ISA = 32 by (auto simp: ISA-def)

lemma address-tag-aligned-plus-iff [simp]:
  fixes  $addr :: 64$  word
  assumes  $int\ (\text{tag-granule } ISA)\ dvd\ i$  and  $0 \leq i$ 
  shows  $\text{address-tag-aligned } ISA\ (\text{unat } (addr + \text{word-of-int } i)) \longleftrightarrow \text{address-tag-aligned}$ 
 $ISA\ (\text{unat } addr)$ 
  using assms
  unfolding address-tag-aligned-def unat-def mod-eq-0-iff-dvd uint-ge-0 [THEN dvd-nat-iff-int-dvd]
  by (auto simp: uint-word-ariths uint-word-of-int mod-add-right-eq dvd-mod-iff
dvd-add-left-iff)

lemma TLBTranslate2-ucast-paddr-eq:
  assumes  $Run\ (TLBTranslate2\ vaddr\ acctype)\ t\ (paddr,\ flag)$ 
  shows  $(\text{ucast } paddr :: 12\ word) = (\text{ucast } vaddr :: 12\ word)$ 
  using assms
  unfolding TLBTranslate2-def Let-def undefined-range-def
  by (auto elim!: Run-bindE Run-ifE split: option.splits
simp: subrange-subrange-concat-ucast-right)

lemma TLBTranslateC-ucast-paddr-eq:
  assumes  $Run\ (TLBTranslateC\ vaddr\ acctype)\ t\ (paddr,\ flag)$ 
  shows  $(\text{ucast } paddr :: 12\ word) = (\text{ucast } vaddr :: 12\ word)$ 
  using assms
  unfolding TLBTranslateC-def Let-def
  by (fastforce elim!: Run-bindE Run-ifE simp: TLBTranslate2-ucast-paddr-eq split:
option.splits bool.splits prod.splits if-splits)

lemma TLBTranslate-ucast-paddr-eq:
  assumes  $Run\ (TLBTranslate\ vaddr\ acctype)\ t\ paddr$ 
  shows  $(\text{ucast } paddr :: 12\ word) = (\text{ucast } vaddr :: 12\ word)$ 
  using assms
  unfolding TLBTranslate-def
  by (auto elim!: Run-bindE simp: TLBTranslateC-ucast-paddr-eq)

```

```

lemma address-tag-aligned-ucast5:
  fixes addr :: 'a::len word
  assumes LENGTH('a) ≥ 5
  shows address-tag-aligned ISA (unat addr)  $\longleftrightarrow$  (ucast addr :: 5 word) = 0
  using assms
  unfolding unat-arith-simps(3)
  by (auto simp: address-tag-aligned-def unat-and-mask min-def)

lemma address-tag-aligned-paddr-iff-vaddr[simp]:
  assumes Run-inv (TLBTranslate vaddr acctype) t paddr regs
  shows address-tag-aligned ISA (unat paddr)  $\longleftrightarrow$  address-tag-aligned ISA (unat
vaddr)
proof –
  have paddr-vaddr: ucast paddr = (ucast vaddr :: 12 word)
    using assms
    by (auto simp: Run-inv-def TLBTranslate-ucast-paddr-eq)
  have address-tag-aligned ISA (unat paddr)  $\longleftrightarrow$  (ucast (ucast paddr :: 12 word)
:: 5 word) = 0
    by (auto simp: address-tag-aligned-ucast5)
  also have ...  $\longleftrightarrow$  address-tag-aligned ISA (unat vaddr)
    unfolding paddr-vaddr
    by (auto simp: address-tag-aligned-ucast5)
  finally show ?thesis .
qed

lemma TLBTranslateC-address-tag-aligned[simp]:
  assumes Run-inv (TLBTranslateC vaddr acctype) t (paddr, noStoreCap) regs
  shows address-tag-aligned ISA (unat paddr)  $\longleftrightarrow$  address-tag-aligned ISA (unat
vaddr)
proof –
  have paddr-vaddr: ucast paddr = (ucast vaddr :: 12 word)
    using assms
    by (auto simp: Run-inv-def TLBTranslateC-ucast-paddr-eq)
  have address-tag-aligned ISA (unat paddr)  $\longleftrightarrow$  (ucast (ucast paddr :: 12 word)
:: 5 word) = 0
    by (auto simp: address-tag-aligned-ucast5)
  also have ...  $\longleftrightarrow$  address-tag-aligned ISA (unat vaddr)
    unfolding paddr-vaddr
    by (auto simp: address-tag-aligned-ucast5)
  finally show ?thesis .
qed

lemma address-tag-aligned-mult-dvd[intro, simp]:
  assumes int (tag-granule ISA) dvd k and 0 ≤ k
  shows address-tag-aligned ISA (nat (k * n))
  using assms
  by (auto simp: address-tag-aligned-def nat-mult-distrib)

```

end

locale *CHERI-MIPS-Mem-Instr-Automaton* = *CHERI-MIPS-Mem-Automaton* **where**
is-fetch = *False*

locale *CHERI-MIPS-Mem-Fetch-Automaton* = *CHERI-MIPS-Mem-Automaton* **where**
is-fetch = *True*

end

theory *CHERI-MIPS-Gen-Lemmas*

imports *CHERI-MIPS-Instantiation*

begin

3.2 Footprint lemmas

context *CHERI-MIPS-Axiom-Inv-Automaton*
begin

lemma *non-cap-regsI*[*intro*, *simp*]:

non-cap-reg BranchPending-ref
non-cap-reg CID-ref
non-cap-reg CP0BadInstr-ref
non-cap-reg CP0BadInstrP-ref
non-cap-reg CP0BadVAddr-ref
non-cap-reg CP0Cause-ref
non-cap-reg CP0Compare-ref
non-cap-reg CP0ConfigK0-ref
non-cap-reg CP0Count-ref
non-cap-reg CP0HWREna-ref
non-cap-reg CP0LLAddr-ref
non-cap-reg CP0LLBit-ref
non-cap-reg CP0Status-ref
non-cap-reg CP0UserLocal-ref
non-cap-reg CurrentInstrBits-ref
non-cap-reg DelayedPC-ref
non-cap-reg GPR-ref
non-cap-reg HI-ref
non-cap-reg InBranchDelay-ref
non-cap-reg LO-ref
non-cap-reg LastInstrBits-ref
non-cap-reg NextInBranchDelay-ref
non-cap-reg NextPC-ref
non-cap-reg PC-ref
non-cap-reg TLBContext-ref
non-cap-reg TLBEntry00-ref
non-cap-reg TLBEntry01-ref
non-cap-reg TLBEntry02-ref
non-cap-reg TLBEntry03-ref
non-cap-reg TLBEntry04-ref

non-cap-reg TLBEntry05-ref
non-cap-reg TLBEntry06-ref
non-cap-reg TLBEntry07-ref
non-cap-reg TLBEntry08-ref
non-cap-reg TLBEntry09-ref
non-cap-reg TLBEntry10-ref
non-cap-reg TLBEntry11-ref
non-cap-reg TLBEntry12-ref
non-cap-reg TLBEntry13-ref
non-cap-reg TLBEntry14-ref
non-cap-reg TLBEntry15-ref
non-cap-reg TLBEntry16-ref
non-cap-reg TLBEntry17-ref
non-cap-reg TLBEntry18-ref
non-cap-reg TLBEntry19-ref
non-cap-reg TLBEntry20-ref
non-cap-reg TLBEntry21-ref
non-cap-reg TLBEntry22-ref
non-cap-reg TLBEntry23-ref
non-cap-reg TLBEntry24-ref
non-cap-reg TLBEntry25-ref
non-cap-reg TLBEntry26-ref
non-cap-reg TLBEntry27-ref
non-cap-reg TLBEntry28-ref
non-cap-reg TLBEntry29-ref
non-cap-reg TLBEntry30-ref
non-cap-reg TLBEntry31-ref
non-cap-reg TLBEntry32-ref
non-cap-reg TLBEntry33-ref
non-cap-reg TLBEntry34-ref
non-cap-reg TLBEntry35-ref
non-cap-reg TLBEntry36-ref
non-cap-reg TLBEntry37-ref
non-cap-reg TLBEntry38-ref
non-cap-reg TLBEntry39-ref
non-cap-reg TLBEntry40-ref
non-cap-reg TLBEntry41-ref
non-cap-reg TLBEntry42-ref
non-cap-reg TLBEntry43-ref
non-cap-reg TLBEntry44-ref
non-cap-reg TLBEntry45-ref
non-cap-reg TLBEntry46-ref
non-cap-reg TLBEntry47-ref
non-cap-reg TLBEntry48-ref
non-cap-reg TLBEntry49-ref
non-cap-reg TLBEntry50-ref
non-cap-reg TLBEntry51-ref
non-cap-reg TLBEntry52-ref
non-cap-reg TLBEntry53-ref

non-cap-reg TLBEntry54-ref
non-cap-reg TLBEntry55-ref
non-cap-reg TLBEntry56-ref
non-cap-reg TLBEntry57-ref
non-cap-reg TLBEntry58-ref
non-cap-reg TLBEntry59-ref
non-cap-reg TLBEntry60-ref
non-cap-reg TLBEntry61-ref
non-cap-reg TLBEntry62-ref
non-cap-reg TLBEntry63-ref
non-cap-reg TLBEntryHi-ref
non-cap-reg TLBEntryLo0-ref
non-cap-reg TLBEntryLo1-ref
non-cap-reg TLBIndex-ref
non-cap-reg TLBPageMask-ref
non-cap-reg TLBProbe-ref
non-cap-reg TLBRandom-ref
non-cap-reg TLBWired-ref
non-cap-reg TLBXContext-ref
non-cap-reg UART-RDATA-ref
non-cap-reg UART-RVALID-ref
non-cap-reg UART-WDATA-ref
non-cap-reg UART-WRITTEN-ref
non-cap-reg InstCount-ref
unfolding *BranchPending-ref-def CID-ref-def CP0BadInstr-ref-def CP0BadInstrP-ref-def*
CP0BadVAddr-ref-def
CP0Cause-ref-def CP0Compare-ref-def CP0ConfigK0-ref-def CP0Count-ref-def
CP0HWREna-ref-def
CP0LLAddr-ref-def CP0LLBit-ref-def CP0Status-ref-def CP0UserLocal-ref-def
CurrentInstrBits-ref-def
DelayedPC-ref-def GPR-ref-def HI-ref-def InBranchDelay-ref-def LO-ref-def
LastInstrBits-ref-def NextInBranchDelay-ref-def NextPC-ref-def PC-ref-def TLBContext-ref-def
TLBEntry00-ref-def TLBEntry01-ref-def TLBEntry02-ref-def TLBEntry03-ref-def
TLBEntry04-ref-def
TLBEntry05-ref-def TLBEntry06-ref-def TLBEntry07-ref-def TLBEntry08-ref-def
TLBEntry09-ref-def
TLBEntry10-ref-def TLBEntry11-ref-def TLBEntry12-ref-def TLBEntry13-ref-def
TLBEntry14-ref-def
TLBEntry15-ref-def TLBEntry16-ref-def TLBEntry17-ref-def TLBEntry18-ref-def
TLBEntry19-ref-def
TLBEntry20-ref-def TLBEntry21-ref-def TLBEntry22-ref-def TLBEntry23-ref-def
TLBEntry24-ref-def
TLBEntry25-ref-def TLBEntry26-ref-def TLBEntry27-ref-def TLBEntry28-ref-def
TLBEntry29-ref-def
TLBEntry30-ref-def TLBEntry31-ref-def TLBEntry32-ref-def TLBEntry33-ref-def
TLBEntry34-ref-def
TLBEntry35-ref-def TLBEntry36-ref-def TLBEntry37-ref-def TLBEntry38-ref-def
TLBEntry39-ref-def
TLBEntry40-ref-def TLBEntry41-ref-def TLBEntry42-ref-def TLBEntry43-ref-def

TLBEntry44-ref-def
TLBEntry45-ref-def TLBEntry46-ref-def TLBEntry47-ref-def TLBEntry48-ref-def
TLBEntry49-ref-def
TLBEntry50-ref-def TLBEntry51-ref-def TLBEntry52-ref-def TLBEntry53-ref-def
TLBEntry54-ref-def
TLBEntry55-ref-def TLBEntry56-ref-def TLBEntry57-ref-def TLBEntry58-ref-def
TLBEntry59-ref-def
TLBEntry60-ref-def TLBEntry61-ref-def TLBEntry62-ref-def TLBEntry63-ref-def
TLBEntryHi-ref-def
TLBEntryLo0-ref-def TLBEntryLo1-ref-def TLBIndex-ref-def TLBPageMask-ref-def
TLBProbe-ref-def
TLBRandom-ref-def TLBWired-ref-def TLBXContext-ref-def UART-RDATA-ref-def
UART-RVALID-ref-def
UART-WDATA-ref-def UART-WRITTEN-ref-def InstCount-ref-def
by (*auto simp: non-cap-reg-def*)

lemmas *non-cap-exp-rw-non-cap-reg[non-cap-expI] =*
non-cap-regsl[THEN non-cap-exp-read-non-cap-reg]
non-cap-regsl[THEN non-cap-exp-write-non-cap-reg]

lemma *non-cap-exp-undefined-option[non-cap-expI]:*
non-cap-exp (undefined-option arg0)
by (*non-cap-expI simp: undefined-option-def*)

lemma *non-cap-exp-undefined-exception[non-cap-expI]:*
non-cap-exp (undefined-exception arg0)
by (*non-cap-expI simp: undefined-exception-def*)

lemma *non-cap-exp-undefined-CauseReg[non-cap-expI]:*
non-cap-exp (undefined-CauseReg arg0)
by (*non-cap-expI simp: undefined-CauseReg-def*)

lemma *non-cap-exp-set-CauseReg-bits[non-cap-expI]:*
assumes *non-cap-reg arg0*
shows *non-cap-exp (set-CauseReg-bits arg0 arg1)*
using *assms*
by (*non-cap-expI simp: set-CauseReg-bits-def*)

lemma *non-cap-exp-set-CauseReg-BD[non-cap-expI]:*
assumes *non-cap-reg arg0*
shows *non-cap-exp (set-CauseReg-BD arg0 arg1)*
using *assms*
by (*non-cap-expI simp: set-CauseReg-BD-def*)

lemma *non-cap-exp-set-CauseReg-CE[non-cap-expI]:*
assumes *non-cap-reg arg0*
shows *non-cap-exp (set-CauseReg-CE arg0 arg1)*
using *assms*
by (*non-cap-expI simp: set-CauseReg-CE-def*)

lemma *non-cap-exp-set-CauseReg-IV*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-CauseReg-IV arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-CauseReg-IV-def*)

lemma *non-cap-exp-set-CauseReg-WP*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-CauseReg-WP arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-CauseReg-WP-def*)

lemma *non-cap-exp-set-CauseReg-IP*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-CauseReg-IP arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-CauseReg-IP-def*)

lemma *non-cap-exp-set-CauseReg-ExcCode*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-CauseReg-ExcCode arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-CauseReg-ExcCode-def*)

lemma *non-cap-exp-undefined-TLBEntryLoReg*[*non-cap-expI*]:
non-cap-exp (undefined-TLBEntryLoReg arg0)
 by (*non-cap-expI simp: undefined-TLBEntryLoReg-def*)

lemma *non-cap-exp-set-TLBEntryLoReg-bits*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-TLBEntryLoReg-bits arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-TLBEntryLoReg-bits-def*)

lemma *non-cap-exp-set-TLBEntryLoReg-CapS*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-TLBEntryLoReg-CapS arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-TLBEntryLoReg-CapS-def*)

lemma *non-cap-exp-set-TLBEntryLoReg-CapL*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-TLBEntryLoReg-CapL arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-TLBEntryLoReg-CapL-def*)

lemma *non-cap-exp-set-TLBEntryLoReg-PFN*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*

shows *non-cap-exp* (*set-TLBEntryLoReg-PFN* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntryLoReg-PFN-def*)

lemma *non-cap-exp-set-TLBEntryLoReg-C*[*non-cap-expI*]:
assumes *non-cap-reg* *arg0*
shows *non-cap-exp* (*set-TLBEntryLoReg-C* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntryLoReg-C-def*)

lemma *non-cap-exp-set-TLBEntryLoReg-D*[*non-cap-expI*]:
assumes *non-cap-reg* *arg0*
shows *non-cap-exp* (*set-TLBEntryLoReg-D* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntryLoReg-D-def*)

lemma *non-cap-exp-set-TLBEntryLoReg-V*[*non-cap-expI*]:
assumes *non-cap-reg* *arg0*
shows *non-cap-exp* (*set-TLBEntryLoReg-V* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntryLoReg-V-def*)

lemma *non-cap-exp-set-TLBEntryLoReg-G*[*non-cap-expI*]:
assumes *non-cap-reg* *arg0*
shows *non-cap-exp* (*set-TLBEntryLoReg-G* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntryLoReg-G-def*)

lemma *non-cap-exp-undefined-TLBEntryHiReg*[*non-cap-expI*]:
non-cap-exp (*undefined-TLBEntryHiReg* *arg0*)
by (*non-cap-expI simp: undefined-TLBEntryHiReg-def*)

lemma *non-cap-exp-set-TLBEntryHiReg-bits*[*non-cap-expI*]:
assumes *non-cap-reg* *arg0*
shows *non-cap-exp* (*set-TLBEntryHiReg-bits* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntryHiReg-bits-def*)

lemma *non-cap-exp-set-TLBEntryHiReg-R*[*non-cap-expI*]:
assumes *non-cap-reg* *arg0*
shows *non-cap-exp* (*set-TLBEntryHiReg-R* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntryHiReg-R-def*)

lemma *non-cap-exp-set-TLBEntryHiReg-VPN2*[*non-cap-expI*]:
assumes *non-cap-reg* *arg0*
shows *non-cap-exp* (*set-TLBEntryHiReg-VPN2* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntryHiReg-VPN2-def*)

lemma *non-cap-exp-set-TLBEntryHiReg-ASID*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-TLBEntryHiReg-ASID arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-TLBEntryHiReg-ASID-def*)

lemma *non-cap-exp-undefined-ContextReg*[*non-cap-expI*]:
non-cap-exp (undefined-ContextReg arg0)
 by (*non-cap-expI simp: undefined-ContextReg-def*)

lemma *non-cap-exp-set-ContextReg-bits*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-ContextReg-bits arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-ContextReg-bits-def*)

lemma *non-cap-exp-set-ContextReg-PTEBase*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-ContextReg-PTEBase arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-ContextReg-PTEBase-def*)

lemma *non-cap-exp-set-ContextReg-BadVPN2*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-ContextReg-BadVPN2 arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-ContextReg-BadVPN2-def*)

lemma *non-cap-exp-undefined-XContextReg*[*non-cap-expI*]:
non-cap-exp (undefined-XContextReg arg0)
 by (*non-cap-expI simp: undefined-XContextReg-def*)

lemma *non-cap-exp-set-XContextReg-bits*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-XContextReg-bits arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-XContextReg-bits-def*)

lemma *non-cap-exp-set-XContextReg-XPTEBase*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-XContextReg-XPTEBase arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-XContextReg-XPTEBase-def*)

lemma *non-cap-exp-set-XContextReg-XR*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-XContextReg-XR arg0 arg1)*
 using *assms*

by (non-cap-expI simp: set-XContextReg-XR-def)

lemma non-cap-exp-set-XContextReg-XBadVPN2[non-cap-expI]:
 assumes non-cap-reg arg0
 shows non-cap-exp (set-XContextReg-XBadVPN2 arg0 arg1)
 using assms
 by (non-cap-expI simp: set-XContextReg-XBadVPN2-def)

lemma non-cap-exp-undefined-TLBEntry[non-cap-expI]:
 non-cap-exp (undefined-TLBEntry arg0)
 by (non-cap-expI simp: undefined-TLBEntry-def)

lemma non-cap-exp-set-TLBEntry-bits[non-cap-expI]:
 assumes non-cap-reg arg0
 shows non-cap-exp (set-TLBEntry-bits arg0 arg1)
 using assms
 by (non-cap-expI simp: set-TLBEntry-bits-def)

lemma non-cap-exp-set-TLBEntry-pagemask[non-cap-expI]:
 assumes non-cap-reg arg0
 shows non-cap-exp (set-TLBEntry-pagemask arg0 arg1)
 using assms
 by (non-cap-expI simp: set-TLBEntry-pagemask-def)

lemma non-cap-exp-set-TLBEntry-r[non-cap-expI]:
 assumes non-cap-reg arg0
 shows non-cap-exp (set-TLBEntry-r arg0 arg1)
 using assms
 by (non-cap-expI simp: set-TLBEntry-r-def)

lemma non-cap-exp-set-TLBEntry-vpn2[non-cap-expI]:
 assumes non-cap-reg arg0
 shows non-cap-exp (set-TLBEntry-vpn2 arg0 arg1)
 using assms
 by (non-cap-expI simp: set-TLBEntry-vpn2-def)

lemma non-cap-exp-set-TLBEntry-asid[non-cap-expI]:
 assumes non-cap-reg arg0
 shows non-cap-exp (set-TLBEntry-asid arg0 arg1)
 using assms
 by (non-cap-expI simp: set-TLBEntry-asid-def)

lemma non-cap-exp-set-TLBEntry-g[non-cap-expI]:
 assumes non-cap-reg arg0
 shows non-cap-exp (set-TLBEntry-g arg0 arg1)
 using assms
 by (non-cap-expI simp: set-TLBEntry-g-def)

lemma non-cap-exp-set-TLBEntry-valid[non-cap-expI]:

```

assumes non-cap-reg arg0
shows non-cap-exp (set-TLBEntry-valid arg0 arg1)
using assms
by (non-cap-expI simp: set-TLBEntry-valid-def)

lemma non-cap-exp-set-TLBEntry-caps1 [non-cap-expI]:
  assumes non-cap-reg arg0
  shows non-cap-exp (set-TLBEntry-caps1 arg0 arg1)
  using assms
  by (non-cap-expI simp: set-TLBEntry-caps1-def)

lemma non-cap-exp-set-TLBEntry-capl1 [non-cap-expI]:
  assumes non-cap-reg arg0
  shows non-cap-exp (set-TLBEntry-capl1 arg0 arg1)
  using assms
  by (non-cap-expI simp: set-TLBEntry-capl1-def)

lemma non-cap-exp-set-TLBEntry-pfn1 [non-cap-expI]:
  assumes non-cap-reg arg0
  shows non-cap-exp (set-TLBEntry-pfn1 arg0 arg1)
  using assms
  by (non-cap-expI simp: set-TLBEntry-pfn1-def)

lemma non-cap-exp-set-TLBEntry-c1 [non-cap-expI]:
  assumes non-cap-reg arg0
  shows non-cap-exp (set-TLBEntry-c1 arg0 arg1)
  using assms
  by (non-cap-expI simp: set-TLBEntry-c1-def)

lemma non-cap-exp-set-TLBEntry-d1 [non-cap-expI]:
  assumes non-cap-reg arg0
  shows non-cap-exp (set-TLBEntry-d1 arg0 arg1)
  using assms
  by (non-cap-expI simp: set-TLBEntry-d1-def)

lemma non-cap-exp-set-TLBEntry-v1 [non-cap-expI]:
  assumes non-cap-reg arg0
  shows non-cap-exp (set-TLBEntry-v1 arg0 arg1)
  using assms
  by (non-cap-expI simp: set-TLBEntry-v1-def)

lemma non-cap-exp-set-TLBEntry-caps0 [non-cap-expI]:
  assumes non-cap-reg arg0
  shows non-cap-exp (set-TLBEntry-caps0 arg0 arg1)
  using assms
  by (non-cap-expI simp: set-TLBEntry-caps0-def)

lemma non-cap-exp-set-TLBEntry-capl0 [non-cap-expI]:
  assumes non-cap-reg arg0

```

shows *non-cap-exp* (*set-TLBEntry-capl0* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntry-capl0-def*)

lemma *non-cap-exp-set-TLBEntry-pfn0*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp* (*set-TLBEntry-pfn0* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntry-pfn0-def*)

lemma *non-cap-exp-set-TLBEntry-c0*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp* (*set-TLBEntry-c0* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntry-c0-def*)

lemma *non-cap-exp-set-TLBEntry-d0*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp* (*set-TLBEntry-d0* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntry-d0-def*)

lemma *non-cap-exp-set-TLBEntry-v0*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp* (*set-TLBEntry-v0* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-TLBEntry-v0-def*)

lemma *non-cap-exp-undefined-StatusReg*[*non-cap-expI*]:
non-cap-exp (*undefined-StatusReg* *arg0*)
by (*non-cap-expI simp: undefined-StatusReg-def*)

lemma *non-cap-exp-set-StatusReg-bits*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp* (*set-StatusReg-bits* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-StatusReg-bits-def*)

lemma *non-cap-exp-set-StatusReg-CU*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp* (*set-StatusReg-CU* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-StatusReg-CU-def*)

lemma *non-cap-exp-set-StatusReg-BEV*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp* (*set-StatusReg-BEV* *arg0* *arg1*)
using *assms*
by (*non-cap-expI simp: set-StatusReg-BEV-def*)

lemma *non-cap-exp-set-StatusReg-IM*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-StatusReg-IM arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-StatusReg-IM-def*)

lemma *non-cap-exp-set-StatusReg-KX*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-StatusReg-KX arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-StatusReg-KX-def*)

lemma *non-cap-exp-set-StatusReg-SX*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-StatusReg-SX arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-StatusReg-SX-def*)

lemma *non-cap-exp-set-StatusReg-UX*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-StatusReg-UX arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-StatusReg-UX-def*)

lemma *non-cap-exp-set-StatusReg-KSU*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-StatusReg-KSU arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-StatusReg-KSU-def*)

lemma *non-cap-exp-set-StatusReg-ERL*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-StatusReg-ERL arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-StatusReg-ERL-def*)

lemma *non-cap-exp-set-StatusReg-EXL*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-StatusReg-EXL arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-StatusReg-EXL-def*)

lemma *non-cap-exp-set-StatusReg-IE*[*non-cap-expI*]:
 assumes *non-cap-reg arg0*
 shows *non-cap-exp (set-StatusReg-IE arg0 arg1)*
 using *assms*
 by (*non-cap-expI simp: set-StatusReg-IE-def*)

lemma *non-cap-exp-execute-branch-mips*[*non-cap-expI*]:
non-cap-exp (*execute-branch-mips* *arg0*)
by (*non-cap-expI simp: execute-branch-mips-def*)

lemma *non-cap-exp-rGPR*[*non-cap-expI*]:
non-cap-exp (*rGPR* *arg0*)
by (*non-cap-expI simp: rGPR-def*)

lemma *non-cap-exp-wGPR*[*non-cap-expI*]:
non-cap-exp (*wGPR* *arg0* *arg1*)
by (*non-cap-expI simp: wGPR-def*)

lemma *non-cap-exp-MEM-sync*[*non-cap-expI*]:
non-cap-exp (*MEM-sync* *arg0*)
by (*non-cap-expI simp: MEM-sync-def*)

lemma *non-cap-exp-undefined-Exception*[*non-cap-expI*]:
non-cap-exp (*undefined-Exception* *arg0*)
by (*non-cap-expI simp: undefined-Exception-def*)

lemma *non-cap-exp-exceptionVectorOffset*[*non-cap-expI*]:
non-cap-exp (*exceptionVectorOffset* *arg0*)
by (*non-cap-expI simp: exceptionVectorOffset-def*)

lemma *non-cap-exp-exceptionVectorBase*[*non-cap-expI*]:
non-cap-exp (*exceptionVectorBase* *arg0*)
by (*non-cap-expI simp: exceptionVectorBase-def*)

lemma *non-cap-exp-updateBadInstr*[*non-cap-expI*]:
non-cap-exp (*updateBadInstr* *arg0*)
by (*non-cap-expI simp: updateBadInstr-def*)

lemma *non-cap-exp-undefined-Capability*[*non-cap-expI*]:
non-cap-exp (*undefined-Capability* *arg0*)
by (*non-cap-expI simp: undefined-Capability-def*)

lemma *non-cap-exp-undefined-MemAccessType*[*non-cap-expI*]:
non-cap-exp (*undefined-MemAccessType* *arg0*)
by (*non-cap-expI simp: undefined-MemAccessType-def*)

lemma *non-cap-exp-undefined-AccessLevel*[*non-cap-expI*]:
non-cap-exp (*undefined-AccessLevel* *arg0*)
by (*non-cap-expI simp: undefined-AccessLevel-def*)

lemma *non-cap-exp-getAccessLevel*[*non-cap-expI*]:
non-cap-exp (*getAccessLevel* *arg0*)
by (*non-cap-expI simp: getAccessLevel-def*)

lemma *non-cap-exp-undefined-CapCauseReg*[*non-cap-expI*]:

non-cap-exp (*undefined-CapCauseReg* *arg0*)
by (*non-cap-expI simp: undefined-CapCauseReg-def*)

lemma *non-cap-exp-set-CapCauseReg-ExcCode*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp (set-CapCauseReg-ExcCode arg0 arg1)*
using *assms*
by (*non-cap-expI simp: set-CapCauseReg-ExcCode-def*)

lemma *non-cap-exp-set-CapCauseReg-RegNum*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp (set-CapCauseReg-RegNum arg0 arg1)*
using *assms*
by (*non-cap-expI simp: set-CapCauseReg-RegNum-def*)

lemma *non-cap-exp-undefined-decode-failure*[*non-cap-expI*]:
non-cap-exp (undefined-decode-failure arg0)
by (*non-cap-expI simp: undefined-decode-failure-def*)

lemma *non-cap-exp-undefined-Comparison*[*non-cap-expI*]:
non-cap-exp (undefined-Comparison arg0)
by (*non-cap-expI simp: undefined-Comparison-def*)

lemma *non-cap-exp-undefined-WordType*[*non-cap-expI*]:
non-cap-exp (undefined-WordType arg0)
by (*non-cap-expI simp: undefined-WordType-def*)

lemma *non-cap-exp-undefined-WordTypeUnaligned*[*non-cap-expI*]:
non-cap-exp (undefined-WordTypeUnaligned arg0)
by (*non-cap-expI simp: undefined-WordTypeUnaligned-def*)

lemma *non-cap-exp-init-cp0-state*[*non-cap-expI*]:
non-cap-exp (init-cp0-state arg0)
by (*non-cap-expI simp: init-cp0-state-def*)

lemma *non-cap-exp-tlbSearch*[*non-cap-expI*]:
non-cap-exp (tlbSearch arg0)
by (*non-cap-expI simp: tlbSearch-def*)

lemma *non-cap-exp-undefined-CPtrCmpOp*[*non-cap-expI*]:
non-cap-exp (undefined-CPtrCmpOp arg0)
by (*non-cap-expI simp: undefined-CPtrCmpOp-def*)

lemma *non-cap-exp-undefined-ClearRegSet*[*non-cap-expI*]:
non-cap-exp (undefined-ClearRegSet arg0)
by (*non-cap-expI simp: undefined-ClearRegSet-def*)

lemma *non-cap-exp-capToString*[*non-cap-expI*]:
non-cap-exp (capToString arg0 arg1)

by (*non-cap-expI simp: capToString-def*)

lemma *non-cap-exp-undefined-CapEx*[*non-cap-expI*]:
non-cap-exp (undefined-CapEx arg0)
by (*non-cap-expI simp: undefined-CapEx-def*)

lemma *non-cap-exp-set-CapCauseReg-bits*[*non-cap-expI*]:
assumes *non-cap-reg arg0*
shows *non-cap-exp (set-CapCauseReg-bits arg0 arg1)*
using *assms*
by (*non-cap-expI simp: set-CapCauseReg-bits-def*)

lemma *non-cap-exp-execute-XORI*[*non-cap-expI*]:
non-cap-exp (execute-XORI arg0 arg1 arg2)
by (*non-cap-expI simp: execute-XORI-def*)

lemma *non-cap-exp-execute-XOR*[*non-cap-expI*]:
non-cap-exp (execute-XOR arg0 arg1 arg2)
by (*non-cap-expI simp: execute-XOR-def*)

lemma *non-cap-exp-execute-SYNC*[*non-cap-expI*]:
non-cap-exp (execute-SYNC arg0)
by (*non-cap-expI simp: execute-SYNC-def*)

lemma *non-cap-exp-execute-SUBU*[*non-cap-expI*]:
non-cap-exp (execute-SUBU arg0 arg1 arg2)
by (*non-cap-expI simp: execute-SUBU-def*)

lemma *non-cap-exp-execute-SRLV*[*non-cap-expI*]:
non-cap-exp (execute-SRLV arg0 arg1 arg2)
by (*non-cap-expI simp: execute-SRLV-def*)

lemma *non-cap-exp-execute-SRL*[*non-cap-expI*]:
non-cap-exp (execute-SRL arg0 arg1 arg2)
by (*non-cap-expI simp: execute-SRL-def*)

lemma *non-cap-exp-execute-SRAV*[*non-cap-expI*]:
non-cap-exp (execute-SRAV arg0 arg1 arg2)
by (*non-cap-expI simp: execute-SRAV-def*)

lemma *non-cap-exp-execute-SRA*[*non-cap-expI*]:
non-cap-exp (execute-SRA arg0 arg1 arg2)
by (*non-cap-expI simp: execute-SRA-def*)

lemma *non-cap-exp-execute-SLTU*[*non-cap-expI*]:
non-cap-exp (execute-SLTU arg0 arg1 arg2)
by (*non-cap-expI simp: execute-SLTU-def*)

lemma *non-cap-exp-execute-SLTIU*[*non-cap-expI*]:

non-cap-exp (*execute-SLTIU* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-SLTIU-def*)

lemma *non-cap-exp-execute-SLTI*[*non-cap-expI*]:
non-cap-exp (*execute-SLTI* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-SLTI-def*)

lemma *non-cap-exp-execute-SLT*[*non-cap-expI*]:
non-cap-exp (*execute-SLT* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-SLT-def*)

lemma *non-cap-exp-execute-SLLV*[*non-cap-expI*]:
non-cap-exp (*execute-SLLV* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-SLLV-def*)

lemma *non-cap-exp-execute-SLL*[*non-cap-expI*]:
non-cap-exp (*execute-SLL* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-SLL-def*)

lemma *non-cap-exp-execute-ORI*[*non-cap-expI*]:
non-cap-exp (*execute-ORI* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-ORI-def*)

lemma *non-cap-exp-execute-OR*[*non-cap-expI*]:
non-cap-exp (*execute-OR* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-OR-def*)

lemma *non-cap-exp-execute-NOR*[*non-cap-expI*]:
non-cap-exp (*execute-NOR* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-NOR-def*)

lemma *non-cap-exp-execute-MULTU*[*non-cap-expI*]:
non-cap-exp (*execute-MULTU* *arg0* *arg1*)
by (*non-cap-expI simp: execute-MULTU-def*)

lemma *non-cap-exp-execute-MULT*[*non-cap-expI*]:
non-cap-exp (*execute-MULT* *arg0* *arg1*)
by (*non-cap-expI simp: execute-MULT-def*)

lemma *non-cap-exp-execute-MUL*[*non-cap-expI*]:
non-cap-exp (*execute-MUL* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-MUL-def*)

lemma *non-cap-exp-execute-MTLO*[*non-cap-expI*]:
non-cap-exp (*execute-MTLO* *arg0*)
by (*non-cap-expI simp: execute-MTLO-def*)

lemma *non-cap-exp-execute-MTHI*[*non-cap-expI*]:
non-cap-exp (*execute-MTHI* *arg0*)

by (*non-cap-expI simp: execute-MTHI-def*)

lemma *non-cap-exp-execute-MSUBU*[*non-cap-expI*]:
non-cap-exp (execute-MSUBU arg0 arg1)
by (*non-cap-expI simp: execute-MSUBU-def*)

lemma *non-cap-exp-execute-MSUB*[*non-cap-expI*]:
non-cap-exp (execute-MSUB arg0 arg1)
by (*non-cap-expI simp: execute-MSUB-def*)

lemma *non-cap-exp-execute-MOVZ*[*non-cap-expI*]:
non-cap-exp (execute-MOVZ arg0 arg1 arg2)
by (*non-cap-expI simp: execute-MOVZ-def*)

lemma *non-cap-exp-execute-MOVN*[*non-cap-expI*]:
non-cap-exp (execute-MOVN arg0 arg1 arg2)
by (*non-cap-expI simp: execute-MOVN-def*)

lemma *non-cap-exp-execute-MFLO*[*non-cap-expI*]:
non-cap-exp (execute-MFLO arg0)
by (*non-cap-expI simp: execute-MFLO-def*)

lemma *non-cap-exp-execute-MFHI*[*non-cap-expI*]:
non-cap-exp (execute-MFHI arg0)
by (*non-cap-expI simp: execute-MFHI-def*)

lemma *non-cap-exp-execute-MADDU*[*non-cap-expI*]:
non-cap-exp (execute-MADDU arg0 arg1)
by (*non-cap-expI simp: execute-MADDU-def*)

lemma *non-cap-exp-execute-MADD*[*non-cap-expI*]:
non-cap-exp (execute-MADD arg0 arg1)
by (*non-cap-expI simp: execute-MADD-def*)

lemma *non-cap-exp-execute-LUI*[*non-cap-expI*]:
non-cap-exp (execute-LUI arg0 arg1)
by (*non-cap-expI simp: execute-LUI-def*)

lemma *non-cap-exp-execute-DSUBU*[*non-cap-expI*]:
non-cap-exp (execute-DSUBU arg0 arg1 arg2)
by (*non-cap-expI simp: execute-DSUBU-def*)

lemma *non-cap-exp-execute-DSRLV*[*non-cap-expI*]:
non-cap-exp (execute-DSRLV arg0 arg1 arg2)
by (*non-cap-expI simp: execute-DSRLV-def*)

lemma *non-cap-exp-execute-DSRL32*[*non-cap-expI*]:
non-cap-exp (execute-DSRL32 arg0 arg1 arg2)
by (*non-cap-expI simp: execute-DSRL32-def*)

lemma *non-cap-exp-execute-DSRL*[*non-cap-expI*]:
non-cap-exp (*execute-DSRL* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-DSRL-def*)

lemma *non-cap-exp-execute-DSRAV*[*non-cap-expI*]:
non-cap-exp (*execute-DSRAV* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-DSRAV-def*)

lemma *non-cap-exp-execute-DSRA32*[*non-cap-expI*]:
non-cap-exp (*execute-DSRA32* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-DSRA32-def*)

lemma *non-cap-exp-execute-DSRA*[*non-cap-expI*]:
non-cap-exp (*execute-DSRA* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-DSRA-def*)

lemma *non-cap-exp-execute-DSLLV*[*non-cap-expI*]:
non-cap-exp (*execute-DSLLV* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-DSLLV-def*)

lemma *non-cap-exp-execute-DSLL32*[*non-cap-expI*]:
non-cap-exp (*execute-DSLL32* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-DSLL32-def*)

lemma *non-cap-exp-execute-DSLL*[*non-cap-expI*]:
non-cap-exp (*execute-DSLL* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-DSLL-def*)

lemma *non-cap-exp-execute-DMULTU*[*non-cap-expI*]:
non-cap-exp (*execute-DMULTU* *arg0* *arg1*)
by (*non-cap-expI simp: execute-DMULTU-def*)

lemma *non-cap-exp-execute-DMULT*[*non-cap-expI*]:
non-cap-exp (*execute-DMULT* *arg0* *arg1*)
by (*non-cap-expI simp: execute-DMULT-def*)

lemma *non-cap-exp-execute-DIVU*[*non-cap-expI*]:
non-cap-exp (*execute-DIVU* *arg0* *arg1*)
by (*non-cap-expI simp: execute-DIVU-def*)

lemma *non-cap-exp-execute-DIV*[*non-cap-expI*]:
non-cap-exp (*execute-DIV* *arg0* *arg1*)
by (*non-cap-expI simp: execute-DIV-def*)

lemma *non-cap-exp-execute-DDIVU*[*non-cap-expI*]:
non-cap-exp (*execute-DDIVU* *arg0* *arg1*)
by (*non-cap-expI simp: execute-DDIVU-def*)

lemma *non-cap-exp-execute-DDIV*[*non-cap-expI*]:
non-cap-exp (*execute-DDIV* *arg0* *arg1*)
by (*non-cap-expI simp: execute-DDIV-def*)

lemma *non-cap-exp-execute-DADDU*[*non-cap-expI*]:
non-cap-exp (*execute-DADDU* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-DADDU-def*)

lemma *non-cap-exp-execute-DADDIU*[*non-cap-expI*]:
non-cap-exp (*execute-DADDIU* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-DADDIU-def*)

lemma *non-cap-exp-execute-ANDI*[*non-cap-expI*]:
non-cap-exp (*execute-ANDI* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-ANDI-def*)

lemma *non-cap-exp-execute-AND*[*non-cap-expI*]:
non-cap-exp (*execute-AND* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-AND-def*)

lemma *non-cap-exp-execute-ADDU*[*non-cap-expI*]:
non-cap-exp (*execute-ADDU* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-ADDU-def*)

lemma *non-cap-exp-execute-ADDIU*[*non-cap-expI*]:
non-cap-exp (*execute-ADDIU* *arg0* *arg1* *arg2*)
by (*non-cap-expI simp: execute-ADDIU-def*)

lemma *non-mem-exp-set-CauseReg-bits*[*non-mem-expI*]:
non-mem-exp (*set-CauseReg-bits* *arg0* *arg1*)
by (*non-mem-expI simp: set-CauseReg-bits-def*)

lemma *non-mem-exp-set-CauseReg-BD*[*non-mem-expI*]:
non-mem-exp (*set-CauseReg-BD* *arg0* *arg1*)
by (*non-mem-expI simp: set-CauseReg-BD-def*)

lemma *non-mem-exp-set-CauseReg-CE*[*non-mem-expI*]:
non-mem-exp (*set-CauseReg-CE* *arg0* *arg1*)
by (*non-mem-expI simp: set-CauseReg-CE-def*)

lemma *non-mem-exp-set-CauseReg-IV*[*non-mem-expI*]:
non-mem-exp (*set-CauseReg-IV* *arg0* *arg1*)
by (*non-mem-expI simp: set-CauseReg-IV-def*)

lemma *non-mem-exp-set-CauseReg-WP*[*non-mem-expI*]:
non-mem-exp (*set-CauseReg-WP* *arg0* *arg1*)
by (*non-mem-expI simp: set-CauseReg-WP-def*)

lemma *non-mem-exp-set-CauseReg-IP*[*non-mem-expI*]:

non-mem-exp (*set-CauseReg-IP* *arg0* *arg1*)
by (*non-mem-expI simp: set-CauseReg-IP-def*)

lemma *non-mem-exp-set-CauseReg-ExcCode*[*non-mem-expI*]:
non-mem-exp (*set-CauseReg-ExcCode* *arg0* *arg1*)
by (*non-mem-expI simp: set-CauseReg-ExcCode-def*)

lemma *non-mem-exp-set-TLBEntryLoReg-bits*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryLoReg-bits* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryLoReg-bits-def*)

lemma *non-mem-exp-set-TLBEntryLoReg-CapS*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryLoReg-CapS* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryLoReg-CapS-def*)

lemma *non-mem-exp-set-TLBEntryLoReg-CapL*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryLoReg-CapL* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryLoReg-CapL-def*)

lemma *non-mem-exp-set-TLBEntryLoReg-PFN*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryLoReg-PFN* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryLoReg-PFN-def*)

lemma *non-mem-exp-set-TLBEntryLoReg-C*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryLoReg-C* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryLoReg-C-def*)

lemma *non-mem-exp-set-TLBEntryLoReg-D*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryLoReg-D* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryLoReg-D-def*)

lemma *non-mem-exp-set-TLBEntryLoReg-V*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryLoReg-V* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryLoReg-V-def*)

lemma *non-mem-exp-set-TLBEntryLoReg-G*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryLoReg-G* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryLoReg-G-def*)

lemma *non-mem-exp-set-TLBEntryHiReg-bits*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryHiReg-bits* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryHiReg-bits-def*)

lemma *non-mem-exp-set-TLBEntryHiReg-R*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryHiReg-R* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntryHiReg-R-def*)

lemma *non-mem-exp-set-TLBEntryHiReg-VPN2*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntryHiReg-VPN2* *arg0* *arg1*)

by (*non-mem-expI simp: set-TLBEntryHiReg-VPN2-def*)

lemma *non-mem-exp-set-TLBEntryHiReg-ASID*[*non-mem-expI*]:
non-mem-exp (set-TLBEntryHiReg-ASID arg0 arg1)
by (*non-mem-expI simp: set-TLBEntryHiReg-ASID-def*)

lemma *non-mem-exp-set-ContextReg-bits*[*non-mem-expI*]:
non-mem-exp (set-ContextReg-bits arg0 arg1)
by (*non-mem-expI simp: set-ContextReg-bits-def*)

lemma *non-mem-exp-set-ContextReg-PTEBase*[*non-mem-expI*]:
non-mem-exp (set-ContextReg-PTEBase arg0 arg1)
by (*non-mem-expI simp: set-ContextReg-PTEBase-def*)

lemma *non-mem-exp-set-ContextReg-BadVPN2*[*non-mem-expI*]:
non-mem-exp (set-ContextReg-BadVPN2 arg0 arg1)
by (*non-mem-expI simp: set-ContextReg-BadVPN2-def*)

lemma *non-mem-exp-set-XContextReg-bits*[*non-mem-expI*]:
non-mem-exp (set-XContextReg-bits arg0 arg1)
by (*non-mem-expI simp: set-XContextReg-bits-def*)

lemma *non-mem-exp-set-XContextReg-XPTEBase*[*non-mem-expI*]:
non-mem-exp (set-XContextReg-XPTEBase arg0 arg1)
by (*non-mem-expI simp: set-XContextReg-XPTEBase-def*)

lemma *non-mem-exp-set-XContextReg-XR*[*non-mem-expI*]:
non-mem-exp (set-XContextReg-XR arg0 arg1)
by (*non-mem-expI simp: set-XContextReg-XR-def*)

lemma *non-mem-exp-set-XContextReg-XBadVPN2*[*non-mem-expI*]:
non-mem-exp (set-XContextReg-XBadVPN2 arg0 arg1)
by (*non-mem-expI simp: set-XContextReg-XBadVPN2-def*)

lemma *non-mem-exp-set-TLBEntry-bits*[*non-mem-expI*]:
non-mem-exp (set-TLBEntry-bits arg0 arg1)
by (*non-mem-expI simp: set-TLBEntry-bits-def*)

lemma *non-mem-exp-set-TLBEntry-pagemask*[*non-mem-expI*]:
non-mem-exp (set-TLBEntry-pagemask arg0 arg1)
by (*non-mem-expI simp: set-TLBEntry-pagemask-def*)

lemma *non-mem-exp-set-TLBEntry-r*[*non-mem-expI*]:
non-mem-exp (set-TLBEntry-r arg0 arg1)
by (*non-mem-expI simp: set-TLBEntry-r-def*)

lemma *non-mem-exp-set-TLBEntry-vpn2*[*non-mem-expI*]:
non-mem-exp (set-TLBEntry-vpn2 arg0 arg1)
by (*non-mem-expI simp: set-TLBEntry-vpn2-def*)

lemma *non-mem-exp-set-TLBEntry-asid*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-asid* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-asid-def*)

lemma *non-mem-exp-set-TLBEntry-g*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-g* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-g-def*)

lemma *non-mem-exp-set-TLBEntry-valid*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-valid* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-valid-def*)

lemma *non-mem-exp-set-TLBEntry-caps1*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-caps1* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-caps1-def*)

lemma *non-mem-exp-set-TLBEntry-capl1*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-capl1* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-capl1-def*)

lemma *non-mem-exp-set-TLBEntry-pfn1*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-pfn1* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-pfn1-def*)

lemma *non-mem-exp-set-TLBEntry-c1*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-c1* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-c1-def*)

lemma *non-mem-exp-set-TLBEntry-d1*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-d1* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-d1-def*)

lemma *non-mem-exp-set-TLBEntry-v1*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-v1* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-v1-def*)

lemma *non-mem-exp-set-TLBEntry-caps0*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-caps0* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-caps0-def*)

lemma *non-mem-exp-set-TLBEntry-capl0*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-capl0* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-capl0-def*)

lemma *non-mem-exp-set-TLBEntry-pfn0*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-pfn0* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-pfn0-def*)

lemma *non-mem-exp-set-TLBEntry-c0*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-c0* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-c0-def*)

lemma *non-mem-exp-set-TLBEntry-d0*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-d0* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-d0-def*)

lemma *non-mem-exp-set-TLBEntry-v0*[*non-mem-expI*]:
non-mem-exp (*set-TLBEntry-v0* *arg0* *arg1*)
by (*non-mem-expI simp: set-TLBEntry-v0-def*)

lemma *non-mem-exp-set-StatusReg-bits*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-bits* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-bits-def*)

lemma *non-mem-exp-set-StatusReg-CU*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-CU* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-CU-def*)

lemma *non-mem-exp-set-StatusReg-BEV*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-BEV* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-BEV-def*)

lemma *non-mem-exp-set-StatusReg-IM*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-IM* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-IM-def*)

lemma *non-mem-exp-set-StatusReg-KX*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-KX* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-KX-def*)

lemma *non-mem-exp-set-StatusReg-SX*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-SX* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-SX-def*)

lemma *non-mem-exp-set-StatusReg-UX*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-UX* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-UX-def*)

lemma *non-mem-exp-set-StatusReg-KSU*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-KSU* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-KSU-def*)

lemma *non-mem-exp-set-StatusReg-ERL*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-ERL* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-ERL-def*)

lemma *non-mem-exp-set-StatusReg-EXL*[*non-mem-expI*]:

non-mem-exp (*set-StatusReg-EXL* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-EXL-def*)

lemma *non-mem-exp-set-StatusReg-IE*[*non-mem-expI*]:
non-mem-exp (*set-StatusReg-IE* *arg0* *arg1*)
by (*non-mem-expI simp: set-StatusReg-IE-def*)

lemma *non-mem-exp-set-CapCauseReg-ExcCode*[*non-mem-expI*]:
non-mem-exp (*set-CapCauseReg-ExcCode* *arg0* *arg1*)
by (*non-mem-expI simp: set-CapCauseReg-ExcCode-def*)

lemma *non-mem-exp-set-CapCauseReg-RegNum*[*non-mem-expI*]:
non-mem-exp (*set-CapCauseReg-RegNum* *arg0* *arg1*)
by (*non-mem-expI simp: set-CapCauseReg-RegNum-def*)

lemma *non-mem-exp-set-next-pcc*[*non-mem-expI*]:
non-mem-exp (*set-next-pcc* *arg0*)
by (*non-mem-expI simp: set-next-pcc-def*)

lemma *non-mem-exp-SignalException*[*non-mem-expI*]:
non-mem-exp (*SignalException* *arg0*)
by (*non-mem-expI simp: SignalException-def*)

lemma *non-mem-exp-SignalExceptionBadAddr*[*non-mem-expI*]:
non-mem-exp (*SignalExceptionBadAddr* *arg0* *arg1*)
by (*non-mem-expI simp: SignalExceptionBadAddr-def*)

lemma *non-mem-exp-SignalExceptionTLB*[*non-mem-expI*]:
non-mem-exp (*SignalExceptionTLB* *arg0* *arg1*)
by (*non-mem-expI simp: SignalExceptionTLB-def*)

lemma *non-mem-exp-pcc-access-system-regs*[*non-mem-expI*]:
non-mem-exp (*pcc-access-system-regs* *arg0*)
by (*non-mem-expI simp: pcc-access-system-regs-def*)

lemma *non-mem-exp-raise-c2-exception8*[*non-mem-expI*]:
non-mem-exp (*raise-c2-exception8* *arg0* *arg1*)
by (*non-mem-expI simp: raise-c2-exception8-def*)

lemma *non-mem-exp-raise-c2-exception-noreg*[*non-mem-expI*]:
non-mem-exp (*raise-c2-exception-noreg* *arg0*)
by (*non-mem-expI simp: raise-c2-exception-noreg-def*)

lemma *non-mem-exp-checkCP0AccessHook*[*non-mem-expI*]:
non-mem-exp (*checkCP0AccessHook* *arg0*)
by (*non-mem-expI simp: checkCP0AccessHook-def*)

lemma *non-mem-exp-checkCP0Access*[*non-mem-expI*]:
non-mem-exp (*checkCP0Access* *arg0*)

by (*non-mem-expI simp: checkCP0Access-def*)

lemma *non-mem-exp-incrementCP0Count*[*non-mem-expI*]:
non-mem-exp (incrementCP0Count arg0)
by (*non-mem-expI simp: incrementCP0Count-def*)

lemma *non-mem-exp-TLBTranslate2*[*non-mem-expI*]:
non-mem-exp (TLBTranslate2 arg0 arg1)
by (*non-mem-expI simp: TLBTranslate2-def*)

lemma *non-mem-exp-TLBTranslateC*[*non-mem-expI*]:
non-mem-exp (TLBTranslateC arg0 arg1)
by (*non-mem-expI simp: TLBTranslateC-def*)

lemma *non-mem-exp-TLBTranslate*[*non-mem-expI*]:
non-mem-exp (TLBTranslate arg0 arg1)
by (*non-mem-expI simp: TLBTranslate-def*)

lemma *non-mem-exp-execute-branch-pcc*[*non-mem-expI*]:
non-mem-exp (execute-branch-pcc arg0)
by (*non-mem-expI simp: execute-branch-pcc-def*)

lemma *non-mem-exp-ERETHook*[*non-mem-expI*]:
non-mem-exp (ERETHook arg0)
by (*non-mem-expI simp: ERETHook-def*)

lemma *non-mem-exp-raise-c2-exception*[*non-mem-expI*]:
non-mem-exp (raise-c2-exception arg0 arg1)
by (*non-mem-expI simp: raise-c2-exception-def*)

lemma *non-mem-exp-checkDDCPperms*[*non-mem-expI*]:
non-mem-exp (checkDDCPperms arg0 arg1)
by (*non-mem-expI simp: checkDDCPperms-def*)

lemma *non-mem-exp-addrWrapper*[*non-mem-expI*]:
non-mem-exp (addrWrapper arg0 arg1 arg2)
by (*non-mem-expI simp: addrWrapper-def*)

lemma *non-mem-exp-addrWrapperUnaligned*[*non-mem-expI*]:
non-mem-exp (addrWrapperUnaligned arg0 arg1 arg2)
by (*non-mem-expI simp: addrWrapperUnaligned-def*)

lemma *non-mem-exp-execute-branch*[*non-mem-expI*]:
non-mem-exp (execute-branch arg0)
by (*non-mem-expI simp: execute-branch-def*)

lemma *non-mem-exp-TranslatePC*[*non-mem-expI*]:
non-mem-exp (TranslatePC arg0)
by (*non-mem-expI simp: TranslatePC-def*)

lemma *non-mem-exp-checkCP2usable*[*non-mem-expI*]:
non-mem-exp (*checkCP2usable* *arg0*)
by (*non-mem-expI simp: checkCP2usable-def*)

lemma *non-mem-exp-get-CP0EPC*[*non-mem-expI*]:
non-mem-exp (*get-CP0EPC* *arg0*)
by (*non-mem-expI simp: get-CP0EPC-def*)

lemma *non-mem-exp-set-CP0EPC*[*non-mem-expI*]:
non-mem-exp (*set-CP0EPC* *arg0*)
by (*non-mem-expI simp: set-CP0EPC-def*)

lemma *non-mem-exp-get-CP0ErrorEPC*[*non-mem-expI*]:
non-mem-exp (*get-CP0ErrorEPC* *arg0*)
by (*non-mem-expI simp: get-CP0ErrorEPC-def*)

lemma *non-mem-exp-set-CP0ErrorEPC*[*non-mem-expI*]:
non-mem-exp (*set-CP0ErrorEPC* *arg0*)
by (*non-mem-expI simp: set-CP0ErrorEPC-def*)

lemma *non-mem-exp-dump-cp2-state*[*non-mem-expI*]:
non-mem-exp (*dump-cp2-state* *arg0*)
by (*non-mem-expI simp: dump-cp2-state-def*)

lemma *non-mem-exp-TLBWriteEntry*[*non-mem-expI*]:
non-mem-exp (*TLBWriteEntry* *arg0*)
by (*non-mem-expI simp: TLBWriteEntry-def*)

lemma *non-mem-exp-execute-WAIT*[*non-mem-expI*]:
non-mem-exp (*execute-WAIT* *arg0*)
by (*non-mem-expI simp: execute-WAIT-def*)

lemma *non-mem-exp-execute-TRAPREG*[*non-mem-expI*]:
non-mem-exp (*execute-TRAPREG* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-TRAPREG-def*)

lemma *non-mem-exp-execute-TRAPIMM*[*non-mem-expI*]:
non-mem-exp (*execute-TRAPIMM* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-TRAPIMM-def*)

lemma *non-mem-exp-execute-TLBWR*[*non-mem-expI*]:
non-mem-exp (*execute-TLBWR* *arg0*)
by (*non-mem-expI simp: execute-TLBWR-def*)

lemma *non-mem-exp-execute-TLBWI*[*non-mem-expI*]:
non-mem-exp (*execute-TLBWI* *arg0*)
by (*non-mem-expI simp: execute-TLBWI-def*)

lemma *non-mem-exp-execute-TLBR*[*non-mem-expI*]:
non-mem-exp (*execute-TLBR* *arg0*)
by (*non-mem-expI simp: execute-TLBR-def*)

lemma *non-mem-exp-execute-TLBP*[*non-mem-expI*]:
non-mem-exp (*execute-TLBP* *arg0*)
by (*non-mem-expI simp: execute-TLBP-def*)

lemma *non-mem-exp-execute-SYSCALL*[*non-mem-expI*]:
non-mem-exp (*execute-SYSCALL* *arg0*)
by (*non-mem-expI simp: execute-SYSCALL-def*)

lemma *non-mem-exp-execute-SUB*[*non-mem-expI*]:
non-mem-exp (*execute-SUB* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-SUB-def*)

lemma *non-mem-exp-execute-RI*[*non-mem-expI*]:
non-mem-exp (*execute-RI* *arg0*)
by (*non-mem-expI simp: execute-RI-def*)

lemma *non-mem-exp-execute-RDHWR*[*non-mem-expI*]:
non-mem-exp (*execute-RDHWR* *arg0* *arg1*)
by (*non-mem-expI simp: execute-RDHWR-def*)

lemma *non-mem-exp-execute-MTC0*[*non-mem-expI*]:
non-mem-exp (*execute-MTC0* *arg0* *arg1* *arg2* *arg3*)
by (*non-mem-expI simp: execute-MTC0-def*)

lemma *non-mem-exp-execute-MFC0*[*non-mem-expI*]:
non-mem-exp (*execute-MFC0* *arg0* *arg1* *arg2* *arg3*)
by (*non-mem-expI simp: execute-MFC0-def*)

lemma *non-mem-exp-execute-JR*[*non-mem-expI*]:
non-mem-exp (*execute-JR* *arg0*)
by (*non-mem-expI simp: execute-JR-def*)

lemma *non-mem-exp-execute-JALR*[*non-mem-expI*]:
non-mem-exp (*execute-JALR* *arg0* *arg1*)
by (*non-mem-expI simp: execute-JALR-def*)

lemma *non-mem-exp-execute-JAL*[*non-mem-expI*]:
non-mem-exp (*execute-JAL* *arg0*)
by (*non-mem-expI simp: execute-JAL-def*)

lemma *non-mem-exp-execute-J*[*non-mem-expI*]:
non-mem-exp (*execute-J* *arg0*)
by (*non-mem-expI simp: execute-J-def*)

lemma *non-mem-exp-execute-ERET*[*non-mem-expI*]:

non-mem-exp (*execute-ERET* *arg0*)
by (*non-mem-expI simp: execute-ERET-def*)

lemma *non-mem-exp-execute-DSUB*[*non-mem-expI*]:
non-mem-exp (*execute-DSUB* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-DSUB-def*)

lemma *non-mem-exp-execute-DADDI*[*non-mem-expI*]:
non-mem-exp (*execute-DADDI* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-DADDI-def*)

lemma *non-mem-exp-execute-DADD*[*non-mem-expI*]:
non-mem-exp (*execute-DADD* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-DADD-def*)

lemma *non-mem-exp-execute-ClearRegs*[*non-mem-expI*]:
non-mem-exp (*execute-ClearRegs* *arg0* *arg1*)
by (*non-mem-expI simp: execute-ClearRegs-def*)

lemma *non-mem-exp-execute-CWriteHwr*[*non-mem-expI*]:
non-mem-exp (*execute-CWriteHwr* *arg0* *arg1*)
by (*non-mem-expI simp: execute-CWriteHwr-def*)

lemma *non-mem-exp-execute-CUnseal*[*non-mem-expI*]:
non-mem-exp (*execute-CUnseal* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-CUnseal-def*)

lemma *non-mem-exp-execute-CToPtr*[*non-mem-expI*]:
non-mem-exp (*execute-CToPtr* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-CToPtr-def*)

lemma *non-mem-exp-execute-CTestSubset*[*non-mem-expI*]:
non-mem-exp (*execute-CTestSubset* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-CTestSubset-def*)

lemma *non-mem-exp-execute-CTSub*[*non-mem-expI*]:
non-mem-exp (*execute-CTSub* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-CTSub-def*)

lemma *non-mem-exp-execute-CSetOffset*[*non-mem-expI*]:
non-mem-exp (*execute-CSetOffset* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-CSetOffset-def*)

lemma *non-mem-exp-execute-CSetFlags*[*non-mem-expI*]:
non-mem-exp (*execute-CSetFlags* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-CSetFlags-def*)

lemma *non-mem-exp-execute-CSetCause*[*non-mem-expI*]:
non-mem-exp (*execute-CSetCause* *arg0*)

by (*non-mem-expI simp: execute-CSetCause-def*)

lemma *non-mem-exp-execute-CSetCID*[*non-mem-expI*]:
non-mem-exp (execute-CSetCID arg0)
by (*non-mem-expI simp: execute-CSetCID-def*)

lemma *non-mem-exp-execute-CSetBoundsImmediate*[*non-mem-expI*]:
non-mem-exp (execute-CSetBoundsImmediate arg0 arg1 arg2)
by (*non-mem-expI simp: execute-CSetBoundsImmediate-def*)

lemma *non-mem-exp-execute-CSetBoundsExact*[*non-mem-expI*]:
non-mem-exp (execute-CSetBoundsExact arg0 arg1 arg2)
by (*non-mem-expI simp: execute-CSetBoundsExact-def*)

lemma *non-mem-exp-execute-CSetBounds*[*non-mem-expI*]:
non-mem-exp (execute-CSetBounds arg0 arg1 arg2)
by (*non-mem-expI simp: execute-CSetBounds-def*)

lemma *non-mem-exp-execute-CSetAddr*[*non-mem-expI*]:
non-mem-exp (execute-CSetAddr arg0 arg1 arg2)
by (*non-mem-expI simp: execute-CSetAddr-def*)

lemma *non-mem-exp-execute-CSeal*[*non-mem-expI*]:
non-mem-exp (execute-CSeal arg0 arg1 arg2)
by (*non-mem-expI simp: execute-CSeal-def*)

lemma *non-mem-exp-execute-CReturn*[*non-mem-expI*]:
non-mem-exp (execute-CReturn arg0)
by (*non-mem-expI simp: execute-CReturn-def*)

lemma *non-mem-exp-execute-CReadHwr*[*non-mem-expI*]:
non-mem-exp (execute-CReadHwr arg0 arg1)
by (*non-mem-expI simp: execute-CReadHwr-def*)

lemma *non-mem-exp-execute-CRAP*[*non-mem-expI*]:
non-mem-exp (execute-CRAP arg0 arg1)
by (*non-mem-expI simp: execute-CRAP-def*)

lemma *non-mem-exp-execute-CRAM*[*non-mem-expI*]:
non-mem-exp (execute-CRAM arg0 arg1)
by (*non-mem-expI simp: execute-CRAM-def*)

lemma *non-mem-exp-execute-CPtrCmp*[*non-mem-expI*]:
non-mem-exp (execute-CPtrCmp arg0 arg1 arg2 arg3)
by (*non-mem-expI simp: execute-CPtrCmp-def*)

lemma *non-mem-exp-execute-CMove*[*non-mem-expI*]:
non-mem-exp (execute-CMove arg0 arg1)
by (*non-mem-expI simp: execute-CMove-def*)

lemma *non-mem-exp-execute-CMOVX*[*non-mem-expI*]:
non-mem-exp (*execute-CMOVX* *arg0 arg1 arg2 arg3*)
by (*non-mem-expI simp: execute-CMOVX-def*)

lemma *non-mem-exp-execute-CJALR*[*non-mem-expI*]:
non-mem-exp (*execute-CJALR* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CJALR-def*)

lemma *non-mem-exp-execute-CIncOffsetImmediate*[*non-mem-expI*]:
non-mem-exp (*execute-CIncOffsetImmediate* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CIncOffsetImmediate-def*)

lemma *non-mem-exp-execute-CIncOffset*[*non-mem-expI*]:
non-mem-exp (*execute-CIncOffset* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CIncOffset-def*)

lemma *non-mem-exp-execute-CGetType*[*non-mem-expI*]:
non-mem-exp (*execute-CGetType* *arg0 arg1*)
by (*non-mem-expI simp: execute-CGetType-def*)

lemma *non-mem-exp-execute-CGetTag*[*non-mem-expI*]:
non-mem-exp (*execute-CGetTag* *arg0 arg1*)
by (*non-mem-expI simp: execute-CGetTag-def*)

lemma *non-mem-exp-execute-CGetSealed*[*non-mem-expI*]:
non-mem-exp (*execute-CGetSealed* *arg0 arg1*)
by (*non-mem-expI simp: execute-CGetSealed-def*)

lemma *non-mem-exp-execute-CGetPerm*[*non-mem-expI*]:
non-mem-exp (*execute-CGetPerm* *arg0 arg1*)
by (*non-mem-expI simp: execute-CGetPerm-def*)

lemma *non-mem-exp-execute-CGetPCCSetOffset*[*non-mem-expI*]:
non-mem-exp (*execute-CGetPCCSetOffset* *arg0 arg1*)
by (*non-mem-expI simp: execute-CGetPCCSetOffset-def*)

lemma *non-mem-exp-execute-CGetPCC*[*non-mem-expI*]:
non-mem-exp (*execute-CGetPCC* *arg0*)
by (*non-mem-expI simp: execute-CGetPCC-def*)

lemma *non-mem-exp-execute-CGetOffset*[*non-mem-expI*]:
non-mem-exp (*execute-CGetOffset* *arg0 arg1*)
by (*non-mem-expI simp: execute-CGetOffset-def*)

lemma *non-mem-exp-execute-CGetLen*[*non-mem-expI*]:
non-mem-exp (*execute-CGetLen* *arg0 arg1*)
by (*non-mem-expI simp: execute-CGetLen-def*)

lemma *non-mem-exp-execute-CGetFlags*[*non-mem-expI*]:
non-mem-exp (*execute-CGetFlags* *arg0* *arg1*)
by (*non-mem-expI simp: execute-CGetFlags-def*)

lemma *non-mem-exp-execute-CGetCause*[*non-mem-expI*]:
non-mem-exp (*execute-CGetCause* *arg0*)
by (*non-mem-expI simp: execute-CGetCause-def*)

lemma *non-mem-exp-execute-CGetCID*[*non-mem-expI*]:
non-mem-exp (*execute-CGetCID* *arg0*)
by (*non-mem-expI simp: execute-CGetCID-def*)

lemma *non-mem-exp-execute-CGetBase*[*non-mem-expI*]:
non-mem-exp (*execute-CGetBase* *arg0* *arg1*)
by (*non-mem-expI simp: execute-CGetBase-def*)

lemma *non-mem-exp-execute-CGetAndAddr*[*non-mem-expI*]:
non-mem-exp (*execute-CGetAndAddr* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-CGetAndAddr-def*)

lemma *non-mem-exp-execute-CGetAddr*[*non-mem-expI*]:
non-mem-exp (*execute-CGetAddr* *arg0* *arg1*)
by (*non-mem-expI simp: execute-CGetAddr-def*)

lemma *non-mem-exp-execute-CFromPtr*[*non-mem-expI*]:
non-mem-exp (*execute-CFromPtr* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-CFromPtr-def*)

lemma *non-mem-exp-execute-CCopyType*[*non-mem-expI*]:
non-mem-exp (*execute-CCopyType* *arg0* *arg1* *arg2*)
by (*non-mem-expI simp: execute-CCopyType-def*)

lemma *non-mem-exp-execute-CClearTag*[*non-mem-expI*]:
non-mem-exp (*execute-CClearTag* *arg0* *arg1*)
by (*non-mem-expI simp: execute-CClearTag-def*)

lemma *non-mem-exp-execute-CCheckType*[*non-mem-expI*]:
non-mem-exp (*execute-CCheckType* *arg0* *arg1*)
by (*non-mem-expI simp: execute-CCheckType-def*)

lemma *non-mem-exp-execute-CCheckTag*[*non-mem-expI*]:
non-mem-exp (*execute-CCheckTag* *arg0*)
by (*non-mem-expI simp: execute-CCheckTag-def*)

lemma *non-mem-exp-execute-CCheckPerm*[*non-mem-expI*]:
non-mem-exp (*execute-CCheckPerm* *arg0* *arg1*)
by (*non-mem-expI simp: execute-CCheckPerm-def*)

lemma *non-mem-exp-execute-CCall*[*non-mem-expI*]:

non-mem-exp (*execute-CCall* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CCall-def*)

lemma *non-mem-exp-execute-CCSeal*[*non-mem-expI*]:
non-mem-exp (*execute-CCSeal* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CCSeal-def*)

lemma *non-mem-exp-execute-CBuildCap*[*non-mem-expI*]:
non-mem-exp (*execute-CBuildCap* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CBuildCap-def*)

lemma *non-mem-exp-execute-CBZ*[*non-mem-expI*]:
non-mem-exp (*execute-CBZ* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CBZ-def*)

lemma *non-mem-exp-execute-CBX*[*non-mem-expI*]:
non-mem-exp (*execute-CBX* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CBX-def*)

lemma *non-mem-exp-execute-CAndPerm*[*non-mem-expI*]:
non-mem-exp (*execute-CAndPerm* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CAndPerm-def*)

lemma *non-mem-exp-execute-CAndAddr*[*non-mem-expI*]:
non-mem-exp (*execute-CAndAddr* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CAndAddr-def*)

lemma *non-mem-exp-execute-CACHE*[*non-mem-expI*]:
non-mem-exp (*execute-CACHE* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-CACHE-def*)

lemma *non-mem-exp-execute-BREAK*[*non-mem-expI*]:
non-mem-exp (*execute-BREAK* *arg0*)
by (*non-mem-expI simp: execute-BREAK-def*)

lemma *non-mem-exp-execute-BEQ*[*non-mem-expI*]:
non-mem-exp (*execute-BEQ* *arg0 arg1 arg2 arg3 arg4*)
by (*non-mem-expI simp: execute-BEQ-def*)

lemma *non-mem-exp-execute-BCMPZ*[*non-mem-expI*]:
non-mem-exp (*execute-BCMPZ* *arg0 arg1 arg2 arg3 arg4*)
by (*non-mem-expI simp: execute-BCMPZ-def*)

lemma *non-mem-exp-execute-ADDI*[*non-mem-expI*]:
non-mem-exp (*execute-ADDI* *arg0 arg1 arg2*)
by (*non-mem-expI simp: execute-ADDI-def*)

lemma *non-mem-exp-execute-ADD*[*non-mem-expI*]:
non-mem-exp (*execute-ADD* *arg0 arg1 arg2*)

by (*non-mem-expI simp: execute-ADD-def*)

lemma *non-mem-exp-cp2-next-pc*[*non-mem-expI*]:
non-mem-exp (cp2-next-pc u)
by (*non-mem-expI simp: cp2-next-pc-def*)

lemma *no-reg-writes-to-undefined-option*[*no-reg-writes-toI, simp*]:
no-reg-writes-to Rs (undefined-option arg0)
unfolding *undefined-option-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MIPS-write*[*no-reg-writes-toI, simp*]:
no-reg-writes-to Rs (MIPS-write arg0 arg1 arg2)
unfolding *MIPS-write-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MIPS-read*[*no-reg-writes-toI, simp*]:
no-reg-writes-to Rs (MIPS-read arg0 arg1)
unfolding *MIPS-read-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-exception*[*no-reg-writes-toI, simp*]:
no-reg-writes-to Rs (undefined-exception arg0)
unfolding *undefined-exception-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-CauseReg*[*no-reg-writes-toI, simp*]:
no-reg-writes-to Rs (undefined-CauseReg arg0)
unfolding *undefined-CauseReg-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CauseReg-bits*[*no-reg-writes-toI, simp*]:
assumes *name arg0 ∉ Rs*
shows *no-reg-writes-to Rs (set-CauseReg-bits arg0 arg1)*
using *assms*
unfolding *set-CauseReg-bits-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CauseReg-BD*[*no-reg-writes-toI, simp*]:
assumes *name arg0 ∉ Rs*
shows *no-reg-writes-to Rs (set-CauseReg-BD arg0 arg1)*
using *assms*
unfolding *set-CauseReg-BD-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CauseReg-CE*[*no-reg-writes-toI, simp*]:
assumes *name arg0 ∉ Rs*
shows *no-reg-writes-to Rs (set-CauseReg-CE arg0 arg1)*
using *assms*

unfolding *set-CauseReg-CE-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CauseReg-IV*[*no-reg-writes-toI, simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-CauseReg-IV arg0 arg1*)
using *assms*
unfolding *set-CauseReg-IV-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CauseReg-WP*[*no-reg-writes-toI, simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-CauseReg-WP arg0 arg1*)
using *assms*
unfolding *set-CauseReg-WP-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CauseReg-IP*[*no-reg-writes-toI, simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-CauseReg-IP arg0 arg1*)
using *assms*
unfolding *set-CauseReg-IP-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CauseReg-ExcCode*[*no-reg-writes-toI, simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-CauseReg-ExcCode arg0 arg1*)
using *assms*
unfolding *set-CauseReg-ExcCode-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-TLBEEntryLoReg*[*no-reg-writes-toI, simp*]:
no-reg-writes-to Rs (*undefined-TLBEEntryLoReg arg0*)
unfolding *undefined-TLBEEntryLoReg-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEEntryLoReg-bits*[*no-reg-writes-toI, simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEEntryLoReg-bits arg0 arg1*)
using *assms*
unfolding *set-TLBEEntryLoReg-bits-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEEntryLoReg-CapS*[*no-reg-writes-toI, simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEEntryLoReg-CapS arg0 arg1*)
using *assms*
unfolding *set-TLBEEntryLoReg-CapS-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntryLoReg-CapL*[*no-reg-writes-toI*, *simp*]:
 assumes *name arg0* \notin *Rs*
 shows *no-reg-writes-to* *Rs* (*set-TLBEntryLoReg-CapL* *arg0* *arg1*)
 using *assms*
 unfolding *set-TLBEntryLoReg-CapL-def* *bind-assoc*
 by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntryLoReg-PFN*[*no-reg-writes-toI*, *simp*]:
 assumes *name arg0* \notin *Rs*
 shows *no-reg-writes-to* *Rs* (*set-TLBEntryLoReg-PFN* *arg0* *arg1*)
 using *assms*
 unfolding *set-TLBEntryLoReg-PFN-def* *bind-assoc*
 by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntryLoReg-C*[*no-reg-writes-toI*, *simp*]:
 assumes *name arg0* \notin *Rs*
 shows *no-reg-writes-to* *Rs* (*set-TLBEntryLoReg-C* *arg0* *arg1*)
 using *assms*
 unfolding *set-TLBEntryLoReg-C-def* *bind-assoc*
 by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntryLoReg-D*[*no-reg-writes-toI*, *simp*]:
 assumes *name arg0* \notin *Rs*
 shows *no-reg-writes-to* *Rs* (*set-TLBEntryLoReg-D* *arg0* *arg1*)
 using *assms*
 unfolding *set-TLBEntryLoReg-D-def* *bind-assoc*
 by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntryLoReg-V*[*no-reg-writes-toI*, *simp*]:
 assumes *name arg0* \notin *Rs*
 shows *no-reg-writes-to* *Rs* (*set-TLBEntryLoReg-V* *arg0* *arg1*)
 using *assms*
 unfolding *set-TLBEntryLoReg-V-def* *bind-assoc*
 by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntryLoReg-G*[*no-reg-writes-toI*, *simp*]:
 assumes *name arg0* \notin *Rs*
 shows *no-reg-writes-to* *Rs* (*set-TLBEntryLoReg-G* *arg0* *arg1*)
 using *assms*
 unfolding *set-TLBEntryLoReg-G-def* *bind-assoc*
 by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-TLBEntryHiReg*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*undefined-TLBEntryHiReg* *arg0*)
 unfolding *undefined-TLBEntryHiReg-def* *bind-assoc*
 by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntryHiReg-bits*[*no-reg-writes-toI*, *simp*]:

assumes $\text{name } arg0 \notin Rs$
shows $\text{no-reg-writes-to } Rs \text{ (set-TLBEntryHiReg-bits } arg0 \text{ } arg1)$
using assms
unfolding $\text{set-TLBEntryHiReg-bits-def bind-assoc}$
by $(\text{no-reg-writes-toI})$

lemma $\text{no-reg-writes-to-set-TLBEntryHiReg-R[no-reg-writes-toI, simp]}:$
assumes $\text{name } arg0 \notin Rs$
shows $\text{no-reg-writes-to } Rs \text{ (set-TLBEntryHiReg-R } arg0 \text{ } arg1)$
using assms
unfolding $\text{set-TLBEntryHiReg-R-def bind-assoc}$
by $(\text{no-reg-writes-toI})$

lemma $\text{no-reg-writes-to-set-TLBEntryHiReg-VPN2[no-reg-writes-toI, simp]}:$
assumes $\text{name } arg0 \notin Rs$
shows $\text{no-reg-writes-to } Rs \text{ (set-TLBEntryHiReg-VPN2 } arg0 \text{ } arg1)$
using assms
unfolding $\text{set-TLBEntryHiReg-VPN2-def bind-assoc}$
by $(\text{no-reg-writes-toI})$

lemma $\text{no-reg-writes-to-set-TLBEntryHiReg-ASID[no-reg-writes-toI, simp]}:$
assumes $\text{name } arg0 \notin Rs$
shows $\text{no-reg-writes-to } Rs \text{ (set-TLBEntryHiReg-ASID } arg0 \text{ } arg1)$
using assms
unfolding $\text{set-TLBEntryHiReg-ASID-def bind-assoc}$
by $(\text{no-reg-writes-toI})$

lemma $\text{no-reg-writes-to-undefined-ContextReg[no-reg-writes-toI, simp]}:$
 $\text{no-reg-writes-to } Rs \text{ (undefined-ContextReg } arg0)$
unfolding $\text{undefined-ContextReg-def bind-assoc}$
by $(\text{no-reg-writes-toI})$

lemma $\text{no-reg-writes-to-set-ContextReg-bits[no-reg-writes-toI, simp]}:$
assumes $\text{name } arg0 \notin Rs$
shows $\text{no-reg-writes-to } Rs \text{ (set-ContextReg-bits } arg0 \text{ } arg1)$
using assms
unfolding $\text{set-ContextReg-bits-def bind-assoc}$
by $(\text{no-reg-writes-toI})$

lemma $\text{no-reg-writes-to-set-ContextReg-PTEBase[no-reg-writes-toI, simp]}:$
assumes $\text{name } arg0 \notin Rs$
shows $\text{no-reg-writes-to } Rs \text{ (set-ContextReg-PTEBase } arg0 \text{ } arg1)$
using assms
unfolding $\text{set-ContextReg-PTEBase-def bind-assoc}$
by $(\text{no-reg-writes-toI})$

lemma $\text{no-reg-writes-to-set-ContextReg-BadVPN2[no-reg-writes-toI, simp]}:$
assumes $\text{name } arg0 \notin Rs$
shows $\text{no-reg-writes-to } Rs \text{ (set-ContextReg-BadVPN2 } arg0 \text{ } arg1)$


```

using assms
unfolding set-ContextReg-BadVPN2-def bind-assoc
by (no-reg-writes-toI)

lemma no-reg-writes-to-undefined-XContextReg[no-reg-writes-toI, simp]:
  no-reg-writes-to Rs (undefined-XContextReg arg0)
unfolding undefined-XContextReg-def bind-assoc
by (no-reg-writes-toI)

lemma no-reg-writes-to-set-XContextReg-bits[no-reg-writes-toI, simp]:
  assumes name arg0 ∉ Rs
  shows no-reg-writes-to Rs (set-XContextReg-bits arg0 arg1)
using assms
unfolding set-XContextReg-bits-def bind-assoc
by (no-reg-writes-toI)

lemma no-reg-writes-to-set-XContextReg-XPTEBase[no-reg-writes-toI, simp]:
  assumes name arg0 ∉ Rs
  shows no-reg-writes-to Rs (set-XContextReg-XPTEBase arg0 arg1)
using assms
unfolding set-XContextReg-XPTEBase-def bind-assoc
by (no-reg-writes-toI)

lemma no-reg-writes-to-set-XContextReg-XR[no-reg-writes-toI, simp]:
  assumes name arg0 ∉ Rs
  shows no-reg-writes-to Rs (set-XContextReg-XR arg0 arg1)
using assms
unfolding set-XContextReg-XR-def bind-assoc
by (no-reg-writes-toI)

lemma no-reg-writes-to-set-XContextReg-XBadVPN2[no-reg-writes-toI, simp]:
  assumes name arg0 ∉ Rs
  shows no-reg-writes-to Rs (set-XContextReg-XBadVPN2 arg0 arg1)
using assms
unfolding set-XContextReg-XBadVPN2-def bind-assoc
by (no-reg-writes-toI)

lemma no-reg-writes-to-undefined-TLBEntry[no-reg-writes-toI, simp]:
  no-reg-writes-to Rs (undefined-TLBEntry arg0)
unfolding undefined-TLBEntry-def bind-assoc
by (no-reg-writes-toI)

lemma no-reg-writes-to-set-TLBEntry-bits[no-reg-writes-toI, simp]:
  assumes name arg0 ∉ Rs
  shows no-reg-writes-to Rs (set-TLBEntry-bits arg0 arg1)
using assms
unfolding set-TLBEntry-bits-def bind-assoc
by (no-reg-writes-toI)

```

lemma *no-reg-writes-to-set-TLBEntry-pagemask*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-pagemask arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-pagemask-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-r*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-r arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-r-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-vpn2*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-vpn2 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-vpn2-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-asid*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-asid arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-asid-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-g*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-g arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-g-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-valid*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-valid arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-valid-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-caps1*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-caps1 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-caps1-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-capl1*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-capl1 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-capl1-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-pfn1*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-pfn1 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-pfn1-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-c1*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-c1 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-c1-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-d1*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-d1 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-d1-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-v1*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-v1 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-v1-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-caps0*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-caps0 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-caps0-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-capl0*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-capl0 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-capl0-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-pfn0*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-pfn0 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-pfn0-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-c0*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-c0 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-c0-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-d0*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-d0 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-d0-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-TLBEntry-v0*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-TLBEntry-v0 arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-v0-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-StatusReg*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (*undefined-StatusReg arg0*)
unfolding *undefined-StatusReg-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-bits*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-StatusReg-bits arg0 arg1*)
using *assms*
unfolding *set-StatusReg-bits-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-CU*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*
shows *no-reg-writes-to Rs* (*set-StatusReg-CU arg0 arg1*)
using *assms*
unfolding *set-StatusReg-CU-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-BEV*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0* \notin *Rs*

shows *no-reg-writes-to* Rs (*set-StatusReg-BEV* $arg0$ $arg1$)
using *assms*
unfolding *set-StatusReg-BEV-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-IM*[*no-reg-writes-toI*, *simp*]:
assumes $name\ arg0 \notin Rs$
shows *no-reg-writes-to* Rs (*set-StatusReg-IM* $arg0$ $arg1$)
using *assms*
unfolding *set-StatusReg-IM-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-KX*[*no-reg-writes-toI*, *simp*]:
assumes $name\ arg0 \notin Rs$
shows *no-reg-writes-to* Rs (*set-StatusReg-KX* $arg0$ $arg1$)
using *assms*
unfolding *set-StatusReg-KX-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-SX*[*no-reg-writes-toI*, *simp*]:
assumes $name\ arg0 \notin Rs$
shows *no-reg-writes-to* Rs (*set-StatusReg-SX* $arg0$ $arg1$)
using *assms*
unfolding *set-StatusReg-SX-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-UX*[*no-reg-writes-toI*, *simp*]:
assumes $name\ arg0 \notin Rs$
shows *no-reg-writes-to* Rs (*set-StatusReg-UX* $arg0$ $arg1$)
using *assms*
unfolding *set-StatusReg-UX-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-KSU*[*no-reg-writes-toI*, *simp*]:
assumes $name\ arg0 \notin Rs$
shows *no-reg-writes-to* Rs (*set-StatusReg-KSU* $arg0$ $arg1$)
using *assms*
unfolding *set-StatusReg-KSU-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-ERL*[*no-reg-writes-toI*, *simp*]:
assumes $name\ arg0 \notin Rs$
shows *no-reg-writes-to* Rs (*set-StatusReg-ERL* $arg0$ $arg1$)
using *assms*
unfolding *set-StatusReg-ERL-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-EXL*[*no-reg-writes-toI*, *simp*]:
assumes $name\ arg0 \notin Rs$

shows *no-reg-writes-to* *Rs* (*set-StatusReg-EXL* *arg0* *arg1*)
using *assms*
unfolding *set-StatusReg-EXL-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-StatusReg-IE*[*no-reg-writes-toI*, *simp*]:
assumes *name* *arg0* \notin *Rs*
shows *no-reg-writes-to* *Rs* (*set-StatusReg-IE* *arg0* *arg1*)
using *assms*
unfolding *set-StatusReg-IE-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-execute-branch-mips*[*no-reg-writes-toI*, *simp*]:
assumes {"*BranchPending*", "*DelayedPC*", "*NextInBranchDelay*'"} \cap *Rs* = {}
shows *no-reg-writes-to* *Rs* (*execute-branch-mips* *arg0*)
using *assms*
unfolding *execute-branch-mips-def* *bind-assoc*
by (*no-reg-writes-toI* *simp*: *BranchPending-ref-def* *DelayedPC-ref-def* *NextInBranchDelay-ref-def*)

lemma *no-reg-writes-to-rGPR*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*rGPR* *arg0*)
unfolding *rGPR-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-wGPR*[*no-reg-writes-toI*, *simp*]:
assumes {"*GPR*'"} \cap *Rs* = {}
shows *no-reg-writes-to* *Rs* (*wGPR* *arg0* *arg1*)
using *assms*
unfolding *wGPR-def* *bind-assoc*
by (*no-reg-writes-toI* *simp*: *GPR-ref-def*)

lemma *no-reg-writes-to-MEMr*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*MEMr* *arg0* *arg1*)
unfolding *MEMr-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMr-reserve*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*MEMr-reserve* *arg0* *arg1*)
unfolding *MEMr-reserve-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEM-sync*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*MEM-sync* *arg0*)
unfolding *MEM-sync-def* *bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMea*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to *Rs* (*MEMea* *arg0* *arg1*)
unfolding *MEMea-def* *bind-assoc*

by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMea-conditional*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (MEMea-conditional arg0 arg1)
unfolding *MEMea-conditional-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMval*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (MEMval arg0 arg1 arg2)
unfolding *MEMval-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMval-conditional*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (MEMval-conditional arg0 arg1 arg2)
unfolding *MEMval-conditional-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-Exception*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (undefined-Exception arg0)
unfolding *undefined-Exception-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-exceptionVectorOffset*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (exceptionVectorOffset arg0)
unfolding *exceptionVectorOffset-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-exceptionVectorBase*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (exceptionVectorBase arg0)
unfolding *exceptionVectorBase-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-updateBadInstr*[*no-reg-writes-toI*, *simp*]:
assumes {"CP0BadInstr", "CP0BadInstrP"} \cap *Rs* = {}
shows *no-reg-writes-to Rs (updateBadInstr arg0)*
using *assms*
unfolding *updateBadInstr-def bind-assoc*
by (*no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def*)

lemma *no-reg-writes-to-undefined-Capability*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (undefined-Capability arg0)
unfolding *undefined-Capability-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-next-pcc*[*no-reg-writes-toI*, *simp*]:
assumes {"DelayedPCC", "NextPCC"} \cap *Rs* = {}
shows *no-reg-writes-to Rs (set-next-pcc arg0)*
using *assms*
unfolding *set-next-pcc-def bind-assoc*

by (no-reg-writes-toI simp: DelayedPCC-ref-def NextPCC-ref-def)

lemma no-reg-writes-to-SignalException[no-reg-writes-toI, simp]:
 assumes {"CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "DelayedPCC",
 "EPCC", "NextPC", "NextPCC"} \cap Rs = {}
 shows no-reg-writes-to Rs (SignalException arg0)
 using assms
 unfolding SignalException-def bind-assoc
 by (no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def
 CP0Status-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def NextPCC-ref-def)

lemma no-reg-writes-to-SignalExceptionBadAddr[no-reg-writes-toI, simp]:
 assumes {"CP0BadInstr", "CP0BadInstrP", "CP0BadVAddr", "CP0Cause",
 "CP0Status", "DelayedPCC", "EPCC", "NextPC", "NextPCC"} \cap Rs = {}
 shows no-reg-writes-to Rs (SignalExceptionBadAddr arg0 arg1)
 using assms
 unfolding SignalExceptionBadAddr-def bind-assoc
 by (no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0BadVAddr-ref-def
 CP0Cause-ref-def CP0Status-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def
 NextPCC-ref-def)

lemma no-reg-writes-to-SignalExceptionTLB[no-reg-writes-toI, simp]:
 assumes {"CP0BadInstr", "CP0BadInstrP", "CP0BadVAddr", "CP0Cause",
 "CP0Status", "DelayedPCC", "EPCC", "NextPC", "NextPCC", "TLBContext",
 "TLBEntryHi", "TLBXContext"} \cap Rs = {}
 shows no-reg-writes-to Rs (SignalExceptionTLB arg0 arg1)
 using assms
 unfolding SignalExceptionTLB-def bind-assoc
 by (no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0BadVAddr-ref-def
 CP0Cause-ref-def CP0Status-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def
 NextPCC-ref-def TLBContext-ref-def TLBEntryHi-ref-def TLBXContext-ref-def)

lemma no-reg-writes-to-undefined-MemAccessType[no-reg-writes-toI, simp]:
 no-reg-writes-to Rs (undefined-MemAccessType arg0)
 unfolding undefined-MemAccessType-def bind-assoc
 by (no-reg-writes-toI)

lemma no-reg-writes-to-undefined-AccessLevel[no-reg-writes-toI, simp]:
 no-reg-writes-to Rs (undefined-AccessLevel arg0)
 unfolding undefined-AccessLevel-def bind-assoc
 by (no-reg-writes-toI)

lemma no-reg-writes-to-getAccessLevel[no-reg-writes-toI, simp]:
 no-reg-writes-to Rs (getAccessLevel arg0)
 unfolding getAccessLevel-def bind-assoc
 by (no-reg-writes-toI)

lemma no-reg-writes-to-pcc-access-system-regs[no-reg-writes-toI, simp]:
 no-reg-writes-to Rs (pcc-access-system-regs arg0)

unfolding *pcc-access-system-regs-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-CapCauseReg[no-reg-writes-toI, simp]:*
no-reg-writes-to Rs (undefined-CapCauseReg arg0)
unfolding *undefined-CapCauseReg-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CapCauseReg-ExcCode[no-reg-writes-toI, simp]:*
assumes *name arg0 \notin Rs*
shows *no-reg-writes-to Rs (set-CapCauseReg-ExcCode arg0 arg1)*
using *assms*
unfolding *set-CapCauseReg-ExcCode-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CapCauseReg-RegNum[no-reg-writes-toI, simp]:*
assumes *name arg0 \notin Rs*
shows *no-reg-writes-to Rs (set-CapCauseReg-RegNum arg0 arg1)*
using *assms*
unfolding *set-CapCauseReg-RegNum-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-raise-c2-exception8[no-reg-writes-toI, simp]:*
assumes *{ "CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "CapCause",
 "DelayedPCC", "EPCC", "NextPC", "NextPCC" } \cap Rs = {}*
shows *no-reg-writes-to Rs (raise-c2-exception8 arg0 arg1)*
using *assms*
unfolding *raise-c2-exception8-def bind-assoc*
by (*no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def
 CP0Status-ref-def CapCause-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def
 NextPCC-ref-def*)

lemma *no-reg-writes-to-raise-c2-exception-noreg[no-reg-writes-toI, simp]:*
assumes *{ "CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "CapCause",
 "DelayedPCC", "EPCC", "NextPC", "NextPCC" } \cap Rs = {}*
shows *no-reg-writes-to Rs (raise-c2-exception-noreg arg0)*
using *assms*
unfolding *raise-c2-exception-noreg-def bind-assoc*
by (*no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def
 CP0Status-ref-def CapCause-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def
 NextPCC-ref-def*)

lemma *no-reg-writes-to-checkCP0AccessHook[no-reg-writes-toI, simp]:*
assumes *{ "CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "CapCause",
 "DelayedPCC", "EPCC", "NextPC", "NextPCC" } \cap Rs = {}*
shows *no-reg-writes-to Rs (checkCP0AccessHook arg0)*
using *assms*
unfolding *checkCP0AccessHook-def bind-assoc*
by (*no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def*)

*CP0Status-ref-def CapCause-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def
NextPCC-ref-def*)

lemma *no-reg-writes-to-checkCP0Access*[*no-reg-writes-toI*, *simp*]:
assumes {"CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "CapCause",
 "DelayedPCC", "EPCC", "NextPC", "NextPCC"} \cap *Rs* = {}
shows *no-reg-writes-to Rs* (*checkCP0Access arg0*)
using *assms*
unfolding *checkCP0Access-def bind-assoc*
by (*no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def
 CP0Status-ref-def CapCause-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def
 NextPCC-ref-def*)

lemma *no-reg-writes-to-incrementCP0Count*[*no-reg-writes-toI*, *simp*]:
assumes {"CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Count", "CP0Status",
 "DelayedPCC", "EPCC", "NextPC", "NextPCC", "TLBRandom"} \cap *Rs* = {}
shows *no-reg-writes-to Rs* (*incrementCP0Count arg0*)
using *assms*
unfolding *incrementCP0Count-def bind-assoc*
by (*no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def
 CP0Count-ref-def CP0Status-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def
 NextPCC-ref-def TLBRandom-ref-def*)

lemma *no-reg-writes-to-undefined-decode-failure*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (*undefined-decode-failure arg0*)
unfolding *undefined-decode-failure-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-Comparison*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (*undefined-Comparison arg0*)
unfolding *undefined-Comparison-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-WordType*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (*undefined-WordType arg0*)
unfolding *undefined-WordType-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-WordTypeUnaligned*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (*undefined-WordTypeUnaligned arg0*)
unfolding *undefined-WordTypeUnaligned-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMr-wrapper*[*no-reg-writes-toI*, *simp*]:
assumes {"UART-RVALID"} \cap *Rs* = {}
shows *no-reg-writes-to Rs* (*MEMr-wrapper arg0 arg1*)
using *assms*
unfolding *MEMr-wrapper-def bind-assoc*
by (*no-reg-writes-toI simp: UART-RVALID-ref-def*)

lemma *no-reg-writes-to-MEMr-reserve-wrapper*[*no-reg-writes-toI*, *simp*]:

no-reg-writes-to *Rs* (*MEMr-reserve-wrapper* *arg0* *arg1*)

unfolding *MEMr-reserve-wrapper-def* *bind-assoc*

by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-init-cp0-state*[*no-reg-writes-toI*, *simp*]:

assumes {"*CP0Status*"} \cap *Rs* = {}

shows *no-reg-writes-to* *Rs* (*init-cp0-state* *arg0*)

using *assms*

unfolding *init-cp0-state-def* *bind-assoc*

by (*no-reg-writes-toI* *simp*: *CP0Status-ref-def*)

lemma *no-reg-writes-to-tlbSearch*[*no-reg-writes-toI*, *simp*]:

no-reg-writes-to *Rs* (*tlbSearch* *arg0*)

unfolding *tlbSearch-def* *bind-assoc*

by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-TLBTranslate2*[*no-reg-writes-toI*, *simp*]:

assumes {"*CP0BadInstr*", "*CP0BadInstrP*", "*CP0BadVAddr*", "*CP0Cause*", "*CP0Status*", "*DelayedPCC*", "*EPCC*", "*NextPC*", "*NextPCC*", "*TLBContext*", "*TLBEntryHi*", "*TLBXContext*"} \cap *Rs* = {}

shows *no-reg-writes-to* *Rs* (*TLBTranslate2* *arg0* *arg1*)

using *assms*

unfolding *TLBTranslate2-def* *bind-assoc*

by (*no-reg-writes-toI* *simp*: *CP0BadInstr-ref-def* *CP0BadInstrP-ref-def* *CP0BadVAddr-ref-def* *CP0Cause-ref-def* *CP0Status-ref-def* *DelayedPCC-ref-def* *EPCC-ref-def* *NextPC-ref-def* *NextPCC-ref-def* *TLBContext-ref-def* *TLBEntryHi-ref-def* *TLBXContext-ref-def*)

lemma *no-reg-writes-to-TLBTranslateC*[*no-reg-writes-toI*, *simp*]:

assumes {"*CP0BadInstr*", "*CP0BadInstrP*", "*CP0BadVAddr*", "*CP0Cause*", "*CP0Status*", "*DelayedPCC*", "*EPCC*", "*NextPC*", "*NextPCC*", "*TLBContext*", "*TLBEntryHi*", "*TLBXContext*"} \cap *Rs* = {}

shows *no-reg-writes-to* *Rs* (*TLBTranslateC* *arg0* *arg1*)

using *assms*

unfolding *TLBTranslateC-def* *bind-assoc*

by (*no-reg-writes-toI* *simp*: *CP0BadInstr-ref-def* *CP0BadInstrP-ref-def* *CP0BadVAddr-ref-def* *CP0Cause-ref-def* *CP0Status-ref-def* *DelayedPCC-ref-def* *EPCC-ref-def* *NextPC-ref-def* *NextPCC-ref-def* *TLBContext-ref-def* *TLBEntryHi-ref-def* *TLBXContext-ref-def*)

lemma *no-reg-writes-to-TLBTranslate*[*no-reg-writes-toI*, *simp*]:

assumes {"*CP0BadInstr*", "*CP0BadInstrP*", "*CP0BadVAddr*", "*CP0Cause*", "*CP0Status*", "*DelayedPCC*", "*EPCC*", "*NextPC*", "*NextPCC*", "*TLBContext*", "*TLBEntryHi*", "*TLBXContext*"} \cap *Rs* = {}

shows *no-reg-writes-to* *Rs* (*TLBTranslate* *arg0* *arg1*)

using *assms*

unfolding *TLBTranslate-def* *bind-assoc*

by (*no-reg-writes-toI* *simp*: *CP0BadInstr-ref-def* *CP0BadInstrP-ref-def* *CP0BadVAddr-ref-def* *CP0Cause-ref-def* *CP0Status-ref-def* *DelayedPCC-ref-def* *EPCC-ref-def* *NextPC-ref-def*)

NextPCC-ref-def TLBContext-ref-def TLBEntryHi-ref-def TLBXContext-ref-def)

lemma *no-reg-writes-to-undefined-CPtrCmpOp*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (undefined-CPtrCmpOp arg0)
unfolding *undefined-CPtrCmpOp-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-ClearRegSet*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (undefined-ClearRegSet arg0)
unfolding *undefined-ClearRegSet-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-capToString*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (capToString arg0 arg1)
unfolding *capToString-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-undefined-CapEx*[*no-reg-writes-toI*, *simp*]:
no-reg-writes-to Rs (undefined-CapEx arg0)
unfolding *undefined-CapEx-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CapCauseReg-bits*[*no-reg-writes-toI*, *simp*]:
assumes *name arg0 ∉ Rs*
shows *no-reg-writes-to Rs (set-CapCauseReg-bits arg0 arg1)*
using *assms*
unfolding *set-CapCauseReg-bits-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-execute-branch-pcc*[*no-reg-writes-toI*, *simp*]:
assumes {"BranchPending", "DelayedPC", "DelayedPCC", "NextInBranchDelay"}
 $\cap Rs = \{\}$
shows *no-reg-writes-to Rs (execute-branch-pcc arg0)*
using *assms*
unfolding *execute-branch-pcc-def bind-assoc*
by (*no-reg-writes-toI simp: BranchPending-ref-def DelayedPC-ref-def DelayedPCC-ref-def NextInBranchDelay-ref-def*)

lemma *no-reg-writes-to-ERETHook*[*no-reg-writes-toI*, *simp*]:
assumes {"DelayedPCC", "NextPCC"} $\cap Rs = \{\}$
shows *no-reg-writes-to Rs (ERETHook arg0)*
using *assms*
unfolding *ERETHook-def bind-assoc*
by (*no-reg-writes-toI simp: DelayedPCC-ref-def NextPCC-ref-def*)

lemma *no-reg-writes-to-raise-c2-exception*[*no-reg-writes-toI*, *simp*]:
assumes {"CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "CapCause",
"DelayedPCC", "EPCC", "NextPC", "NextPCC"} $\cap Rs = \{\}$
shows *no-reg-writes-to Rs (raise-c2-exception arg0 arg1)*

using *assms*
unfolding *raise-c2-exception-def bind-assoc*
by (*no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def CP0Status-ref-def CapCause-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def NextPCC-ref-def*)

lemma *no-reg-writes-to-MEMr-tagged[no-reg-writes-toI, simp]:*
no-reg-writes-to Rs (MEMr-tagged arg0 arg1 arg2)
unfolding *MEMr-tagged-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMr-tagged-reserve[no-reg-writes-toI, simp]:*
no-reg-writes-to Rs (MEMr-tagged-reserve arg0 arg1 arg2)
unfolding *MEMr-tagged-reserve-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMw-tagged[no-reg-writes-toI, simp]:*
no-reg-writes-to Rs (MEMw-tagged arg0 arg1 arg2 arg3)
unfolding *MEMw-tagged-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMw-tagged-conditional[no-reg-writes-toI, simp]:*
no-reg-writes-to Rs (MEMw-tagged-conditional arg0 arg1 arg2 arg3)
unfolding *MEMw-tagged-conditional-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-MEMw-wrapper[no-reg-writes-toI, simp]:*
assumes {"UART-WDATA", "UART-WRITTEN"} \cap *Rs* = {}
shows *no-reg-writes-to Rs (MEMw-wrapper arg0 arg1 arg2)*
using *assms*
unfolding *MEMw-wrapper-def bind-assoc*
by (*no-reg-writes-toI simp: UART-WDATA-ref-def UART-WRITTEN-ref-def*)

lemma *no-reg-writes-to-MEMw-conditional-wrapper[no-reg-writes-toI, simp]:*
no-reg-writes-to Rs (MEMw-conditional-wrapper arg0 arg1 arg2)
unfolding *MEMw-conditional-wrapper-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-checkDDCPerms[no-reg-writes-toI, simp]:*
assumes {"CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "CapCause", "DelayedPCC", "EPCC", "NextPC", "NextPCC"} \cap *Rs* = {}
shows *no-reg-writes-to Rs (checkDDCPerms arg0 arg1)*
using *assms*
unfolding *checkDDCPerms-def bind-assoc*
by (*no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def CP0Status-ref-def CapCause-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def NextPCC-ref-def*)

lemma *no-reg-writes-to-addrWrapper[no-reg-writes-toI, simp]:*

assumes {"CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "CapCause",
 "DelayedPCC", "EPCC", "NextPC", "NextPCC"} $\cap Rs = \{\}$
shows no-reg-writes-to Rs (addrWrapper arg0 arg1 arg2)
using *assms*
unfolding addrWrapper-def bind-assoc
by (no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def
 CP0Status-ref-def CapCause-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def
 NextPCC-ref-def)

lemma no-reg-writes-to-addrWrapperUnaligned[no-reg-writes-toI, simp]:
assumes {"CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "CapCause",
 "DelayedPCC", "EPCC", "NextPC", "NextPCC"} $\cap Rs = \{\}$
shows no-reg-writes-to Rs (addrWrapperUnaligned arg0 arg1 arg2)
using *assms*
unfolding addrWrapperUnaligned-def bind-assoc
by (no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def
 CP0Status-ref-def CapCause-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def
 NextPCC-ref-def)

lemma no-reg-writes-to-execute-branch[no-reg-writes-toI, simp]:
assumes {"BranchPending", "CP0BadInstr", "CP0BadInstrP", "CP0Cause",
 "CP0Status", "CapCause", "DelayedPC", "DelayedPCC", "EPCC", "NextInBranchDelay",
 "NextPC", "NextPCC"} $\cap Rs = \{\}$
shows no-reg-writes-to Rs (execute-branch arg0)
using *assms*
unfolding execute-branch-def bind-assoc
by (no-reg-writes-toI simp: BranchPending-ref-def CP0BadInstr-ref-def CP0BadInstrP-ref-def
 CP0Cause-ref-def CP0Status-ref-def CapCause-ref-def DelayedPC-ref-def DelayedPCC-ref-def
 EPCC-ref-def NextInBranchDelay-ref-def NextPC-ref-def NextPCC-ref-def)

lemma no-reg-writes-to-TranslatePC[no-reg-writes-toI, simp]:
assumes {"CP0BadInstr", "CP0BadInstrP", "CP0BadVAddr", "CP0Cause",
 "CP0Count", "CP0Status", "CapCause", "DelayedPCC", "EPCC", "NextPC",
 "NextPCC", "TLBContext", "TLBEntryHi", "TLBRandom", "TLBXContext"}
 $\cap Rs = \{\}$
shows no-reg-writes-to Rs (TranslatePC arg0)
using *assms*
unfolding TranslatePC-def bind-assoc
by (no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0BadVAddr-ref-def
 CP0Cause-ref-def CP0Count-ref-def CP0Status-ref-def CapCause-ref-def DelayedPCC-ref-def
 EPCC-ref-def NextPC-ref-def NextPCC-ref-def TLBContext-ref-def TLBEntryHi-ref-def
 TLBRandom-ref-def TLBXContext-ref-def)

lemma no-reg-writes-to-checkCP2usable[no-reg-writes-toI, simp]:
assumes {"CP0BadInstr", "CP0BadInstrP", "CP0Cause", "CP0Status", "DelayedPCC",
 "EPCC", "NextPC", "NextPCC"} $\cap Rs = \{\}$
shows no-reg-writes-to Rs (checkCP2usable arg0)
using *assms*
unfolding checkCP2usable-def bind-assoc

by (*no-reg-writes-toI simp: CP0BadInstr-ref-def CP0BadInstrP-ref-def CP0Cause-ref-def CP0Status-ref-def DelayedPCC-ref-def EPCC-ref-def NextPC-ref-def NextPCC-ref-def*)

lemma *no-reg-writes-to-get-CP0EPC*[*no-reg-writes-toI, simp*]:
no-reg-writes-to Rs (get-CP0EPC arg0)
unfolding *get-CP0EPC-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CP0EPC*[*no-reg-writes-toI, simp*]:
assumes {"EPCC"} $\cap Rs = \{\}$
shows *no-reg-writes-to Rs (set-CP0EPC arg0)*
using *assms*
unfolding *set-CP0EPC-def bind-assoc*
by (*no-reg-writes-toI simp: EPCC-ref-def*)

lemma *no-reg-writes-to-get-CP0ErrorEPC*[*no-reg-writes-toI, simp*]:
no-reg-writes-to Rs (get-CP0ErrorEPC arg0)
unfolding *get-CP0ErrorEPC-def bind-assoc*
by (*no-reg-writes-toI*)

lemma *no-reg-writes-to-set-CP0ErrorEPC*[*no-reg-writes-toI, simp*]:
assumes {"ErrorEPCC"} $\cap Rs = \{\}$
shows *no-reg-writes-to Rs (set-CP0ErrorEPC arg0)*
using *assms*
unfolding *set-CP0ErrorEPC-def bind-assoc*
by (*no-reg-writes-toI simp: ErrorEPCC-ref-def*)

lemma *no-reg-writes-to-dump-cp2-state*[*no-reg-writes-toI, simp*]:
no-reg-writes-to Rs (dump-cp2-state arg0)
unfolding *dump-cp2-state-def bind-assoc*
by (*no-reg-writes-toI*)

end

end

theory *CHERI-MIPS-Reg-Axioms*
imports *CHERI-MIPS-Gen-Lemmas*
begin

3.3 Register and capability derivability properties of instructions

context *CHERI-MIPS-Reg-Automaton*
begin

lemma *preserves-invariant-write-non-inv-regs*[*preserves-invariantI*]:
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg BranchPending-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg C01-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg C02-ref } v)$

$\wedge v.$ *traces-preserve-invariant* (write-reg C03-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C04-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C05-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C06-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C07-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C08-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C09-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C10-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C11-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C12-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C13-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C14-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C15-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C16-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C17-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C18-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C19-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C20-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C21-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C22-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C23-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C24-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C25-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C26-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C27-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C28-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C29-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C30-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg C31-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CID-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0BadInstr-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0BadInstrP-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0BadVAddr-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0Cause-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0Compare-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0ConfigK0-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0Count-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0HWRena-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0LLAddr-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0LLBit-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0Status-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CP0UserLocal-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CPLR-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CULR-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CapCause-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg CurrentInstrBits-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg DDC-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg DelayedPC-ref v)
 $\wedge v.$ *traces-preserve-invariant* (write-reg DelayedPCC-ref v)

[illegible]

C20-ref-def C21-ref-def C22-ref-def C23-ref-def C24-ref-def
C25-ref-def C26-ref-def C27-ref-def C28-ref-def C29-ref-def
C30-ref-def C31-ref-def CID-ref-def CP0BadInstr-ref-def CP0BadInstrP-ref-def
CP0BadVAddr-ref-def CP0Cause-ref-def CP0Compare-ref-def CP0ConfigK0-ref-def
CP0Count-ref-def
CP0HWREna-ref-def CP0LLAddr-ref-def CP0LLBit-ref-def CP0Status-ref-def
CP0UserLocal-ref-def
CPLR-ref-def CULR-ref-def CapCause-ref-def CurrentInstrBits-ref-def DDC-ref-def
DelayedPC-ref-def DelayedPCC-ref-def EPCC-ref-def ErrorEPCC-ref-def GPR-ref-def
HI-ref-def InBranchDelay-ref-def KCC-ref-def KDC-ref-def KR1C-ref-def
KR2C-ref-def LO-ref-def LastInstrBits-ref-def NextInBranchDelay-ref-def NextPC-ref-def
NextPCC-ref-def PC-ref-def TLBContext-ref-def TLBEntry00-ref-def TLBEntry01-ref-def
TLBEntry02-ref-def TLBEntry03-ref-def TLBEntry04-ref-def TLBEntry05-ref-def
TLBEntry06-ref-def
TLBEntry07-ref-def TLBEntry08-ref-def TLBEntry09-ref-def TLBEntry10-ref-def
TLBEntry11-ref-def
TLBEntry12-ref-def TLBEntry13-ref-def TLBEntry14-ref-def TLBEntry15-ref-def
TLBEntry16-ref-def
TLBEntry17-ref-def TLBEntry18-ref-def TLBEntry19-ref-def TLBEntry20-ref-def
TLBEntry21-ref-def
TLBEntry22-ref-def TLBEntry23-ref-def TLBEntry24-ref-def TLBEntry25-ref-def
TLBEntry26-ref-def
TLBEntry27-ref-def TLBEntry28-ref-def TLBEntry29-ref-def TLBEntry30-ref-def
TLBEntry31-ref-def
TLBEntry32-ref-def TLBEntry33-ref-def TLBEntry34-ref-def TLBEntry35-ref-def
TLBEntry36-ref-def
TLBEntry37-ref-def TLBEntry38-ref-def TLBEntry39-ref-def TLBEntry40-ref-def
TLBEntry41-ref-def
TLBEntry42-ref-def TLBEntry43-ref-def TLBEntry44-ref-def TLBEntry45-ref-def
TLBEntry46-ref-def
TLBEntry47-ref-def TLBEntry48-ref-def TLBEntry49-ref-def TLBEntry50-ref-def
TLBEntry51-ref-def
TLBEntry52-ref-def TLBEntry53-ref-def TLBEntry54-ref-def TLBEntry55-ref-def
TLBEntry56-ref-def
TLBEntry57-ref-def TLBEntry58-ref-def TLBEntry59-ref-def TLBEntry60-ref-def
TLBEntry61-ref-def
TLBEntry62-ref-def TLBEntry63-ref-def TLBEntryHi-ref-def TLBEntryLo0-ref-def
TLBEntryLo1-ref-def
TLBIndex-ref-def TLBPageMask-ref-def TLBProbe-ref-def TLBRandom-ref-def
TLBWired-ref-def
TLBXContext-ref-def UART-RDATA-ref-def UART-RVALID-ref-def UART-WDATA-ref-def
UART-WRITTEN-ref-def
InstCount-ref-def
by (*intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp*)**+**

lemma *preserves-invariant-no-writes-to-inv-regs*[*preserves-invariantI*]:
 $\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant (MIPS-write arg0 arg1 arg2)}$

$\bigwedge \text{arg0 arg1. traces-preserve-invariant (MIPS-read arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-CauseReg-BD arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-CauseReg-CE arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-CauseReg-IV arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-CauseReg-IP arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-CauseReg-ExcCode arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryLoReg-bits arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryLoReg-CapS arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryLoReg-CapL arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryLoReg-PFN arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryLoReg-C arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryLoReg-D arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryLoReg-V arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryLoReg-G arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryHiReg-R arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryHiReg-VPN2 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntryHiReg-ASID arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-ContextReg-PTEBase arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-ContextReg-BadVPN2 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-XContextReg-XPTEBase arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-XContextReg-XR arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-XContextReg-XBadVPN2 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-pagemask arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-r arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-vpn2 arg0 arg1)}$

[illegible]

arg0 arg1)
 $\bigwedge \text{arg0. traces-preserve-invariant (execute-branch-mips arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (rGPR arg0)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (wGPR arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (MEMr arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (MEMr-reserve arg0 arg1)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (MEM-sync arg0)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (MEMea arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (MEMea-conditional arg0 arg1)}$
 $\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMval arg0 arg1 arg2)}$
 $\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMval-conditional arg0 arg1 arg2)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (exceptionVectorOffset arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (exceptionVectorBase arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (updateBadInstr arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (set-next-pcc arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (SignalException arg0)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (SignalExceptionBadAddr arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (SignalExceptionTLB arg0 arg1)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (getAccessLevel arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (pcc-access-system-regs arg0)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-CapCauseReg-ExcCode arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{inv-regs} \implies \text{traces-preserve-invariant (set-CapCauseReg-RegNum arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (raise-c2-exception8 arg0 arg1)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (raise-c2-exception-noreg arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (checkCP0AccessHook arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (checkCP0Access arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (incrementCP0Count arg0)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (MEMr-wrapper arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (MEMr-reserve-wrapper arg0 arg1)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (tlbSearch arg0)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (TLBTranslate2 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (TLBTranslateC arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (TLBTranslate arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (capToString arg0 arg1)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (execute-branch-pcc arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (ERETHook arg0)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (raise-c2-exception arg0 arg1)}$
 $\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMr-tagged arg0 arg1 arg2)}$
 $\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMr-tagged-reserve arg0 arg1 arg2)}$
 $\bigwedge \text{arg0 arg1 arg2 arg3. traces-preserve-invariant (MEMw-tagged arg0 arg1 arg2 arg3)}$
 $\bigwedge \text{arg0 arg1 arg2 arg3. traces-preserve-invariant (MEMw-tagged-conditional arg0 arg1 arg2 arg3)}$
 $\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMw-wrapper arg0 arg1 arg2)}$
 $\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMw-conditional-wrapper arg0 arg1 arg2)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (checkDDCPerms arg0 arg1)}$

$\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant (addrWrapper arg0 arg1 arg2)}$
 $\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant (addrWrapperUnaligned arg0 arg1 arg2)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (execute-branch arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (checkCP2usable arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (get-CP0EPC arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (set-CP0EPC arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (get-CP0ErrorEPC arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (set-CP0ErrorEPC arg0)}$
by (intro no-reg-writes-traces-preserve-invariantI no-reg-writes-toI; simp)+

lemma *preserves-invariant-write-reg*[*preserves-invariantI*]:
 assumes *name r* \notin *inv-regs*
 shows *traces-preserve-invariant* (*write-reg r v*)
 using *assms*
by (intro no-reg-writes-traces-preserve-invariantI no-reg-writes-toI)

lemma *preserves-invariant-TLBWriteEntry*[*preserves-invariantI*]:
traces-preserve-invariant (*TLBWriteEntry idx*)
unfolding *TLBWriteEntry-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-option*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-option arg0*)
unfolding *undefined-option-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-exception*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-exception arg0*)
unfolding *undefined-exception-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-CauseReg*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-CauseReg arg0*)
unfolding *undefined-CauseReg-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-set-CauseReg-bits*[*preserves-invariantI*]:
 assumes *name arg0* \notin *inv-regs*
 shows *runs-preserve-invariant* (*set-CauseReg-bits arg0 arg1*)
 using *assms*
unfolding *set-CauseReg-bits-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-set-CauseReg-WP*[*preserves-invariantI*]:
 assumes *name arg0* \notin *inv-regs*
 shows *runs-preserve-invariant* (*set-CauseReg-WP arg0 arg1*)
 using *assms*
unfolding *set-CauseReg-WP-def* *bind-assoc*

by *preserves-invariantI*

lemma *preserves-invariant-undefined-TLBEntryLoReg*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-TLBEntryLoReg* *arg0*)
unfolding *undefined-TLBEntryLoReg-def* *bind-assoc*
 by *preserves-invariantI*

lemma *preserves-invariant-undefined-TLBEntryHiReg*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-TLBEntryHiReg* *arg0*)
unfolding *undefined-TLBEntryHiReg-def* *bind-assoc*
 by *preserves-invariantI*

lemma *preserves-invariant-set-TLBEntryHiReg-bits*[*preserves-invariantI*]:
assumes *name arg0* \notin *inv-regs*
shows *runs-preserve-invariant* (*set-TLBEntryHiReg-bits* *arg0* *arg1*)
using *assms*
unfolding *set-TLBEntryHiReg-bits-def* *bind-assoc*
 by *preserves-invariantI*

lemma *preserves-invariant-undefined-ContextReg*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-ContextReg* *arg0*)
unfolding *undefined-ContextReg-def* *bind-assoc*
 by *preserves-invariantI*

lemma *preserves-invariant-set-ContextReg-bits*[*preserves-invariantI*]:
assumes *name arg0* \notin *inv-regs*
shows *runs-preserve-invariant* (*set-ContextReg-bits* *arg0* *arg1*)
using *assms*
unfolding *set-ContextReg-bits-def* *bind-assoc*
 by *preserves-invariantI*

lemma *preserves-invariant-undefined-XContextReg*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-XContextReg* *arg0*)
unfolding *undefined-XContextReg-def* *bind-assoc*
 by *preserves-invariantI*

lemma *preserves-invariant-set-XContextReg-bits*[*preserves-invariantI*]:
assumes *name arg0* \notin *inv-regs*
shows *runs-preserve-invariant* (*set-XContextReg-bits* *arg0* *arg1*)
using *assms*
unfolding *set-XContextReg-bits-def* *bind-assoc*
 by *preserves-invariantI*

lemma *preserves-invariant-undefined-TLBEntry*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-TLBEntry* *arg0*)
unfolding *undefined-TLBEntry-def* *bind-assoc*
 by *preserves-invariantI*

lemma *preserves-invariant-set-TLBEntry-bits*[*preserves-invariantI*]:

assumes *name arg0* \notin *inv-regs*
shows *runs-preserve-invariant* (*set-TLBEntry-bits arg0 arg1*)
using *assms*
unfolding *set-TLBEntry-bits-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-StatusReg*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-StatusReg arg0*)
unfolding *undefined-StatusReg-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-set-StatusReg-bits*[*preserves-invariantI*]:
assumes *name arg0* \notin *inv-regs*
shows *runs-preserve-invariant* (*set-StatusReg-bits arg0 arg1*)
using *assms*
unfolding *set-StatusReg-bits-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-Exception*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-Exception arg0*)
unfolding *undefined-Exception-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-Capability*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-Capability arg0*)
unfolding *undefined-Capability-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-MemAccessType*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-MemAccessType arg0*)
unfolding *undefined-MemAccessType-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-AccessLevel*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-AccessLevel arg0*)
unfolding *undefined-AccessLevel-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-CapCauseReg*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-CapCauseReg arg0*)
unfolding *undefined-CapCauseReg-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-decode-failure*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-decode-failure arg0*)
unfolding *undefined-decode-failure-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-Comparison*[*preserves-invariantI*]:

runs-preserve-invariant (*undefined-Comparison* *arg0*)
unfolding *undefined-Comparison-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-WordType*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-WordType* *arg0*)
unfolding *undefined-WordType-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-WordTypeUnaligned*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-WordTypeUnaligned* *arg0*)
unfolding *undefined-WordTypeUnaligned-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-init-cp0-state*[*preserves-invariantI*]:
runs-preserve-invariant (*init-cp0-state* *arg0*)
unfolding *init-cp0-state-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-CPtrCmpOp*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-CPtrCmpOp* *arg0*)
unfolding *undefined-CPtrCmpOp-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-ClearRegSet*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-ClearRegSet* *arg0*)
unfolding *undefined-ClearRegSet-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-undefined-CapEx*[*preserves-invariantI*]:
runs-preserve-invariant (*undefined-CapEx* *arg0*)
unfolding *undefined-CapEx-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-set-CapCauseReg-bits*[*preserves-invariantI*]:
assumes *name* *arg0* \notin *inv-regs*
shows *runs-preserve-invariant* (*set-CapCauseReg-bits* *arg0* *arg1*)
using *assms*
unfolding *set-CapCauseReg-bits-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-TranslatePC*[*preserves-invariantI*]:
runs-preserve-invariant (*TranslatePC* *arg0*)
unfolding *TranslatePC-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-dump-cp2-state*[*preserves-invariantI*]:
runs-preserve-invariant (*dump-cp2-state* *arg0*)
unfolding *dump-cp2-state-def* *bind-assoc*

by *preserves-invariantI*

lemma *preserves-invariant-execute-XORI*[*preserves-invariantI*]:
runs-preserve-invariant (execute-XORI arg0 arg1 arg2)
unfolding *execute-XORI-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-XOR*[*preserves-invariantI*]:
runs-preserve-invariant (execute-XOR arg0 arg1 arg2)
unfolding *execute-XOR-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-WAIT*[*preserves-invariantI*]:
runs-preserve-invariant (execute-WAIT arg0)
unfolding *execute-WAIT-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-TRAPREG*[*preserves-invariantI*]:
runs-preserve-invariant (execute-TRAPREG arg0 arg1 arg2)
unfolding *execute-TRAPREG-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-TRAPIMM*[*preserves-invariantI*]:
runs-preserve-invariant (execute-TRAPIMM arg0 arg1 arg2)
unfolding *execute-TRAPIMM-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-TLBWR*[*preserves-invariantI*]:
runs-preserve-invariant (execute-TLBWR arg0)
unfolding *execute-TLBWR-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-TLBWI*[*preserves-invariantI*]:
runs-preserve-invariant (execute-TLBWI arg0)
unfolding *execute-TLBWI-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-TLBR*[*preserves-invariantI*]:
runs-preserve-invariant (execute-TLBR arg0)
unfolding *execute-TLBR-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-TLBP*[*preserves-invariantI*]:
runs-preserve-invariant (execute-TLBP arg0)
unfolding *execute-TLBP-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-Store*[*preserves-invariantI*]:
runs-preserve-invariant (execute-Store arg0 arg1 arg2 arg3 arg4)

unfolding *execute-Store-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SYSCALL*[*preserves-invariantI*]:
runs-preserve-invariant (execute-SYSCALL arg0)
unfolding *execute-SYSCALL-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SYNC*[*preserves-invariantI*]:
runs-preserve-invariant (execute-SYNC arg0)
unfolding *execute-SYNC-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SWR*[*preserves-invariantI*]:
runs-preserve-invariant (execute-SWR arg0 arg1 arg2)
unfolding *execute-SWR-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SWL*[*preserves-invariantI*]:
runs-preserve-invariant (execute-SWL arg0 arg1 arg2)
unfolding *execute-SWL-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SUBU*[*preserves-invariantI*]:
runs-preserve-invariant (execute-SUBU arg0 arg1 arg2)
unfolding *execute-SUBU-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SUB*[*preserves-invariantI*]:
runs-preserve-invariant (execute-SUB arg0 arg1 arg2)
unfolding *execute-SUB-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SRLV*[*preserves-invariantI*]:
runs-preserve-invariant (execute-SRLV arg0 arg1 arg2)
unfolding *execute-SRLV-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SRL*[*preserves-invariantI*]:
runs-preserve-invariant (execute-SRL arg0 arg1 arg2)
unfolding *execute-SRL-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SRAV*[*preserves-invariantI*]:
runs-preserve-invariant (execute-SRAV arg0 arg1 arg2)
unfolding *execute-SRAV-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SRA*[*preserves-invariantI*]:

runs-preserve-invariant (*execute-SRA* *arg0* *arg1* *arg2*)
unfolding *execute-SRA-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SLTU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-SLTU* *arg0* *arg1* *arg2*)
unfolding *execute-SLTU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SLTIU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-SLTIU* *arg0* *arg1* *arg2*)
unfolding *execute-SLTIU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SLTI*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-SLTI* *arg0* *arg1* *arg2*)
unfolding *execute-SLTI-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SLT*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-SLT* *arg0* *arg1* *arg2*)
unfolding *execute-SLT-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SLLV*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-SLLV* *arg0* *arg1* *arg2*)
unfolding *execute-SLLV-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SLL*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-SLL* *arg0* *arg1* *arg2*)
unfolding *execute-SLL-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SDR*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-SDR* *arg0* *arg1* *arg2*)
unfolding *execute-SDR-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-SDL*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-SDL* *arg0* *arg1* *arg2*)
unfolding *execute-SDL-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-RI*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-RI* *arg0*)
unfolding *execute-RI-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-RDHWR*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-RDHWR* *arg0* *arg1*)
unfolding *execute-RDHWR-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-ORI*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-ORI* *arg0* *arg1* *arg2*)
unfolding *execute-ORI-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-OR*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-OR* *arg0* *arg1* *arg2*)
unfolding *execute-OR-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-NOR*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-NOR* *arg0* *arg1* *arg2*)
unfolding *execute-NOR-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MULTU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MULTU* *arg0* *arg1*)
unfolding *execute-MULTU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MULT*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MULT* *arg0* *arg1*)
unfolding *execute-MULT-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MUL*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MUL* *arg0* *arg1* *arg2*)
unfolding *execute-MUL-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MTLO*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MTLO* *arg0*)
unfolding *execute-MTLO-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MTHI*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MTHI* *arg0*)
unfolding *execute-MTHI-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MTC0*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MTC0* *arg0* *arg1* *arg2* *arg3*)
unfolding *execute-MTC0-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MSUBU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MSUBU* *arg0* *arg1*)
unfolding *execute-MSUBU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MSUB*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MSUB* *arg0* *arg1*)
unfolding *execute-MSUB-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MOVZ*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MOVZ* *arg0* *arg1* *arg2*)
unfolding *execute-MOVZ-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MOVN*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MOVN* *arg0* *arg1* *arg2*)
unfolding *execute-MOVN-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MFLO*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MFLO* *arg0*)
unfolding *execute-MFLO-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MFHI*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MFHI* *arg0*)
unfolding *execute-MFHI-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MFC0*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MFC0* *arg0* *arg1* *arg2* *arg3*)
unfolding *execute-MFC0-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MADDU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MADDU* *arg0* *arg1*)
unfolding *execute-MADDU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-MADD*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-MADD* *arg0* *arg1*)
unfolding *execute-MADD-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-Load*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-Load* *arg0* *arg1* *arg2* *arg3* *arg4* *arg5*)
unfolding *execute-Load-def* *bind-assoc*

by *preserves-invariantI*

lemma *preserves-invariant-execute-LWR*[*preserves-invariantI*]:
runs-preserve-invariant (execute-LWR arg0 arg1 arg2)
unfolding *execute-LWR-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-LWL*[*preserves-invariantI*]:
runs-preserve-invariant (execute-LWL arg0 arg1 arg2)
unfolding *execute-LWL-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-LUI*[*preserves-invariantI*]:
runs-preserve-invariant (execute-LUI arg0 arg1)
unfolding *execute-LUI-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-LDR*[*preserves-invariantI*]:
runs-preserve-invariant (execute-LDR arg0 arg1 arg2)
unfolding *execute-LDR-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-LDL*[*preserves-invariantI*]:
runs-preserve-invariant (execute-LDL arg0 arg1 arg2)
unfolding *execute-LDL-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-JR*[*preserves-invariantI*]:
runs-preserve-invariant (execute-JR arg0)
unfolding *execute-JR-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-JALR*[*preserves-invariantI*]:
runs-preserve-invariant (execute-JALR arg0 arg1)
unfolding *execute-JALR-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-JAL*[*preserves-invariantI*]:
runs-preserve-invariant (execute-JAL arg0)
unfolding *execute-JAL-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-J*[*preserves-invariantI*]:
runs-preserve-invariant (execute-J arg0)
unfolding *execute-J-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-ERET*[*preserves-invariantI*]:
runs-preserve-invariant (execute-ERET arg0)

unfolding *execute-ERET-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSUBU*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DSUBU arg0 arg1 arg2)
unfolding *execute-DSUBU-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSUB*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DSUB arg0 arg1 arg2)
unfolding *execute-DSUB-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSRLV*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DSRLV arg0 arg1 arg2)
unfolding *execute-DSRLV-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSRL32*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DSRL32 arg0 arg1 arg2)
unfolding *execute-DSRL32-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSRL*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DSRL arg0 arg1 arg2)
unfolding *execute-DSRL-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSRAV*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DSRAV arg0 arg1 arg2)
unfolding *execute-DSRAV-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSRA32*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DSRA32 arg0 arg1 arg2)
unfolding *execute-DSRA32-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSRA*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DSRA arg0 arg1 arg2)
unfolding *execute-DSRA-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSLLV*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DSLLV arg0 arg1 arg2)
unfolding *execute-DSLLV-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSLL32*[*preserves-invariantI*]:

runs-preserve-invariant (*execute-DSLL32* *arg0 arg1 arg2*)
unfolding *execute-DSLL32-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DSLL*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-DSLL* *arg0 arg1 arg2*)
unfolding *execute-DSLL-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DMULTU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-DMULTU* *arg0 arg1*)
unfolding *execute-DMULTU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DMULT*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-DMULT* *arg0 arg1*)
unfolding *execute-DMULT-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DIVU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-DIVU* *arg0 arg1*)
unfolding *execute-DIVU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DIV*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-DIV* *arg0 arg1*)
unfolding *execute-DIV-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DDIVU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-DDIVU* *arg0 arg1*)
unfolding *execute-DDIVU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DDIV*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-DDIV* *arg0 arg1*)
unfolding *execute-DDIV-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DADDU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-DADDU* *arg0 arg1 arg2*)
unfolding *execute-DADDU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DADDIU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-DADDIU* *arg0 arg1 arg2*)
unfolding *execute-DADDIU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DADDI*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DADDI arg0 arg1 arg2)
unfolding *execute-DADDI-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-DADD*[*preserves-invariantI*]:
runs-preserve-invariant (execute-DADD arg0 arg1 arg2)
unfolding *execute-DADD-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-ClearRegs*[*preserves-invariantI*]:
runs-preserve-invariant (execute-ClearRegs arg0 arg1)
unfolding *execute-ClearRegs-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CWriteHwr*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CWriteHwr arg0 arg1)
unfolding *execute-CWriteHwr-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CUnseal*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CUnseal arg0 arg1 arg2)
unfolding *execute-CUnseal-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CToPtr*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CToPtr arg0 arg1 arg2)
unfolding *execute-CToPtr-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CTestSubset*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CTestSubset arg0 arg1 arg2)
unfolding *execute-CTestSubset-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSub*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CSub arg0 arg1 arg2)
unfolding *execute-CSub-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CStoreConditional*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CStoreConditional arg0 arg1 arg2 arg3)
unfolding *execute-CStoreConditional-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CStore*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CStore arg0 arg1 arg2 arg3 arg4)
unfolding *execute-CStore-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSetOffset*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSetOffset* *arg0* *arg1* *arg2*)
unfolding *execute-CSetOffset-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSetFlags*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSetFlags* *arg0* *arg1* *arg2*)
unfolding *execute-CSetFlags-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSetCause*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSetCause* *arg0*)
unfolding *execute-CSetCause-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSetCID*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSetCID* *arg0*)
unfolding *execute-CSetCID-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSetBoundsImmediate*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSetBoundsImmediate* *arg0* *arg1* *arg2*)
unfolding *execute-CSetBoundsImmediate-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSetBoundsExact*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSetBoundsExact* *arg0* *arg1* *arg2*)
unfolding *execute-CSetBoundsExact-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSetBounds*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSetBounds* *arg0* *arg1* *arg2*)
unfolding *execute-CSetBounds-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSetAddr*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSetAddr* *arg0* *arg1* *arg2*)
unfolding *execute-CSetAddr-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSeal*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSeal* *arg0* *arg1* *arg2*)
unfolding *execute-CSeal-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CSCC*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CSCC* *arg0* *arg1* *arg2*)
unfolding *execute-CSCC-def* *bind-assoc*

by *preserves-invariantI*

lemma *preserves-invariant-execute-CSC*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CSC arg0 arg1 arg2 arg3)
unfolding *execute-CSC-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CReturn*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CReturn arg0)
unfolding *execute-CReturn-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CReadHwr*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CReadHwr arg0 arg1)
unfolding *execute-CReadHwr-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CRAP*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CRAP arg0 arg1)
unfolding *execute-CRAP-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CRAM*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CRAM arg0 arg1)
unfolding *execute-CRAM-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CPtrCmp*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CPtrCmp arg0 arg1 arg2 arg3)
unfolding *execute-CPtrCmp-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CMove*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CMove arg0 arg1)
unfolding *execute-CMove-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CMOVX*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CMOVX arg0 arg1 arg2 arg3)
unfolding *execute-CMOVX-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CLoadTags*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CLoadTags arg0 arg1)
unfolding *execute-CLoadTags-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CLoadLinked*[*preserves-invariantI*]:
runs-preserve-invariant (execute-CLoadLinked arg0 arg1 arg2 arg3)

unfolding *execute-CLoadLinked-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CLoad[preserves-invariantI]:*
runs-preserve-invariant (execute-CLoad arg0 arg1 arg2 arg3 arg4 arg5)
unfolding *execute-CLoad-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CLLC[preserves-invariantI]:*
runs-preserve-invariant (execute-CLLC arg0 arg1)
unfolding *execute-CLLC-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CLCBI[preserves-invariantI]:*
runs-preserve-invariant (execute-CLCBI arg0 arg1 arg2)
unfolding *execute-CLCBI-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CLC[preserves-invariantI]:*
runs-preserve-invariant (execute-CLC arg0 arg1 arg2 arg3)
unfolding *execute-CLC-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CJALR[preserves-invariantI]:*
runs-preserve-invariant (execute-CJALR arg0 arg1 arg2)
unfolding *execute-CJALR-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CIncOffsetImmediate[preserves-invariantI]:*
runs-preserve-invariant (execute-CIncOffsetImmediate arg0 arg1 arg2)
unfolding *execute-CIncOffsetImmediate-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CIncOffset[preserves-invariantI]:*
runs-preserve-invariant (execute-CIncOffset arg0 arg1 arg2)
unfolding *execute-CIncOffset-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetType[preserves-invariantI]:*
runs-preserve-invariant (execute-CGetType arg0 arg1)
unfolding *execute-CGetType-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetTag[preserves-invariantI]:*
runs-preserve-invariant (execute-CGetTag arg0 arg1)
unfolding *execute-CGetTag-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetSealed*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetSealed* *arg0* *arg1*)
unfolding *execute-CGetSealed-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetPerm*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetPerm* *arg0* *arg1*)
unfolding *execute-CGetPerm-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetPCCSetOffset*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetPCCSetOffset* *arg0* *arg1*)
unfolding *execute-CGetPCCSetOffset-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetPCC*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetPCC* *arg0*)
unfolding *execute-CGetPCC-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetOffset*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetOffset* *arg0* *arg1*)
unfolding *execute-CGetOffset-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetLen*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetLen* *arg0* *arg1*)
unfolding *execute-CGetLen-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetFlags*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetFlags* *arg0* *arg1*)
unfolding *execute-CGetFlags-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetCause*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetCause* *arg0*)
unfolding *execute-CGetCause-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetCID*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetCID* *arg0*)
unfolding *execute-CGetCID-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetBase*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetBase* *arg0* *arg1*)
unfolding *execute-CGetBase-def* *bind-assoc*

by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetAndAddr*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetAndAddr* *arg0* *arg1* *arg2*)
unfolding *execute-CGetAndAddr-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CGetAddr*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CGetAddr* *arg0* *arg1*)
unfolding *execute-CGetAddr-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CFromPtr*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CFromPtr* *arg0* *arg1* *arg2*)
unfolding *execute-CFromPtr-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CCopyType*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CCopyType* *arg0* *arg1* *arg2*)
unfolding *execute-CCopyType-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CClearTag*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CClearTag* *arg0* *arg1*)
unfolding *execute-CClearTag-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CCheckType*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CCheckType* *arg0* *arg1*)
unfolding *execute-CCheckType-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CCheckTag*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CCheckTag* *arg0*)
unfolding *execute-CCheckTag-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CCheckPerm*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CCheckPerm* *arg0* *arg1*)
unfolding *execute-CCheckPerm-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CCall*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CCall* *arg0* *arg1* *arg2*)
unfolding *execute-CCall-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CCSeal*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-CCSeal* *arg0* *arg1* *arg2*)

unfolding *execute-CCSeal-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CBuildCap[preserves-invariantI]:*
runs-preserve-invariant (execute-CBuildCap arg0 arg1 arg2)
unfolding *execute-CBuildCap-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CBZ[preserves-invariantI]:*
runs-preserve-invariant (execute-CBZ arg0 arg1 arg2)
unfolding *execute-CBZ-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CBX[preserves-invariantI]:*
runs-preserve-invariant (execute-CBX arg0 arg1 arg2)
unfolding *execute-CBX-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CAndPerm[preserves-invariantI]:*
runs-preserve-invariant (execute-CAndPerm arg0 arg1 arg2)
unfolding *execute-CAndPerm-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CAndAddr[preserves-invariantI]:*
runs-preserve-invariant (execute-CAndAddr arg0 arg1 arg2)
unfolding *execute-CAndAddr-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-CACHE[preserves-invariantI]:*
runs-preserve-invariant (execute-CACHE arg0 arg1 arg2)
unfolding *execute-CACHE-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-BREAK[preserves-invariantI]:*
runs-preserve-invariant (execute-BREAK arg0)
unfolding *execute-BREAK-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-BEQ[preserves-invariantI]:*
runs-preserve-invariant (execute-BEQ arg0 arg1 arg2 arg3 arg4)
unfolding *execute-BEQ-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-BCMPZ[preserves-invariantI]:*
runs-preserve-invariant (execute-BCMPZ arg0 arg1 arg2 arg3 arg4)
unfolding *execute-BCMPZ-def bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-ANDI[preserves-invariantI]:*

runs-preserve-invariant (*execute-ANDI* *arg0* *arg1* *arg2*)
unfolding *execute-ANDI-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-AND*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-AND* *arg0* *arg1* *arg2*)
unfolding *execute-AND-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-ADDU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-ADDU* *arg0* *arg1* *arg2*)
unfolding *execute-ADDU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-ADDIU*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-ADDIU* *arg0* *arg1* *arg2*)
unfolding *execute-ADDIU-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-ADDI*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-ADDI* *arg0* *arg1* *arg2*)
unfolding *execute-ADDI-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute-ADD*[*preserves-invariantI*]:
runs-preserve-invariant (*execute-ADD* *arg0* *arg1* *arg2*)
unfolding *execute-ADD-def* *bind-assoc*
by *preserves-invariantI*

lemma *preserves-invariant-execute*[*preserves-invariantI*]:
runs-preserve-invariant (*execute instr*)
by (*cases instr rule: execute.cases; simp; preserves-invariantI*)

lemma *traces-enabled-write-cap-regs*[*traces-enabledI*]:
assumes *c* \in *derivable-caps* *s*
shows *traces-enabled* (*write-reg C01-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C02-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C03-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C04-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C05-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C06-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C07-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C08-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C09-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C10-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C11-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C12-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C13-ref c*) *s* *regs*
and *traces-enabled* (*write-reg C14-ref c*) *s* *regs*

```

and traces-enabled (write-reg C15-ref c) s regs
and traces-enabled (write-reg C16-ref c) s regs
and traces-enabled (write-reg C17-ref c) s regs
and traces-enabled (write-reg C18-ref c) s regs
and traces-enabled (write-reg C19-ref c) s regs
and traces-enabled (write-reg C20-ref c) s regs
and traces-enabled (write-reg C21-ref c) s regs
and traces-enabled (write-reg C22-ref c) s regs
and traces-enabled (write-reg C23-ref c) s regs
and traces-enabled (write-reg C24-ref c) s regs
and traces-enabled (write-reg C25-ref c) s regs
and traces-enabled (write-reg C26-ref c) s regs
and traces-enabled (write-reg C27-ref c) s regs
and traces-enabled (write-reg C28-ref c) s regs
and traces-enabled (write-reg C29-ref c) s regs
and traces-enabled (write-reg C30-ref c) s regs
and traces-enabled (write-reg C31-ref c) s regs
and traces-enabled (write-reg CPLR-ref c) s regs
and traces-enabled (write-reg CULR-ref c) s regs
and traces-enabled (write-reg DDC-ref c) s regs
and traces-enabled (write-reg DelayedPCC-ref c) s regs
and traces-enabled (write-reg EPCC-ref c) s regs
and traces-enabled (write-reg ErrorEPCC-ref c) s regs
and traces-enabled (write-reg KCC-ref c) s regs
and traces-enabled (write-reg KDC-ref c) s regs
and traces-enabled (write-reg KR1C-ref c) s regs
and traces-enabled (write-reg KR2C-ref c) s regs
and traces-enabled (write-reg NextPCC-ref c) s regs
and traces-enabled (write-reg PCC-ref c) s regs
using assms
by (intro traces-enabled-write-reg; auto simp: register-defs derivable-caps-def)+

lemma traces-enabled-write-reg-CapCause[traces-enabledI]:
  traces-enabled (write-reg CapCause-ref c) s regs
by (intro traces-enabled-write-reg; auto simp: register-defs derivable-caps-def)+

lemma traces-enabled-read-cap-regs[traces-enabledI]:
  traces-enabled (read-reg C01-ref) s regs
  traces-enabled (read-reg C02-ref) s regs
  traces-enabled (read-reg C03-ref) s regs
  traces-enabled (read-reg C04-ref) s regs
  traces-enabled (read-reg C05-ref) s regs
  traces-enabled (read-reg C06-ref) s regs
  traces-enabled (read-reg C07-ref) s regs
  traces-enabled (read-reg C08-ref) s regs
  traces-enabled (read-reg C09-ref) s regs
  traces-enabled (read-reg C10-ref) s regs
  traces-enabled (read-reg C11-ref) s regs
  traces-enabled (read-reg C12-ref) s regs

```

$\text{traces-enabled (read-reg C13-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C14-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C15-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C16-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C17-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C18-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C19-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C20-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C21-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C22-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C23-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C24-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C25-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C26-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C27-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C28-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C29-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C30-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg C31-ref) } s \text{ regs}$
 $\text{system-reg-access } s \vee \text{ex-traces} \implies \text{traces-enabled (read-reg CPLR-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg CULR-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg DDC-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg DelayedPCC-ref) } s \text{ regs}$
 $\text{system-reg-access } s \vee \text{ex-traces} \implies \text{traces-enabled (read-reg EPCC-ref) } s \text{ regs}$
 $\text{system-reg-access } s \vee \text{ex-traces} \implies \text{traces-enabled (read-reg ErrorEPCC-ref) } s \text{ regs}$
 regs
 $\text{system-reg-access } s \vee \text{ex-traces} \implies \text{traces-enabled (read-reg KCC-ref) } s \text{ regs}$
 $\text{system-reg-access } s \vee \text{ex-traces} \implies \text{traces-enabled (read-reg KDC-ref) } s \text{ regs}$
 $\text{system-reg-access } s \vee \text{ex-traces} \implies \text{traces-enabled (read-reg KR1C-ref) } s \text{ regs}$
 $\text{system-reg-access } s \vee \text{ex-traces} \implies \text{traces-enabled (read-reg KR2C-ref) } s \text{ regs}$
 $\text{system-reg-access } s \vee \text{ex-traces} \implies \text{traces-enabled (read-reg CapCause-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg NextPCC-ref) } s \text{ regs}$
 $\text{traces-enabled (read-reg PCC-ref) } s \text{ regs}$
by (intro traces-enabled-read-reg; auto simp: register-defs)+

lemma *read-cap-regs-derivable*[*derivable-capsE*]:

$\bigwedge t \ c \ \text{regs } s. \text{Run-inv (read-reg C01-ref) } t \ c \ \text{regs} \implies \{ "C01" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps (run } s \ t)$
 $\bigwedge t \ c \ \text{regs } s. \text{Run-inv (read-reg C02-ref) } t \ c \ \text{regs} \implies \{ "C02" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps (run } s \ t)$
 $\bigwedge t \ c \ \text{regs } s. \text{Run-inv (read-reg C03-ref) } t \ c \ \text{regs} \implies \{ "C03" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps (run } s \ t)$
 $\bigwedge t \ c \ \text{regs } s. \text{Run-inv (read-reg C04-ref) } t \ c \ \text{regs} \implies \{ "C04" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps (run } s \ t)$
 $\bigwedge t \ c \ \text{regs } s. \text{Run-inv (read-reg C05-ref) } t \ c \ \text{regs} \implies \{ "C05" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps (run } s \ t)$
 $\bigwedge t \ c \ \text{regs } s. \text{Run-inv (read-reg C06-ref) } t \ c \ \text{regs} \implies \{ "C06" \} \subseteq \text{accessible-regs}$

$\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } C31\text{-ref}) \ t \ c \ \text{regs} \implies \{ "C31" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } CPLR\text{-ref}) \ t \ c \ \text{regs} \implies \{ "CPLR" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } CULR\text{-ref}) \ t \ c \ \text{regs} \implies \{ "CULR" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } DDC\text{-ref}) \ t \ c \ \text{regs} \implies \{ "DDC" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } DelayedPCC\text{-ref}) \ t \ c \ \text{regs} \implies \{ "DelayedPCC" \}$
 $\subseteq \text{accessible-regs} \ s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } EPCC\text{-ref}) \ t \ c \ \text{regs} \implies \{ "EPCC" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } ErrorEPCC\text{-ref}) \ t \ c \ \text{regs} \implies \{ "ErrorEPCC" \} \subseteq$
 $\text{accessible-regs} \ s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } KCC\text{-ref}) \ t \ c \ \text{regs} \implies \{ "KCC" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } KDC\text{-ref}) \ t \ c \ \text{regs} \implies \{ "KDC" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } KR1C\text{-ref}) \ t \ c \ \text{regs} \implies \{ "KR1C" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } KR2C\text{-ref}) \ t \ c \ \text{regs} \implies \{ "KR2C" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } NextPCC\text{-ref}) \ t \ c \ \text{regs} \implies \{ "NextPCC" \} \subseteq$
 $\text{accessible-regs} \ s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
 $\bigwedge t \ c \ \text{regs} \ s. \text{Run-inv}(\text{read-reg } PCC\text{-ref}) \ t \ c \ \text{regs} \implies \{ "PCC" \} \subseteq \text{accessible-regs}$
 $s \implies c \in \text{derivable-caps}(\text{run } s \ t)$
unfolding *C01-ref-def C02-ref-def C03-ref-def C04-ref-def C05-ref-def*
C06-ref-def C07-ref-def C08-ref-def C09-ref-def C10-ref-def
C11-ref-def C12-ref-def C13-ref-def C14-ref-def C15-ref-def
C16-ref-def C17-ref-def C18-ref-def C19-ref-def C20-ref-def
C21-ref-def C22-ref-def C23-ref-def C24-ref-def C25-ref-def
C26-ref-def C27-ref-def C28-ref-def C29-ref-def C30-ref-def
C31-ref-def CPLR-ref-def CULR-ref-def DDC-ref-def DelayedPCC-ref-def
EPCC-ref-def ErrorEPCC-ref-def KCC-ref-def KDC-ref-def KR1C-ref-def
KR2C-ref-def NextPCC-ref-def PCC-ref-def Run-inv-def derivable-caps-def
by (*auto elim!; Run-read-regE intro!; derivable.Copy*)

end

context *CHERI-MIPS-Reg-Automaton*

begin

lemmas *non-cap-exp-traces-enabled[traces-enabledI] = non-cap-expI[THEN non-cap-exp-traces-enabledI]*

lemma *traces-enabled-MIPS-write[traces-enabledI]:*

shows *traces-enabled (MIPS-write arg0 arg1 arg2) s regs*

unfolding *MIPS-write-def bind-assoc*

by (*traces-enabledI*)

lemma *traces-enabled-MIPS-read*[*traces-enabledI*]:
shows *traces-enabled* (*MIPS-read* *arg0* *arg1*) *s* *regs*
unfolding *MIPS-read-def* *bind-assoc*
by (*traces-enabledI*)

lemma *traces-enabled-MEMr*[*traces-enabledI*]:
shows *traces-enabled* (*MEMr* *arg0* *arg1*) *s* *regs*
unfolding *MEMr-def* *bind-assoc*
by (*traces-enabledI*)

lemma *traces-enabled-MEMr-reserve*[*traces-enabledI*]:
shows *traces-enabled* (*MEMr-reserve* *arg0* *arg1*) *s* *regs*
unfolding *MEMr-reserve-def* *bind-assoc*
by (*traces-enabledI*)

lemma *traces-enabled-MEMea*[*traces-enabledI*]:
shows *traces-enabled* (*MEMea* *arg0* *arg1*) *s* *regs*
unfolding *MEMea-def* *bind-assoc*
by (*traces-enabledI*)

lemma *traces-enabled-MEMea-conditional*[*traces-enabledI*]:
shows *traces-enabled* (*MEMea-conditional* *arg0* *arg1*) *s* *regs*
unfolding *MEMea-conditional-def* *bind-assoc*
by (*traces-enabledI*)

lemma *traces-enabled-MEMval*[*traces-enabledI*]:
shows *traces-enabled* (*MEMval* *arg0* *arg1* *arg2*) *s* *regs*
unfolding *MEMval-def* *bind-assoc*
by (*traces-enabledI*)

lemma *traces-enabled-MEMval-conditional*[*traces-enabledI*]:
shows *traces-enabled* (*MEMval-conditional* *arg0* *arg1* *arg2*) *s* *regs*
unfolding *MEMval-conditional-def* *bind-assoc*
by (*traces-enabledI*)

lemma *traces-enabled-set-next-pcc*[*traces-enabledI*]:
assumes *arg0* \in *derivable-caps* *s*
shows *traces-enabled* (*set-next-pcc* *arg0*) *s* *regs*
unfolding *set-next-pcc-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *Run-inv-read-reg-PCC-not-sealed*:
assumes *Run-inv* (*read-reg* *PCC-ref*) *t* *c* *regs*
shows *Capability-sealed* *c* = *False*
using *assms*
unfolding *Run-inv-def*
by (*auto elim!*: *Run-read-regE simp: PCC-ref-def get-regval-def regval-of-Capability-def*)

```

lemma traces-enabled-SignalException[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s
  shows traces-enabled (SignalException arg0) s regs
proof cases
  assume ex: ex-traces
  note [derivable-capsE] = read-reg-KCC-exception-targets
  show ?thesis
    unfolding SignalException-def bind-assoc
    by (traces-enabledI assms: assms intro: traces-enabled-set-next-pcc-ex ex simp:
Run-inv-read-reg-PCC-not-sealed)
next
  assume  $\neg$ ex-traces
  then show ?thesis
    unfolding traces-enabled-def finished-def isException-def
    by auto
qed

lemma traces-enabled-SignalExceptionBadAddr[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s
  shows traces-enabled (SignalExceptionBadAddr arg0 arg1) s regs
  unfolding SignalExceptionBadAddr-def bind-assoc
  by (traces-enabledI assms: assms)

lemma traces-enabled-SignalExceptionTLB[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s
  shows traces-enabled (SignalExceptionTLB arg0 arg1) s regs
  unfolding SignalExceptionTLB-def bind-assoc
  by (traces-enabledI assms: assms)

lemma traces-enabled-pcc-access-system-regs[traces-enabledI]:
  shows traces-enabled (pcc-access-system-regs arg0) s regs
  unfolding pcc-access-system-regs-def bind-assoc
  by (traces-enabledI)

lemma Run-raise-c2-exception8-False[simp]: Run (raise-c2-exception8 arg0 arg1)
t a  $\longleftrightarrow$  False
  unfolding raise-c2-exception8-def
  by (auto elim!: Run-bindE)

lemma traces-enabled-raise-c2-exception8[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s
  shows traces-enabled (raise-c2-exception8 arg0 arg1) s regs
proof cases
  assume ex: ex-traces
  have set-ExcCode: traces-enabled (set-CapCauseReg-ExcCode CapCause-ref exc)
s regs for exc s regs
  unfolding set-CapCauseReg-ExcCode-def
  by (traces-enabledI intro: ex)

```


have *set-RegNum*: *traces-enabled* (*set-CapCauseReg-RegNum* *CapCause-ref* *r*) *s*
regs **for** *r s* *regs*
 unfolding *set-CapCauseReg-RegNum-def*
 by (*traces-enabledI* *intro*: *ex*)
 show ?*thesis*
 unfolding *raise-c2-exception8-def* *bind-assoc*
 by (*traces-enabledI* *intro*: *set-ExcCode* *set-RegNum* *assms*: *assms simp*: *CapCause-ref-def*)
next
 assume $\neg ex$ -*traces*
 then show ?*thesis*
 unfolding *traces-enabled-def* *finished-def* *isException-def*
 by *auto*
qed

lemma *traces-enabled-raise-c2-exception-noreg*[*traces-enabledI*]:
 assumes {"*PCC*"} \subseteq *accessible-regs* *s*
 shows *traces-enabled* (*raise-c2-exception-noreg* *arg0*) *s* *regs*
 unfolding *raise-c2-exception-noreg-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-checkCP0AccessHook*[*traces-enabledI*]:
 assumes {"*PCC*"} \subseteq *accessible-regs* *s*
 shows *traces-enabled* (*checkCP0AccessHook* *arg0*) *s* *regs*
 unfolding *checkCP0AccessHook-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-checkCP0Access*[*traces-enabledI*]:
 assumes {"*PCC*"} \subseteq *accessible-regs* *s*
 shows *traces-enabled* (*checkCP0Access* *arg0*) *s* *regs*
 unfolding *checkCP0Access-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-incrementCP0Count*[*traces-enabledI*]:
 assumes {"*PCC*"} \subseteq *accessible-regs* *s*
 shows *traces-enabled* (*incrementCP0Count* *arg0*) *s* *regs*
 unfolding *incrementCP0Count-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMr-wrapper*[*traces-enabledI*]:
 shows *traces-enabled* (*MEMr-wrapper* *arg0* *arg1*) *s* *regs*
 unfolding *MEMr-wrapper-def* *bind-assoc*
 by (*traces-enabledI*)

lemma *traces-enabled-MEMr-reserve-wrapper*[*traces-enabledI*]:
 shows *traces-enabled* (*MEMr-reserve-wrapper* *arg0* *arg1*) *s* *regs*
 unfolding *MEMr-reserve-wrapper-def* *bind-assoc*
 by (*traces-enabledI*)

lemma *traces-enabled-TLBTranslate2*[*traces-enabledI*]:

assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{TLBTranslate2 } \text{arg0 } \text{arg1}) \ s \ \text{regs}$
unfolding $\text{TLBTranslate2-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-TLBTranslateC}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{TLBTranslateC } \text{arg0 } \text{arg1}) \ s \ \text{regs}$
unfolding $\text{TLBTranslateC-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-TLBTranslate}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{TLBTranslate } \text{arg0 } \text{arg1}) \ s \ \text{regs}$
unfolding $\text{TLBTranslate-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-execute-branch-pcc}[\text{traces-enabledI}]$:
assumes $\text{arg0} \in \text{derivable-caps } s$
shows $\text{traces-enabled } (\text{execute-branch-pcc } \text{arg0}) \ s \ \text{regs}$
unfolding $\text{execute-branch-pcc-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-ERETHook}[\text{traces-enabledI}]$:
assumes $\{ "EPCC", "ErrorEPCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{ERETHook } \text{arg0}) \ s \ \text{regs}$
unfolding $\text{ERETHook-def bind-assoc}$
by $(\text{traces-enabledI assms: assms simp: accessible-regs-def})$

lemma $\text{traces-enabled-raise-c2-exception}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{raise-c2-exception } \text{arg0 } \text{arg1}) \ s \ \text{regs}$
unfolding $\text{raise-c2-exception-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-MEMr-tagged}[\text{traces-enabledI}]$:
shows $\text{traces-enabled } (\text{MEMr-tagged } \text{arg0 } \text{arg1 } \text{arg2}) \ s \ \text{regs}$
unfolding $\text{MEMr-tagged-def bind-assoc}$
by (traces-enabledI)

lemma $\text{traces-enabled-MEMr-tagged-reserve}[\text{traces-enabledI}]$:
shows $\text{traces-enabled } (\text{MEMr-tagged-reserve } \text{arg0 } \text{arg1 } \text{arg2}) \ s \ \text{regs}$
unfolding $\text{MEMr-tagged-reserve-def bind-assoc}$
by (traces-enabledI)

lemma $\text{traces-enabled-MEMw-tagged}[\text{traces-enabledI}]$:
assumes $\text{memBitsToCapability tag } (\text{ucast } v) \in \text{derivable-caps } s$
shows $\text{traces-enabled } (\text{MEMw-tagged } \text{addr } \text{sz } \text{tag } v) \ s \ \text{regs}$
unfolding $\text{MEMw-tagged-def bind-assoc}$

by (traces-enabledI assms: assms)

lemma traces-enabled-MEMw-tagged-conditional[traces-enabledI]:
 assumes memBitsToCapability tag (ucast v) ∈ derivable-caps s
 shows traces-enabled (MEMw-tagged-conditional addr sz tag v) s regs
 unfolding MEMw-tagged-conditional-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-MEMw-wrapper[traces-enabledI]:
 shows traces-enabled (MEMw-wrapper arg0 arg1 arg2) s regs
 unfolding MEMw-wrapper-def bind-assoc
 by (traces-enabledI)

lemma traces-enabled-MEMw-conditional-wrapper[traces-enabledI]:
 shows traces-enabled (MEMw-conditional-wrapper arg0 arg1 arg2) s regs
 unfolding MEMw-conditional-wrapper-def bind-assoc
 by (traces-enabledI)

lemma traces-enabled-checkDDCPerms[traces-enabledI]:
 assumes {"PCC"} ⊆ accessible-regs s and arg0 ∈ derivable-caps s
 shows traces-enabled (checkDDCPerms arg0 arg1) s regs
 unfolding checkDDCPerms-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-addrWrapper[traces-enabledI]:
 assumes {"DDC", "PCC"} ⊆ accessible-regs s
 shows traces-enabled (addrWrapper arg0 arg1 arg2) s regs
 unfolding addrWrapper-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-addrWrapperUnaligned[traces-enabledI]:
 assumes {"DDC", "PCC"} ⊆ accessible-regs s
 shows traces-enabled (addrWrapperUnaligned arg0 arg1 arg2) s regs
 unfolding addrWrapperUnaligned-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-execute-branch[traces-enabledI]:
 assumes {"PCC"} ⊆ accessible-regs s
 shows traces-enabled (execute-branch arg0) s regs
 unfolding execute-branch-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-TranslatePC[traces-enabledI]:
 assumes {"PCC"} ⊆ accessible-regs s
 shows traces-enabled (TranslatePC arg0) s regs
 unfolding TranslatePC-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-checkCP2usable[traces-enabledI]:

assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{checkCP2usable } \text{arg0}) \ s \ \text{regs}$
unfolding $\text{checkCP2usable-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms})$

lemma $\text{traces-enabled-get-CP0EPC}[\text{traces-enabledI}]$:
assumes $\{ "EPCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{get-CP0EPC } \text{arg0}) \ s \ \text{regs}$
unfolding $\text{get-CP0EPC-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms simp: accessible-regs-def})$

lemma $\text{traces-enabled-set-CP0EPC}[\text{traces-enabledI}]$:
assumes $\{ "EPCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{set-CP0EPC } \text{arg0}) \ s \ \text{regs}$
unfolding $\text{set-CP0EPC-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms simp: accessible-regs-def})$

lemma $\text{traces-enabled-get-CP0ErrorEPC}[\text{traces-enabledI}]$:
assumes $\{ "ErrorEPCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{get-CP0ErrorEPC } \text{arg0}) \ s \ \text{regs}$
unfolding $\text{get-CP0ErrorEPC-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms simp: accessible-regs-def})$

lemma $\text{traces-enabled-set-CP0ErrorEPC}[\text{traces-enabledI}]$:
assumes $\{ "ErrorEPCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{set-CP0ErrorEPC } \text{arg0}) \ s \ \text{regs}$
unfolding $\text{set-CP0ErrorEPC-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms simp: accessible-regs-def system-reg-access-run})$

lemma $\text{traces-enabled-TLBWriteEntry}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{TLBWriteEntry } \text{arg0}) \ s \ \text{regs}$
unfolding $\text{TLBWriteEntry-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms})$

lemma $\text{traces-enabled-execute-WAIT}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-WAIT } \text{arg0}) \ s \ \text{regs}$
unfolding $\text{execute-WAIT-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms})$

lemma $\text{traces-enabled-execute-TRAPREG}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-TRAPREG } \text{arg0 } \text{arg1 } \text{arg2}) \ s \ \text{regs}$
unfolding $\text{execute-TRAPREG-def bind-assoc}$

by (traces-enabledI assms: assms)

lemma traces-enabled-execute-TRAPIMM[traces-enabledI]:
 assumes {"PCC"} \subseteq accessible-regs s
 shows traces-enabled (execute-TRAPIMM arg0 arg1 arg2) s regs
 unfolding execute-TRAPIMM-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-execute-TLBWR[traces-enabledI]:
 assumes {"PCC"} \subseteq accessible-regs s
 shows traces-enabled (execute-TLBWR arg0) s regs
 unfolding execute-TLBWR-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-execute-TLBWI[traces-enabledI]:
 assumes {"PCC"} \subseteq accessible-regs s
 shows traces-enabled (execute-TLBWI arg0) s regs
 unfolding execute-TLBWI-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-execute-TLBR[traces-enabledI]:
 assumes {"PCC"} \subseteq accessible-regs s
 shows traces-enabled (execute-TLBR arg0) s regs
 unfolding execute-TLBR-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-execute-TLBP[traces-enabledI]:
 assumes {"PCC"} \subseteq accessible-regs s
 shows traces-enabled (execute-TLBP arg0) s regs
 unfolding execute-TLBP-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-execute-Store[traces-enabledI]:
 assumes {"DDC", "PCC"} \subseteq accessible-regs s
 shows traces-enabled (execute-Store arg0 arg1 arg2 arg3 arg4) s regs
 unfolding execute-Store-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-execute-SYSCALL[traces-enabledI]:
 assumes {"PCC"} \subseteq accessible-regs s
 shows traces-enabled (execute-SYSCALL arg0) s regs
 unfolding execute-SYSCALL-def bind-assoc
 by (traces-enabledI assms: assms)

lemma traces-enabled-execute-SWR[traces-enabledI]:
 assumes {"DDC", "PCC"} \subseteq accessible-regs s
 shows traces-enabled (execute-SWR arg0 arg1 arg2) s regs
 unfolding execute-SWR-def bind-assoc
 by (traces-enabledI assms: assms)

lemma *traces-enabled-execute-SWL*[*traces-enabledI*]:
assumes {"DDC", "PCC"} \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-SWL* *arg0 arg1 arg2*) *s regs*
unfolding *execute-SWL-def bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-SUB*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-SUB* *arg0 arg1 arg2*) *s regs*
unfolding *execute-SUB-def bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-SDR*[*traces-enabledI*]:
assumes {"DDC", "PCC"} \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-SDR* *arg0 arg1 arg2*) *s regs*
unfolding *execute-SDR-def bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-SDL*[*traces-enabledI*]:
assumes {"DDC", "PCC"} \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-SDL* *arg0 arg1 arg2*) *s regs*
unfolding *execute-SDL-def bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-RI*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-RI* *arg0*) *s regs*
unfolding *execute-RI-def bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-RDHWR*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-RDHWR* *arg0 arg1*) *s regs*
unfolding *execute-RDHWR-def bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *Run-inv-pcc-access-system-regs-accessible-regs*:
assumes *Run-inv* (*pcc-access-system-regs u*) *t a regs* **and** *a*
and *Rs* \cap *written-regs s* = {} **and** {"PCC"} \subseteq *accessible-regs s*
shows *Rs* \subseteq *accessible-regs* (*run s t*)
using *assms*
by (*auto simp: accessible-regs-def system-reg-access-run pcc-access-system-regs-allows-system-reg-access*
runs-no-reg-writes-written-regs-eq no-reg-writes-runs-no-reg-writes)

lemmas *Run-inv-pcc-access-system-regs-privileged-regs-accessible*[*accessible-regsE*]
= *Run-inv-pcc-access-system-regs-accessible-regs*[**where** *Rs* = {"KCC"}]
Run-inv-pcc-access-system-regs-accessible-regs[**where** *Rs* = {"KDC"}]

$\text{Run-inv-pcc-access-system-regs-accessible-regs}[\text{where } Rs = \{\text{"EPCC"}\}]$
 $\text{Run-inv-pcc-access-system-regs-accessible-regs}[\text{where } Rs = \{\text{"ErrorEPCC"}\}]$
 $\text{Run-inv-pcc-access-system-regs-accessible-regs}[\text{where } Rs = \{\text{"KR1C"}\}]$
 $\text{Run-inv-pcc-access-system-regs-accessible-regs}[\text{where } Rs = \{\text{"KR2C"}\}]$
 $\text{Run-inv-pcc-access-system-regs-accessible-regs}[\text{where } Rs = \{\text{"CapCause"}\}]$
 $\text{Run-inv-pcc-access-system-regs-accessible-regs}[\text{where } Rs = \{\text{"CPLR"}\}]$

lemma $\text{Run-inv-SignalException-False}[\text{simp}]$: $\text{Run-inv } (\text{SignalException } \text{exc}) \ t \ a$
 $\text{regs} \longleftrightarrow \text{False}$
unfolding Run-inv-def
by auto

lemma $\text{Run-inv-checkCP0Access-accessible-regs}$:
assumes $\text{Run-inv } (\text{checkCP0Access } u) \ t \ a \ \text{regs}$
and $R_s \cap \text{written-regs } s = \{\}$ **and** $\{\text{"PCC"}\} \subseteq \text{accessible-regs } s$
shows $R_s \subseteq \text{accessible-regs } (\text{run } s \ t)$
using assms
unfolding $\text{checkCP0Access-def checkCP0AccessHook-def bind-assoc}$
by $(\text{auto elim!} : \text{Run-inv-bindE intro!} : \text{preserves-invariantI traces-runs-preserve-invariantI}$
 $\text{split} : \text{if-splits simp} : \text{CP0Cause-ref-def})$
 $(\text{auto simp} : \text{accessible-regs-def runs-no-reg-writes-written-regs-eq no-reg-writes-runs-no-reg-writes}$
 $\text{system-reg-access-run pcc-access-system-regs-allows-system-reg-access})$

lemmas $\text{Run-inv-checkCP0Access-privileged-regs-accessible}[\text{accessible-regsE}] =$
 $\text{Run-inv-checkCP0Access-accessible-regs}[\text{where } Rs = \{\text{"KCC"}\}]$
 $\text{Run-inv-checkCP0Access-accessible-regs}[\text{where } Rs = \{\text{"KDC"}\}]$
 $\text{Run-inv-checkCP0Access-accessible-regs}[\text{where } Rs = \{\text{"EPCC"}\}]$
 $\text{Run-inv-checkCP0Access-accessible-regs}[\text{where } Rs = \{\text{"ErrorEPCC"}\}]$
 $\text{Run-inv-checkCP0Access-accessible-regs}[\text{where } Rs = \{\text{"KR1C"}\}]$
 $\text{Run-inv-checkCP0Access-accessible-regs}[\text{where } Rs = \{\text{"KR2C"}\}]$
 $\text{Run-inv-checkCP0Access-accessible-regs}[\text{where } Rs = \{\text{"CapCause"}\}]$
 $\text{Run-inv-checkCP0Access-accessible-regs}[\text{where } Rs = \{\text{"CPLR"}\}]$

lemma $\text{Run-inv-no-reg-writes-written-regs}[\text{accessible-regsE}]$:
assumes $\text{Run-inv } m \ t \ a \ \text{regs}$
and $\text{runs-no-reg-writes-to } R_s \ m$ **and** $R_s \cap \text{written-regs } s = \{\}$
shows $R_s \cap \text{written-regs } (\text{run } s \ t) = \{\}$
using assms
by $(\text{auto simp} : \text{runs-no-reg-writes-written-regs-eq runs-no-reg-writes-to-def})$

lemma $\text{Run-inv-assert-exp-iff}[\text{iff}]$:
 $\text{Run-inv } (\text{assert-exp } c \ \text{msg}) \ t \ a \ \text{regs} \longleftrightarrow c \wedge t = [] \wedge \text{invariant } \text{regs}$
unfolding Run-inv-def
by auto

lemma $\text{throw-bind-eq}[\text{simp}]$: $(\text{throw } e \gg m) = \text{throw } e$
by $(\text{auto simp} : \text{throw-def})$

lemma $\text{SignalException-bind-eq}[\text{simp}]$: $(\text{SignalException } ex \gg m) = \text{SignalEx-}$

ception ex

unfolding *SignalException-def Let-def bind-assoc throw-bind-eq ..*

lemma *runs-no-reg-writes-to-checkCP2usable[runs-no-reg-writes-toI, simp]:*

shows *runs-no-reg-writes-to Rs (checkCP2usable u)*

unfolding *checkCP2usable-def runs-no-reg-writes-to-def*

by *(auto elim!: Run-bindE Run-read-regE split: if-splits)*

lemma *traces-enabled-execute-MTC0[traces-enabledI]:*

assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\{"EPCC", "ErrorEPCC"\} \cap \text{written-regs } s = \{\}$

shows *traces-enabled (execute-MTC0 arg0 arg1 arg2 arg3) s regs*

unfolding *execute-MTC0-def bind-assoc*

by *(intro traces-enabled-if-ignore-cond traces-enabledI preserves-invariantI traces-runs-preserve-invariantI; accessible-regsI assms: assms)*

lemma *traces-enabled-execute-MFC0[traces-enabledI]:*

assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\{"EPCC", "ErrorEPCC"\} \cap \text{written-regs } s = \{\}$

shows *traces-enabled (execute-MFC0 arg0 arg1 arg2 arg3) s regs*

unfolding *execute-MFC0-def bind-assoc*

by *(intro traces-enabled-if-ignore-cond traces-enabledI preserves-invariantI traces-runs-preserve-invariantI conjI allI impI; accessible-regsI assms: assms)*

lemma *traces-enabled-execute-Load[traces-enabledI]:*

assumes $\{"DDC", "PCC"\} \subseteq \text{accessible-regs } s$

shows *traces-enabled (execute-Load arg0 arg1 arg2 arg3 arg4 arg5) s regs*

unfolding *execute-Load-def bind-assoc*

by *(traces-enabledI assms: assms)*

lemma *traces-enabled-execute-LWR[traces-enabledI]:*

assumes $\{"DDC", "PCC"\} \subseteq \text{accessible-regs } s$

shows *traces-enabled (execute-LWR arg0 arg1 arg2) s regs*

unfolding *execute-LWR-def bind-assoc*

by *(traces-enabledI assms: assms)*

lemma *traces-enabled-execute-LWL[traces-enabledI]:*

assumes $\{"DDC", "PCC"\} \subseteq \text{accessible-regs } s$

shows *traces-enabled (execute-LWL arg0 arg1 arg2) s regs*

unfolding *execute-LWL-def bind-assoc*

by *(traces-enabledI assms: assms)*

lemma *traces-enabled-execute-LDR[traces-enabledI]:*

assumes $\{"DDC", "PCC"\} \subseteq \text{accessible-regs } s$

shows *traces-enabled (execute-LDR arg0 arg1 arg2) s regs*

unfolding *execute-LDR-def bind-assoc*

by *(traces-enabledI assms: assms)*

lemma *traces-enabled-execute-LDL[traces-enabledI]:*

assumes $\{"DDC", "PCC"\} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-LDL } \text{arg0 } \text{arg1 } \text{arg2}) \text{ } s \text{ } \text{regs}$
unfolding $\text{execute-LDL-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms})$

lemma $\text{traces-enabled-execute-JR}[\text{traces-enabledI}]$:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-JR } \text{arg0}) \text{ } s \text{ } \text{regs}$
unfolding $\text{execute-JR-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms})$

lemma $\text{traces-enabled-execute-JALR}[\text{traces-enabledI}]$:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-JALR } \text{arg0 } \text{arg1}) \text{ } s \text{ } \text{regs}$
unfolding $\text{execute-JALR-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms})$

lemma $\text{traces-enabled-execute-JAL}[\text{traces-enabledI}]$:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-JAL } \text{arg0}) \text{ } s \text{ } \text{regs}$
unfolding $\text{execute-JAL-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms})$

lemma $\text{traces-enabled-execute-J}[\text{traces-enabledI}]$:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-J } \text{arg0}) \text{ } s \text{ } \text{regs}$
unfolding $\text{execute-J-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms})$

lemma $\text{runs-no-reg-writes-to-checkCP0Access}[\text{runs-no-reg-writes-toI}, \text{simp}]$:
 $\text{runs-no-reg-writes-to } \text{Rs } (\text{checkCP0Access } u)$
using $\text{no-reg-writes-to-pcc-access-system-regs no-reg-writes-to-getAccessLevel no-reg-writes-to-read-reg}[\text{where } r = \text{CP0Status-ref}]$
unfolding $\text{checkCP0Access-def checkCP0AccessHook-def runs-no-reg-writes-to-def no-reg-writes-to-def and-boolM-def}$
by $(\text{fastforce elim!: Run-bindE split: if-splits})$

lemma $\text{traces-enabled-execute-ERET}[\text{traces-enabledI}]$:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\{"EPCC", "ErrorEPCC"\} \cap \text{written-regs } s = \{\}$
shows $\text{traces-enabled } (\text{execute-ERET } \text{arg0}) \text{ } s \text{ } \text{regs}$
unfolding $\text{execute-ERET-def bind-assoc}$
by $(\text{traces-enabledI } \text{assms: } \text{assms checkCP0Access-system-reg-access simp: accessible-regs-def runs-no-reg-writes-written-regs-eq no-reg-writes-runs-no-reg-writes system-reg-access-run})$

lemma $\text{traces-enabled-execute-DSUB}[\text{traces-enabledI}]$:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-DSUB } \text{arg0 } \text{arg1 } \text{arg2}) \text{ } s \text{ } \text{regs}$
unfolding $\text{execute-DSUB-def bind-assoc}$

```

by (traces-enabledI assms: assms)

lemma traces-enabled-execute-DADDI[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-DADDI arg0 arg1 arg2) s regs
  unfolding execute-DADDI-def bind-assoc
  by (traces-enabledI assms: assms)

lemma traces-enabled-execute-DADD[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-DADD arg0 arg1 arg2) s regs
  unfolding execute-DADD-def bind-assoc
  by (traces-enabledI assms: assms)

declare traces-enabled-foreachM-inv[where P =  $\lambda$ -. True, traces-enabledI]

lemma traces-enabled-execute-ClearRegs[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-ClearRegs arg0 arg1) s regs
  unfolding execute-ClearRegs-def bind-assoc
  by (traces-enabledI assms: assms)

lemma traces-enabled-execute-CWriteHwr[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CWriteHwr arg0 arg1) s regs
  unfolding execute-CWriteHwr-def bind-assoc
  by (traces-enabledI assms: assms)

lemma unsealCap-derivable-caps[derivable-capsI]:
  assumes c  $\in$  derivable-caps s and c'  $\in$  derivable-caps s
    and Capability-tag c and Capability-tag c'
    and Capability-sealed c and  $\neg$ Capability-sealed c'
    and Capability-permit-unseal c'
    and getCapCursor c' = uint (Capability-otype c)
  shows (unsealCap c)( $\downarrow$ Capability-global := Capability-global c  $\wedge$  Capability-global
c')  $\in$  derivable-caps s
    (is ?unseal c c'  $\in$  derivable-caps s)
proof -
  have unseal CC c (get-global-method CC c')  $\in$  derivable (accessed-caps s)
    using assms
  by (intro derivable.Unseal) (auto simp: derivable-caps-def unat-def[symmetric]
get-cap-perms-def)
  then have ?unseal c c'  $\in$  derivable (accessed-caps s)
    by (elim derivable.Restrict)
    (auto simp: leq-cap-def unseal-def unsealCap-def getCapBase-def getCapTop-def
get-cap-perms-def)
  then show ?thesis
    by (auto simp: derivable-caps-def)
qed

```

lemma *traces-enabled-execute-CUnseal*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CUnseal* *arg0 arg1 arg2*) *s* *regs*
 unfolding *execute-CUnseal-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CToPtr*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CToPtr* *arg0 arg1 arg2*) *s* *regs*
 unfolding *execute-CToPtr-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CTestSubset*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CTestSubset* *arg0 arg1 arg2*) *s* *regs*
 unfolding *execute-CTestSubset-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CSub*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CSub* *arg0 arg1 arg2*) *s* *regs*
 unfolding *execute-CSub-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CStoreConditional*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CStoreConditional* *arg0 arg1 arg2 arg3*) *s* *regs*
 unfolding *execute-CStoreConditional-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CStore*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CStore* *arg0 arg1 arg2 arg3 arg4*) *s* *regs*
 unfolding *execute-CStore-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CSetOffset*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CSetOffset* *arg0 arg1 arg2*) *s* *regs*
 unfolding *execute-CSetOffset-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *setCapFlags-derivable-caps*[*derivable-capsI*]:
 assumes $c \in \text{derivable-caps } s$
 shows *setCapFlags* *c* $f \in \text{derivable-caps } s$
 using *assms*
 by (*auto simp*: *setCapFlags-def*)

lemma *traces-enabled-execute-CSetFlags*[*traces-enabledI*]:
assumes $\{\text{"PCC"}\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CSetFlags* *arg0 arg1 arg2*) *s* *regs*
unfolding *execute-CSetFlags-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-set-CapCauseReg-ExcCode*:
assumes *system-reg-access* *s* \vee *ex-traces*
shows *traces-enabled* (*set-CapCauseReg-ExcCode* *CapCause-ref exc*) *s* *regs*
unfolding *set-CapCauseReg-ExcCode-def*
by (*traces-enabledI* *intro*: *traces-enabled-read-reg* *traces-enabled-write-reg simp*:
CapCause-ref-def *assms*: *assms*)

lemma *traces-enabled-set-CapCauseReg-RegNum*:
assumes *system-reg-access* *s* \vee *ex-traces*
shows *traces-enabled* (*set-CapCauseReg-RegNum* *CapCause-ref exc*) *s* *regs*
unfolding *set-CapCauseReg-RegNum-def*
by (*traces-enabledI* *intro*: *traces-enabled-read-reg* *traces-enabled-write-reg simp*:
CapCause-ref-def *assms*: *assms*)

lemma *system-reg-access-run-ex-tracesI*[*accessible-regsI*]:
assumes $\neg \text{trace-allows-system-reg-access } (\text{accessible-regs } s) \ t \implies \text{system-reg-access}$
 $s \vee \text{ex-traces}$
shows *system-reg-access* (*run* *s* *t*) \vee *ex-traces*
using *assms*
by (*auto simp*: *system-reg-access-run*)

lemma *pcc-access-system-regs-allows-system-reg-access-ex-tracesI*[*accessible-regsE*]:
assumes *Run-inv* (*pcc-access-system-regs* *u*) *t a* *regs* **and** *a* **and** $\{\text{"PCC"}\} \subseteq$
 $\text{accessible-regs } s$
shows *system-reg-access* (*run* *s* *t*) \vee *ex-traces*
using *assms*
by (*auto simp*: *system-reg-access-run* *pcc-access-system-regs-allows-system-reg-access*)

lemma *traces-enabled-execute-CSetCause*[*traces-enabledI*]:
assumes $\{\text{"PCC"}\} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CSetCause* *arg0*) *s* *regs*
unfolding *execute-CSetCause-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms* *intro*: *traces-enabled-set-CapCauseReg-ExcCode*
traces-enabled-set-CapCauseReg-RegNum simp: *CapCause-ref-def*)

lemma *traces-enabled-execute-CSetCID*[*traces-enabledI*]:
assumes $\{\text{"PCC"}\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CSetCID* *arg0*) *s* *regs*
unfolding *execute-CSetCID-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *unat-add-nat-uint-add*: *unat* *a* + *unat* *b* = *nat* (*uint* *a* + *uint* *b*)
by (*auto simp*: *unat-def* *nat-add-distrib*)

lemma *[simp]*: $0 \leq i \implies \text{nat } i \leq \text{unat } j \longleftrightarrow i \leq \text{uint } j$
by (*auto simp: unat-def nat-le-eq-zle*)

lemma *setCapBounds-derivable-caps*[*derivable-capsE*]:
assumes *setCapBounds* *c b t = (e, c')*
and *c* \in *derivable-caps s* **and** \neg *Capability-sealed c*
and *getCapBase c* \leq *uint b* **and** *uint b* \leq *uint t* **and** *uint t* \leq *getCapTop c*
shows *c' \in derivable-caps s*
proof –
have *getCapTop c' \leq uint t*
using *assms Divides.mod-less-eq-dividend* [**where** *a = uint (t – ucast b)* **and**
b = 2 ^ 64]
unfolding *setCapBounds-def getCapTop-def getCapBase-def*
by (*auto simp: uint-and-mask uint-sub-if-size*)
then have *leq-cap CC c' c*
using *assms*
by (*auto simp: leq-cap-def setCapBounds-def getCapBase-def getCapTop-def*
nat-le-eq-zle get-cap-perms-def)
from *derivable.Restrict[OF - this]*
show *?thesis*
using *assms*
by (*auto simp: derivable-caps-def setCapBounds-def*)
qed

lemma *to-bits-uint-ucast*[*simp*]:
 $n = \text{int } (\text{LENGTH } ('a)) \implies \text{to-bits } n \ (\text{uint } w) = (\text{ucast } w :: 'a :: \text{len word})$
by (*auto simp: to-bits-def of-bl-bin-word-of-int ucast-def*)

lemma *to-bits-add*[*simp*]:
 $n = \text{int } (\text{LENGTH } ('a)) \implies \text{to-bits } n \ (a + b) = (\text{to-bits } n \ a + \text{to-bits } n \ b :: 'a :: \text{len word})$
by (*auto simp: to-bits-def of-bl-bin-word-of-int wi-hom-syms*)

lemma *to-bits-64-getCapCursor*[*simp*]: *to-bits 64 (getCapCursor c) = Capability-address*
c
by (*auto simp: getCapCursor-def*)

lemma *traces-enabled-execute-CSetBoundsImmediate*[*traces-enabledI*]:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** *CapRegs-names* \subseteq *accessible-regs s*
shows *traces-enabled (execute-CSetBoundsImmediate arg0 arg1 arg2) s regs*
unfolding *execute-CSetBoundsImmediate-def bind-assoc*
by (*traces-enabledI assms: assms simp: getCapCursor-def getCapTop-def*)

lemma *traces-enabled-execute-CSetBoundsExact*[*traces-enabledI*]:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** *CapRegs-names* \subseteq *accessible-regs s*
shows *traces-enabled (execute-CSetBoundsExact arg0 arg1 arg2) s regs*
unfolding *execute-CSetBoundsExact-def bind-assoc*
by (*traces-enabledI assms: assms simp: getCapCursor-def getCapTop-def*)

lemma *traces-enabled-execute-CSetBounds*[*traces-enabledI*]:
assumes $\{\text{"PCC"}\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CSetBounds* *arg0 arg1 arg2*) *s regs*
unfolding *execute-CSetBounds-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms simp*: *getCapCursor-def* *getCapTop-def*)

lemma *setCapAddr-derivable-caps*[*derivable-capsE*]:
assumes *setCapAddr* *c a'* = (*success*, *c'*)
and $c \in \text{derivable-caps } s$
and $\text{Capability-tag } c \longrightarrow \neg \text{Capability-sealed } c$
shows $c' \in \text{derivable-caps } s$
proof –
have *leq-cap* *CC c' c* **and** $\text{Capability-tag } c' \longleftrightarrow \text{Capability-tag } c$
using *assms*
by (*auto simp*: *setCapAddr-def* *leq-cap-def* *getCapBase-def* *getCapTop-def* *get-cap-perms-def*)
then show *?thesis*
using *assms*
by (*auto simp*: *derivable-caps-def* *elim*: *derivable.Restrict*)
qed

lemma *traces-enabled-execute-CSetAddr*[*traces-enabledI*]:
assumes $\{\text{"PCC"}\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CSetAddr* *arg0 arg1 arg2*) *s regs*
unfolding *execute-CSetAddr-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *sealCap-derivable-caps*[*derivable-capsE*]:
assumes *sealCap* *c* (*to-bits* 24 (*getCapCursor* *c'*)) = (*success*, *c''*)
and $c \in \text{derivable-caps } s$ **and** $c' \in \text{derivable-caps } s$
and $\text{Capability-tag } c$ **and** $\text{Capability-tag } c'$
and $\neg \text{Capability-sealed } c$ **and** $\neg \text{Capability-sealed } c'$
and $\text{Capability-permit-seal } c'$
shows $c'' \in \text{derivable-caps } s$
proof –
have *seal* *CC c* (*get-cursor-method* *CC c'*) $\in \text{derivable (accessed-caps } s)$
using *assms*
by (*intro* *derivable.Seal*) (*auto simp*: *derivable-caps-def* *get-cap-perms-def*)
moreover have *seal* *CC c* (*get-cursor-method* *CC c'*) = *c''*
using *assms*
by (*cases* *c*)
(auto simp: *sealCap-def* *seal-def* *to-bits-def* *getCapCursor-def* *of-bl-bin-word-of-int*
word-of-int-nat)
ultimately show *?thesis*
by (*simp add*: *derivable-caps-def*)
qed

lemma *traces-enabled-execute-CSeal*[*traces-enabledI*]:
assumes $\{\text{"PCC"}\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$

shows *traces-enabled* (*execute-CSeal* *arg0 arg1 arg2*) *s regs*
unfolding *execute-CSeal-def* *bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-CSCC*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s* **and** *CapRegs-names* \subseteq *accessible-reg*s *s*
shows *traces-enabled* (*execute-CSCC* *arg0 arg1 arg2*) *s regs*
unfolding *execute-CSCC-def* *bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-CSC*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s* **and** *CapRegs-names* \subseteq *accessible-reg*s *s*
shows *traces-enabled* (*execute-CSC* *arg0 arg1 arg2 arg3*) *s regs*
unfolding *execute-CSC-def* *bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-CReturn*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s*
shows *traces-enabled* (*execute-CReturn* *arg0*) *s regs*
unfolding *execute-CReturn-def* *bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-CReadHwr*[*traces-enabledI*]:
assumes {"CULR", "DDC", "PCC"} \subseteq *accessible-reg*s *s*
and *privileged-reg*s *ISA* \cap *written-reg*s *s* = {}
shows *traces-enabled* (*execute-CReadHwr* *arg0 arg1*) *s regs*
proof –
have *wint* *arg1* \in {0..31}
by *auto*
then show ?thesis
unfolding *upto-31-unfold* *execute-CReadHwr-def* *bind-assoc*
by (*elim insertE*; *simp cong: if-cong*; *use nothing* **in** (*traces-enabledI* *assms: assms*))
qed

lemma *traces-enabled-execute-CRAP*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s*
shows *traces-enabled* (*execute-CRAP* *arg0 arg1*) *s regs*
unfolding *execute-CRAP-def* *bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-CRAM*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s*
shows *traces-enabled* (*execute-CRAM* *arg0 arg1*) *s regs*
unfolding *execute-CRAM-def* *bind-assoc*
by (*traces-enabledI* *assms: assms*)

lemma *traces-enabled-execute-CPtrCmp*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s* **and** *CapRegs-names* \subseteq *accessible-reg*s *s*

shows *traces-enabled* (*execute-CPtrCmp* *arg0 arg1 arg2 arg3*) *s regs*
unfolding *execute-CPtrCmp-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CMove*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs s* **and** *CapRegs-names* \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-CMove* *arg0 arg1*) *s regs*
unfolding *execute-CMove-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CMOVX*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs s* **and** *CapRegs-names* \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-CMOVX* *arg0 arg1 arg2 arg3*) *s regs*
unfolding *execute-CMOVX-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CLoadTags*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs s* **and** *CapRegs-names* \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-CLoadTags* *arg0 arg1*) *s regs*
unfolding *execute-CLoadTags-def bind-assoc*
by (*traces-enabledI intro: traces-enabled-foreachM-inv*[**where** *s = s* **and** *P =*
 $\lambda vars\ s'\ regs'. \{ "PCC" \} \subseteq accessible-regs\ s' \text{ for } s]$ *assms: assms*)

lemma *traces-enabled-execute-CLoadLinked*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs s* **and** *CapRegs-names* \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-CLoadLinked* *arg0 arg1 arg2 arg3*) *s regs*
unfolding *execute-CLoadLinked-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CLoad*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs s* **and** *CapRegs-names* \subseteq *accessible-regs s*
shows *traces-enabled* (*execute-CLoad* *arg0 arg1 arg2 arg3 arg4 arg5*) *s regs*
unfolding *execute-CLoad-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *Run-inv-read-memt-derivable-caps*[*derivable-capsE*]:
assumes *Run-inv* (*read-memt BCa BC-mword rk addr sz*) *t a regs*
and *tag* \longrightarrow *a = (mem, B1)*
shows *memBitsToCapability tag mem* \in *derivable-caps* (*run s t*)
using *assms*
unfolding *Run-inv-def read-memt-def read-memt-bytes-def maybe-fail-def bind-assoc*
by (*cases tag*)
(auto simp: derivable-caps-def BC-mword-defs memBitsToCapability-def capBitsToCapability-def
elim!: Run-bindE intro!: derivable.Copy elim: Traces-cases split: op-
tion.splits)

lemma *Run-inv-maybe-fail-iff*[*simp*]:
Run-inv (*maybe-fail msg x*) *t a regs* \longleftrightarrow (*x = Some a* \wedge *t = []* \wedge *invariant regs*)
by (*auto simp: Run-inv-def maybe-fail-def split: option.splits*)

lemma *Run-inv-MEMr-tagged-reserve-derivable-caps*[*derivable-capsE*]:
assumes *Run-inv* (*MEMr-tagged-reserve* *addr sz allow-tag*) *t a regs*
and *tag* \longrightarrow *a* = (*True*, *mem*)
shows *memBitsToCapability* *tag mem* \in *derivable-caps* (*run s t*)
using *assms*
unfolding *MEMr-tagged-reserve-def*
by (*auto elim!*: *Run-inv-bindE Run-inv-read-memt-derivable-caps intro: preserves-invariantI*
traces-runs-preserve-invariantI
split: option.splits bitU.splits if-splits simp: bool-of-bitU-def)

lemma *Run-inv-MEMr-tagged-derivable-caps*[*derivable-capsE*]:
assumes *Run-inv* (*MEMr-tagged* *addr sz allow-tag*) *t a regs*
and *tag* \longrightarrow *a* = (*True*, *mem*)
shows *memBitsToCapability* *tag mem* \in *derivable-caps* (*run s t*)
using *assms*
unfolding *MEMr-tagged-def*
by (*auto elim!*: *Run-inv-bindE Run-inv-read-memt-derivable-caps intro: preserves-invariantI*
traces-runs-preserve-invariantI
split: option.splits bitU.splits if-splits simp: bool-of-bitU-def)

lemma *traces-enabled-execute-CLLC*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s* **and** *CapRegs-names* \subseteq *accessible-reg*s *s*
shows *traces-enabled* (*execute-CLLC* *arg0 arg1*) *s regs*
unfolding *execute-CLLC-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CLCBI*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s* **and** *CapRegs-names* \subseteq *accessible-reg*s *s*
shows *traces-enabled* (*execute-CLCBI* *arg0 arg1 arg2*) *s regs*
unfolding *execute-CLCBI-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CLC*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s* **and** *CapRegs-names* \subseteq *accessible-reg*s *s*
shows *traces-enabled* (*execute-CLC* *arg0 arg1 arg2 arg3*) *s regs*
unfolding *execute-CLC-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CJALR*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-reg*s *s* **and** *CapRegs-names* \subseteq *accessible-reg*s *s*
shows *traces-enabled* (*execute-CJALR* *arg0 arg1 arg2*) *s regs*
unfolding *execute-CJALR-def bind-assoc*
by (*traces-enabledI assms: assms simp: Run-inv-read-reg-PCC-not-sealed*)

lemma *incCapOffset-derivable-caps*[*derivable-capsE*]:
assumes *c'*: *incCapOffset* *c i* = (*success*, *c'*)

and $c: c \in \text{derivable-caps } s$ **and** $\text{noseal}: \text{Capability-tag } c \wedge i \neq 0 \longrightarrow \neg \text{Capability-sealed } c$
shows $c' \in \text{derivable-caps } s$
proof –
have $\text{leq-cap } CC \ c' \ c$ **if** $\text{Capability-tag } c$
using $\text{that } c'[\text{symmetric}] \text{ noseal}$
by $(\text{auto simp: leq-cap-def incCapOffset-def getCapTop-def getCapBase-def get-cap-perms-def})$
moreover have $\text{Capability-tag } c' \longleftrightarrow \text{Capability-tag } c$
using c'
by $(\text{auto simp: incCapOffset-def})$
ultimately show $?thesis$
using c
unfolding $\text{derivable-caps-def}$
by $(\text{auto elim: derivable.Restrict})$
qed

lemma $\text{traces-enabled-execute-CIncOffsetImmediate}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-CIncOffsetImmediate } \text{arg0 } \text{arg1 } \text{arg2}) \ s \ \text{regs}$
unfolding $\text{execute-CIncOffsetImmediate-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-execute-CIncOffset}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-CIncOffset } \text{arg0 } \text{arg1 } \text{arg2}) \ s \ \text{regs}$
unfolding $\text{execute-CIncOffset-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-execute-CGetType}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-CGetType } \text{arg0 } \text{arg1}) \ s \ \text{regs}$
unfolding $\text{execute-CGetType-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-execute-CGetTag}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-CGetTag } \text{arg0 } \text{arg1}) \ s \ \text{regs}$
unfolding $\text{execute-CGetTag-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-execute-CGetSealed}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{execute-CGetSealed } \text{arg0 } \text{arg1}) \ s \ \text{regs}$
unfolding $\text{execute-CGetSealed-def bind-assoc}$
by $(\text{traces-enabledI assms: assms})$

lemma $\text{traces-enabled-execute-CGetPerm}[\text{traces-enabledI}]$:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$

shows *traces-enabled* (*execute-CGetPerm* *arg0* *arg1*) *s* *regs*
unfolding *execute-CGetPerm-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CGetPCCSetOffset*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs* *s*
shows *traces-enabled* (*execute-CGetPCCSetOffset* *arg0* *arg1*) *s* *regs*
unfolding *execute-CGetPCCSetOffset-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms* *simp*: *Run-inv-read-reg-PCC-not-sealed*)

lemma *traces-enabled-execute-CGetPCC*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs* *s*
shows *traces-enabled* (*execute-CGetPCC* *arg0*) *s* *regs*
unfolding *execute-CGetPCC-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms* *simp*: *Run-inv-read-reg-PCC-not-sealed*)

lemma *traces-enabled-execute-CGetOffset*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs* *s* **and** *CapRegs-names* \subseteq *accessible-regs* *s*
shows *traces-enabled* (*execute-CGetOffset* *arg0* *arg1*) *s* *regs*
unfolding *execute-CGetOffset-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CGetLen*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs* *s* **and** *CapRegs-names* \subseteq *accessible-regs* *s*
shows *traces-enabled* (*execute-CGetLen* *arg0* *arg1*) *s* *regs*
unfolding *execute-CGetLen-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CGetFlags*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs* *s* **and** *CapRegs-names* \subseteq *accessible-regs* *s*
shows *traces-enabled* (*execute-CGetFlags* *arg0* *arg1*) *s* *regs*
unfolding *execute-CGetFlags-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CGetCause*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs* *s*
shows *traces-enabled* (*execute-CGetCause* *arg0*) *s* *regs*
unfolding *execute-CGetCause-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CGetCID*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs* *s*
shows *traces-enabled* (*execute-CGetCID* *arg0*) *s* *regs*
unfolding *execute-CGetCID-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CGetBase*[*traces-enabledI*]:
assumes {"PCC"} \subseteq *accessible-regs* *s* **and** *CapRegs-names* \subseteq *accessible-regs* *s*
shows *traces-enabled* (*execute-CGetBase* *arg0* *arg1*) *s* *regs*

unfolding *execute-CGetBase-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CGetAndAddr[traces-enabledI]:*
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled (execute-CGetAndAddr arg0 arg1 arg2) s regs*
unfolding *execute-CGetAndAddr-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CGetAddr[traces-enabledI]:*
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled (execute-CGetAddr arg0 arg1) s regs*
unfolding *execute-CGetAddr-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CFromPtr[traces-enabledI]:*
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled (execute-CFromPtr arg0 arg1 arg2) s regs*
unfolding *execute-CFromPtr-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *update-Capability-address-derivable-caps[derivable-capsI]:*
assumes $c \in \text{derivable-caps } s$ **and** $\neg \text{Capability-sealed } c$
shows $c(\text{Capability-address} := a) \in \text{derivable-caps } s$
proof –
have *leq-cap CC (c(Capability-address := a)) c*
using *assms*
by (*auto simp: leq-cap-def getCapBase-def getCapTop-def get-cap-perms-def*)
then show *?thesis*
using *assms*
by (*auto simp: derivable-caps-def elim: derivable.Restrict*)
qed

lemma *null-cap-not-sealed[simp, intro]: $\neg \text{Capability-sealed null-cap}$*
by (*auto simp: null-cap-def*)

declare *null-cap-derivable[derivable-capsI]*

lemma *traces-enabled-execute-CCopyType[traces-enabledI]:*
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled (execute-CCopyType arg0 arg1 arg2) s regs*
unfolding *execute-CCopyType-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CClearTag[traces-enabledI]:*
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled (execute-CClearTag arg0 arg1) s regs*
unfolding *execute-CClearTag-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-execute-CCheckType*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CCheckType* *arg0* *arg1*) *s* *regs*
 unfolding *execute-CCheckType-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CCheckTag*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CCheckTag* *arg0*) *s* *regs*
 unfolding *execute-CCheckTag-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CCheckPerm*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-CCheckPerm* *arg0* *arg1*) *s* *regs*
 unfolding *execute-CCheckPerm-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *set-next-pcc-invoked-caps*:
 assumes $cc \in \text{derivable-caps } s$
 and $\exists cd. cd \in \text{derivable-caps } s \wedge \text{invokable } CC \ cc \ cd$ **and** *invocation-traces*
 shows *traces-enabled* (*set-next-pcc* (*unsealCap* *cc*)) *s* *regs*
proof –
 have *leq-cap* *CC* (*unsealCap* *cc*) (*unseal* *CC* *cc* *True*)
 by (*auto simp*: *leq-cap-def* *unsealCap-def* *unseal-def* *getCapBase-def* *getCapTop-def* *get-cap-perms-def*)
 moreover obtain *cd*
 where $cc: cc \in \text{derivable } (\text{accessed-caps } s)$ **and** $cd: cd \in \text{derivable } (\text{accessed-caps } s)$
 and *Capability-tag* *cc* **and** *Capability-tag* *cd* **and** *invokable* *CC* *cc* *cd*
 using *assms*
 by (*auto simp*: *derivable-caps-def* *invokable-def* *get-cap-perms-def*)
 moreover have $cc \in \text{derivable } (\text{accessed-caps } (\text{run } s \ t))$ **and** $cd \in \text{derivable } (\text{accessed-caps } (\text{run } s \ t))$ **for** *t*
 using *cc* *cd* *derivable-mono*[*OF* *accessed-caps-run-mono*]
 by *auto*
 ultimately show *?thesis*
 unfolding *set-next-pcc-def*
 by (*intro* *traces-enabled-write-reg* *traces-enabledI* *preserves-invariantI* *traces-runs-preserve-invariantI*)
 (*auto simp* *add*: *NextPCC-ref-def* *DelayedPCC-ref-def* *derivable-caps-def* *intro*:
invocation-traces)
qed

lemma *write-reg-C26-invoked-caps*:
 assumes $cd \in \text{derivable-caps } s$
 and $\exists cc. cc \in \text{derivable-caps } s \wedge \text{invokable } CC \ cc \ cd$ **and** *invocation-traces*
 shows *traces-enabled* (*write-reg* *C26-ref* (*unsealCap* *cd*)) *s* *regs*
proof –

have *leq-cap CC* (*unsealCap cd*) (*unseal CC cd True*)
by (*auto simp: leq-cap-def unsealCap-def unseal-def getCapBase-def getCapTop-def get-cap-perms-def*)
moreover obtain *cc*
where *cc*: *cc* \in *derivable* (*accessed-caps s*) **and** *cd*: *cd* \in *derivable* (*accessed-caps s*)
and *Capability-tag cc* **and** *Capability-tag cd* **and** *invokable CC cc cd*
using *assms*
by (*auto simp: derivable-caps-def invokable-def get-cap-perms-def*)
ultimately show *?thesis*
by (*intro traces-enabled-write-reg*)
(auto simp: C26-ref-def derivable-caps-def intro: invocation-traces)
qed

lemma *getCapCursor-nonneg[simp]*: $0 \leq \text{getCapCursor } c$
by (*auto simp: getCapCursor-def*)

lemma *getCapTop-nonneg[simp]*: $0 \leq \text{getCapTop } c$
by (*auto simp: getCapTop-def*)

lemma *invokable-data-cap-derivable*:
assumes $\neg \text{Capability-permit-execute } cd$ **and** *Capability-permit-execute cc*
and *Capability-tag cd* **and** *Capability-tag cc*
and *Capability-sealed cd* **and** *Capability-sealed cc*
and *Capability-permit-ccall cd* **and** *Capability-permit-ccall cc*
and *Capability-otype cc = Capability-otype cd*
and *getCapBase cc* \leq *getCapCursor cc*
and *getCapCursor cc* $<$ *getCapTop cc*
and *cd* \in *derivable-caps s*
shows $\exists cd. cd \in \text{derivable-caps } s \wedge \text{invokable } CC \text{ } cc \text{ } cd$
using *assms getCapCursor-nonneg[of cc] getCapTop-nonneg[of cc]*
unfolding *le-less*
by (*auto simp: invokable-def nat-le-eq-zle get-cap-perms-def*)

lemma *invokable-code-cap-derivable*:
assumes $\neg \neg \text{Capability-permit-execute } cc$ **and** $\neg \text{Capability-permit-execute } cd$
and *Capability-tag cd* **and** *Capability-tag cc*
and *Capability-sealed cd* **and** *Capability-sealed cc*
and *Capability-permit-ccall cd* **and** *Capability-permit-ccall cc*
and *Capability-otype cc = Capability-otype cd*
and *getCapBase cc* \leq *getCapCursor cc*
and *getCapCursor cc* $<$ *getCapTop cc*
and *cc* \in *derivable-caps s*
shows $\exists cc. cc \in \text{derivable-caps } s \wedge \text{invokable } CC \text{ } cc \text{ } cd$
using *assms getCapCursor-nonneg[of cc] getCapTop-nonneg[of cc]*
unfolding *le-less*
by (*auto simp: invokable-def nat-le-eq-zle get-cap-perms-def*)

lemma *traces-enabled-execute-CCall[traces-enabledI]*:

assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
and *invocation-traces*
shows *traces-enabled* (*execute-CCall* *arg0 arg1 arg2*) *s regs*
unfolding *execute-CCall-def bind-assoc*
by (*traces-enabledI* *assms: assms intro: set-next-pcc-invoked-caps write-reg-C26-invoked-caps*
elim: invokable-data-cap-derivable invokable-code-cap-derivable)

lemma *traces-enabled-execute-CCSeal*[*traces-enabledI*]:
assumes $\{ "PCC" \} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CCSeal* *arg0 arg1 arg2*) *s regs*
unfolding *execute-CCSeal-def bind-assoc*
by (*traces-enabledI* *assms: assms*)

definition *perms-of-bits* :: $31 \text{ word} \Rightarrow \text{perms}$ **where**

perms-of-bits *p* =
 \langle *permit-ccall* $= p !! 8,$
 $\text{permit-execute} = p !! 1,$
 $\text{permit-load} = p !! 2,$
 $\text{permit-load-capability} = p !! 4,$
 $\text{permit-seal} = p !! 7,$
 $\text{permit-store} = p !! 3,$
 $\text{permit-store-capability} = p !! 5,$
 $\text{permit-store-local-capability} = p !! 6,$
 $\text{permit-system-access} = p !! 10,$
 $\text{permit-unseal} = p !! 9 \rangle$

definition *and-perms* :: $\text{perms} \Rightarrow \text{perms} \Rightarrow \text{perms}$ **where**

and-perms *p1 p2* =
 \langle *permit-ccall* $= \text{permit-ccall } p1 \wedge \text{permit-ccall } p2,$
 $\text{permit-execute} = \text{permit-execute } p1 \wedge \text{permit-execute } p2,$
 $\text{permit-load} = \text{permit-load } p1 \wedge \text{permit-load } p2,$
 $\text{permit-load-capability} = \text{permit-load-capability } p1 \wedge \text{permit-load-capability } p2,$
 $\text{permit-seal} = \text{permit-seal } p1 \wedge \text{permit-seal } p2,$
 $\text{permit-store} = \text{permit-store } p1 \wedge \text{permit-store } p2,$
 $\text{permit-store-capability} = \text{permit-store-capability } p1 \wedge \text{permit-store-capability } p2,$
 $\text{permit-store-local-capability} = \text{permit-store-local-capability } p1 \wedge \text{permit-store-local-capability } p2,$
 $\text{permit-system-access} = \text{permit-system-access } p1 \wedge \text{permit-system-access } p2,$
 $\text{permit-unseal} = \text{permit-unseal } p1 \wedge \text{permit-unseal } p2 \rangle$

lemma *setCapPerms-derivable-caps*[*derivable-capsI*]:

assumes $c \in \text{derivable-caps } s$ **and** $\text{Capability-tag } c \longrightarrow \text{leq-perms } (\text{perms-of-bits } p)$
 $(\text{get-cap-perms } c) \wedge \neg \text{Capability-sealed } c \wedge (p !! 0 \longrightarrow \text{Capability-global } c)$
shows $\text{setCapPerms } c \ p \in \text{derivable-caps } s$

proof –

have $\text{leq-cap } CC (\text{setCapPerms } c \ p) \ c$ **and** $\text{Capability-tag } (\text{setCapPerms } c \ p) =$

```

Capability-tag c
  using assms
  by (auto simp: setCapPerms-def leq-cap-def getCapBase-def getCapTop-def
perms-of-bits-def get-cap-perms-def)
  then show ?thesis
  using assms
  by (auto simp: derivable-caps-def elim!: derivable.Restrict)
qed

```

```

lemma bool-to-bits-nth[simp]: bool-to-bits b !! n  $\longleftrightarrow$  b  $\wedge$  n = 0
by (auto simp: bool-to-bits-def)

```

```

lemma perms-of-bits-getCapPerms-get-cap-perms[simp]:
perms-of-bits (getCapPerms c) = get-cap-perms c
by (auto simp: perms-of-bits-def getCapPerms-def getCapHardPerms-def get-cap-perms-def
test-bit-cat nth-ucast)

```

```

lemma getCapPerms-0th-iff-global[simp]:
getCapPerms c !! 0 = Capability-global c
by (auto simp: getCapPerms-def getCapHardPerms-def test-bit-cat nth-ucast)

```

```

lemma perms-of-bits-AND-and-perms[simp]:
perms-of-bits (x AND y) = and-perms (perms-of-bits x) (perms-of-bits y)
by (auto simp: perms-of-bits-def and-perms-def word-ao-nth)

```

```

lemma leq-perms-and-perms[simp, intro]:
leq-perms (and-perms p1 p2) p1
by (auto simp: leq-perms-def and-perms-def)

```

```

lemma traces-enabled-execute-CBuildCap[traces-enabledI]:
assumes  $\{ "PCC" \} \subseteq \text{accessible-regs } s$  and  $\text{CapRegs-names} \subseteq \text{accessible-regs } s$ 
shows traces-enabled (execute-CBuildCap arg0 arg1 arg2) s regs

```

proof –

```

  have [simp]:
    Capability-global c' = Capability-global c
    Capability-sealed c'  $\longleftrightarrow$  Capability-sealed c
    get-cap-perms c' = get-cap-perms c
    if setCapOffset c offset = (success, c') for c c' offset success
    using that
    by (auto simp: setCapOffset-def get-cap-perms-def)

```

```

  have [simp]:
    Capability-global c' = Capability-global c
    Capability-sealed c'  $\longleftrightarrow$  Capability-sealed c
    get-cap-perms c' = get-cap-perms c
    if setCapBounds c t b = (success, c') for c c' t b success
    using that
    by (auto simp: setCapBounds-def get-cap-perms-def)

```



```

have [simp]: Capability-global c  $\longrightarrow$  Capability-global c'
  if getCapPerms c AND getCapPerms c' = getCapPerms c for c c'
  unfolding getCapPerms-0th-iff-global[symmetric]
  by (subst that[symmetric]) (auto simp add: word-ao-nth simp del: getCapPerms-0th-iff-global)
have [elim]: leq-perms (get-cap-perms c) (get-cap-perms c')
  if getCapPerms c AND getCapPerms c' = getCapPerms c for c c'
  unfolding perms-of-bits-getCapPerms-get-cap-perms[symmetric]
  by (subst that[symmetric]) (auto simp add: leq-perms-def perms-of-bits-def
word-ao-nth)
show ?thesis
  unfolding execute-CBuildCap-def bind-assoc
  by (traces-enabledI assms: assms simp: getCapBase-def getCapTop-def)
qed

```

```

lemma traces-enabled-execute-CBZ[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CBZ arg0 arg1 arg2) s regs
  unfolding execute-CBZ-def bind-assoc
  by (traces-enabledI assms: assms)

```

```

lemma traces-enabled-execute-CBX[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CBX arg0 arg1 arg2) s regs
  unfolding execute-CBX-def bind-assoc
  by (traces-enabledI assms: assms)

```

```

lemma traces-enabled-execute-CAndPerm[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CAndPerm arg0 arg1 arg2) s regs
  unfolding execute-CAndPerm-def bind-assoc
  by (traces-enabledI assms: assms simp: word-ao-nth)

```

```

lemma traces-enabled-execute-CAndAddr[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CAndAddr arg0 arg1 arg2) s regs
  unfolding execute-CAndAddr-def bind-assoc
  by (traces-enabledI assms: assms)

```

```

lemma traces-enabled-execute-CACHE[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CACHE arg0 arg1 arg2) s regs
  unfolding execute-CACHE-def bind-assoc
  by (traces-enabledI assms: assms)

```

```

lemma traces-enabled-execute-BREAK[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-BREAK arg0) s regs
  unfolding execute-BREAK-def bind-assoc
  by (traces-enabledI assms: assms)

```

lemma *traces-enabled-execute-BEQ*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-BEQ* *arg0 arg1 arg2 arg3 arg4*) *s* *regs*
 unfolding *execute-BEQ-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-BCMPZ*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-BCMPZ* *arg0 arg1 arg2 arg3 arg4*) *s* *regs*
 unfolding *execute-BCMPZ-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-ADDI*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-ADDI* *arg0 arg1 arg2*) *s* *regs*
 unfolding *execute-ADDI-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-ADD*[*traces-enabledI*]:
 assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$
 shows *traces-enabled* (*execute-ADD* *arg0 arg1 arg2*) *s* *regs*
 unfolding *execute-ADD-def* *bind-assoc*
 by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-instr-sem*[*traces-enabledI*]:
 assumes $\{"CULR", "DDC", "PCC"\} \subseteq \text{accessible-regs } s$
 and *CapRegs-names* $\subseteq \text{accessible-regs } s$
 and *privileged-regs* *ISA* \cap *written-regs* *s* = {}
 and *invokes-caps* *ISA* *instr* [] \longrightarrow *invocation-traces*
 shows *traces-enabled* (*instr-sem* *ISA* *instr*) *s* *regs*
 by (*cases* *instr* *rule*: *execute.cases*; *simp*; *use nothing* **in** (*traces-enabledI* *assms*:
assms))

lemma *hasTrace-instr-reg-axioms*:
 assumes *hasTrace* *t* (*instr-sem* *ISA* *instr*)
 and *reads-regs-from* *inv-regs* *t* *regs* **and** *invariant* *regs*
 and *hasException* *t* (*instr-sem* *ISA* *instr*) \vee *hasFailure* *t* (*instr-sem* *ISA* *instr*)
 \longrightarrow *ex-traces*
 and *invokes-caps* *ISA* *instr* *t* \longrightarrow *invocation-traces*
 shows *store-cap-reg-axiom* *CC* *ISA* *ex-traces* *invocation-traces* *t*
 and *store-cap-mem-axiom* *CC* *ISA* *t*
 and *read-reg-axiom* *CC* *ISA* *ex-traces* *t*
 using *assms*
 by (*intro* *traces-enabled-reg-axioms* [**where** *m* = *instr-sem* *ISA* *instr* **and** *regs* =
regs] *traces-enabled-instr-sem*; *auto*)**+**

lemma *preserves-invariant-write-reg-PCC*[*preserves-invariantI*]:
 assumes *Capability-tag* *c* **and** \neg *Capability-sealed* *c*

```

shows traces-preserve-invariant (write-reg PCC-ref c)
using assms
unfolding traces-preserve-invariant-def trace-preserves-invariant-def
by (auto simp: write-reg-def register-defs elim: Write-reg-TracesE)

end

```

```

end
theory CHERI-MIPS-Mem-Axioms
imports CHERI-MIPS-Gen-Lemmas
begin

```

3.4 Memory access properties of instructions

```

context CHERI-MIPS-Mem-Automaton
begin

```

lemma *preserves-invariant-write-non-inv-regs*[*preserves-invariantI*]:

```

   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg BranchPending-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg C26-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CID-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0BadInstr-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0BadInstrP-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0BadVAddr-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0Cause-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0Compare-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0ConfigK0-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0Count-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0HWREna-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0LLAddr-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0LLBit-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CP0UserLocal-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CPLR-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CULR-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CapCause-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg CurrentInstrBits-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg DDC-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg DelayedPC-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg DelayedPCC-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg EPCC-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg ErrorEPCC-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg GPR-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg HI-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg InBranchDelay-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg KCC-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg KDC-ref } v)$ 
   $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg KR1C-ref } v)$ 

```

$\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } KR2C\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } LO\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } LastInstrBits\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } NextInBranchDelay\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } NextPC\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } NextPCC\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } PC\text{-ref } v)$

$\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } TLBEntryLo0\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } TLBEntryLo1\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } TLBIndex\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } TLBPageMask\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } TLBProbe\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } TLBRandom\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } TLBWired\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } UART\text{-RDATA}\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } UART\text{-RVALID}\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } UART\text{-WDATA}\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } UART\text{-WRITTEN}\text{-ref } v)$
 $\bigwedge v. \text{traces-preserve-invariant } (\text{write-reg } InstCount\text{-ref } v)$

unfolding BranchPending-ref-def C26-ref-def CID-ref-def CP0BadInstr-ref-def
CP0BadInstrP-ref-def
CP0BadVAddr-ref-def CP0Cause-ref-def CP0Compare-ref-def CP0ConfigK0-ref-def
CP0Count-ref-def
CP0HWRena-ref-def CP0LLAddr-ref-def CP0LLBit-ref-def CP0UserLocal-ref-def
CPLR-ref-def
CULR-ref-def CapCause-ref-def CurrentInstrBits-ref-def DDC-ref-def DelayedPC-ref-def
DelayedPCC-ref-def EPCC-ref-def ErrorEPCC-ref-def GPR-ref-def HI-ref-def
InBranchDelay-ref-def KCC-ref-def KDC-ref-def KR1C-ref-def KR2C-ref-def
LO-ref-def LastInstrBits-ref-def NextInBranchDelay-ref-def NextPC-ref-def NextPCC-ref-def
PC-ref-def PCC-ref-def TLBEntryLo0-ref-def TLBEntryLo1-ref-def TLBIndex-ref-def
TLBPageMask-ref-def TLBProbe-ref-def TLBRandom-ref-def TLBWired-ref-def
UART-RDATA-ref-def
UART-RVALID-ref-def UART-WDATA-ref-def UART-WRITTEN-ref-def InstCount-ref-def
by (intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg; simp
add: trans-regs-def)+

declare MemAccessType.split[**where** $P = \lambda m. \text{runs-preserve-invariant } m$, **THEN**
iffD2, preserves-invariantI]

lemma preserves-invariant-no-writes-to-inv-regs[preserves-invariantI]:
 $\bigwedge \text{arg0 arg1 arg2. traces-preserve-invariant } (MIPS\text{-write } \text{arg0 } \text{arg1 } \text{arg2})$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant } (MIPS\text{-read } \text{arg0 } \text{arg1})$
 $\bigwedge \text{arg0 arg1. name } \text{arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant } (\text{set-CauseReg-BD } \text{arg0 } \text{arg1})$
 $\bigwedge \text{arg0 arg1. name } \text{arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant } (\text{set-CauseReg-CE } \text{arg0 } \text{arg1})$
 $\bigwedge \text{arg0 arg1. name } \text{arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant } (\text{set-CauseReg-IV } \text{arg0 } \text{arg1})$

[illegible]

arg0 arg1)
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-capl1 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-pfn1 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-c1 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-d1 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-v1 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-caps0 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-capl0 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-pfn0 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-c0 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-d0 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-TLBEntry-v0 arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-CU arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-BEV arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-IM arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-KX arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-SX arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-UX arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-KSU arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-ERL arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-EXL arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-StatusReg-IE arg0 arg1)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (execute-branch-mips arg0)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (rGPR arg0)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (wGPR arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (MEMr arg0 arg1)}$
 $\bigwedge \text{arg0 arg1. traces-preserve-invariant (MEMr-reserve arg0 arg1)}$
 $\bigwedge \text{arg0. traces-preserve-invariant (MEM-sync arg0)}$

$\wedge \text{arg0 arg1. traces-preserve-invariant (MEMea arg0 arg1)}$
 $\wedge \text{arg0 arg1. traces-preserve-invariant (MEMea-conditional arg0 arg1)}$
 $\wedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMval arg0 arg1 arg2)}$
 $\wedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMval-conditional arg0 arg1 arg2)}$
 $\wedge \text{arg0. traces-preserve-invariant (exceptionVectorOffset arg0)}$
 $\wedge \text{arg0. traces-preserve-invariant (exceptionVectorBase arg0)}$
 $\wedge \text{arg0. traces-preserve-invariant (updateBadInstr arg0)}$
 $\wedge \text{arg0. traces-preserve-invariant (set-next-pcc arg0)}$
 $\wedge \text{arg0. traces-preserve-invariant (getAccessLevel arg0)}$
 $\wedge \text{arg0. traces-preserve-invariant (pcc-access-system-regs arg0)}$
 $\wedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-CapCauseReg-ExcCode arg0 arg1)}$
 $\wedge \text{arg0 arg1. name arg0} \notin \text{trans-regs} \implies \text{traces-preserve-invariant (set-CapCauseReg-RegNum arg0 arg1)}$
 $\wedge \text{arg0 arg1. traces-preserve-invariant (MEMr-wrapper arg0 arg1)}$
 $\wedge \text{arg0 arg1. traces-preserve-invariant (MEMr-reserve-wrapper arg0 arg1)}$
 $\wedge \text{arg0. traces-preserve-invariant (tlbSearch arg0)}$
 $\wedge \text{arg0 arg1. traces-preserve-invariant (capToString arg0 arg1)}$
 $\wedge \text{arg0. traces-preserve-invariant (execute-branch-pcc arg0)}$
 $\wedge \text{arg0. traces-preserve-invariant (ERETHook arg0)}$
 $\wedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMr-tagged arg0 arg1 arg2)}$
 $\wedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMr-tagged-reserve arg0 arg1 arg2)}$
 $\wedge \text{arg0 arg1 arg2 arg3. traces-preserve-invariant (MEMw-tagged arg0 arg1 arg2 arg3)}$
 $\wedge \text{arg0 arg1 arg2 arg3. traces-preserve-invariant (MEMw-tagged-conditional arg0 arg1 arg2 arg3)}$
 $\wedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMw-wrapper arg0 arg1 arg2)}$
 $\wedge \text{arg0 arg1 arg2. traces-preserve-invariant (MEMw-conditional-wrapper arg0 arg1 arg2)}$
 $\wedge \text{arg0. traces-preserve-invariant (get-CP0EPC arg0)}$
 $\wedge \text{arg0. traces-preserve-invariant (set-CP0EPC arg0)}$
 $\wedge \text{arg0. traces-preserve-invariant (get-CP0ErrorEPC arg0)}$
 $\wedge \text{arg0. traces-preserve-invariant (set-CP0ErrorEPC arg0)}$
by (intro no-reg-writes-traces-preserve-invariantI no-reg-writes-toI; simp add: trans-regs-def)+

lemma preserves-invariant-undefined-option[preserves-invariantI]:
shows runs-preserve-invariant (undefined-option arg0)
unfolding undefined-option-def bind-assoc
by (preserves-invariantI)

lemma preserves-invariant-undefined-exception[preserves-invariantI]:
shows runs-preserve-invariant (undefined-exception arg0)
unfolding undefined-exception-def bind-assoc
by (preserves-invariantI)

lemma preserves-invariant-undefined-CauseReg[preserves-invariantI]:
shows runs-preserve-invariant (undefined-CauseReg arg0)
unfolding undefined-CauseReg-def bind-assoc

by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-TLBEntryLoReg*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*undefined-TLBEntryLoReg* *arg0*)
 unfolding *undefined-TLBEntryLoReg-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-TLBEntryHiReg*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*undefined-TLBEntryHiReg* *arg0*)
 unfolding *undefined-TLBEntryHiReg-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-ContextReg*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*undefined-ContextReg* *arg0*)
 unfolding *undefined-ContextReg-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-XContextReg*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*undefined-XContextReg* *arg0*)
 unfolding *undefined-XContextReg-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-TLBEntry*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*undefined-TLBEntry* *arg0*)
 unfolding *undefined-TLBEntry-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-StatusReg*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*undefined-StatusReg* *arg0*)
 unfolding *undefined-StatusReg-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-Exception*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*undefined-Exception* *arg0*)
 unfolding *undefined-Exception-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-Capability*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-Capability* *arg0*)
unfolding *undefined-Capability-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-MemAccessType*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-MemAccessType* *arg0*)
unfolding *undefined-MemAccessType-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-AccessLevel*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-AccessLevel* *arg0*)
unfolding *undefined-AccessLevel-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-CapCauseReg*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-CapCauseReg* *arg0*)
unfolding *undefined-CapCauseReg-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *trans-regs-non-members*[*simp*]:
name CP0Cause-ref \notin *trans-regs*
name CapCause-ref \notin *trans-regs*
by (*auto simp: trans-regs-def CP0Cause-ref-def CapCause-ref-def*)

lemma *preserves-invariant-incrementCP0Count*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*incrementCP0Count* *arg0*)
unfolding *incrementCP0Count-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-decode-failure*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-decode-failure* *arg0*)
unfolding *undefined-decode-failure-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-Comparison*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-Comparison* *arg0*)
unfolding *undefined-Comparison-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-WordType*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-WordType* *arg0*)
unfolding *undefined-WordType-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-WordTypeUnaligned*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-WordTypeUnaligned* *arg0*)
unfolding *undefined-WordTypeUnaligned-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-TLBTranslate2*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*TLBTranslate2* *arg0* *arg1*)
unfolding *TLBTranslate2-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-TLBTranslateC*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*TLBTranslateC* *arg0* *arg1*)
unfolding *TLBTranslateC-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-TLBTranslate*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*TLBTranslate* *arg0* *arg1*)
unfolding *TLBTranslate-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-CPtrCmpOp*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-CPtrCmpOp* *arg0*)
unfolding *undefined-CPtrCmpOp-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-ClearRegSet*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-ClearRegSet* *arg0*)
unfolding *undefined-ClearRegSet-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-undefined-CapEx*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*undefined-CapEx* *arg0*)
unfolding *undefined-CapEx-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-set-CapCauseReg-bits*[*preserves-invariantI*]:
assumes *name* *arg0* \notin *trans-regs*
shows *runs-preserve-invariant* (*set-CapCauseReg-bits* *arg0* *arg1*)
using *assms*
unfolding *set-CapCauseReg-bits-def* *bind-assoc*
by (*preserves-invariantI*; *intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-write-reg*)

lemma *preserves-invariant-raise-c2-exception*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*raise-c2-exception* *arg0* *arg1*)
unfolding *raise-c2-exception-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-checkDDCPerms*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*checkDDCPerms* *arg0* *arg1*)
unfolding *checkDDCPerms-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-addrWrapper*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*addrWrapper* *arg0* *arg1* *arg2*)
unfolding *addrWrapper-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-addrWrapperUnaligned*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*addrWrapperUnaligned* *arg0* *arg1* *arg2*)
unfolding *addrWrapperUnaligned-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-branch*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-branch* *arg0*)
unfolding *execute-branch-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-TranslatePC*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*TranslatePC* *arg0*)
unfolding *TranslatePC-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-checkCP2usable*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*checkCP2usable* *arg0*)
unfolding *checkCP2usable-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-dump-cp2-state*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*dump-cp2-state* *arg0*)
unfolding *dump-cp2-state-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-XORI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-XORI* *arg0* *arg1* *arg2*)
unfolding *execute-XORI-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-XOR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-XOR* *arg0* *arg1* *arg2*)
unfolding *execute-XOR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-WAIT*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-WAIT* *arg0*)

unfolding *execute-WAIT-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-TRAPREG*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-TRAPREG* *arg0* *arg1* *arg2*)
unfolding *execute-TRAPREG-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-TRAPIMM*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-TRAPIMM* *arg0* *arg1* *arg2*)
unfolding *execute-TRAPIMM-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-bind-checkCP0Access*:
runs-preserve-invariant (*checkCP0Access* *u* \gg *m*)
using *Run-inv-checkCP0Access-False*
unfolding *Run-inv-def runs-preserve-invariant-def trace-preserves-invariant-def*
by (*auto simp: regstate-simp elim!: Run-bindE*)

lemma *preserves-invariant-execute-TLBWR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-TLBWR* *arg0*)
unfolding *execute-TLBWR-def bind-assoc*
by (*intro preserves-invariant-bind-checkCP0Access*)

lemma *preserves-invariant-execute-TLBWI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-TLBWI* *arg0*)
unfolding *execute-TLBWI-def bind-assoc*
by (*intro preserves-invariant-bind-checkCP0Access*)

lemma *preserves-invariant-execute-TLBR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-TLBR* *arg0*)
unfolding *execute-TLBR-def bind-assoc*
by (*intro preserves-invariant-bind-checkCP0Access*)

lemma *preserves-invariant-execute-TLBP*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-TLBP* *arg0*)
unfolding *execute-TLBP-def bind-assoc*
by (*intro preserves-invariant-bind-checkCP0Access*)

lemma *preserves-invariant-execute-Store*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-Store* *arg0* *arg1* *arg2* *arg3* *arg4*)
unfolding *execute-Store-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SYSCALL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SYSCALL* *arg0*)
unfolding *execute-SYSCALL-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SYNC*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SYNC* *arg0*)
unfolding *execute-SYNC-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SWR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SWR* *arg0* *arg1* *arg2*)
unfolding *execute-SWR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SWL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SWL* *arg0* *arg1* *arg2*)
unfolding *execute-SWL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SUBU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SUBU* *arg0* *arg1* *arg2*)
unfolding *execute-SUBU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SUB*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SUB* *arg0* *arg1* *arg2*)
unfolding *execute-SUB-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SRLV*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SRLV* *arg0* *arg1* *arg2*)
unfolding *execute-SRLV-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SRL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SRL* *arg0* *arg1* *arg2*)
unfolding *execute-SRL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SRAV*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SRAV* *arg0* *arg1* *arg2*)
unfolding *execute-SRAV-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SRA*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SRA* *arg0* *arg1* *arg2*)
unfolding *execute-SRA-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SLTU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SLTU* *arg0* *arg1* *arg2*)
unfolding *execute-SLTU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SLTIU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SLTIU* *arg0* *arg1* *arg2*)
unfolding *execute-SLTIU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SLTI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SLTI* *arg0* *arg1* *arg2*)
unfolding *execute-SLTI-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SLT*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SLT* *arg0* *arg1* *arg2*)
unfolding *execute-SLT-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SLLV*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SLLV* *arg0* *arg1* *arg2*)
unfolding *execute-SLLV-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SLL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SLL* *arg0* *arg1* *arg2*)
unfolding *execute-SLL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SDR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SDR* *arg0* *arg1* *arg2*)
unfolding *execute-SDR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-SDL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-SDL* *arg0* *arg1* *arg2*)
unfolding *execute-SDL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-RI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-RI* *arg0*)
unfolding *execute-RI-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-RDHWR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-RDHWR* *arg0* *arg1*)
unfolding *execute-RDHWR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-ORI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-ORI* *arg0* *arg1* *arg2*)
unfolding *execute-ORI-def* *bind-assoc*

by (*preserves-invariantI*)

lemma *preserves-invariant-execute-OR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-OR* *arg0* *arg1* *arg2*)
unfolding *execute-OR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-NOR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-NOR* *arg0* *arg1* *arg2*)
unfolding *execute-NOR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MULTU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MULTU* *arg0* *arg1*)
unfolding *execute-MULTU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MULT*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MULT* *arg0* *arg1*)
unfolding *execute-MULT-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MUL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MUL* *arg0* *arg1* *arg2*)
unfolding *execute-MUL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MTLO*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MTLO* *arg0*)
unfolding *execute-MTLO-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MTHI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MTHI* *arg0*)
unfolding *execute-MTHI-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MTC0*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MTC0* *arg0* *arg1* *arg2* *arg3*)
unfolding *execute-MTC0-def* *bind-assoc*
by (*intro preserves-invariant-bind-checkCP0Access*)

lemma *preserves-invariant-execute-MSUBU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MSUBU* *arg0* *arg1*)
unfolding *execute-MSUBU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MSUB*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MSUB* *arg0* *arg1*)

unfolding *execute-MSUB-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MOVZ*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MOVZ* *arg0* *arg1* *arg2*)
unfolding *execute-MOVZ-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MOVN*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MOVN* *arg0* *arg1* *arg2*)
unfolding *execute-MOVN-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MFLO*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MFLO* *arg0*)
unfolding *execute-MFLO-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MFHI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MFHI* *arg0*)
unfolding *execute-MFHI-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MFC0*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MFC0* *arg0* *arg1* *arg2* *arg3*)
unfolding *execute-MFC0-def bind-assoc*
by (*intro preserves-invariant-bind-checkCP0Access*)

lemma *preserves-invariant-execute-MADDU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MADDU* *arg0* *arg1*)
unfolding *execute-MADDU-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-MADD*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-MADD* *arg0* *arg1*)
unfolding *execute-MADD-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-Load*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-Load* *arg0* *arg1* *arg2* *arg3* *arg4* *arg5*)
unfolding *execute-Load-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-LWR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-LWR* *arg0* *arg1* *arg2*)
unfolding *execute-LWR-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-LWL*[*preserves-invariantI*]:

shows *runs-preserve-invariant* (*execute-LWL* *arg0* *arg1* *arg2*)
unfolding *execute-LWL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-LUI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-LUI* *arg0* *arg1*)
unfolding *execute-LUI-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-LDR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-LDR* *arg0* *arg1* *arg2*)
unfolding *execute-LDR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-LDL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-LDL* *arg0* *arg1* *arg2*)
unfolding *execute-LDL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-JR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-JR* *arg0*)
unfolding *execute-JR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-JALR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-JALR* *arg0* *arg1*)
unfolding *execute-JALR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-JAL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-JAL* *arg0*)
unfolding *execute-JAL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-J*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-J* *arg0*)
unfolding *execute-J-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-ERET*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-ERET* *arg0*)
unfolding *execute-ERET-def* *bind-assoc*
by (*intro preserves-invariant-bind-checkCP0Access*)

lemma *preserves-invariant-execute-DSUBU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSUBU* *arg0* *arg1* *arg2*)
unfolding *execute-DSUBU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSUB*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSUB* *arg0* *arg1* *arg2*)
unfolding *execute-DSUB-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSRLV*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSRLV* *arg0* *arg1* *arg2*)
unfolding *execute-DSRLV-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSRL32*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSRL32* *arg0* *arg1* *arg2*)
unfolding *execute-DSRL32-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSRL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSRL* *arg0* *arg1* *arg2*)
unfolding *execute-DSRL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSRAV*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSRAV* *arg0* *arg1* *arg2*)
unfolding *execute-DSRAV-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSRA32*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSRA32* *arg0* *arg1* *arg2*)
unfolding *execute-DSRA32-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSRA*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSRA* *arg0* *arg1* *arg2*)
unfolding *execute-DSRA-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSLLV*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSLLV* *arg0* *arg1* *arg2*)
unfolding *execute-DSLLV-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSLL32*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSLL32* *arg0* *arg1* *arg2*)
unfolding *execute-DSLL32-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DSLL*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DSLL* *arg0* *arg1* *arg2*)
unfolding *execute-DSLL-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DMULTU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DMULTU* *arg0* *arg1*)
unfolding *execute-DMULTU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DMULT*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DMULT* *arg0* *arg1*)
unfolding *execute-DMULT-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DIVU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DIVU* *arg0* *arg1*)
unfolding *execute-DIVU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DIV*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DIV* *arg0* *arg1*)
unfolding *execute-DIV-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DDIVU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DDIVU* *arg0* *arg1*)
unfolding *execute-DDIVU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DDIV*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DDIV* *arg0* *arg1*)
unfolding *execute-DDIV-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DADDU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DADDU* *arg0* *arg1* *arg2*)
unfolding *execute-DADDU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DADDIU*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DADDIU* *arg0* *arg1* *arg2*)
unfolding *execute-DADDIU-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DADDI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DADDI* *arg0* *arg1* *arg2*)
unfolding *execute-DADDI-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-DADD*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-DADD* *arg0* *arg1* *arg2*)
unfolding *execute-DADD-def* *bind-assoc*

by (preserves-invariantI)

lemma preserves-invariant-writeCapReg[preserves-invariantI]:
 shows traces-preserve-invariant (writeCapReg n v)
 by (intro no-reg-writes-traces-preserve-invariantI no-reg-writes-to-writeCapReg)
 (simp add: CapRegs-names-def trans-regs-def)

lemma preserves-invariant-execute-ClearRegs[preserves-invariantI]:
 shows runs-preserve-invariant (execute-ClearRegs arg0 arg1)
 unfolding execute-ClearRegs-def bind-assoc
 by (preserves-invariantI)

lemma preserves-invariant-execute-CWriteHwr[preserves-invariantI]:
 shows runs-preserve-invariant (execute-CWriteHwr arg0 arg1)
 unfolding execute-CWriteHwr-def bind-assoc
 by (preserves-invariantI)

lemma preserves-invariant-execute-CUnseal[preserves-invariantI]:
 shows runs-preserve-invariant (execute-CUnseal arg0 arg1 arg2)
 unfolding execute-CUnseal-def bind-assoc
 by (preserves-invariantI)

lemma preserves-invariant-execute-CToPtr[preserves-invariantI]:
 shows runs-preserve-invariant (execute-CToPtr arg0 arg1 arg2)
 unfolding execute-CToPtr-def bind-assoc
 by (preserves-invariantI)

lemma preserves-invariant-execute-CTestSubset[preserves-invariantI]:
 shows runs-preserve-invariant (execute-CTestSubset arg0 arg1 arg2)
 unfolding execute-CTestSubset-def bind-assoc
 by (preserves-invariantI)

lemma preserves-invariant-execute-CSub[preserves-invariantI]:
 shows runs-preserve-invariant (execute-CSub arg0 arg1 arg2)
 unfolding execute-CSub-def bind-assoc
 by (preserves-invariantI)

lemma preserves-invariant-execute-CStoreConditional[preserves-invariantI]:
 shows runs-preserve-invariant (execute-CStoreConditional arg0 arg1 arg2 arg3)
 unfolding execute-CStoreConditional-def bind-assoc
 by (preserves-invariantI)

lemma preserves-invariant-execute-CStore[preserves-invariantI]:
 shows runs-preserve-invariant (execute-CStore arg0 arg1 arg2 arg3 arg4)
 unfolding execute-CStore-def bind-assoc
 by (preserves-invariantI)

lemma preserves-invariant-execute-CSetOffset[preserves-invariantI]:
 shows runs-preserve-invariant (execute-CSetOffset arg0 arg1 arg2)

unfolding *execute-CSetOffset-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSetFlags*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CSetFlags* *arg0* *arg1* *arg2*)
unfolding *execute-CSetFlags-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSetCause*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CSetCause* *arg0*)
unfolding *execute-CSetCause-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSetCID*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CSetCID* *arg0*)
unfolding *execute-CSetCID-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSetBoundsImmediate*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CSetBoundsImmediate* *arg0* *arg1* *arg2*)
unfolding *execute-CSetBoundsImmediate-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSetBoundsExact*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CSetBoundsExact* *arg0* *arg1* *arg2*)
unfolding *execute-CSetBoundsExact-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSetBounds*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CSetBounds* *arg0* *arg1* *arg2*)
unfolding *execute-CSetBounds-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSetAddr*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CSetAddr* *arg0* *arg1* *arg2*)
unfolding *execute-CSetAddr-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSeal*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CSeal* *arg0* *arg1* *arg2*)
unfolding *execute-CSeal-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSCC*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CSCC* *arg0* *arg1* *arg2*)
unfolding *execute-CSCC-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CSC*[*preserves-invariantI*]:

shows *runs-preserve-invariant* (*execute-CSC* *arg0* *arg1* *arg2* *arg3*)
unfolding *execute-CSC-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CReturn*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CReturn* *arg0*)
unfolding *execute-CReturn-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CReadHwr*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CReadHwr* *arg0* *arg1*)
unfolding *execute-CReadHwr-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CRAP*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CRAP* *arg0* *arg1*)
unfolding *execute-CRAP-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CRAM*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CRAM* *arg0* *arg1*)
unfolding *execute-CRAM-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CPtrCmp*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CPtrCmp* *arg0* *arg1* *arg2* *arg3*)
unfolding *execute-CPtrCmp-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CMove*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CMove* *arg0* *arg1*)
unfolding *execute-CMove-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CMOVX*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CMOVX* *arg0* *arg1* *arg2* *arg3*)
unfolding *execute-CMOVX-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CLoadTags*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CLoadTags* *arg0* *arg1*)
unfolding *execute-CLoadTags-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CLoadLinked*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CLoadLinked* *arg0* *arg1* *arg2* *arg3*)
unfolding *execute-CLoadLinked-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CLoad*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CLoad* *arg0* *arg1* *arg2* *arg3* *arg4* *arg5*)
unfolding *execute-CLoad-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CLLC*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CLLC* *arg0* *arg1*)
unfolding *execute-CLLC-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CLCBI*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CLCBI* *arg0* *arg1* *arg2*)
unfolding *execute-CLCBI-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CLC*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CLC* *arg0* *arg1* *arg2* *arg3*)
unfolding *execute-CLC-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CJALR*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CJALR* *arg0* *arg1* *arg2*)
unfolding *execute-CJALR-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CIncOffsetImmediate*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CIncOffsetImmediate* *arg0* *arg1* *arg2*)
unfolding *execute-CIncOffsetImmediate-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CIncOffset*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CIncOffset* *arg0* *arg1* *arg2*)
unfolding *execute-CIncOffset-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetType*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetType* *arg0* *arg1*)
unfolding *execute-CGetType-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetTag*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetTag* *arg0* *arg1*)
unfolding *execute-CGetTag-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetSealed*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetSealed* *arg0* *arg1*)

unfolding *execute-CGetSealed-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetPerm*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetPerm* *arg0* *arg1*)
unfolding *execute-CGetPerm-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetPCCSetOffset*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetPCCSetOffset* *arg0* *arg1*)
unfolding *execute-CGetPCCSetOffset-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetPCC*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetPCC* *arg0*)
unfolding *execute-CGetPCC-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetOffset*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetOffset* *arg0* *arg1*)
unfolding *execute-CGetOffset-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetLen*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetLen* *arg0* *arg1*)
unfolding *execute-CGetLen-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetFlags*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetFlags* *arg0* *arg1*)
unfolding *execute-CGetFlags-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetCause*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetCause* *arg0*)
unfolding *execute-CGetCause-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetCID*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetCID* *arg0*)
unfolding *execute-CGetCID-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetBase*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetBase* *arg0* *arg1*)
unfolding *execute-CGetBase-def bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetAndAddr*[*preserves-invariantI*]:

shows *runs-preserve-invariant* (*execute-CGetAndAddr* *arg0* *arg1* *arg2*)
unfolding *execute-CGetAndAddr-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CGetAddr*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CGetAddr* *arg0* *arg1*)
unfolding *execute-CGetAddr-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CFromPtr*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CFromPtr* *arg0* *arg1* *arg2*)
unfolding *execute-CFromPtr-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CCopyType*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CCopyType* *arg0* *arg1* *arg2*)
unfolding *execute-CCopyType-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CClearTag*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CClearTag* *arg0* *arg1*)
unfolding *execute-CClearTag-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CCheckType*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CCheckType* *arg0* *arg1*)
unfolding *execute-CCheckType-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CCheckTag*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CCheckTag* *arg0*)
unfolding *execute-CCheckTag-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CCheckPerm*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CCheckPerm* *arg0* *arg1*)
unfolding *execute-CCheckPerm-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CCall*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CCall* *arg0* *arg1* *arg2*)
unfolding *execute-CCall-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CCSeal*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CCSeal* *arg0* *arg1* *arg2*)
unfolding *execute-CCSeal-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CBuildCap*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CBuildCap* *arg0* *arg1* *arg2*)
unfolding *execute-CBuildCap-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CBZ*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CBZ* *arg0* *arg1* *arg2*)
unfolding *execute-CBZ-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CBX*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CBX* *arg0* *arg1* *arg2*)
unfolding *execute-CBX-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CAndPerm*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CAndPerm* *arg0* *arg1* *arg2*)
unfolding *execute-CAndPerm-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-CAndAddr*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CAndAddr* *arg0* *arg1* *arg2*)
unfolding *execute-CAndAddr-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-checkCP0Access*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*checkCP0Access* *u*)
using *Run-inv-checkCP0Access-False*
unfolding *runs-preserve-invariant-def* *trace-preserves-invariant-def* *Run-inv-def*
by *auto*

lemma *preserves-invariant-execute-CACHE*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-CACHE* *arg0* *arg1* *arg2*)
unfolding *execute-CACHE-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-BREAK*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-BREAK* *arg0*)
unfolding *execute-BREAK-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-BEQ*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-BEQ* *arg0* *arg1* *arg2* *arg3* *arg4*)
unfolding *execute-BEQ-def* *bind-assoc*
by (*preserves-invariantI*)

lemma *preserves-invariant-execute-BCMPZ*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (*execute-BCMPZ* *arg0* *arg1* *arg2* *arg3* *arg4*)
unfolding *execute-BCMPZ-def* *bind-assoc*

by (*preserves-invariantI*)

lemma *preserves-invariant-execute-ANDI*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*execute-ANDI* *arg0* *arg1* *arg2*)
 unfolding *execute-ANDI-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-execute-AND*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*execute-AND* *arg0* *arg1* *arg2*)
 unfolding *execute-AND-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-execute-ADDU*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*execute-ADDU* *arg0* *arg1* *arg2*)
 unfolding *execute-ADDU-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-execute-ADDIU*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*execute-ADDIU* *arg0* *arg1* *arg2*)
 unfolding *execute-ADDIU-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-execute-ADDI*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*execute-ADDI* *arg0* *arg1* *arg2*)
 unfolding *execute-ADDI-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-execute-ADD*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*execute-ADD* *arg0* *arg1* *arg2*)
 unfolding *execute-ADD-def* *bind-assoc*
 by (*preserves-invariantI*)

lemma *preserves-invariant-execute*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*execute instr*)
 by (*cases instr rule: execute.cases; simp; preserves-invariantI*)

lemma *preserves-invariant-write-reg-PCC*[*preserves-invariantI*]:
traces-preserve-invariant (*write-reg PCC-ref v*)
 by (*auto simp: write-reg-def traces-preserve-invariant-def elim!: Write-reg-TracesE*)
 (*auto simp: trace-preserves-invariant-def trans-inv-def register-defs split: option.splits*)

lemma *preserves-invariant-cp2-next-pc*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*cp2-next-pc u*)
 unfolding *cp2-next-pc-def*
 by (*preserves-invariantI*)

lemma *preserves-invariant-fetch*[*preserves-invariantI*]:
 shows *runs-preserve-invariant* (*fetch u*)

```

    unfolding fetch-def
    by (preserves-invariantI)

end

context CHERI-MIPS-Mem-Instr-Automaton
begin

lemmas non-cap-exp-traces-enabled[traces-enabledI] = non-cap-expI[THEN non-cap-exp-traces-enabledI]

lemmas non-mem-exp-traces-enabled[traces-enabledI] = non-mem-expI[THEN non-mem-exp-traces-enabledI]

lemma notnotE[derivable-capsE]:
  assumes  $\neg\neg P$ 
  obtains  $P$ 
  using assms
  by blast

lemma getCapCursor-mod-pow2-64[simp]:
  getCapCursor c mod 18446744073709551616 = getCapCursor c
  using uint-idem[of Capability-address c]
  by (auto simp: getCapCursor-def)

lemma mem-val-is-local-cap-Capability-global[simp]:
  mem-val-is-local-cap CC ISA (mem-bytes-of-word (capToMemBits c)) tag  $\longleftrightarrow$ 
 $\neg$ Capability-global c  $\wedge$  tag  $\neq$  BU
  by (cases tag) (auto simp: mem-val-is-local-cap-def bind-eq-Some-conv)

declare cap-size-def[simp]

lemma access-enabled-Store[derivable-capsE]:
  assumes Capability-permit-store c
    and tag  $\neq$  B0  $\longrightarrow$  Capability-permit-store-cap c
    and mem-val-is-local-cap CC ISA v tag  $\wedge$  tag = B1  $\longrightarrow$  Capability-permit-store-local-cap
  c
    and Capability-tag c and  $\neg$ Capability-sealed c
    and paddr-in-mem-region c Store paddr sz
    and  $c \in$  derivable-caps s
    and tag = B0  $\vee$  tag = B1 and length v = sz
    and tag  $\neq$  B0  $\longrightarrow$  address-tag-aligned ISA paddr  $\wedge$  sz = tag-granule ISA
  shows access-enabled s Store paddr sz v tag
  using assms
  unfolding access-enabled-def authorises-access-def has-access-permission-def
  by (auto simp: get-cap-perms-def derivable-caps-def)

lemma access-enabled-Load[derivable-capsE]:
  assumes Capability-permit-load c
    and tag  $\neq$  B0  $\longrightarrow$  Capability-permit-load-cap c

```

```

and Capability-tag c and  $\neg$ Capability-sealed c
and paddr-in-mem-region c Load paddr sz
and  $c \in \text{derivable-caps } s$ 
and  $\text{tag} \neq B0 \longrightarrow \text{address-tag-aligned } ISA \text{ paddr} \wedge \text{sz} = \text{tag-granule } ISA$ 
shows access-enabled s Load paddr sz v tag
using assms
unfolding access-enabled-def authorises-access-def has-access-permission-def
by (auto simp: get-cap-perms-def derivable-caps-def)

lemma [simp]: isa.translate-address ISA = translate-address
by (auto simp: ISA-def)

fun acctype-of-bool where
  acctype-of-bool True = LoadData
| acctype-of-bool False = StoreData

lemma Run-raise-c2-exception-False[simp]:
  Run (raise-c2-exception ex r) t a  $\longleftrightarrow$  False
  Run-inv (raise-c2-exception ex r) t a regs  $\longleftrightarrow$  False
unfolding Run-inv-def
by (auto simp: raise-c2-exception-def raise-c2-exception8-def elim!: Run-bindE)

lemma Run-if-then-raise-c2-exception-else[simp]:
  Run (if c then raise-c2-exception ex r else m) t a  $\longleftrightarrow$   $\neg c \wedge \text{Run } m \text{ } t \text{ } a$ 
  Run-inv (if c then raise-c2-exception ex r else m) t a regs  $\longleftrightarrow$   $\neg c \wedge \text{Run-inv } m$ 
t a regs
by auto

lemma no-translation-tables[simp]: translation-tables ISA t = {}
by (auto simp: ISA-def)

lemma Run-read-reg-DDC-derivable-caps:
  assumes Run (read-reg DDC-ref) t c and  $\{"DDC"\} \subseteq \text{accessible-regs } s$ 
shows  $c \in \text{derivable-caps } (\text{run } s \text{ } t)$ 
using assms
by (auto elim!: Run-read-regE simp: DDC-ref-def derivable-caps-def intro!: derivable.Copy)

abbreviation empty-trace :: register-value trace where empty-trace  $\equiv []$ 

lemma Run-inv-addrWrapper-access-enabled[derivable-capsE]:
  assumes Run-inv (addrWrapper addr acctype width) t vaddr regs
  and translate-address (unat vaddr) acctype' empty-trace = Some paddr
  and  $\{"DDC'\} \subseteq \text{accessible-regs } s$ 
  and acctype = MemAccessType-of-acctype acctype'
  and acctype' = Store  $\longrightarrow \text{length } v = \text{nat } sz$ 
  and  $sz = \text{wordWidthBytes } width$ 
shows access-enabled (run s t) acctype' paddr (nat sz) v B0
using assms

```

```

unfolding Run-inv-def addrWrapper-def checkDDCPerms-def Let-def
unfolding access-enabled-def authorises-access-def has-access-permission-def paddr-in-mem-region-def
apply (cases acctype')
apply (auto elim!: Run-bindE simp: get-cap-perms-def getCapBounds-def address-range-def
derivable-caps-def dest!: Run-read-reg-DDC-derivable-caps)
subgoal for c
apply (rule beXI[where x = c])
apply (clarify)
apply (rule exI[where x = unat vaddr])
by auto
subgoal for c
apply (rule beXI[where x = c])
apply (clarify)
apply (rule exI[where x = unat vaddr])
by auto
done

```

```

lemma Run-read-reg-DDC-access-enabled:
assumes Run (read-reg DDC-ref) t c
and  $\{\text{"DDC"}\} \subseteq \text{accessible-regs } s$ 
and Capability-tag c and  $\neg$ Capability-sealed c
and paddr-in-mem-region c acctype paddr sz
and acctype = Store  $\longrightarrow$  length v = nat sz
and acctype = Load  $\wedge$  Capability-permit-load c  $\vee$  acctype = Store  $\wedge$  Capability-permit-store
c
shows access-enabled (run s t) acctype paddr sz v B0
using assms
unfolding access-enabled-def authorises-access-def has-access-permission-def
by (auto simp: get-cap-perms-def derivable-caps-def dest!: Run-read-reg-DDC-derivable-caps)

```

```

lemma translate-address-paddr-in-mem-region:
assumes translate-address (nat vaddr) is-load empty-trace = Some paddr
and getCapBase c  $\leq$  vaddr and  $vaddr + sz \leq$  getCapTop c
and 0  $\leq$  vaddr
shows paddr-in-mem-region c is-load paddr (nat sz)
using assms
unfolding paddr-in-mem-region-def
by (intro exI[where x = nat vaddr])
(auto simp: paddr-in-mem-region-def address-range-def simp flip: nat-add-distrib)

```

```

lemma pos-mod-le[simp]:
 $0 < b \implies a \bmod b \leq (b :: \text{int})$ 
by (auto simp: le-less)

```

```

lemma mod-diff-mod-eq:
fixes a b c :: int
assumes c dvd b and 0 < b and 0 < c
shows (a mod b - a mod c) mod b = a mod b - a mod c
using assms

```

```

apply (auto simp: dvd-def)
by (smt Divides.pos-mod-bound assms(1) int-mod-eq' mod-mod-cancel unique-euclidean-semiring-numeral-cla

lemma mod-le-dvd-divisor:
  fixes a b c :: int
  assumes c dvd b and 0 < b and 0 < c
  shows a mod c ≤ a mod b
  using assms
  apply (auto simp: dvd-def)
  by (metis assms(1) assms(2) mod-mod-cancel pos-mod-conj zmod-le-nonneg-dividend)

lemma Run-inv-addrWrapperUnaligned-access-enabled[derivable-capsE]:
  assumes Run-inv (addrWrapperUnaligned addr acctype width) t (vaddr, sz) regs
    and translate-address (unat vaddr) acctype' empty-trace = Some paddr
    and {"DDC"} ⊆ accessible-regs s
    and acctype = MemAccessType-of-acctype acctype'
    and acctype' = Store ⟶ length v = nat sz
  shows access-enabled (run s t) acctype' paddr (nat sz) v B0
  using assms
  unfolding Run-inv-def addrWrapperUnaligned-def unalignedBytesTouched-def checkDDCPerms-def
  Let-def
  by (cases width; cases acctype';
    auto elim!: Run-bindE Run-read-reg-DDC-access-enabled translate-address-paddr-in-mem-region
    simp: getCapBounds-def mod-mod-cancel mod-diff-mod-eq mod-le-dvd-divisor)

lemma access-enabled-run-mono:
  assumes access-enabled s is-load paddr sz v tag
  shows access-enabled (run s t) is-load paddr sz v tag
  using assms derivable-mono[OF accessed-caps-run-mono[where s = s and t =
t]]
  unfolding access-enabled-def
  by blast

declare Run-inv-addrWrapperUnaligned-access-enabled[THEN access-enabled-run-mono,
derivable-capsE]

lemma TLBTranslateC-translate-address-eq[simp]:
  assumes Run-inv (TLBTranslateC vaddr acctype) t (paddr, noStoreCap) regs
    and acctype = MemAccessType-of-acctype acctype'
  shows translate-address (unat vaddr) acctype' t' = Some (unat paddr)
proof –
  from assms have Run-inv (translate-addressM (unat vaddr) acctype') t (unat
paddr) regs
    unfolding translate-addressM-def TLBTranslate-def bind-assoc Run-inv-def
  by (auto simp flip: uint-nat intro: Traces-bindI[of - t - [], simplified])
  then show ?thesis

```

using *determ-runs-translate-addressM*
 by (auto simp: *translate-address-def determ-the-result-eq*)
 qed

lemma *TLBTranslate-translate-address-eq[simp]*:
 assumes *Run-inv* (*TLBTranslate vaddr acctype*) *t paddr regs*
 and *acctype* = *MemAccessType-of-acctype acctype'*
 shows *translate-address* (*unat vaddr*) *acctype'* *t'* = *Some* (*unat paddr*)
proof –
 from *assms* have *Run-inv* (*translate-addressM* (*unat vaddr*) *acctype'*) *t* (*unat paddr*) *regs*
 unfolding *translate-addressM-def bind-assoc Run-inv-def*
 by (auto simp flip: *uint-nat intro: Traces-bindI[of - t - [], simplified]*)
 then show ?thesis
 using *determ-runs-translate-addressM*
 by (auto simp: *translate-address-def determ-the-result-eq*)
 qed

lemma *traces-enabled-bind-prod-split[traces-enabled-combinatorI]*:
 assumes $\bigwedge t a b. \text{Run-inv } m \ t \ (a, b) \ \text{regs} \implies \text{traces-enabled } (f \ a \ b) \ (\text{run } s \ t)$
 (the (*updates-regs trans-regs t regs*))
 and *runs-preserve-invariant m* and *traces-enabled m s regs*
 shows *traces-enabled* (*m* \gg ($\lambda \text{vars}. \text{let } (a, b) = \text{vars in } f \ a \ b$)) *s regs*
 using *assms*
 by (auto intro: *traces-enabled-bind*)

lemma *TLBTranslate-paddr-in-mem-region[derivable-capsE]*:
 assumes *Run-inv* (*TLBTranslate vaddr acctype*) *t paddr regs*
 and *getCapBase c* \leq *uint vaddr* and *uint vaddr* + *sz* \leq *getCapTop c* and *0*
 \leq *sz*
 and *acctype* = *MemAccessType-of-acctype acctype'*
 shows *paddr-in-mem-region c acctype'* (*unat paddr*) (*nat sz*)
 using *assms TLBTranslate-translate-address-eq[OF assms(1), where t' = []]*
 unfolding *paddr-in-mem-region-def*
 by (intro exI[where *x* = *unat vaddr*])
 (auto simp add: *address-range-def unat-def simp flip: nat-add-distrib*)

lemma *TLBTranslateC-paddr-in-mem-region[derivable-capsE]*:
 assumes *Run-inv* (*TLBTranslateC vaddr acctype*) *t (paddr, noStoreCap) regs*
 and *getCapBase c* \leq *uint vaddr* and *uint vaddr* + *sz* \leq *getCapTop c* and *0*
 \leq *sz*
 and *acctype* = *MemAccessType-of-acctype acctype'*
 shows *paddr-in-mem-region c acctype'* (*unat paddr*) (*nat sz*)
 using *assms TLBTranslateC-translate-address-eq[OF assms(1), where t' = []]*
 unfolding *paddr-in-mem-region-def*
 by (intro exI[where *x* = *unat vaddr*])
 (auto simp add: *address-range-def unat-def simp flip: nat-add-distrib*)

lemma *non-cap-exp-MEMea*[*non-cap-expI*]:
non-cap-exp (*MEMea addr sz*)
unfolding *MEMea-def write-mem-ea-def maybe-fail-def*
by (*auto simp: non-cap-exp-def elim: Traces-cases*)

lemma *non-cap-exp-MEMea-conditional*[*non-cap-expI*]:
non-cap-exp (*MEMea-conditional addr sz*)
unfolding *MEMea-conditional-def write-mem-ea-def maybe-fail-def*
by (*auto simp: non-cap-exp-def elim: Traces-cases*)

lemma *traces-enabled-write-mem-ea*[*traces-enabledI*]:
shows *traces-enabled* (*write-mem-ea BC-mword wk addr-sz addr sz*) *s regs*
by (*auto simp: write-mem-ea-def maybe-fail-def traces-enabled-def split: option.splits*
elim: Traces-cases)

lemma *traces-enabled-write-mem*[*traces-enabledI*]:
assumes *access-enabled s Store (unat addr) (nat sz) (mem-bytes-of-word v) B0*
shows *traces-enabled* (*write-mem BC-mword BC-mword wk addr-sz addr sz v*) *s*
regs
using *assms*
by (*auto simp: write-mem-def traces-enabled-def split: option.splits elim: Traces-cases*)

lemma *traces-enabled-write-memt*[*traces-enabledI*]:
assumes *access-enabled s Store (unat addr) (nat sz) (mem-bytes-of-word v) tag*
shows *traces-enabled* (*write-memt BC-mword BC-mword wk addr sz v tag*) *s regs*
using *assms*
by (*auto simp: write-memt-def traces-enabled-def split: option.splits elim: Traces-cases*)

lemma *traces-enabled-read-mem-bytes*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes. } \text{access-enabled } s \text{ Load } (\text{unat } \text{addr}) (\text{nat } \text{sz}) \text{ bytes } B0$
shows *traces-enabled* (*read-mem-bytes BC-mword BC-mword rk addr sz*) *s regs*
using *assms*
by (*auto simp: read-mem-bytes-def maybe-fail-def traces-enabled-def split: option.splits elim: Traces-cases*)

lemma *traces-enabled-read-mem*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes. } \text{access-enabled } s \text{ Load } (\text{unat } \text{addr}) (\text{nat } \text{sz}) \text{ bytes } B0$
shows *traces-enabled* (*read-mem BC-mword BC-mword rk addr-sz addr sz*) *s regs*
unfolding *read-mem-def*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-read-memt-bytes*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes tag. } \text{access-enabled } s \text{ Load } (\text{unat } \text{addr}) (\text{nat } \text{sz}) \text{ bytes tag}$
shows *traces-enabled* (*read-memt-bytes BC-mword BC-mword rk addr sz*) *s regs*

using *assms*
by (*auto simp: read-memt-bytes-def maybe-fail-def traces-enabled-def split: option.splits elim: Traces-cases*)

lemma *traces-enabled-read-memt*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes tag. access-enabled } s \text{ Load (unat addr) (nat sz) bytes tag}$
shows *traces-enabled* (*read-memt BC-mword BC-mword rk addr sz*) *s regs*
unfolding *read-memt-def*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-MEMea*[*traces-enabledI*]:
shows *traces-enabled* (*MEMea arg0 arg1*) *s regs*
unfolding *MEMea-def bind-assoc*
by (*traces-enabledI*)

lemma *traces-enabled-MEMea-conditional*[*traces-enabledI*]:
shows *traces-enabled* (*MEMea-conditional arg0 arg1*) *s regs*
unfolding *MEMea-conditional-def bind-assoc*
by (*traces-enabledI*)

lemma *traces-enabled-MEMval*[*traces-enabledI*]:
assumes *access-enabled s Store (unat addr) (nat sz) (mem-bytes-of-word v) B0*
shows *traces-enabled* (*MEMval addr sz v*) *s regs*
unfolding *MEMval-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-MEMr*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes. access-enabled } s \text{ Load (unat addr) (nat sz) bytes B0}$
shows *traces-enabled* (*MEMr addr sz*) *s regs*
unfolding *MEMr-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-MIPS-write*[*traces-enabledI*]:
assumes *access-enabled s Store (unat addr) (nat sz) (mem-bytes-of-word v) B0*
shows *traces-enabled* (*MIPS-write addr sz v*) *s regs*
unfolding *MIPS-write-def write-ram-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-MIPS-read*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes. access-enabled } s \text{ Load (unat addr) (nat sz) bytes B0}$
shows *traces-enabled* (*MIPS-read addr sz*) *s regs*
unfolding *MIPS-read-def read-ram-def bind-assoc*
by (*traces-enabledI assms: assms*)

lemma *traces-enabled-MEMr-reserve*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes. access-enabled } s \text{ Load (unat addr) (nat sz) bytes B0}$
shows *traces-enabled* (*MEMr-reserve addr sz*) *s regs*
unfolding *MEMr-reserve-def bind-assoc*

by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMval-conditional*[*traces-enabledI*]:
assumes *access-enabled s Store (unat addr) (nat sz) (mem-bytes-of-word v) B0*
shows *traces-enabled (MEMval-conditional addr sz v) s regs*
unfolding *MEMval-conditional-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMr-wrapper*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes. } \text{access-enabled } s \text{ Load } (\text{unat } \text{addr}) (\text{nat } \text{sz}) \text{ bytes } B0$
shows *traces-enabled (MEMr-wrapper addr sz) s regs*
unfolding *MEMr-wrapper-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMr-reserve-wrapper*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes. } \text{access-enabled } s \text{ Load } (\text{unat } \text{addr}) (\text{nat } \text{sz}) \text{ bytes } B0$
shows *traces-enabled (MEMr-reserve-wrapper addr sz) s regs*
unfolding *MEMr-reserve-wrapper-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMr-tagged*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes } \text{tag. } \text{tag} \neq B0 \longrightarrow \text{allow-tag} \implies \text{access-enabled } s \text{ Load } (\text{unat } \text{addr}) (\text{nat } \text{sz}) \text{ bytes } \text{tag}$
shows *traces-enabled (MEMr-tagged addr sz allow-tag) s regs*
unfolding *MEMr-tagged-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMr-tagged-reserve*[*traces-enabledI*]:
assumes $\bigwedge \text{bytes } \text{tag. } \text{tag} \neq B0 \longrightarrow \text{allow-tag} \implies \text{access-enabled } s \text{ Load } (\text{unat } \text{addr}) (\text{nat } \text{sz}) \text{ bytes } \text{tag}$
shows *traces-enabled (MEMr-tagged-reserve addr sz allow-tag) s regs*
unfolding *MEMr-tagged-reserve-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMw-tagged*[*traces-enabledI*]:
assumes *access-enabled s Store (unat addr) (nat sz) (mem-bytes-of-word v)*
(bitU-of-bool tag)
shows *traces-enabled (MEMw-tagged addr sz tag v) s regs*
unfolding *MEMw-tagged-def MEMval-tagged-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMw-tagged-conditional*[*traces-enabledI*]:
assumes *access-enabled s Store (unat addr) (nat sz) (mem-bytes-of-word v)*
(bitU-of-bool tag)
shows *traces-enabled (MEMw-tagged-conditional addr sz tag v) s regs*
unfolding *MEMw-tagged-conditional-def MEMval-tagged-conditional-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMw-wrapper*[*traces-enabledI*]:

assumes *access-enabled s Store (unat addr) (nat sz) (mem-bytes-of-word v) B0*
shows *traces-enabled (MEMw-wrapper addr sz v) s regs*
unfolding *MEMw-wrapper-def bind-assoc*
by *(traces-enabledI assms: assms)*

lemma *traces-enabled-MEMw-conditional-wrapper[traces-enabledI]:*
assumes *access-enabled s Store (unat addr) (nat sz) (mem-bytes-of-word v) B0*
shows *traces-enabled (MEMw-conditional-wrapper addr sz v) s regs*
unfolding *MEMw-conditional-wrapper-def bind-assoc*
by *(traces-enabledI assms: assms)*

declare *Run-inv-addrWrapper-access-enabled[THEN access-enabled-run-mono, derivable-capsE]*

lemma *traces-enabled-execute-Store[traces-enabledI]:*
assumes *{"DDC", "PCC"} ⊆ accessible-regs s*
shows *traces-enabled (execute-Store arg0 arg1 arg2 arg3 arg4) s regs*
unfolding *execute-Store-def bind-assoc*
by *(traces-enabledI assms: assms)*

lemma *traces-enabled-execute-SWR[traces-enabledI]:*
assumes *{"DDC", "PCC"} ⊆ accessible-regs s*
shows *traces-enabled (execute-SWR arg0 arg1 arg2) s regs*
unfolding *execute-SWR-def bind-assoc*
by *(traces-enabledI assms: assms)*

lemma *traces-enabled-execute-SWL[traces-enabledI]:*
assumes *{"DDC", "PCC"} ⊆ accessible-regs s*
shows *traces-enabled (execute-SWL arg0 arg1 arg2) s regs*
unfolding *execute-SWL-def bind-assoc*
by *(traces-enabledI assms: assms)*

lemma *traces-enabled-execute-SDR[traces-enabledI]:*
assumes *{"DDC", "PCC"} ⊆ accessible-regs s*
shows *traces-enabled (execute-SDR arg0 arg1 arg2) s regs*
unfolding *execute-SDR-def bind-assoc*
by *(traces-enabledI assms: assms)*

lemma *traces-enabled-execute-SDL[traces-enabledI]:*
assumes *{"DDC", "PCC"} ⊆ accessible-regs s*
shows *traces-enabled (execute-SDL arg0 arg1 arg2) s regs*
unfolding *execute-SDL-def bind-assoc*
by *(traces-enabledI assms: assms)*

lemma *traces-enabled-execute-Load[traces-enabledI]:*
assumes *{"DDC", "PCC"} ⊆ accessible-regs s*
shows *traces-enabled (execute-Load arg0 arg1 arg2 arg3 arg4 arg5) s regs*

unfolding *execute-Load-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-LWR*[*traces-enabledI*]:
assumes $\{"DDC", "PCC"\} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-LWR* *arg0* *arg1* *arg2*) *s* *regs*
unfolding *execute-LWR-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-LWL*[*traces-enabledI*]:
assumes $\{"DDC", "PCC"\} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-LWL* *arg0* *arg1* *arg2*) *s* *regs*
unfolding *execute-LWL-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-LDR*[*traces-enabledI*]:
assumes $\{"DDC", "PCC"\} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-LDR* *arg0* *arg1* *arg2*) *s* *regs*
unfolding *execute-LDR-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-LDL*[*traces-enabledI*]:
assumes $\{"DDC", "PCC"\} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-LDL* *arg0* *arg1* *arg2*) *s* *regs*
unfolding *execute-LDL-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CStoreConditional*[*traces-enabledI*]:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CStoreConditional* *arg0* *arg1* *arg2* *arg3*) *s* *regs*
unfolding *execute-CStoreConditional-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CStore*[*traces-enabledI*]:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CStore* *arg0* *arg1* *arg2* *arg3* *arg4*) *s* *regs*
unfolding *execute-CStore-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CSCC*[*traces-enabledI*]:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CSCC* *arg0* *arg1* *arg2*) *s* *regs*
unfolding *execute-CSCC-def bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-execute-CSC*[*traces-enabledI*]:
assumes $\{"PCC"\} \subseteq \text{accessible-regs } s$ **and** $\text{CapRegs-names} \subseteq \text{accessible-regs } s$
shows *traces-enabled* (*execute-CSC* *arg0* *arg1* *arg2* *arg3*) *s* *regs*
unfolding *execute-CSC-def bind-assoc*

```

by (traces-enabledI assms: assms)

declare traces-enabled-foreachM-inv[where  $P = \lambda vars\ s\ regs.$  True, simplified,
traces-enabledI]
thm traces-enabled-foreachM-inv[where  $s = s$  and  $P = \lambda vars\ s'\ regs'.$  derivable-caps
 $s \subseteq derivable-caps\ s'$  for  $s$ ]

lemma uint-cacheline-plus-cap-size:
  assumes getCapCursor  $c = 128 * q$  and  $0 \leq x$  and  $x \leq 3$ 
  shows uint (to-bits 64 128 * to-bits 64  $q + (word-of-int\ (x * 32) :: 64\ word)$ )
= 128 *  $q + x * 32$ 
proof -
  have  $128 * q < 2^{64}$  and *:  $0 \leq 128 * q$ 
  using uint-bounded[of Capability-address  $c$ ]
  unfolding assms(1)[symmetric] getCapCursor-def
  by (auto)
  moreover have  $0 \leq q$ 
  using *
  by auto
  ultimately show ?thesis
  using assms
  by (auto simp: uint-word-ariths getCapCursor-def uint-word-of-int)
qed

lemma traces-enabled-execute-CLoadTags[traces-enabledI]:
  assumes {"PCC"}  $\subseteq accessible-regs\ s$  and CapRegs-names  $\subseteq accessible-regs\ s$ 
  shows traces-enabled (execute-CLoadTags  $arg0\ arg1$ )  $s\ regs$ 
  unfolding execute-CLoadTags-def bind-assoc
  apply (traces-enabledI-with  $\langle - \rangle$  intro: traces-enabled-foreachM-inv[where  $s = s$ 
and  $P = \lambda vars\ s'\ regs'.$  derivable-caps  $s \subseteq derivable-caps\ s'$  for  $s$ ])
  apply (derivable-caps-step)
  apply (derivable-caps-step)
  apply (derivable-caps-step)
  apply (derivable-caps-step)
  apply (derivable-caps-step)
  apply (auto)[]
  apply (auto)[]
  apply (auto)[]

  apply (derivable-caps-step)
  apply (auto simp: caps-per-cacheline-def uint-cacheline-plus-cap-size)[]
  apply (auto simp: caps-per-cacheline-def uint-cacheline-plus-cap-size)[]
  apply (auto simp: caps-per-cacheline-def)[]
  apply (auto simp: caps-per-cacheline-def)[]

  apply (derivable-caps-step)

```

```

apply (elim set-mp)
apply (derivable-capsI assms: assms)[]
apply (auto simp: caps-per-cacheline-def)[]

apply (elim subset-trans)
apply (intro derivable-caps-run-mono)
apply (auto simp: caps-per-cacheline-def)[]
done

lemma traces-enabled-execute-CLoadLinked[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CLoadLinked arg0 arg1 arg2 arg3) s regs
  unfolding execute-CLoadLinked-def bind-assoc
  by (traces-enabledI assms: assms)

lemma [simp]: integerOfString "18446744073709551616" = 18446744073709551616
  by eval

lemma traces-enabled-execute-CLoad[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CLoad arg0 arg1 arg2 arg3 arg4 arg5) s regs
  unfolding execute-CLoad-def bind-assoc
  by (traces-enabledI assms: assms)

lemma traces-enabled-execute-CLLC[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CLLC arg0 arg1) s regs
  unfolding execute-CLLC-def bind-assoc
  by (traces-enabledI assms: assms)

lemma traces-enabled-execute-CLCBI[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CLCBI arg0 arg1 arg2) s regs
  unfolding execute-CLCBI-def bind-assoc
  by (traces-enabledI assms: assms)

lemma traces-enabled-execute-CLC[traces-enabledI]:
  assumes {"PCC"}  $\subseteq$  accessible-regs s and CapRegs-names  $\subseteq$  accessible-regs s
  shows traces-enabled (execute-CLC arg0 arg1 arg2 arg3) s regs
  unfolding execute-CLC-def bind-assoc
  by (traces-enabledI assms: assms)

lemma traces-enabled-instr-sem[traces-enabledI]:
  assumes {"DDC", "PCC"}  $\subseteq$  accessible-regs s
  and CapRegs-names  $\subseteq$  accessible-regs s

```

shows *traces-enabled* (*instr-sem ISA instr*) *s regs*
by (*cases instr rule: execute.cases; simp; use nothing in* \langle *traces-enabledI assms:*
assms \rangle)

lemma *hasTrace-instr-mem-axioms:*
assumes *hasTrace t (instr-sem ISA instr)*
and *reads-regs-from trans-regs t regs and trans-inv regs*
and *instr-raises-ex ISA instr t \longrightarrow ex-traces*
shows *store-mem-axiom CC ISA t*
and *store-tag-axiom CC ISA t*
and *load-mem-axiom CC ISA False t*
using *assms*
by (*intro traces-enabled-mem-axioms[where m = instr-sem ISA instr and regs*
 $=$ *regs] traces-enabled-instr-sem;*
auto) $+$

end

3.5 Instruction fetch properties

context *CHERI-MIPS-Mem-Fetch-Automaton*
begin

lemmas *non-cap-exp-traces-enabled[traces-enabledI] = non-cap-expI[THEN non-cap-exp-traces-enabledI]*

lemmas *non-mem-exp-traces-enabled[traces-enabledI] = non-mem-expI[THEN non-mem-exp-traces-enabledI]*

thm *Run-bind-trace-enabled traces-enabled-bind*

lemma *Run-inv-bind-trace-enabled:*
assumes *Run-inv (m \gg f) t a regs and runs-preserve-invariant m*
and $\bigwedge tm\ tf\ am. t = tm @ tf \implies Run\text{-}inv\ m\ tm\ am\ regs \implies trace\text{-}enabled\ s$
 tm
and $\bigwedge tm\ tf\ am. t = tm @ tf \implies Run\text{-}inv\ m\ tm\ am\ regs \implies Run\text{-}inv\ (f\ am)$
 $tf\ a\ (the\ (updates\text{-}regs\ trans\text{-}regs\ tm\ regs)) \implies trace\text{-}enabled\ (run\ s\ tm)\ tf$
shows *trace-enabled s t*
using *assms*
by (*elim Run-inv-bindE*) (*auto simp: trace-enabled-append-iff*)

lemma *traces-enabled-read-mem-bytes[traces-enabledI]:*
assumes $\bigwedge bytes. access\text{-}enabled\ s\ Fetch\ (unat\ addr)\ (nat\ sz)\ bytes\ B0$
shows *traces-enabled (read-mem-bytes BC-mword BC-mword rk addr sz) s regs*
using *assms*
by (*auto simp: read-mem-bytes-def maybe-fail-def traces-enabled-def split: op-*
tion.splits elim: Traces-cases)

lemma *traces-enabled-MEMr-wrapper[traces-enabledI]:*

assumes $\bigwedge \text{bytes}. \text{access-enabled } s \text{ Fetch } (\text{unat } \text{addr}) (\text{nat } \text{sz}) \text{ bytes } B0$
shows $\text{traces-enabled } (\text{MEMr-wrapper } \text{addr } \text{sz}) s \text{ regs}$
unfolding $\text{MEMr-wrapper-def MEMr-def read-mem-def}$
by $(\text{traces-enabledI assms: assms})$

lemma $[\text{simp}]$: $\text{translation-tables ISA } t = \{\}$
by $(\text{auto simp: ISA-def})$

lemma $[\text{simp}]$: $\text{isa.translate-address ISA vaddr Fetch } t = \text{translate-address vaddr Fetch } t$
by $(\text{auto simp: ISA-def})$

lemma $\text{access-enabled-FetchI}$:
assumes $c \in \text{derivable-caps } s$ **and** $\text{Capability-tag } c$ **and** $\neg \text{Capability-sealed } c$
and $\text{translate-address vaddr Fetch } ([\] :: \text{register-value trace}) = \text{Some paddr}$
and $\text{vaddr} \geq \text{nat } (\text{getCapBase } c)$ **and** $\text{vaddr} + \text{sz} \leq \text{nat } (\text{getCapTop } c)$
and $\text{Capability-permit-execute } c$ **and** $\text{sz} > 0$
shows $\text{access-enabled } s \text{ Fetch paddr sz bytes } B0$
using assms
by $(\text{auto simp: access-enabled-defs derivable-caps-def address-range-def get-cap-perms-def})$

lemma $\text{Run-inv-no-reg-writes-to-updates-regs-inv}[\text{simp}]$:
assumes $\text{Run-inv } m \text{ t a regs}$ **and** $\text{no-reg-writes-to } Rs \text{ m}$
shows $\text{updates-regs } Rs \text{ t regs}' = \text{Some regs}'$
using assms
unfolding Run-inv-def
by auto

lemma Run-inv-read-regE :
assumes $\text{Run-inv } (\text{read-reg } r) \text{ t v regs}$
obtains rv **where** $t = [E\text{-read-reg } (\text{name } r) \text{ rv}]$ **and** $\text{of-regval } r \text{ rv} = \text{Some } v$
using assms
unfolding Run-inv-def
by $(\text{auto elim!: Run-read-regE})$

lemma $[\text{simp}]$: $\text{Run-inv } (\text{SignalExceptionBadAddr ex badAddr}) \text{ t a regs} \longleftrightarrow \text{False}$
by $(\text{auto simp: Run-inv-def})$

lemma $[\text{simp}]$: $"PCC" \in \text{trans-regs}$
by $(\text{auto simp: trans-regs-def})$

lemma $\text{runs-no-reg-writes-to-incrementCP0Count}[\text{runs-no-reg-writes-toI}, \text{simp}]$:
assumes $\{"TLB\text{Random}", "CP0\text{Count}", "CP0\text{Cause}"\} \cap Rs = \{\}$
shows $\text{runs-no-reg-writes-to } Rs (\text{incrementCP0Count } u)$
using assms
unfolding $\text{incrementCP0Count-def Let-def bind-assoc}$
by $(\text{no-reg-writes-toI simp: register-defs})$

```

lemma [simp]: runs-no-reg-writes-to trans-regs (incrementCP0Count u)
  by (auto simp: trans-regs-def)
find-theorems updates-regs no-reg-writes-to

lemma Run-inv-runs-no-reg-writes-to-updates-regs-inv[simp]:
  assumes Run-inv m t a regs
  and runs-no-reg-writes-to trans-regs m
  shows updates-regs trans-regs t regs = Some regs
  using assms
proof -
  have  $\forall r \in \text{trans-regs}. \forall v. E\text{-write-reg } r \ v \notin \text{set } t$ 
  using assms
  by (auto simp: runs-no-reg-writes-to-def Run-inv-def)
  then show updates-regs trans-regs t regs = Some regs
  by (induction trans-regs t regs rule: updates-regs.induct) auto
qed

lemma Run-inv-read-reg-PCC[simp]:
  assumes Run-inv (read-reg PCC-ref) t c regs
  shows regstate.PCC regs = c
  using assms
  by (auto simp: Run-inv-def register-defs regval-of-Capability-def elim!: Run-read-regE)

lemma foo:
  assumes  $\neg \text{getCapTop } c < \text{getCapBase } c + \text{uint } vaddr + 4$  and  $\text{getCapTop } c \leq \text{pow2 } 64$ 
  shows  $\text{unat } (\text{to-bits } 64 (\text{getCapBase } c + \text{uint } vaddr) :: 64 \text{ word}) = \text{nat } (\text{getCapBase } c + \text{uint } vaddr) \wedge \text{nat } (\text{getCapBase } c) \leq \text{nat } (\text{getCapBase } c + \text{uint } vaddr) \wedge \text{nat } (\text{getCapBase } c + \text{uint } vaddr) + 4 = \text{nat } (\text{getCapBase } c + \text{uint } vaddr + 4)$ 
  using assms
  by (auto simp: nat-add-distrib getCapBase-def)

lemma Run-inv-TranslatePC-access-enabled-Fetch:
  assumes Run-inv (TranslatePC vaddr) t paddr regs
  and regstate.PCC regs  $\in$  derivable-caps s
  shows access-enabled (run s t) Fetch (unat paddr) (nat 4) bytes B0
proof -
  { fix c
    assume  $\neg \text{getCapTop } c < \text{getCapBase } c + \text{uint } vaddr + 4$  and  $\text{getCapTop } c \leq \text{pow2 } 64$ 
    then have  $\text{unat } (\text{to-bits } 64 (\text{getCapBase } c + \text{uint } vaddr) :: 64 \text{ word}) = \text{nat } (\text{getCapBase } c + \text{uint } vaddr) \wedge \text{nat } (\text{getCapBase } c) \leq \text{nat } (\text{getCapBase } c + \text{uint } vaddr) \wedge \text{nat } (\text{getCapBase } c + \text{uint } vaddr) + 4 = \text{nat } (\text{getCapBase } c + \text{uint } vaddr + 4)$ 
    by (auto simp: nat-add-distrib getCapBase-def)
  }
from this[of regstate.PCC regs]
show ?thesis

```

using *assms*
unfolding *TranslatePC-def bind-assoc Let-def*
by (*intro access-enabled-FetchI*[**where** $c = \text{regstate.PCC regs}$ **and** $\text{vaddr} = \text{unat (to-bits 64 (getCapBase (regstate.PCC regs) + uint vaddr) :: 64 word))}$]
(auto elim!: Run-inv-bindE Run-inv-ifE intro!: preserves-invariantI intro: traces-runs-preserve-invariantI derivable-caps-run-imp simp add: getCapBounds-def simp del: unat-to-bits dest!: TLBTranslate-Instruction-translate-address-eq[**where** $t' = [] :: \text{register-value trace}$])
qed

lemma [*simp*]:
name UART-WRITTEN-ref $\notin \text{trans-regs}$
name InstCount-ref $\notin \text{trans-regs}$
name NextPCC-ref $\notin \text{trans-regs}$
by (*auto simp: trans-regs-def register-defs*)

lemma *Run-write-regE*:
assumes *Run* (*write-reg* $r v$) $t a$
obtains $t = [E\text{-write-reg (name } r) (\text{regval-of } r v)]$
using *assms*
by (*auto simp: write-reg-def elim!: Write-reg-TracesE*)

lemma *Run-inv-write-reg-PCC-updates-regs*[*simp*]:
assumes *Run-inv* (*write-reg* $PCC\text{-ref } c$) $t a \text{ regs}$
shows *updates-regs trans-regs* $t \text{ regs}' = \text{Some (regs'(|regstate.PCC := c))}$
using *assms*
unfolding *Run-inv-def*
by (*auto simp: register-defs elim: Run-write-regE*)

lemma *Run-inv-read-reg-NextPCC-derivable-caps*[*derivable-capsE*]:
assumes *Run-inv* (*read-reg* *NextPCC-ref*) $t c \text{ regs}$
and $\{\text{"NextPCC"}\} \subseteq \text{accessible-regs } s$
shows $c \in \text{derivable-caps (run } s t)$
using *assms*
by (*auto simp: step-defs register-defs derivable-caps-def intro: derivable.Copy elim!: Run-inv-read-regE*)

lemma *Run-inv-cp2-next-pc-PCC-derivable*:
assumes *Run-inv* (*cp2-next-pc* ()) $t a \text{ regs}$
and $\{\text{"NextPCC"}\} \subseteq \text{accessible-regs } s$
shows $\text{regstate.PCC (the (updates-regs trans-regs } t \text{ regs))} \in \text{derivable-caps (run } s t)$
using *assms*(1)
unfolding *cp2-next-pc-def*
by (*auto elim!: Run-inv-bindE Run-inv-ifE intro: preserves-invariantI traces-runs-preserve-invariantI simp: regstate-simp*)
(derivable-capsI assms: assms(2))+

lemma *traces-enabled-fetch*[*traces-enabledI*]:

assumes $\{\text{"NextPCC"}\} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{fetch } u) \ s \ \text{regs}$
unfolding $\text{fetch-def bind-assoc}$
by $(\text{traces-enabledI elim: Run-inv-TranslatePC-access-enabled-Fetch Run-inv-cp2-next-pc-PCC-derivable assms: assms})$

lemma $\text{traces-enabled-instr-fetch}[\text{traces-enabledI}]$:
assumes $\{\text{"NextPCC"}\} \subseteq \text{accessible-regs } s$
shows $\text{traces-enabled } (\text{instr-fetch ISA}) \ s \ \text{regs}$
unfolding ISA-simps
by $(\text{traces-enabledI assms: assms})$

lemma $\text{hasTrace-fetch-mem-axioms}$:
assumes $\text{hasTrace } t \ (\text{instr-fetch ISA})$
and $\text{reads-regs-from trans-regs } t \ \text{regs}$ **and** trans-inv regs
and $\text{fetch-raises-ex ISA } t \longrightarrow \text{ex-traces}$
shows $\text{store-mem-axiom CC ISA } t$
and $\text{store-tag-axiom CC ISA } t$
and $\text{load-mem-axiom CC ISA True } t$
using assms
by $(\text{intro traces-enabled-mem-axioms}[\text{where } m = \text{instr-fetch ISA and regs = regs}] \text{ traces-enabled-instr-fetch; auto})+$

end

locale $\text{CHERI-MIPS-Reg-Fetch-Automaton} = \text{CHERI-MIPS-Fixed-Trans} +$
fixes $\text{ex-traces} :: \text{bool}$
begin

sublocale $\text{Reg-Automaton?} : \text{Write-Cap-Inv-Automaton CC ISA ex-traces False}$
 $\text{get-regval set-regval trans-inv trans-regs ..}$

sublocale $\text{CHERI-MIPS-Axiom-Inv-Automaton}$ **where** $\text{enabled} = \text{enabled}$ **and**
 $\text{invariant} = \text{trans-inv}$ **and** $\text{inv-regs} = \text{trans-regs}$ **and** $\text{translate-address} = \text{translate-address}$
..

sublocale $\text{Mem-Automaton} : \text{CHERI-MIPS-Mem-Fetch-Automaton trans-regstate}$
 ex-traces ..

lemmas $\text{non-cap-exp-traces-enabled}[\text{traces-enabledI}] = \text{non-cap-expI}[\text{THEN non-cap-exp-traces-enabledI}]$

definition $\text{PCC-accessible } s \ \text{regs} \equiv \text{"PCC"} \in \text{accessible-regs } s \vee \text{regstate.PCC regs}$
 $\in \text{derivable-caps } s$

lemma
assumes $\text{Run-inv } (\text{read-reg PCC-ref}) \ t \ c \ \text{regs}$ **and** $\text{PCC-accessible } s \ \text{regs}$
shows $c \in \text{derivable-caps } (\text{run } s \ t)$

using *assms derivable-mono*[where $C = C$ and $C' = C \cup C'$ for $C \ C'$]
 unfolding *Run-inv-def PCC-accessible-def derivable-caps-def*
 by (*fastforce simp: register-defs regval-of-Capability-def elim!: Run-read-regE intro: derivable.Copy*)

lemma *traces-enabled-write-cap-regs*[*traces-enabledI*]:

assumes $c \in \text{derivable-caps } s$

shows *traces-enabled* (*write-reg C01-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C02-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C03-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C04-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C05-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C06-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C07-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C08-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C09-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C10-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C11-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C12-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C13-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C14-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C15-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C16-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C17-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C18-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C19-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C20-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C21-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C22-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C23-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C24-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C25-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C26-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C27-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C28-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C29-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C30-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg C31-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg CPLR-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg CULR-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg DDC-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg DelayedPCC-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg EPCC-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg ErrorEPCC-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg KCC-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg KDC-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg KR1C-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg KR2C-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg NextPCC-ref c*) $s \text{ regs}$

and *traces-enabled* (*write-reg PCC-ref c*) *s regs*
using *assms*
by (*intro traces-enabled-write-reg; auto simp: register-defs derivable-caps-def*)+

lemma *traces-enabled-write-reg-CapCause*[*traces-enabledI*]:
traces-enabled (*write-reg CapCause-ref c*) *s regs*
by (*intro traces-enabled-write-reg; auto simp: register-defs derivable-caps-def*)+

lemma *traces-enabled-read-cap-regs*[*traces-enabledI*]:
traces-enabled (*read-reg C01-ref*) *s regs*
traces-enabled (*read-reg C02-ref*) *s regs*
traces-enabled (*read-reg C03-ref*) *s regs*
traces-enabled (*read-reg C04-ref*) *s regs*
traces-enabled (*read-reg C05-ref*) *s regs*
traces-enabled (*read-reg C06-ref*) *s regs*
traces-enabled (*read-reg C07-ref*) *s regs*
traces-enabled (*read-reg C08-ref*) *s regs*
traces-enabled (*read-reg C09-ref*) *s regs*
traces-enabled (*read-reg C10-ref*) *s regs*
traces-enabled (*read-reg C11-ref*) *s regs*
traces-enabled (*read-reg C12-ref*) *s regs*
traces-enabled (*read-reg C13-ref*) *s regs*
traces-enabled (*read-reg C14-ref*) *s regs*
traces-enabled (*read-reg C15-ref*) *s regs*
traces-enabled (*read-reg C16-ref*) *s regs*
traces-enabled (*read-reg C17-ref*) *s regs*
traces-enabled (*read-reg C18-ref*) *s regs*
traces-enabled (*read-reg C19-ref*) *s regs*
traces-enabled (*read-reg C20-ref*) *s regs*
traces-enabled (*read-reg C21-ref*) *s regs*
traces-enabled (*read-reg C22-ref*) *s regs*
traces-enabled (*read-reg C23-ref*) *s regs*
traces-enabled (*read-reg C24-ref*) *s regs*
traces-enabled (*read-reg C25-ref*) *s regs*
traces-enabled (*read-reg C26-ref*) *s regs*
traces-enabled (*read-reg C27-ref*) *s regs*
traces-enabled (*read-reg C28-ref*) *s regs*
traces-enabled (*read-reg C29-ref*) *s regs*
traces-enabled (*read-reg C30-ref*) *s regs*
traces-enabled (*read-reg C31-ref*) *s regs*
system-reg-access s \vee *ex-traces* \implies *traces-enabled* (*read-reg CPLR-ref*) *s regs*
traces-enabled (*read-reg CULR-ref*) *s regs*
traces-enabled (*read-reg DDC-ref*) *s regs*
traces-enabled (*read-reg DelayedPCC-ref*) *s regs*
system-reg-access s \vee *ex-traces* \implies *traces-enabled* (*read-reg EPCC-ref*) *s regs*
system-reg-access s \vee *ex-traces* \implies *traces-enabled* (*read-reg ErrorEPCC-ref*) *s*
regs
system-reg-access s \vee *ex-traces* \implies *traces-enabled* (*read-reg KCC-ref*) *s regs*
system-reg-access s \vee *ex-traces* \implies *traces-enabled* (*read-reg KDC-ref*) *s regs*

$system-reg-access\ s \vee ex-traces \implies traces-enabled\ (read-reg\ KR1C-ref)\ s\ regs$
 $system-reg-access\ s \vee ex-traces \implies traces-enabled\ (read-reg\ KR2C-ref)\ s\ regs$
 $system-reg-access\ s \vee ex-traces \implies traces-enabled\ (read-reg\ CapCause-ref)\ s\ regs$
 $traces-enabled\ (read-reg\ NextPCC-ref)\ s\ regs$
 $traces-enabled\ (read-reg\ PCC-ref)\ s\ regs$
by (intro traces-enabled-read-reg; auto simp: register-defs)+

lemma read-cap-regs-derivable[derivable-capsE]:

$\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C01-ref)\ t\ c\ regs \implies \{ "C01" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C02-ref)\ t\ c\ regs \implies \{ "C02" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C03-ref)\ t\ c\ regs \implies \{ "C03" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C04-ref)\ t\ c\ regs \implies \{ "C04" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C05-ref)\ t\ c\ regs \implies \{ "C05" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C06-ref)\ t\ c\ regs \implies \{ "C06" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C07-ref)\ t\ c\ regs \implies \{ "C07" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C08-ref)\ t\ c\ regs \implies \{ "C08" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C09-ref)\ t\ c\ regs \implies \{ "C09" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C10-ref)\ t\ c\ regs \implies \{ "C10" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C11-ref)\ t\ c\ regs \implies \{ "C11" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C12-ref)\ t\ c\ regs \implies \{ "C12" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C13-ref)\ t\ c\ regs \implies \{ "C13" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C14-ref)\ t\ c\ regs \implies \{ "C14" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C15-ref)\ t\ c\ regs \implies \{ "C15" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C16-ref)\ t\ c\ regs \implies \{ "C16" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C17-ref)\ t\ c\ regs \implies \{ "C17" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C18-ref)\ t\ c\ regs \implies \{ "C18" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C19-ref)\ t\ c\ regs \implies \{ "C19" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C20-ref)\ t\ c\ regs \implies \{ "C20" \} \subseteq accessible-regs$
 $s \implies c \in derivable-caps\ (run\ s\ t)$
 $\bigwedge t\ c\ regs\ s. Run-inv\ (read-reg\ C21-ref)\ t\ c\ regs \implies \{ "C21" \} \subseteq accessible-regs$

[illegible]

C21-ref-def C22-ref-def C23-ref-def C24-ref-def C25-ref-def
C26-ref-def C27-ref-def C28-ref-def C29-ref-def C30-ref-def
C31-ref-def CPLR-ref-def CULR-ref-def DDC-ref-def DelayedPCC-ref-def
EPCC-ref-def ErrorEPCC-ref-def KCC-ref-def KDC-ref-def KR1C-ref-def
KR2C-ref-def NextPCC-ref-def PCC-ref-def Run-inv-def derivable-caps-def
by (*auto elim!*: *Run-read-regE intro!*: *derivable.Copy*)

lemma *traces-enabled-cp2-next-pc*[*traces-enabledI*]:
assumes {"*DelayedPCC*", "*NextPCC*"} \subseteq *accessible-regs s*
shows *traces-enabled* (*cp2-next-pc u*) *s regs*
unfolding *cp2-next-pc-def bind-assoc*
by (*traces-enabledI assms: assms simp: register-defs*)

lemma *traces-enabled-set-next-pcc-ex*:
assumes *arg0*: *arg0* \in *exception-targets ISA* (*read-from-KCC s*) **and** *ex*: *ex-traces*
shows *traces-enabled* (*set-next-pcc arg0*) *s regs*
unfolding *set-next-pcc-def bind-assoc*
by (*traces-enabledI intro: traces-enabled-write-reg assms exception-targets-run-imp simp: register-defs*)

lemma *read-reg-PCC-from-iff*:
assumes *reads-regs-from trans-regs t regs*
defines *pcc* \equiv *regstate.PCC regs*
and *e* \equiv *E-read-reg "PCC" (Regval-Capability (regstate.PCC regs))*
shows *Run* (*read-reg PCC-ref*) *t c* \longleftrightarrow (*c* = *pcc* \wedge *t* = [*e*])
using *assms*
by (*auto simp: read-reg-def register-defs regval-of-Capability-def elim!*: *Read-reg-TracesE*)

lemma *read-reg-PCC-from-bind-iff*:
assumes *reads-regs-from trans-regs t regs*
defines *pcc* \equiv *regstate.PCC regs*
and *e* \equiv *E-read-reg "PCC" (Regval-Capability (regstate.PCC regs))*
shows *Run* (*read-reg PCC-ref*) *t a* \longleftrightarrow (\exists *tf*. *t* = *e* # *tf* \wedge *Run* (*f pcc*) *tf*
a)
using *assms*
by (*auto elim!*: *Run-bindE simp: read-reg-PCC-from-iff regstate-simp intro!*: *Traces-bindI*[*of read-reg PCC-ref [e], unfolded e-def, simplified*])

lemmas *read-reg-PCC-from-iffs* = *read-reg-PCC-from-iff read-reg-PCC-from-bind-iff*

lemma *Run-read-accessible-PCC-derivable*:
assumes *Run* (*read-reg PCC-ref*) *t c* **and** *reads-regs-from trans-regs t regs* **and**
PCC-accessible s regs
shows *c* \in *derivable-caps* (*run s t*)
using *assms derivable-mono*[*OF Un-upper1, THEN in-mono*]
by (*auto simp: register-defs regval-of-Capability-def derivable-caps-def PCC-accessible-def elim!*: *Run-read-regE intro: derivable.Copy*)

lemma *Run-write-derivable-PCC-accessible*:

assumes *Run* (*write-reg PCC-ref c*) *t a* **and** *reads-regs-from Rs t regs* **and**
"PCC" ∈ Rs
and *c ∈ derivable-caps s*
shows *PCC-accessible* (*run s t*) (*the* (*updates-regs Rs t regs*))
using *assms*
by (*auto simp: PCC-accessible-def register-defs derivable-caps-def elim!: Mem-Automaton.Run-write-regE*)

lemma *Run-PCC-accessible-run*:
assumes *Run m t a* **and** *runs-no-reg-writes-to {"PCC"} m* **and** *PCC-accessible*
s regs
shows *PCC-accessible* (*run s t*) *regs*
using *assms derivable-caps-run-mono[of s t]*
by (*auto simp: PCC-accessible-def accessible-regs-def Run-runs-no-reg-writes-written-regs-eq*)

lemmas *Run-inv-PCC-accessible-run = Run-inv-RunI[THEN Run-PCC-accessible-run]*

lemma *Run-runs-no-reg-writes-to-updates-regs-inv[simp]*:
assumes *Run m t a* **and** *reads-regs-from Rs t regs* **and** *runs-no-reg-writes-to Rs*
m
shows *updates-regs Rs t regs = Some regs*
proof –
have $\forall r \in Rs. \forall v. E\text{-write-reg } r \ v \notin \text{set } t$
using *assms*
by (*auto simp: runs-no-reg-writes-to-def Run-inv-def*)
then show *updates-regs Rs t regs = Some regs*
by (*induction Rs t regs rule: updates-regs.induct*) *auto*
qed

lemma *Run-runs-no-reg-writes-to-get-regval-eq[simp]*:
assumes *Run m t a* **and** *reads-regs-from Rs t regs* **and** *runs-no-reg-writes-to {r}*
m
shows *get-regval r* (*the* (*updates-regs Rs t regs*)) = *get-regval r regs*
proof –
have $\forall v. E\text{-write-reg } r \ v \notin \text{set } t$
using *assms*
by (*auto simp: runs-no-reg-writes-to-def Run-inv-def*)
then show *?thesis*
using *assms(2)*
by (*induction Rs t regs rule: updates-regs.induct*)
(auto split: Option.bind-splits if-splits simp: get-ignore-set-regval)
qed

lemma *Run-PCC-accessible-update*:
assumes *Run m t a* **and** *reads-regs-from Rs t regs* **and** *runs-no-reg-writes-to*
{"PCC"} m
and *PCC-accessible s regs*
shows *PCC-accessible s* (*the* (*updates-regs Rs t regs*))
proof –

```

have get-regval "PCC" (the (updates-regs Rs t regs)) = get-regval "PCC" regs
  using assms
  by auto
then show ?thesis
  using ⟨PCC-accessible s regs⟩
  by (auto simp: PCC-accessible-def register-defs regval-of-Capability-def)
qed

```

```

lemma Run-inv-PCC-accessible-update:
  assumes Run-inv m t a regs and runs-no-reg-writes-to {"PCC"} m
    and PCC-accessible s regs
  shows PCC-accessible s (the (updates-regs trans-regs t regs))
  using assms
  by (intro Run-PCC-accessible-update) (auto simp: Run-inv-def)

```

```

lemma Run-PCC-accessible-run-update:
  assumes Run m t a and reads-regs-from Rs t regs and runs-no-reg-writes-to
    {"PCC"} m
    and PCC-accessible s regs
  shows PCC-accessible (run s t) (the (updates-regs Rs t regs))
  using assms
  by (blast intro: Run-PCC-accessible-run Run-PCC-accessible-update)

```

```

lemma Run-inv-PCC-accessible-run-update:
  assumes Run-inv m t a regs and runs-no-reg-writes-to {"PCC"} m
    and PCC-accessible s regs
  shows PCC-accessible (run s t) (the (updates-regs trans-regs t regs))
  using assms
  by (blast intro: Run-inv-PCC-accessible-update Run-inv-PCC-accessible-run)

```

```

lemmas Run-PCC-accessibleE[derivable-capsE] =
  Run-PCC-accessible-run-update Run-PCC-accessible-update Run-PCC-accessible-run
  Run-inv-PCC-accessible-run-update Run-inv-PCC-accessible-update Run-inv-PCC-accessible-run

```

```

lemma (in Register-State) reads-regs-bind-updates-regs-the[simp]:
  assumes reads-regs-from R t s
  shows Option.bind (updates-regs R t s) f = f (the (updates-regs R t s))
  using assms
  by (elim reads-regs-from-updates-regs-Some) auto

```

```

find-theorems NextPCC-ref derivable-caps

```

```

lemma Run-inv-cp2-next-pc-PCC-accessible:
  assumes Run-inv (cp2-next-pc u) t a regs and {"NextPCC"} ⊆ accessible-regs
    s
  shows PCC-accessible (run s t) (the (updates-regs trans-regs t regs))
proof -
  have *: PCC-accessible s (regs⟦regstate.PCC := c⟧) if c ∈ derivable-caps s for
    s c and regs :: regstate

```

```

    using that
    by (auto simp: PCC-accessible-def)
show ?thesis
    using assms
    unfolding cp2-next-pc-def bind-assoc
    by (auto elim!: Run-inv-bindE Run-inv-ifE intro: preserves-invariantI traces-runs-preserve-invariantI
intro!: * simp: regstate-simp)
    (derivable-capsI)+
qed

```

lemma *SignalException-trace-enabled:*

assumes $(\text{SignalException } \arg0, t, m') \in \text{Traces}$ **and** $\text{reads-regs-from trans-regs } t \text{ regs}$

and $\text{PCC-accessible } s \text{ regs}$ **and** $ex: \text{ex-traces}$

shows $\text{trace-enabled } s \ t$

proof –

note $[\text{trace-elim}] = \text{non-cap-expI}[\text{THEN non-cap-exp-trace-enabledI}]$

have $[\text{trace-elim}]: (\text{read-reg PCC-ref } c, t, m') \in \text{Traces} \implies \text{trace-enabled } s \ t$ **for** $s \ t$ **and** $m' :: \text{Capability } M$

by $(\text{elim read-reg-trace-enabled; auto simp: register-defs})$

have $[\text{trace-elim}]: (\text{read-reg KCC-ref } c, t, m') \in \text{Traces} \implies \text{trace-enabled } s \ t$ **for** $s \ t$ **and** $m' :: \text{Capability } M$

by $(\text{elim read-reg-trace-enabled; auto simp: register-defs intro: ex})$

have $[\text{trace-elim}]: (\text{write-reg EPCC-ref } c, t, m') \in \text{Traces} \implies c \in \text{derivable-caps}$
 $s \implies \text{trace-enabled } s \ t$ **for** $s \ t \ c$ **and** $m' :: \text{unit } M$

by $(\text{elim write-reg-trace-enabled})$ $(\text{auto simp: derivable-caps-def register-defs})$

have $[\text{trace-elim}]: (\text{set-next-pcc } c, t, m') \in \text{Traces} \implies c \in \text{exception-targets ISA}$
 $(\text{read-from-KCC } s) \implies \text{trace-enabled } s \ t$ **for** $s \ t \ c$ **and** $m' :: \text{unit } M$

unfolding set-next-pcc-def

by $(\text{elim trace-elim write-reg-trace-enabled})$

$(\text{auto simp: register-defs intro: ex exception-targets-run-imp})$

have $[\text{trace-elim}]: (\text{set-CauseReg-BD } \text{CP0Cause-ref } x, t, m') \in \text{Traces} \implies \text{trace-enabled}$
 $s \ t$ **for** $s \ t \ x \ m'$

unfolding $\text{set-CauseReg-BD-def}$

by (elim trace-elim)

have $[\text{trace-elim}]: (\text{set-CauseReg-ExcCode } \text{CP0Cause-ref } x, t, m') \in \text{Traces} \implies$
 $\text{trace-enabled } s \ t$ **for** $s \ t \ x \ m'$

unfolding $\text{set-CauseReg-ExcCode-def}$

by (elim trace-elim)

have $[\text{trace-elim}]: (\text{set-StatusReg-EXL } \text{CP0Status-ref } x, t, m') \in \text{Traces} \implies$
 $\text{trace-enabled } s \ t$ **for** $s \ t \ x \ m'$

unfolding $\text{set-StatusReg-EXL-def}$

by (elim trace-elim)

have $\text{read-KCC-ex-target}: c \in \text{exception-targets ISA } (\text{read-from-KCC } (\text{Mem-Automaton.run } s \ t))$

if $\text{Run } (\text{read-reg KCC-ref}) \ t \ c$ **for** $s \ t \ c$

using *that*

by $(\text{auto elim!: Run-read-regE simp: register-defs})$

note $[\text{derivable-capsE}] = \text{Run-read-accessible-PCC-derivable}[\text{where } \text{regs} = \text{regs}]$

```

show ?thesis
  using assms(1-3)
  unfolding SignalException-def bind-assoc
  by (elim trace-elim read-KCC-ex-target)
      (derivable-capsI simp: regstate-simp read-reg-PCC-from-iffs)
qed

```

```

lemma traces-enabled-SignalException[traces-enabledI]:
  assumes PCC-accessible s regs
  shows traces-enabled (SignalException arg0 :: 'a M) s regs
proof cases
  assume ex: ex-traces
  then show ?thesis
    using assms SignalException-trace-enabled
    unfolding traces-enabled-def
    by blast
next
  assume  $\neg ex\text{-traces}$ 
  then show ?thesis
    unfolding traces-enabled-def finished-def isException-def
    by auto
qed

```

```

lemma [simp]:
  name CP0Count-ref = "CP0Count"
  name TLBRandom-ref = "TLBRandom"
  name CP0BadVAddr-ref = "CP0BadVAddr"
  name CapCause-ref = "CapCause"
  name BranchPending-ref = "BranchPending"
  name NextInBranchDelay-ref = "NextInBranchDelay"
  name InBranchDelay-ref = "InBranchDelay"
  name PC-ref = "PC"
  name NextPC-ref = "NextPC"
  name InstCount-ref = "InstCount"
  name CurrentInstrBits-ref = "CurrentInstrBits"
  by (auto simp: register-defs)

```

```

lemma [simp]:
  "CP0Count" \notin trans-regs
  "TLBRandom" \notin trans-regs
  "CP0BadVAddr" \notin trans-regs
  "CapCause" \notin trans-regs
  "BranchPending" \notin trans-regs
  "NextInBranchDelay" \notin trans-regs
  "InBranchDelay" \notin trans-regs
  "PC" \notin trans-regs
  "NextPC" \notin trans-regs
  "InstCount" \notin trans-regs
  "CurrentInstrBits" \notin trans-regs

```

by (auto simp: trans-regs-def)

lemma *traces-enabled-SignalExceptionBadAddr*[*traces-enabledI*]:
 assumes *PCC-accessible s regs*
 shows *traces-enabled (SignalExceptionBadAddr arg0 arg1) s regs*
 unfolding *SignalExceptionBadAddr-def*
 by (traces-enabledI assms: assms)

lemma *SignalExceptionTLB-trace-enabled*:
 assumes (*SignalExceptionTLB arg0 arg1 :: 'a M, t, m' ∈ Traces* and *reads-regs-from trans-regs t regs*
 and *PCC-accessible s regs* and *ex: ex-traces*
 shows *trace-enabled s t*
proof –
 have [trace-elim]: (*write-reg CP0BadVAddr-ref v, t, m' ∈ Traces* \implies *trace-enabled s t* for *s t v* and *m' :: unit M*
 by (auto elim!: *write-reg-trace-enabled simp: register-defs*)
 have [trace-elim]: (*set-ContextReg-BadVPN2 TLBContext-ref v, t, m' ∈ Traces*
 \implies *trace-enabled s t* for *s t v* and *m' :: unit M*
 by (auto elim!: *trace-elim read-reg-trace-enabled write-reg-trace-enabled simp: set-ContextReg-BadVPN2-def register-defs*)
 have [trace-elim]: (*set-XContextReg-XBadVPN2 TLBXContext-ref v, t, m' ∈ Traces*
 \implies *trace-enabled s t* for *s t v* and *m' :: unit M*
 by (auto elim!: *trace-elim read-reg-trace-enabled write-reg-trace-enabled simp: set-XContextReg-XBadVPN2-def register-defs*)
 have [trace-elim]: (*set-XContextReg-XR TLBXContext-ref v, t, m' ∈ Traces*
 \implies *trace-enabled s t* for *s t v* and *m' :: unit M*
 by (auto elim!: *trace-elim read-reg-trace-enabled write-reg-trace-enabled simp: set-XContextReg-XR-def register-defs*)
 have [trace-elim]: (*set-TLBEEntryHiReg-R TLBEEntryHi-ref v, t, m' ∈ Traces*
 \implies *trace-enabled s t* for *s t v* and *m' :: unit M*
 by (auto elim!: *trace-elim read-reg-trace-enabled write-reg-trace-enabled simp: set-TLBEEntryHiReg-R-def register-defs*)
 have [trace-elim]: (*set-TLBEEntryHiReg-VPN2 TLBEEntryHi-ref v, t, m' ∈ Traces*
 \implies *trace-enabled s t* for *s t v* and *m' :: unit M*
 by (auto elim!: *trace-elim read-reg-trace-enabled write-reg-trace-enabled simp: set-TLBEEntryHiReg-VPN2-def register-defs*)
 note [*derivable-capsI*] = *ex*
 show ?thesis
 using *assms(1–3)*
 unfolding *SignalExceptionTLB-def bind-assoc*
 by (elim trace-elim *SignalException-trace-enabled*)
 (*derivable-capsI simp: regstate-simp*)
qed

lemma *traces-enabled-SignalExceptionTLB*[*traces-enabledI*]:
 assumes *PCC-accessible s regs*
 shows *traces-enabled (SignalExceptionTLB arg0 arg1) s regs*
proof *cases*

```

assume ex: ex-traces
show ?thesis
  unfolding traces-enabled-def
  using assms
  by (auto elim!: SignalExceptionTLB-trace-enabled intro: ex)
next
  assume  $\neg$ ex-traces
  then show ?thesis
    unfolding traces-enabled-def finished-def isException-def
    by auto
qed

lemma traces-enabled-incrementCP0Count[traces-enabledI]:
  assumes PCC-accessible s regs
  shows traces-enabled (incrementCP0Count u) s regs
  unfolding incrementCP0Count-def bind-assoc
  by (traces-enabledI assms: assms)

lemma traces-enabled-raise-c2-exception8[traces-enabledI]:
  assumes PCC-accessible s regs
  shows traces-enabled (raise-c2-exception8 arg0 arg1) s regs
proof cases
  assume ex: ex-traces
  have 1: traces-enabled (set-CapCauseReg-ExcCode CapCause-ref x) s regs for x
s regs
    unfolding set-CapCauseReg-ExcCode-def
    by (traces-enabledI assms: ex)
  have 2: traces-enabled (set-CapCauseReg-RegNum CapCause-ref x) s regs for x
s regs
    unfolding set-CapCauseReg-RegNum-def
    by (traces-enabledI assms: ex)
  show ?thesis
    unfolding raise-c2-exception8-def bind-assoc
    by (traces-enabledI assms: assms intro: 1 2)
next
  assume  $\neg$  ex-traces
  then show ?thesis
    unfolding traces-enabled-def finished-def isException-def
    by auto
qed

lemma traces-enabled-raise-c2-exception-noreg[traces-enabledI]:
  assumes PCC-accessible s regs
  shows traces-enabled (raise-c2-exception-noreg arg0) s regs
  unfolding raise-c2-exception-noreg-def
  by (traces-enabledI assms: assms)

lemma traces-enabled-TLBTranslate2[traces-enabledI]:
  assumes PCC-accessible s regs

```

shows *traces-enabled* (*TLBTranslate2* *arg0* *arg1*) *s* *regs*
unfolding *TLBTranslate2-def*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-TLBTranslateC*[*traces-enabledI*]:
assumes *PCC-accessible* *s* *regs*
shows *traces-enabled* (*TLBTranslateC* *arg0* *arg1*) *s* *regs*
unfolding *TLBTranslateC-def*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-TLBTranslate*[*traces-enabledI*]:
assumes *PCC-accessible* *s* *regs*
shows *traces-enabled* (*TLBTranslate* *arg0* *arg1*) *s* *regs*
unfolding *TLBTranslate-def*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-TranslatePC*[*traces-enabledI*]:
assumes *PCC-accessible* *s* *regs*
shows *traces-enabled* (*TranslatePC* *vaddr*) *s* *regs*
unfolding *TranslatePC-def* *bind-assoc*
by (*traces-enabledI* *assms*: *assms*)

lemma *traces-enabled-MEMr*[*traces-enabledI*]:
shows *traces-enabled* (*MEMr* *arg0* *arg1*) *s* *regs*
unfolding *MEMr-def* *read-mem-def* *read-mem-bytes-def* *maybe-fail-def* *bind-assoc*
by (*auto simp*: *traces-enabled-def* *elim*!: *bind-Traces-cases* *split*: *option.splits* *elim*:
Traces-cases)

lemma *traces-enabled-MEMr-wrapper*[*traces-enabledI*]:
shows *traces-enabled* (*MEMr-wrapper* *arg0* *arg1*) *s* *regs*
unfolding *MEMr-wrapper-def* *bind-assoc*
by (*traces-enabledI-with* $\langle _ \rangle$)

lemma *traces-enabled-fetch*[*traces-enabledI*]:
assumes $\{ "DelayedPCC", "NextPCC", "PCC" \} \subseteq accessible_regs\ s$
shows *traces-enabled* (*fetch* *u*) *s* *regs*
unfolding *fetch-def* *bind-assoc*
by (*traces-enabledI* *elim*: *Run-inv-cp2-next-pc-PCC-accessible* *assms*: *assms* *simp*:
register-defs)

lemma *traces-enabled-instr-fetch*[*traces-enabledI*]:
assumes $\{ "DelayedPCC", "NextPCC", "PCC" \} \subseteq accessible_regs\ s$
shows *traces-enabled* (*instr-fetch* *ISA*) *s* *regs*
unfolding *ISA-simps*
by (*traces-enabledI* *assms*: *assms*)

lemma *hasTrace-fetch-reg-axioms*:
assumes *hasTrace* *t* (*instr-fetch* *ISA*)
and *reads-regs-from* *trans-regs* *t* *regs* **and** *trans-inv* *regs*


```

    and fetch-raises-ex ISA t  $\longrightarrow$  ex-traces
  shows store-cap-reg-axiom CC ISA ex-traces False t
    and store-cap-mem-axiom CC ISA t
    and read-reg-axiom CC ISA ex-traces t
  using assms
  by (intro traces-enabled-reg-axioms [where m = instr-fetch ISA and regs = regs]
    traces-enabled-instr-fetch; auto) +

end

end

theory CHERI-MIPS-Properties
imports CHERI-MIPS-Reg-Axioms CHERI-MIPS-Mem-Axioms Properties
begin

```

3.6 Instantiation of monotonicity result

```

context CHERI-MIPS-Reg-Automaton
begin

lemma runs-no-reg-writes-to-incrementCP0Count [runs-no-reg-writes-toI]:
  assumes {"TLBRandom", "CP0Count", "CP0Cause"}  $\cap$  Rs = {}
  shows runs-no-reg-writes-to Rs (incrementCP0Count u)
  using assms
  unfolding incrementCP0Count-def bind-assoc Let-def
  by (no-reg-writes-toI simp: TLBRandom-ref-def CP0Count-ref-def CP0Cause-ref-def)

lemma TranslatePC-establishes-inv:
  assumes Run (TranslatePC vaddr) t a and reads-regs-from {"PCC"} t s
  shows invariant s
  using assms
  unfolding TranslatePC-def bind-assoc Let-def
  by (auto elim!: Run-bindE Run-read-regE split: if-splits
    simp: regstate-simp register-defs regval-of-Capability-def)

lemma not-PCC-regs [simp]:
  name PC-ref  $\neq$  "PCC"
  name InBranchDelay-ref  $\neq$  "PCC"
  name NextPC-ref  $\neq$  "PCC"
  name NextInBranchDelay-ref  $\neq$  "PCC"
  name BranchPending-ref  $\neq$  "PCC"
  name CurrentInstrBits-ref  $\neq$  "PCC"
  name LastInstrBits-ref  $\neq$  "PCC"
  name UART-WRITTEN-ref  $\neq$  "PCC"
  name InstCount-ref  $\neq$  "PCC"
  by (auto simp: register-defs)

lemma fetch-establishes-inv:
  assumes Run (fetch u) t a and reads-regs-from {"PCC"} t s

```

shows *invariant* (the (updates-regs {"PCC"} t s))
using *assms*
unfolding *fetch-def bind-assoc Let-def*
by (auto elim!: Run-bindE simp: regstate-simp dest: TranslatePC-establishes-inv)

lemma *instr-fetch-establishes-inv*:
assumes *Run* (instr-fetch ISA) t a **and** *reads-regs-from* {"PCC"} t s
shows *invariant* (the (updates-regs {"PCC"} t s))
using *assms*
by (auto simp: ISA-def elim!: Run-bindE split: option.splits dest: fetch-establishes-inv)

end

lemma (in *CHERI-MIPS-Mem-Automaton*) *preserves-invariant-instr-fetch*[*preserves-invariantI*]:
shows *runs-preserve-invariant* (instr-fetch ISA)
by (auto simp: ISA-def intro!: *preserves-invariantI*; simp add: *runs-preserve-invariant-def*)

context *CHERI-MIPS-Fixed-Trans*
begin

definition *state-assms* t reg-s mem-s \equiv *reads-regs-from* *trans-regs* t mem-s \wedge *reads-regs-from* {"PCC"} t reg-s \wedge *trans-inv* mem-s

definition *fetch-assms* t \equiv (\exists reg-s mem-s. *state-assms* t reg-s mem-s)

definition *instr-assms* t \equiv (\exists reg-s mem-s. *state-assms* t reg-s mem-s \wedge *CHERI-MIPS-Reg-Automaton.invariant* reg-s)

sublocale *CHERI-ISA* **where** *CC* = *CC* **and** *ISA* = *ISA* **and** *fetch-assms* =
fetch-assms **and** *instr-assms* = *instr-assms*

proof

fix t *instr*

interpret *Reg-Axioms*: *CHERI-MIPS-Reg-Automaton*

where *ex-traces* = *instr-raises-ex* ISA *instr* t

and *invocation-traces* = *invokes-caps* ISA *instr* t

and *translate-address* = *translate-address* .

interpret *Mem-Axioms*: *CHERI-MIPS-Mem-Instr-Automaton* *trans-regstate* *instr-raises-ex* ISA *instr* t

by *unfold-locales*

assume t: *hasTrace* t (*instr-sem* ISA *instr*) **and** *instr-assms* t

then obtain reg-s mem-s

where *reg-assms*: *reads-regs-from* {"PCC"} t reg-s *Reg-Axioms.invariant* reg-s

and *mem-assms*: *reads-regs-from* *trans-regs* t mem-s *trans-inv* mem-s

by (auto simp: *instr-assms-def* *state-assms-def*)

show *cheri-axioms* CC ISA False (*instr-raises-ex* ISA *instr* t)

(*invokes-caps* ISA *instr* t) t

unfolding *cheri-axioms-def*

using *Reg-Axioms.hasTrace-instr-reg-axioms*[OF t reg-assms]

using *Mem-Axioms.hasTrace-instr-mem-axioms*[OF t mem-assms]

by *auto*

next

```

fix t
interpret Fetch-Axioms: CHERI-MIPS-Reg-Fetch-Automaton trans-regstate fetch-raises-ex
ISA t ..
  assume t: hasTrace t (instr-fetch ISA) and fetch-assms t
  then obtain regs where *: reads-regs-from trans-regs t regs trans-inv regs
    by (auto simp: fetch-assms-def state-assms-def)
  then show cheri-axioms CC ISA True (fetch-raises-ex ISA t) False t
    unfolding cheri-axioms-def
    using Fetch-Axioms.hasTrace-fetch-reg-axioms[OF t *]
    using Fetch-Axioms.Mem-Automaton.hasTrace-fetch-mem-axioms[OF t *]
    by auto
next
fix t t' instr
interpret Mem-Axioms: CHERI-MIPS-Mem-Instr-Automaton trans-regstate by
unfold-locales
  assume *: instr-assms (t @ t') and **: Run (instr-sem ISA instr) t ()
  have trans-inv (the (updates-regs trans-regs t mem-s))
    if trans-inv mem-s and reads-regs-from trans-regs t mem-s for mem-s
    using Mem-Axioms.preserves-invariant-execute[of instr] that **
  by (elim runs-preserve-invariantE[where t = t and s = mem-s and a = ()])
    (auto simp: instr-assms-def state-assms-def regstate-simp)
  with * show instr-assms t ∧ fetch-assms t'
    by (auto simp: instr-assms-def fetch-assms-def state-assms-def regstate-simp)
next
fix t t' instr
interpret Reg-Axioms: CHERI-MIPS-Reg-Automaton
  where ex-traces = fetch-raises-ex ISA t
  and invocation-traces = False
  and translate-address = translate-address .
interpret Mem-Axioms: CHERI-MIPS-Mem-Automaton trans-regstate by unfold-locales
  assume *: fetch-assms (t @ t') and **: Run (instr-fetch ISA) t instr
  have ***: trans-inv (the (updates-regs trans-regs t regs))
    if reads-regs-from trans-regs t regs and trans-inv regs for regs
    using ** that
  by (elim runs-preserve-invariantE[OF Mem-Axioms.preserves-invariant-instr-fetch])
auto
  show fetch-assms t ∧ instr-assms t'
    using * **
    unfolding fetch-assms-def instr-assms-def state-assms-def
    by (fastforce simp: regstate-simp elim!: Run-bindE split: option.splits
        dest: Reg-Axioms.fetch-establishes-inv ***)
qed

lemma translate-address-tag-aligned:
  fixes s :: regstate sequential-state
  assumes translate-address vaddr acctype s = Some paddr
  shows address-tag-aligned ISA paddr = address-tag-aligned ISA vaddr
    (is ?aligned paddr = ?aligned vaddr)
proof -

```

```

interpret CHERI-MIPS-Mem-Automaton ..
have [simp]: ?aligned (unat (word-of-int (int vaddr) :: 64 word))  $\longleftrightarrow$  ?aligned
vaddr
  unfolding address-tag-aligned-def
  by (auto simp: unat-def uint-word-of-int nat-mod-distrib nat-power-eq mod-mod-cancel)
from assms obtain t regs where Run-inv (translate-addressM vaddr acctype) t
paddr regs
  by (auto simp: translate-address-def determ-the-result-eq[OF determ-runs-translate-addressM]
      split: if-splits)
then show ?thesis
  by (auto simp: translate-addressM-def elim!: Run-inv-bindE intro: preserves-invariantI)
qed

sublocale CHERI-ISA-State where CC = CC and ISA = ISA
and read-regval = get-regval and write-regval = set-regval
and fetch-assms = fetch-assms and instr-assms = instr-assms
and s-translation-tables =  $\lambda\cdot. \{\}$  and s-translate-address = translate-address
using get-absorb-set-regval get-ignore-set-regval translate-address-tag-aligned
by unfold-locales (auto simp: ISA-def translate-address-def)

thm reachable-caps-instrs-trace-intradomain-monotonicity

lemma regstate-put-mem-bytes-eq[simp]:
  regstate (put-mem-bytes addr sz v tag s) = regstate s
  by (auto simp: put-mem-bytes-def Let-def)

lemma set-regval-Some-Some:
  assumes set-regval r v s = Some s1
  obtains s1' where set-regval r v s' = Some s1'
  using assms
  by (elim set-regval-cases of-regval-SomeE) (auto simp: register-defs)

lemma get-regval-eq-reads-regs-imp:
  assumes  $\forall r \in Rs. \text{get-regval } r \text{ } s = \text{get-regval } r \text{ } s'$ 
  and reads-regs-from Rs t s'
  shows reads-regs-from Rs t s
proof (use assms in (induction t arbitrary: s s'))
  case (Cons e t)
  then show ?case
  proof (cases e)
    fix r v
    assume e: e = E-write-reg r v
    with Cons show ?thesis
  proof cases
    assume r: r  $\in$  Rs
    with Cons.prems e obtain s1' where s1': set-regval r v s' = Some s1' and
  *: reads-regs-from Rs t s1'
    by (auto split: if-splits option.splits)
    moreover obtain s1 where s1: set-regval r v s = Some s1

```

```

    by (rule set-regval-Some-Some[OF s1'])
  have **:  $\forall r' \in Rs. \text{get-regval } r' s1 = \text{get-regval } r' s1'$ 
proof
  fix r'
  assume  $r' \in Rs$ 
  then show  $\text{get-regval } r' s1 = \text{get-regval } r' s1'$ 
    using s1 s1' Cons.prem
    by (cases  $r' = r$ ) (auto simp: get-absorb-set-regval get-ignore-set-regval)
qed
show ?thesis
  using e r s1 Cons.IH[OF ** *]
  by auto
qed auto
qed (auto split: if-splits option.splits)
qed auto

```

```

lemma set-other-reg-reads-regs-iff:
  assumes  $\text{set-regval } r v s = \text{Some } s'$  and  $r \notin Rs$ 
  shows  $\text{reads-regs-from } Rs t s' = \text{reads-regs-from } Rs t s$ 
proof -
  have  $\forall r' \in Rs. \text{get-regval } r' s = \text{get-regval } r' s'$ 
    using assms get-ignore-set-regval
    by fastforce
  then show ?thesis
    using get-regval-eq-reads-regs-imp[of Rs s s' t]
    using get-regval-eq-reads-regs-imp[of Rs s' s t]
    by auto
qed

```

```

lemma reads-regs-from-mono:
  assumes  $\text{reads-regs-from } Rs t s$ 
    and  $Rs' \subseteq Rs$ 
  shows  $\text{reads-regs-from } Rs' t s$ 
  using assms
  by (induction  $Rs t s$  rule: reads-regs-from.induct)
    (auto split: if-splits option.splits dest: set-other-reg-reads-regs-iff[where  $Rs = Rs'$ ])

```

```

lemma s-invariant-trivial:
  assumes  $t: s\text{-allows-trace } t s$  and  $f: \bigwedge s'. f s' = f s$ 
  shows  $s\text{-invariant } f t s$ 
proof -
  have  $f s1 = f s2$  for  $s1 s2$ 
    using f[of s1] f[of s2, symmetric]
    by auto
  obtain s' where  $s\text{-run-trace } t s = \text{Some } s'$ 
    using t

```

```

    by blast
  then show s-invariant f t s
    by (induction (get-regval, set-regval) t s rule: runTraceS.induct)
      (auto split: Option.bind-splits intro: f)
qed

theorem cheri-mips-cap-monotonicity:
  assumes t: hasTrace t (fetch-execute-loop ISA n)
    and s: s-run-trace t s = Some s'
    and regs: reads-regs-from trans-regs t trans-regstate — Fixes contents of address
translation control registers and implies that we are in user mode via the assumption
noCP0Access trans-regstate.
    and no-ex:  $\neg$  instrs-raise-ex ISA n t
    and no-ccall:  $\neg$  instrs-invoke-caps ISA n t
  shows reachable-caps s' ⊆ reachable-caps s
proof (intro reachable-caps-instrs-trace-intradomain-monotonicity[OF t - s no-ex
no-ccall])
  have state-assms t trans-regstate trans-regstate
    using regs
  by (auto simp: state-assms-def trans-regs-def elim: reads-regs-from-mono)
  then show fetch-assms t
    by (auto simp: fetch-assms-def)
  show s-invariant ( $\lambda s'$  addr load. local.translate-address addr load s') t s
    and s-invariant ( $\lambda -. \{\}$ ) t s
    and s-invariant-holds no-caps-in-translation-tables t s
  using s
  by (auto simp: translate-address-def no-caps-in-translation-tables-def intro:
s-invariant-trivial)
qed

end

end

```