Variable aleatoria bidimensional discreta

Función de masa de probabilidad conjunta

$$p_{X,Y}(x,y) = P(X=x, Y=y)$$

$$0 \le p_{X,Y}(x,y) \le 1$$

$$P(a \le X \le b, c \le Y \le d) = \sum_{x=a}^{b} \sum_{y=c}^{d} p_{X,Y}(x,y)$$

$$\sum_{x \in R_X} \sum_{y \in R_Y} p_{X,Y}(x,y) = 1$$

Variable aleatoria bidimensional discreta

Función de probabilidad acumulada conjunta

$$F_{X,Y}(x,y) = P(X \le x, Y \le y)$$

$$F_{X,Y}(x,y) = \sum_{\alpha \le x} \sum_{\beta \le y} p_{X,Y}(\alpha,\beta)$$

- $$0 \le F_{X,Y}(x,y) \le 1$$
- Monótonamente no decreciente en cada variable
- Continua por derecha en cada variable

Variable aleatoria bidimensional discreta

Función de masa de probabilidad marginal

$$p_Y(y) = \sum_{x \in R_X} p_{X,Y}(x, y)$$

$$p_X(x) = \sum_{y \in R_Y} p_{X,Y}(x, y)$$

Variable aleatoria bidimensional discreta

Función de masa de probabilidad condicional

$$p_{X \vee Y}(x \vee y) = \frac{p_{X,Y}(x, y)}{p_Y(y)}$$

$$p_{Y \vee X}(y \vee x) = \frac{p_{X,Y}(x, y)}{p_X(x)}$$

Variable aleatoria bidimensional discreta

Variables aleatorias independientes

$$p_{X,Y}(x,y)=p_X(x)\,p_Y(y)$$

$$p_{X\vee Y}(x\vee y)=p_X(x)$$

$$p_{Y\vee X}(y\vee x)=p_Y(y)$$

Variable aleatoria bidimensional discreta

Valor esperado

$$E\left[g\left(X,Y\right)\right]=\sum_{x\in R_X}\sum_{y\in R_Y} g\left(x,y\right)p_{X,Y}\left(x,y\right)$$

Covarianza

$$\mathrm{Cov}(X,Y)=E[XY]-E[X]E[Y]$$

Variable aleatoria bidimensional discreta

Variables independientes

$$E\left[g_1(X)g_2(Y)\right]=\sum_{x\in R_X}\sum_{y\in R_Y} g_1(x)g_2(y)p_{X,Y}(x,y)$$

$$E\left[g_1(X)g_2(Y)\right]=\sum_{x\in R_X}\sum_{y\in R_Y} g_1(x)g_2(y)p_X(x)p_Y(y)$$

$$E\left[g_1(X)g_2(Y)\right]=\sum_{x\in R_X} g_1(x)p_X(x)\sum_{y\in R_Y} g_2(y)p_Y(y)$$

$$E\left[g_1(X)g_2(Y)\right]=E\left[g_1(X)\right]E\left[g_2(Y)\right]$$

Variable aleatoria bidimensional discreta

Covarianza

$$\mathrm{Cov}(X,Y) = E[XY] - E[X]E[Y]$$

Variables independientes

¡OJO! En general:

/

Variable aleatoria bidimensional discreta

$$Y = a\,X + b \quad \text{con} \quad a \neq 0$$

$$\mu_Y = a\,\mu_X + b$$

$$\sigma_Y^2 = a^2 \sigma_X^2 \Rightarrow \sigma_Y = |a|\,\sigma_X$$

$$E[XY] = E\left[a X^2 + bX\right] = aE\left[X^2\right] + bE[X]$$

$$E[X]\,E[Y] = a\,\mu_X^2 + b\,\mu_X$$

$$\mathbf{Cov}(X, Y) = a\,\sigma_X^2$$

Relación lineal

Variable aleatoria bidimensional discreta

Correlación

$$\varrho_{X,Y} = \frac{\mathrm{Cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$\varrho_{X,Y} \in [-1,+1]$$

Relación lineal

Variable aleatoria bidimensional discreta

Experimento:

Una urna contiene 10 bolillas blancas y 5 negras. Se tira una moneda al aire. Si sale cara se agregan 10 bolillas negras a la urna; si sale ceca se agregan 10 bolillas blancas. Se saca una bolilla al azar y se observa el color.

Se definen dos variables aleatorias discretas:

: vale 0 si sale ceca al arrojar la moneda y 1 si sale cara

: vale 0 si la bolilla extraída es negra y 1 si es blanca.

Variable aleatoria bidimensional discreta

: probabilidad condicional de dado

X = 0

20 blancas
5 negras

X = 1

15 negras
10 blancas

		negra	blanca
		0	1
ceca	0	0.2	0.8
cara	1	0.6	0.4

$$p_{Y \vee X}(0|0)=\frac{5}{25}, p_{Y \vee X}(1|0)=\frac{20}{25} \qquad p_{Y \vee X}(0|1)=\frac{15}{25}, p_{Y \vee X}(1|1)=\frac{10}{25}$$

Variable aleatoria bidimensional discreta

: probabilidad conjunta de e

,: probabilidades marginales de e

	0	1	$p_X(x)$
0	0.1	0.4	0.5
1	0.3	0.2	0.5
$p_Y(y)$	0.4	0.6	

Variable aleatoria bidimensional discreta

No son independientes

	0	1
0	0.1	0.4
1	0.3	0.2

$p_X(x)$
0.5
0.5

$p_Y(y)$		
	0.4	0.6

Variable aleatoria bidimensional discreta

$$\mu_X = E[X] = 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5$$

$$E[X^2] = 0^2 \cdot 0.5 + 1^2 \cdot 0.5 = 0.5$$

$$\sigma_X^2 = 0.5 - 0.25 = 0.25$$

	0	1	$p_X(x)$
0	0.1	0.4	0.5
1	0.3	0.2	0.5
$p_Y(y)$	0.4	0.6	

Variable aleatoria bidimensional discreta

$$\mu_Y = E[Y] = 0 \cdot 0.4 + 1 \cdot 0.6 = 0.6$$

$$E[Y^2] = 0^2 \cdot 0.4 + 1^2 \cdot 0.6 = 0.6$$

$$\sigma_Y^2 = 0.6 - 0.36 = 0.24$$

	0	1	$p_X(x)$
0	0.1	0.4	0.5
1	0.3	0.2	0.5
$p_Y(y)$	0.4	0.6	

Variable aleatoria bidimensional discreta

$$E[XY] = 0\cdot 0.1 + 0\cdot 0.4 + 0\cdot 0.3 + 1\cdot 0.2 = 0.2$$

	0	1
0	0.1	0.4
1	0.3	0.2

$p_X(x)$

0.5
0.5

$p_Y(y)$		
	0.4	**0.6**

Variable aleatoria bidimensional discreta

$$\mu_X = 0.5, \sigma_X^2 = 0.25$$

$$\mu_Y = 0.6, \sigma_Y^2 = 0.24$$

$$E[XY] = 0.2$$

No son variables independientes

No hay una relación lineal

Variable aleatoria bidimensional discreta

Experimento

Se tira un dado dos veces y se anotan los resultados X_1 y X_2.

Se definen dos variables aleatorias discretas:

X_1

$| X_1 – X_2|$

Variable aleatoria bidimensional discreta

El recorrido de

$$R_X = \{1,2,3,4,5,6\}$$

El recorrido de

$$R_Y = \{0,1,2,3,4,5\}$$

El recorrido de

$$R_{X,Y} = R_X \times R_Y$$

Variable aleatoria bidimensional discreta

X_1 \ X_2	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Variable aleatoria bidimensional discreta

X_1 \ X_2	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Variable aleatoria bidimensional discreta

: probabilidad conjunta de e

	0	1	2	3	4	5
1						
2						
3						
4						
5						
6						

Variable aleatoria bidimensional discreta

	0	1	2	3	4	5
1						
2						
3						
4						
5						
6						

	0	1	2	3	4	5
1						
2						
3						
4						
5						
6						

Variable aleatoria bidimensional discreta

	0	1	2	3	4	5
1						
2						
3						
4						
5						
6						

	0	1	2	3	4	5
1						
2						
3						
4						
5						
6						

Variable aleatoria bidimensional discreta

$$P\left(X=1 \middle| Y=2\right) \quad ¿\frac{P\left(X=1, Y=2\right)}{P\left(Y=2\right)}$$

$$¿\frac{\frac{1}{36}}{\frac{8}{36}}$$

$$¿\frac{1}{8} \quad \neq P\left(X=1\right)=\frac{1}{6}$$

No son variables aleatorias independientes

Variable aleatoria bidimensional discreta

$$p_{X \vee Y}\left(1 \middle| 2\right) \text{¿} \frac{p_{X,Y}(1,2)}{p_Y(2)}$$

$$\text{¿} \frac{\frac{1}{36}}{\frac{8}{36}}$$

$$\text{¿} \frac{1}{8} \quad \neq p_X(1) = \frac{1}{6}$$