AST4007W - Computational Methods
Project 4: Integrating Acceleration

The Problem

In this project you will be working with simulated data for a water rocket. That is a rocket that is propelled by the release of water, pushed out by compressed air. The basic form is highlighted in Figure 1 below, if you aren't already familiar with these rockets you can find many video recordings on YouTube.

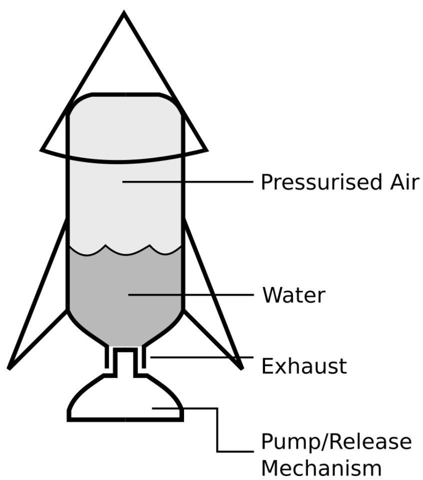

Figure 1: A typical basic water rocket.

You are provided with the linear acceleration measurements on the vertical axis for this simulation (this may be difficult to measure in reality). Your task is to reconstruct the linear vertical motion of the rocket (velocity and displacement) using numerical integration techniques, highlighted in the section below.

You can assume that the rocket either maintains it's orientation, or rather that the launch height of the measurement instrument is the same as it's height on impact with the ground (the final data point).

Background

You are given a data set of acceleration measurements over time ($a(t)$) for the vertical (y) axis. You are required to determine the position over time, given the initial position and the initial velocity of the body ($y(0) = 0$ m and $v(0) = 0$ m.s^{-2} respectively).

You can do this by integrating the equations:

$$\frac{dv(t)}{dt} = a(t) \quad \text{and} \quad \frac{dy(t)}{dt} = v(t)$$

numerically, though instead of a single value of the integral over the measured interval, you want a collection of values along the interval. To this end consider the differential equation:

$$\frac{dF}{dx} = f(x)$$

Using the fundamental theorem of calculus:

$$F(x) = F(x_0) + \int_{x_0}^{x} f(s)ds$$

For a data set of values (x_i, y_i), F can be found for each point by integrating up to $x = x_i$:

$$F(x_i) = F(x_0) + \int_{x_0}^{x_i} f(s)ds$$

Alternatively you can integrate from the previous data point:

$$F(x_i) = F(x_{i-1}) + \int_{x_{i-1}}^{x_i} f(s)ds$$

which is more efficient for numerical solutions.

It is advised that you use either the trapezoidal or Simpson's rule for approximating the integral. Note that using Simpson's rule will result in approximately half the number of original data points for $F(x_i)$.

Your Task

Your task is to reconstruct the motion (vertical velocity and vertical displacement) of the rocket using the measured acceleration data. You are to acheive this using either trapezoidal or Simpson's rule. Program your own implementation of the integration techniques, do not use built-in functions from a package such as SciPy.

Before you apply your techniques to the given data, generate your own data set from a function with a non-constant second derivative and test it on this. Plot the data set and integrated data set, comparing it to the known function. Make sure to mention what function you are using in the write-up.

Read in and plot the original acceleration data, as well as the calculated velocity and displacement curves.

Answer the following questions in your write-up:

- In what time interval does the rocket run out of water? How do you know?
- What is the maximum height that the rocket achieves and at what time, according to your calculations? You need not determine any uncertainties for your calculations.

Submission Guidelines

As well as your source code (Python scripts or IPython notebooks), you will be required to submit a short write-up of your methodology for performing the data analysis above. Record/plot any intermediate results you get as well as your final result. **Justify** any decisions you make in the analysis.

The write-up must be typeset in LaTeX and the **.tex** file(s) along with figures used in the document must be submitted in a compressed file. Your source code may be in the same file or a separate file, depending on the structure of your project directory.

The marking will follow the rubric:

	Item	**Mark Allocation**
Code	Good programming practices	3
	Reading relevant data from files	2
	Integrating data to find velocity and displacement curves.	8
Write-up	Good LaTeX practices followed.	2
	Summary of methodology, including justifications for decisions made.	2
	Presentation of intermediate and final results, including tables and plots	4
	Answering the questions asked in the brief.	4
	Total	**25**