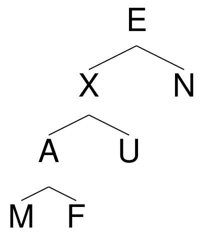
DSA Homework #4

4.1 Trees

1.



2. In the tree T, there are n nodes $a_1, a_2, a_3, ..., a_n$.
Claim:for every node $a_i(0 < i < n)$, $f(a_i) \leq 2^n - 1$.

   - For $n = 1$:
     The only element must be the root element. From the definition given in page 295, its level numbering is 1.
     Because $1 \leq 2^1 - 1$, the claim is true for $n = 1$.

   - Assume for $n = k$, $f(a_i) \leq 2^n - 1$ $(0 < i < n)$

   - For $n = k + 1$,

$$f(a_{k+1}) = \begin{cases} f(a_{k+1}) = 2f(a_i) & \text{if v is the left child of node } a_i \\ f(a_{k+1}) = 2f(a_i) + 1 & \text{if v is the right child of node } a_i \end{cases}$$

$$\begin{aligned} a_{k+1} &\leq 2f(a_i) + 1 \\ &\leq 2(2^n - 1) + 1 \\ &= 2^{n+1} - 1 \end{aligned}$$

     Therefore, the claim is true when $k = n + 1$.

   From mathematical induction, it prove that for every node $a_i(0 < i < n)$,
   $f(a_i) \leq 2^n - 1$.
   That is, for every node v of T , $f(v) \leq 2^n - 1$.

3. Because in a postorder travelsal, the node v is traveled before all its ascendant and after all its descendant. In a preorder travelsal, the node v is traveled before all its descendant and after all its ascendant. Therefore, we can derive the formula:
$post(v) = pre(v) - depth(v) + desc(v) + 1$