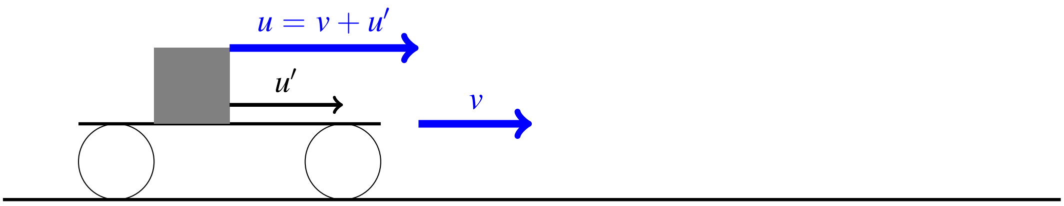
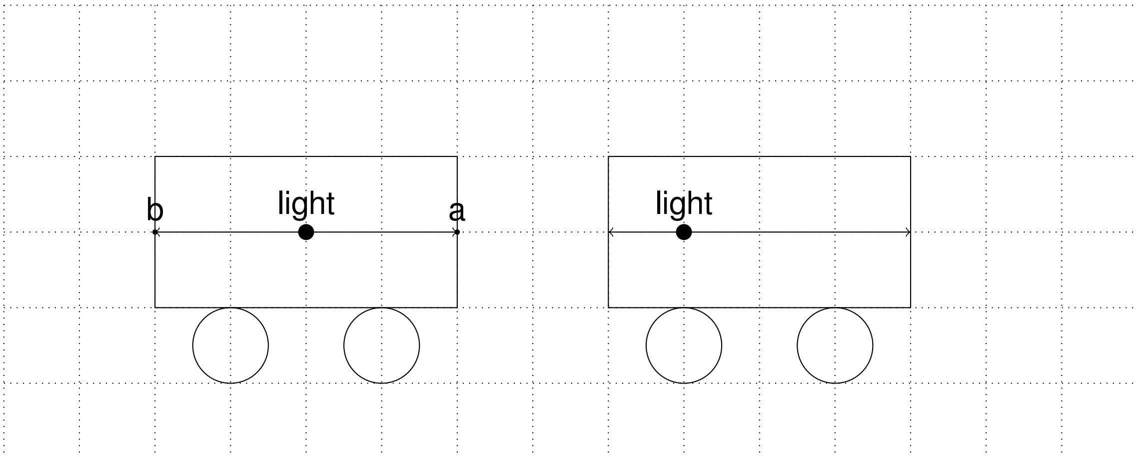


Galileo's velocity addiction rule

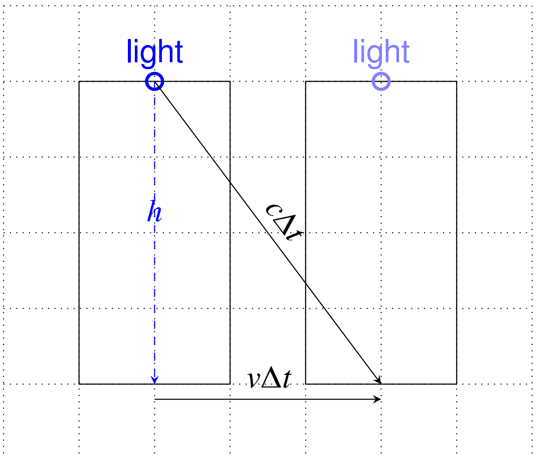
$$u = u' + v$$

Einstein's velocity addiction rule

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

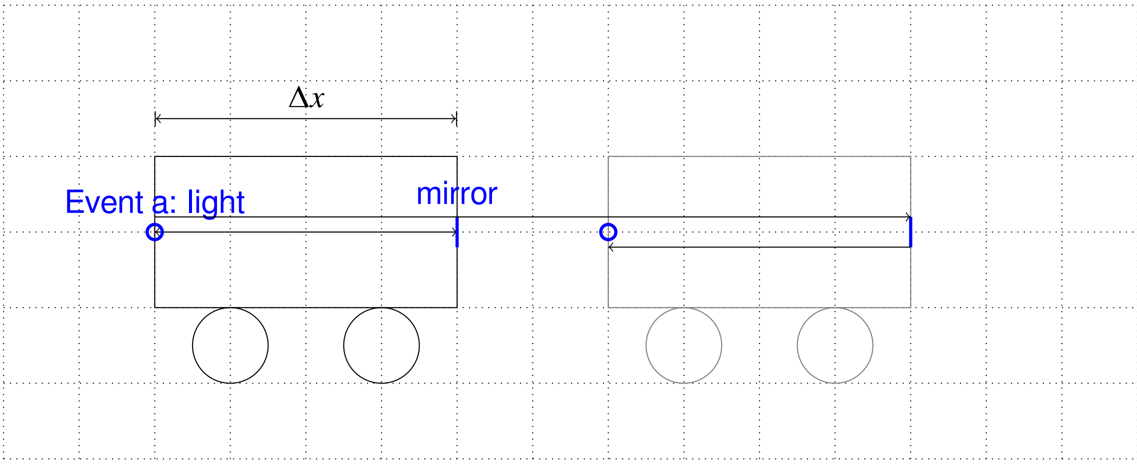


for people at stationary coordination, b happens first
for people on the car, a and b happens at the same time.



$$\begin{aligned}
(c\Delta t)^2 &= h^2 + (v\Delta t)^2 \\
h^2 &= (c^2 - v^2)\Delta t \\
c^2\Delta t'^2 &= (c^2 - v^2)\Delta t \\
\Delta t' &= \sqrt{1 - \frac{v^2}{c^2}}\Delta t \\
\Delta t' &< \Delta t
\end{aligned}$$

Moving clocks run slow

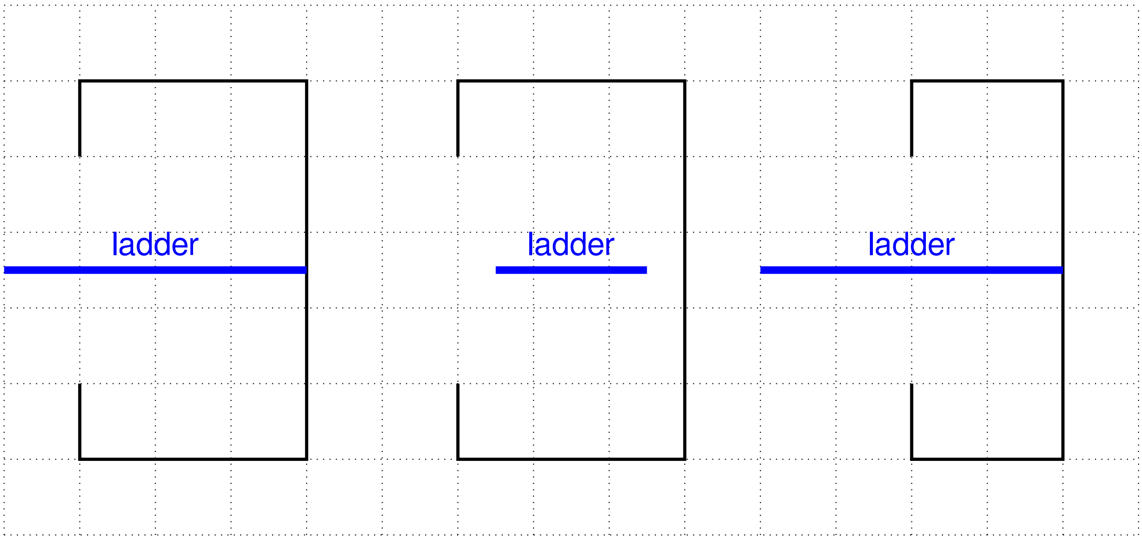


$$
\begin{aligned}
\Delta t' &= 2\frac{\Delta x'}{c} \\
\Delta t_1 &= \frac{\Delta x + v\Delta t_1}{c} \implies \Delta t_1 = \frac{\Delta x}{c-v} \\
\Delta t_2 &= \frac{\Delta x - v\Delta t_2}{c} \implies \Delta t_2 = \frac{\Delta x}{c+v} \\
\Delta t &= \Delta t_1 + \Delta t_2 = \frac{2\Delta x}{c}\frac{1}{1-\frac{v^2}{c^2}} \\
\Delta x' &= \frac{c}{2}\Delta t' \\
&= \frac{c}{2}\Delta t\sqrt{1-\frac{v^2}{c^2}} \\
&= \frac{c}{2}\frac{2\Delta x}{c}\frac{1}{1-\frac{v^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}} \\
&= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta x \\
&\implies \Delta x' > \Delta x
\end{aligned}
$$

0.1 The barn and ladder paracby

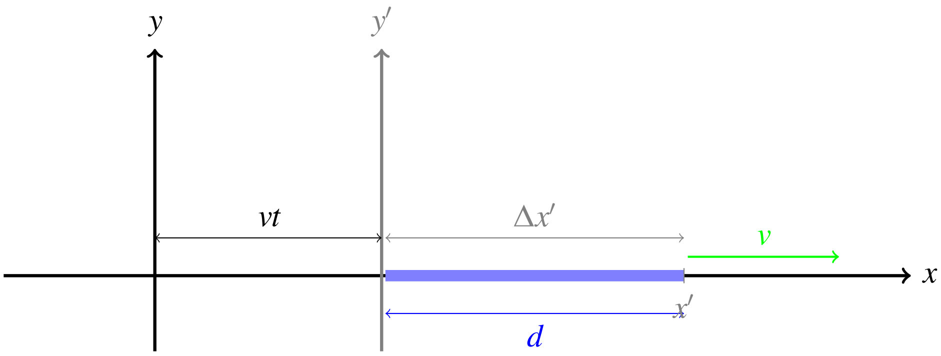
(A) Back end of the ladder makes it in the door
(B) Front end of the ladder hits the wall of the born

Farmer: (A) before (B)
Son: (B) before (A)



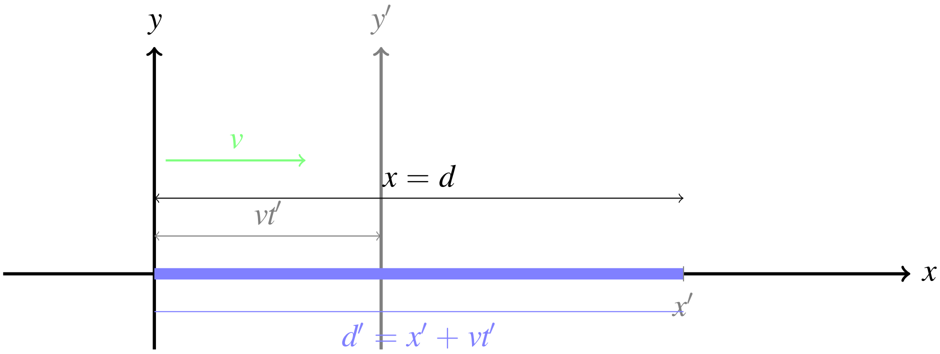
0.2 The Lorentz Transformations

- **Event:** Something that take place at a specific location at precise time.
- Knowing (x, y, z, t), what is (x', y', z', t') of the same event
- Let x be the same position as x'.
  The stick's length measured in the stationary coordiate be d.



$$
\begin{aligned}
d &= \sqrt{1-\frac{v^2}{c^2}}\Delta x' \\
(x - vt) &= \sqrt{1-\frac{v^2}{c^2}}x' \\
x' &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}(x - vt)
\end{aligned}
$$

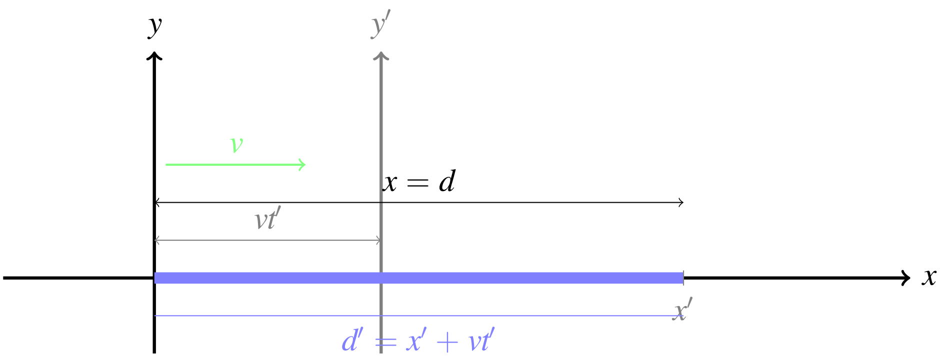
- Let x be the same position as x'.
  The stick's length measured in the stationary coordiate be d.



$$
\begin{aligned}
x' &= d' - vt' \\
d' &= \sqrt{1 - \frac{v^2}{c^2}}x \\
x' &= \sqrt{1 - \frac{v^2}{c^2}}x - vt' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}(x - vt) \\
vt' &= (\sqrt{1 - \frac{v^2}{c^2}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}})x - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}vt \\
&= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}(\frac{v}{c^2}x - vt)
\end{aligned}
$$

$-vt)$

- Let x be the same position as x'.
  The stick's length measured in the stationary coordiate be d.



$$
\begin{aligned}
x' &= d' - vt' \\
d' &= \sqrt{1-\frac{v^2}{c^2}}x \\
x' &= \sqrt{1-\frac{v^2}{c^2}}x - vt' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}(x-vt) \\
vt' &= (\sqrt{1-\frac{v^2}{c^2}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}})x - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}vt \\
&= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}(vt - \frac{v^2}{c^2}x) \\
t' &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}(t - \frac{v}{c^2}x)
\end{aligned}
$$

- **Check Simutanuously:**
  S:
  Event A $(x = 0, t = 0)$
  Event B $(x = a, t = 0)$
  S':
  Event A $(x' = 0, t' = 0)$
  Event B $(x' = \gamma a, t' = -\gamma(\frac{v}{c^2})a)$

- **Time Dimention:**
  $\Delta x' = 0 = \gamma(\Delta x - v\Delta t) \implies \Delta x = v\Delta t$

$$
\begin{aligned}
\Delta t' &= \gamma(\Delta t - \frac{v}{c^2}\Delta x) \\
&= \gamma(\Delta t - \frac{v^2}{c^2}\Delta t) \\
&= \sqrt{1-\frac{v^2}{c^2}}\Delta t
\end{aligned}
$$

-

$$
\begin{aligned}
{u_x}' &= \frac{\Delta x'}{\Delta t'} \\
&= \frac{\gamma(\Delta x - v\Delta t)}{\gamma(\Delta t - \frac{v}{c^2}\Delta x)} \\
&= \frac{u_x - v}{1 - \frac{v}{c^2}V_x} \\
{u_y}' &= \frac{\Delta y'}{\Delta t'} \\
&= \frac{\Delta y}{\gamma(\Delta t - \frac{v}{c^2}\Delta x)} \\
&= \frac{u_y}{\gamma(1 - \frac{v}{c^2})}
\end{aligned}
$$

- **Structure of Spacetime:**
  - **Four vectors:**
    (Time is the 0th-dimension.)

$$
\begin{aligned}
x^0 &= ct \\
x^1 &= x \\
x^2 &= y \\
x^3 &= z
\end{aligned}
$$

Loranze transformation:

$$
\begin{aligned}
x^{0\prime} &= \gamma(x^0 - \beta x^1) \\
x^{1\prime} &= \gamma(x^1 - \beta x^0) \\
x^{2\prime} &= x^2 \\
x^{3\prime} &= x^3
\end{aligned}
$$

$$
\begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
\begin{aligned}
& (x^\mu)' = \sum_{\beta=0}^{\delta} (\Lambda^\alpha_\beta x^\beta) \\
\implies & (x^\mu)' = \Lambda^\mu_\nu X^\nu
\end{aligned}
$$

0.3 HW note

- $$f' = f\sqrt{\frac{1-\beta}{1+\beta}}$$

- $$v = \frac{\Delta\lambda}{\lambda}c$$