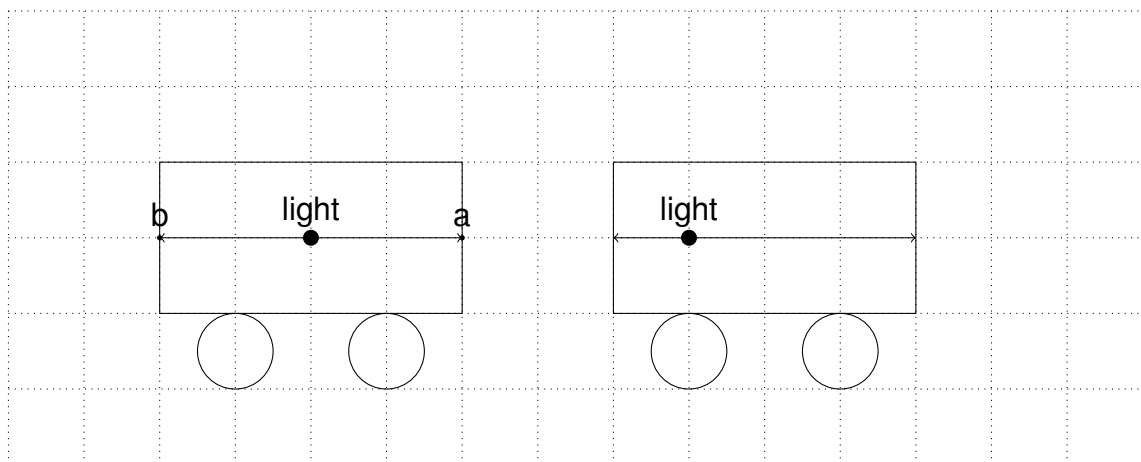


Galileo's velocity addition rule

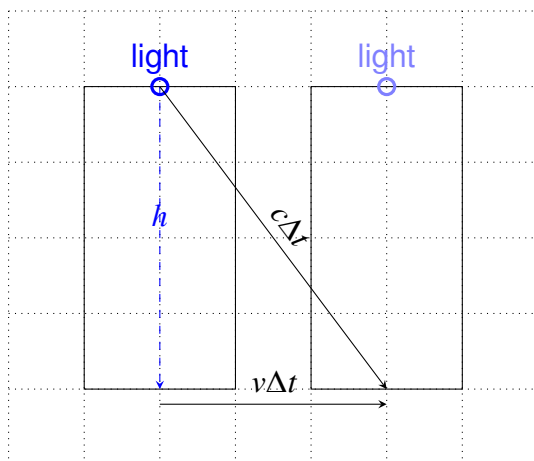
$$u = u' + v$$

Einstein's velocity addition rule

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$



for people at stationary coordination, b happens first
for people on the car, a and b happens at the same time.



Moving clocks run slow

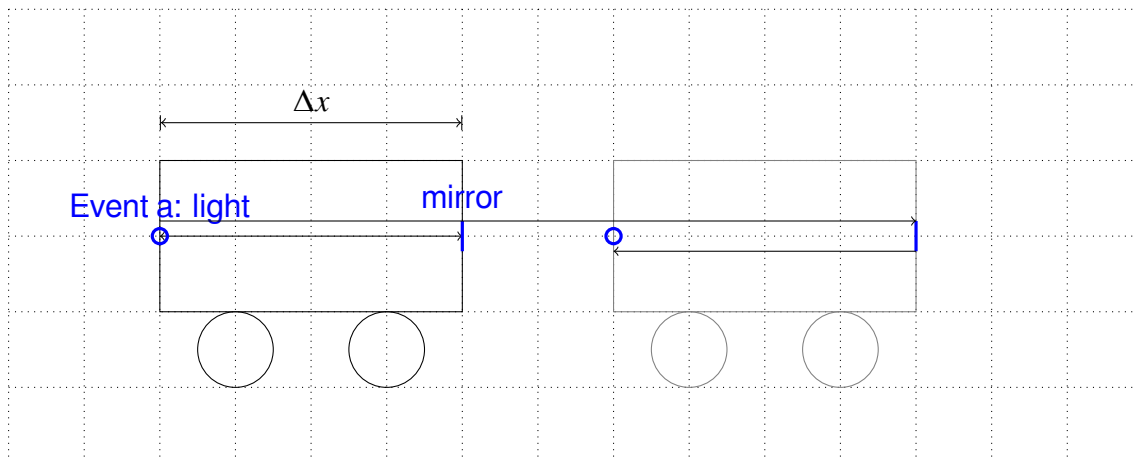
$$(c\Delta t')^2 = h^2 + (v\Delta t)^2$$

$$h^2 = (c^2 - v^2)\Delta t^2$$

$$c^2\Delta t'^2 = (c^2 - v^2)\Delta t^2$$

$$\Delta t' = \sqrt{1 - \frac{v^2}{c^2}}\Delta t$$

$$\Delta t' < \Delta t$$



$$\Delta t' = 2 \frac{\Delta x'}{c}$$

$$\Delta t_1 = \frac{\Delta x + v \Delta t_1}{c} \implies \Delta t_1 = \frac{\Delta x}{c - v}$$

$$\Delta t_2 = \frac{\Delta x - v \Delta t_2}{c} \implies \Delta t_2 = \frac{\Delta x}{c + v}$$

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{2\Delta x}{c} \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\Delta x' = \frac{c}{2} \Delta t'$$

$$= \frac{c}{2} \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{c}{2} \frac{2\Delta x}{c} \frac{1}{1 - \frac{v^2}{c^2}} \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Delta x$$

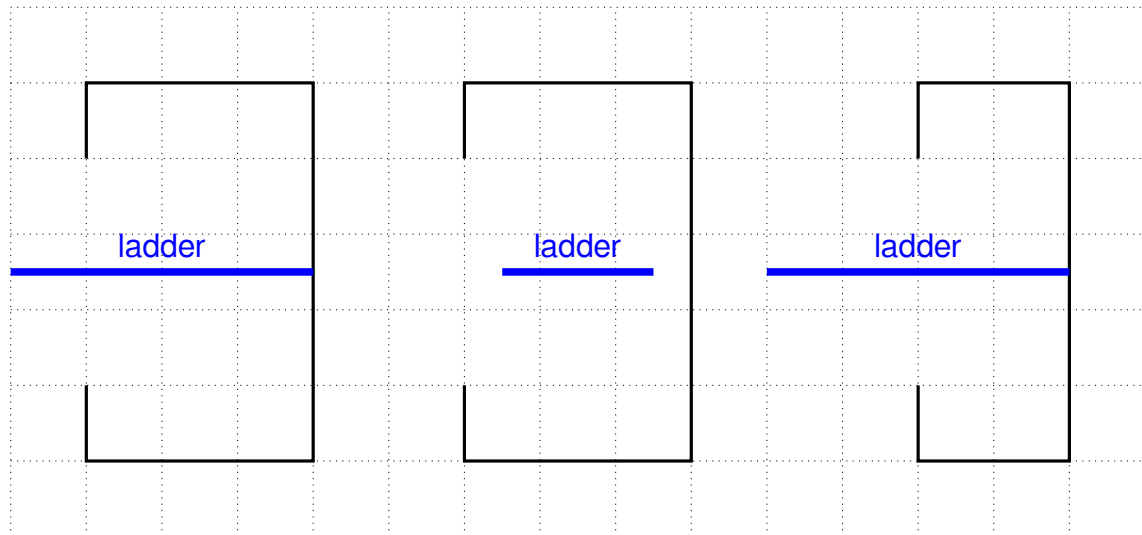
$$\implies \Delta x' > \Delta x$$

0.1 The barn and ladder paracby

- (A) Back end of the ladder makes it in the door
- (B) Front end of the ladder hits the wall of the born

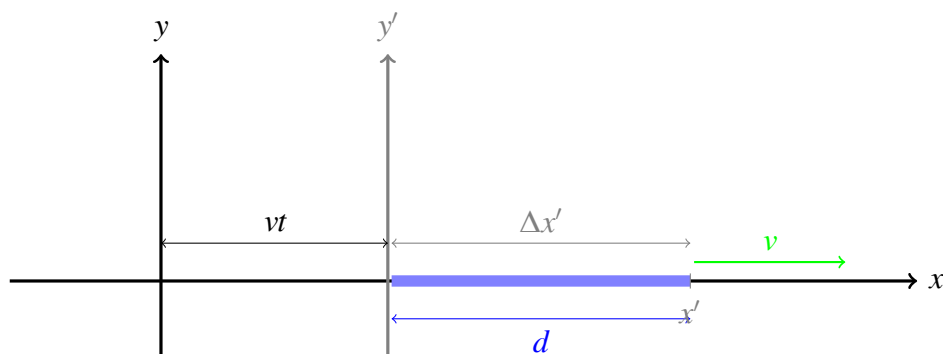
Farmer: (A) before (B)

Son: (B) before (A)



0.2 The Lorentz Transformations

- **Event:** Something that take place at a specific location at precise time.
- Knowing (x, y, z, t) , what is (x', y', z', t') of the same event
- Let x be the same position as x' .
The stick's length measured in the stationary coordinate be d .

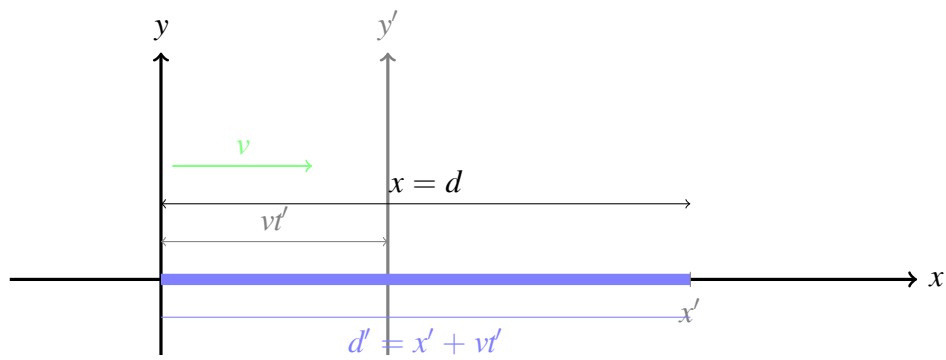


$$d = \sqrt{1 - \frac{v^2}{c^2}} \Delta x'$$

$$(x - vt) = \sqrt{1 - \frac{v^2}{c^2}} x'$$

$$x' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} (x - vt)$$

- Let x be the same position as x' .
The stick's length measured in the stationary coordinate be d .



$$x' = d' - vt'$$

$$d' = \sqrt{1 - \frac{v^2}{c^2}} x$$

$$x' = \sqrt{1 - \frac{v^2}{c^2}} x - vt' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} (x - vt')$$

$$vt' = \left(\sqrt{1 - \frac{v^2}{c^2}} - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right) x - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} vt$$

$$= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(\frac{v}{c^2} x - vt \right)$$