ISOM 677 Project 2
Crispy Critters

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Problem Summary

1. **Company Overview**

Crispy Critters Inc. is a biotech startup that has developed unique genetically engineered critters. Their production process yields two types of critters - the beneficial car-eating ones and harmful critters that prey on the beneficial ones.

2. **Challenge**

There are 4 combinations of growing methods and chemical agent used. Each has different costs and characteristics. The primary challenge is to optimize the production process to ensure a viable final product. This product must contain a minimum of 40% beneficial critters and no more than 7% harmful critters.

3. **Objective**

Formulate and solve an optimization problem that maximizes the production of beneficial critters while adhering to the constraints of cost, production environment efficiencies, technician capacity, and the final product requirements regarding the proportion of good and bad critters.

Workflow for Drafting Optimization of Critter Production

Step 1: Harvesting

Every gram of environment A produces 0.44 grams of the critter mix.

Every gram of environment B produces 0.72 grams of the critter mix.

<u>Overtime costs for harvesting:</u> If processing exceeds 1000 grams, overtime costs are incurred at a rate of $4 per additional gram, up to 1400 grams.

Step 2: Combining Chemical Agents & Baking

Agent 1 is applied to the critter mix at a cost of $18 per gram.

Agent 2 is applied to the critter mix at a cost of $12.5 per gram.

<u>Overtime costs for baking:</u> If processing exceeds 800 grams, overtime costs are incurred at a rate of $6.5 per additional gram, up to 1100 grams.

Workflow for Drafting Optimization of Critter Production

Step 3: Blending Problem

Consider each combination of environment and agent that could yield a different mix of critters

From Environment A with Agent 1: 50% good, 5% bad critters.

From Environment A with Agent 2: 40% good, 6% bad critters.

From Environment B with Agent 1: 60% good, 10% bad critters.

From Environment B with Agent 2: 30% good, 6% bad critters.

Step 4: Combine the results into one final vat of critters

Ensure the final product meets the success criteria:

At least 40% good critters & No more than 7% bad critters.

The final product is sold at $100 per gram.

Question 1 - Formulation

For the Crispy Critters Inc. (CCI) case, we can create a Linear Programming (LP) formulation starting with defining the following components:

1. **Variables** - they indicate the amounts of the critter mix from environments A and B processed with agents 1 and 2, and the total amounts of environments A and B used, as well as whether overtime is needed in the harvesting and baking steps
2. **Constraints** - they ensure the percentage of critter mix harvested from each environment as well as limitation on overtime
3. **Objective Function** - maximize profit, which is calculated as the revenue from selling the critter mix minus all associated costs

Question 1 - LP formulation

Define the variables

A/B: the number of grams in environment A/B

A1/A2: the harvest from environment A then using agent1/agent2

B1/B2: the harvest from environment B then using agent1/agent2

y1(Binary variable): whether technicians will work overtime on the harvest step

y2(Binary variable): whether technicians will work overtime on the baking step

Question 1 - LP formulation

Constraints

1. **Composition Constraints**
   The final mix must have at least 40% good critters and no more than 7% bad critters.
2. **Conversion Rates**
   Conversion rates of 0.44 and 0.72 for environments A and B, respectively, need to be applied to calculate the actual amount of critter mix obtained.
3. **Overtime Constraints**
   The overtime work in harvesting and baking should not exceed the specified extra capacities

Question 1 - LP formulation

Constraints Formulation

The proportion between step one and step two:

1. A1 + A2 = 0.44 * A
2. B1 + B2 = 0.72 * B

Overtime output limit:

M = 100000 # A large constant

3. A + B <= 1000 + M * y1; A + B >= 1000 - M * (1 - y1);
4. A1+A2+B1+B2>=800*y2; A1+A2+B1+B2<=1100*y2

Critters composition:

5. 0.5A1+0.4A2+0.6B1+0.3B2>=0.4(A1+A2+B1+B2)
6. 0.05A1+0.06A2+0.1B1+0.06B2＜=0.07(A1+A2+B1+B2)

Question 1 - LP formulation

Objective Function

Maximize Profits= Revenue -Cost

Revenue = 100(A1+A2+B1+B2)

Cost: 1. Direct cost:

11A+17.5B

18A1+12.5A2+18B1+12.5B2

2. Overtime work cost:

4*y1*(A+B-1000)

6.5*y2*(A1+A2+B1+B2-800)

Question 2 - Optimal Solution

A1	86.625
A2	0
B1	259.875
B2	606.375
A	196.875
B	1203.125
y1	1
y2	1

1400 grams of raw material in total are required

196.875 for environment **A** & 1203.125 for environment **B**

*Harvest from Environment A using Agent 1(**A1**): 86.625 units.*

*No harvest from Environment A using Agent 2 (**A2**).*

*Harvest from Environment B using Agent 1(**B1**): 259.875 units.*

*Harvest from Environment B using Agent 2(**B2**): 606.375 units.*

Technicians need to work over time for both harvesting and baking

400 grams of overtime needed for harvesting.

152.875 grams of overtime needed for baking.

The optimal solution brings a profit of **55656.813**

Question 2 - Optimal Solution

Revenue:

100(A1+A2+B1+B2)

Cost:

1. Direct cost:

    11A+17.5B

    18A1+12.5A2+18B1+12.5B2

2. Overtime work cost:

    4*y1*(A+B-1000)

    6.5*y2*(A1+A2+B1+B2-800)

A1	86.625
A2	0
B1	259.875
B2	606.375
A	196.875
B	1203.125
y1	1
y2	1

Revenue	
100(A1+A2+B1+B2)	95287.49989
direct cost	
11A+17.5B	23220.3125
18A1+12.5A2+18B1+12.5B2	13816.68748
overtime work cost	
4*y1*(A+B-1000)	1600
6.5*y2*(A1+A2+B1+B2)	6193.687493
Max Profit	
=Revenue-direct-overtime	55656.8125

Question 3 - Sensitivity Analysis

- **Objective:** To find the coefficients are sensitive enough for further investigation

- **Method:** Looking at the rc(Reduced Cost for an additional unit of variable) and the range from Objlow and Objup, which represent the range that objective function coefficient can vary without changing the solution.

Sensitivity Analysis Output Table

Total Profit	55656.8125				
	X	Obj	**rc**	**Objlow**	**ObjUp**
A1	86.625	75.5	0	28.17647059	112.9394
A2	0	81	-18.8555	#NAME?	99.85547
B1	259.875	75.5	0	32.89655172	546
B2	606.375	81	0	57.1039604	inf
A	196.875	-15	0	-250.9711111	-1.20857
B	1203.125	-21.5	0	-35.29142857	inf

- **Answer:**As the range between Objlow and Objup are all wide as the range of variation, it could infer that these objective function coefficients are not that sensitive. However, the reduced cost for A2 is -18.8555 which would lead to change in objective function when X shift from 0. **Hence, we still need to investigate more on A2.**

Question 3 - Sensitivity Analysis

- **Objective:** To explain what the shadow prices infer from the solution's sensitivity to the assumption

- **Method:** Looking into pi(The Shadow Price) from each constraint to explain how an additional value add to RHS for the constraint is going to affect the Total Profit.

Sensitivity Analysis Output Table

	RHS	Slack	pi	RHSlow	RHSup
0.44A=A1+A2	0	0	109.5078	-98.82828283	95.11111
0.72B=B1+B2	0	0	75.94922	-161.7190083	155.6364
A+B>1000	1000	-400	0	#NAME?	1400
A+B<1400	1400	0	33.18344	1175.390266	1616.162
A1+A2+B1+B2>800	800	-152.875	0	#NAME?	952.875
A1+A2+B1+B2<1100	1100	147.125	0	952.875	inf
Good Critters	0	0	-147.0313	-25.2	69.88571
Bad Critters	0	0	-965.2344	-3.36	9.318095

- **Answer:**From the shadow prices, **the Good Critters and Bad Critter constraint influence the Total Profit negatively with huge sensitivity** since they have -147.031 and -965.234 for their shadow price. The first three constraints with highlight would positively affect the Total Profit with relatively high sensitivity base on their shadow price. These remaining constraint would not influence the objective value based on the solution we have for now.

Question 4 - Change in Harvest Yield

Increase A yield by an an additional 10%

- **Baseline Yield:** Starting with a baseline yield of 0.44 grams of critter mix per gram of Environment A.
- **10% Increase Scenario:** Projected outcomes show a significant increase in harvest yield, leading to improved production efficiency.
- **Incremental Gains:** Detailed analysis for each 1% increment in yield from Environment A shows a positive trend in harvest output, with potential for substantial gains in profitability.

The optimization process enhances profitability by exploiting the increased yield, which decreases the marginal cost per unit and, assuming stable prices, expands the profit margin for product A.

Initial Proportions

Product A: A1 + A2 = 0.44 * Total Quantity of A

Adjusted for Yield Increases

For product A: Adjusted Proportion A = 0.44 * (1 + % in Yield of A)

Proportion	% Change	Harvest Yield
A	0.01	55750.86538
	0.02	55843.32903
	0.03	55934.24341
	0.04	56023.64713
	0.05	56111.57753
	0.06	56198.07073
	0.07	56283.16168
	0.08	56366.88421
	0.09	56449.27107
	0.1	56530.35396

Question 4 - Change in Harvest Yield

Increase B yield by an an additional 5%

- **Baseline Yield:** Starting with a baseline yield of 0.72 grams of critter mix per gram of Environment B.
- **10% Increase Scenario:** Projections reveal a notable harvest yield increase, enhancing production output significantly.
- **Incremental Gains:** For each 1% improvement in yield from Environment B, analysis shows a consistent upward trend in harvested critter mix, indicating potential for marked profitability improvements.

Initial Proportions

Product B: B1 + B2 = 0.72 * Total Quantity of B

Adjusted for Yield Increases

For product B: Adjusted Proportion B = 0.72 * (1 + % in Yield of B)

Proportion	% Change	Harvest Yield
B	0.01	56313.79872
	0.02	56968.94235
	0.03	57622.25113
	0.04	58273.73275
	0.05	58923.39488

The optimization process capitalizes on these yield increases, which lowers the cost per unit of harvested mix. With steady sale prices, this leads to a wider profit margin for product B.

Question 5 - Impact of Critter Percentage on Profits

Impact of Increasing Allowable Bad Critters on Profits

Increasing the allowance of bad critters from 7% to 10% does not impact the profits, which remain stable at $58,900.00. This indicates that within this range, the cost of additional bad critters does not significantly affect profit margins.

profits	good critters	bad critters
55656.81	40	7
58900.00	40	8
58900.00	40	9
58900.00	40	10

Question 5 - Impact of Critter Percentage on Profits

Impact of Reducing Good Critters on Profits

Reducing the required percentage of good critters from 40% to 35% results in a downward trend in profits. The exact impact per 1% reduction needs to be calculated based on the optimization model outputs. This trend suggests that maintaining higher standards for good critters is financially beneficial.

profits	good critters	bad critters
55656.81	40	7
57089.17	39	7
58587.06	38	7
59454.40	37	7
59639.20	36	7
59824.00	35	7

Question 6 Change in Staffing Levels

Increase Profits by assigning new employees

- **Objective:** Find the best number of new harvesting and baking staff to make more profits without overspending.
- **Variables:**
  - Harvesting Team: How many workers are in the harvest area.
  - Baking Team: How many workers are in the bake area.
- **Constraints :**
  - Harvest Work Limit: Harvesters can process more than 1400 grams weekly.
  - Baking Work Limit: Bakers can handle more than 1100 grams of mix weekly.

We'll balance the extra cost of hiring against the money we can make from more product, use gurobipy (linear programming) to find the number of workers that brings in the most profit.

Question 6 Change in Staffing Levels

New employees would increase the fixed cost by $30 per gram for harvesting and $20 per gram for baking.

Define two new variables:

1. h: the number of grams that new employees produce in harvesting
2. b: the number of grams that new employees produce in baking

Change the constraints :

1. A + B >= (1000+h)* y1; A + B <= (1400+h)* y1
2. A1+A2+B1+B2>=(800+b)*y2; A1+A2+B1+B2<=(1100+b)*y2

Change the objective function:

New overwork cost: **4*y1*(A+B-1000-h)** | **6.5*y2*(A1+A2+B1+B2-800-b)**

New objective function: Profits-Direct cost-New overwork cost-30*h-20*b

Question 6 Change in Staffing Levels

A1	100
A2	0
B1	300
B2	700
h	**216.1616**
b	**0**
A	227.2727
B	1388.889
y1	1
y2	1
Max Profit	**57209.6**

After the calculation, we can assign new employees to produce **216.16** grams of environment in harvest department and 0 grams of harvested mix treated in baking department.

After the inclusion of new employees, profits experienced a notable increase, rising from **$55,656.80** to **$57,209.60**. This boost of **$1,552.78** can be attributed primarily to the revenue generated from the products produced by new team members, after accounting for fixed costs

Conclusion - What We Have Done

1. **LP Formulation for CCI:**
   - Developed a Linear Programming model to maximize profits.
   - Decision variables included the quantities of environments A and B used, and the amounts of Agents 1 and 2 applied.
   - Constraints incorporated the harvesting and baking capacities, overtime possibilities, and quality requirements for the final product.
2. **Optimal Solution Presentation:**
   - Provided clear guidance on raw material procurement and technician scheduling.
   - Ensured the production department understood the quantities needed for each component.
3. **Sensitivity Analysis:**
   - Conducted sensitivity analysis to identify which objective coefficients significantly impact the solution.
   - Analyzed shadow prices to understand the constraints' sensitivity and their impact on the optimal solution.

Conclusion - What We Have Done

4. Yield Improvement Analysis

- Evaluated the effects of increasing the yields for both A and B harvests.
- Presented a detailed table showing the impact of each 1% change in yields on overall profitability.

5. Quality Requirements Impact:

- Analyzed how changes in the quality requirements (percentage of good and bad critters) affect profits.
- Prepared tables to illustrate the effects of varying these percentages.

6. Staffing Level Optimization:

- Assessed the impact of increasing staffing in harvesting and baking departments.
- Recommended optimal staffing levels considering the increased fixed costs and existing overtime policies.

Conclusion - Key Takeaways & Business Insight

1. **Cost Management in Production**
   - **Strategic Handling of Variable Costs**: The analysis sheds light on managing variable costs effectively, such as the cost differences between environments A and B and the agents used for processing.
   - **Overtime Work Optimization**: The model offers a strategic approach to handling overtime costs for technicians. By understanding when and how much overtime is beneficial, CCI can manage labor costs more effectively.

2. **Quality Control for Market Success**
   - **Ensuring Product Efficacy**: Adherence to quality constraints, such as maintaining a minimum percentage of good critters, is crucial for the efficacy of the product in decomposing abandoned automobiles.
   - **Market Reputation and Compliance**: Maintaining a low percentage of bad critters is vital for market reputation and potential regulatory compliance, ensuring the product is safe and effective.

Conclusion - Key Takeaways & Business Insight

3.Impact of Production Changes

- Yield Improvement Analysis: The model's analysis of how yield improvements in environments A and B impact overall profitability provides a roadmap for where technological improvements could be most beneficial.
- Quality Requirement Adjustments: Understanding the impact of changing the ratio of good to bad critters in the final product allows CCI to explore different market positions or respond to regulatory changes.

4.Workforce and Capacity Planning

- Efficient Staffing Strategies: to optimal staffing levels, considering the potential increase in fixed costs due to new hires, enable CCI to make informed decisions about expanding their workforce.
- Balancing Capacity and Demand: The model aids in aligning harvesting and baking capacities with market demand, ensuring that CCI can meet customer needs without excessive expenditure on resources.

Thanks for Watching!