Estimating Marginal Treatment Effects using Subjective Beliefs

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Note: Paper will be available soon

Introduction

Treatment Effects and Policy Evaluation

- Estimating the causal effect of being treated on an outcome is a central problem in applied economics
- Estimating causal effects is complicated because treatment effects can vary across individuals and selection into treatment is often not random
- Selection into treatment may be driven by unobserved factors that are correlated with the potential outcomes
- Fundamental identification problem: Individuals are only observed in one treatment state
- Thus, quasi-experimental methods (e.g., IV) are used to estimate treatment effects
- However, these treatment effect estimates are potentially specific to the subpopulation for whom the quasi-experiment generates variation (see LATE)
- The external validity of treatment effect estimates is a key concern

Marginal Treatment Effects (MTEs)

- Heckman & Vytlacil (1999, 2005, 2007) introduce marginal treatment effects (MTEs). Weighted sums of MTEs equal other treatment effects (e.g., ATE, LATE, ATT, ATU)
- MTE is basically the LATE that is local to a part of the distribution of the unobserved propensity to select into treatment
- Generally hard to identify in naturally occurring data.
  - IV estimation requires a "super-instrument" that induces orthogonal variation in treatment probabilities continuously across the full distribution of unobserved selection propensity
- Can point-estimate MTEs without a super-instrument with parametric assumptions:
  - Parametric distributions (e.g., Bjorklund & Moffitt (1987), Aakvik Heckman & Vytlacil (2005)), shape (e.g. Manski & Pepper (2000)), independence (Carneiro, Heckman & Vytlacil (2011)), additive separability (Brinch, Mogstad & Wiswall (2017))
- Can provide bounds on the MTEs:
  - See, e.g., Manski (1990), Manski (1997), Manski (2003), Heckman & Vytlacil (2007b), Mogstad, Santos, & Torgovitsky (2018)

This paper

Contribution: New strategy for estimating *MTEs* using survey data on subjective expectations

Key idea: Individuals have private information regarding selection into and effects of treatment

Start from the widely used Roy model and collect data that identifies key elements of model

- Subjective probability of treatment identifies the latent determinant of treatment selection

$\Rightarrow$ Estimate ex ante *MTE*s (2 methods depending on data)

  - Subjective probability of treatment and realized outcomes
  - Subjective probability of treatment and subjective beliefs about outcomes

$\Rightarrow$ Evaluate policies that have not yet occurred and test policy invariance of *MTEs*

Outline

1. Roy model and marginal treatment effects with survey data
2. Application 1: The expected effect of childbirth on female labor supply in Denmark
   - Custom survey in Denmark: measures beliefs regarding childbirth and labor supply $\Rightarrow$ estimate ex ante MTEs
   - Merge survey data with registry data to compare survey responses to measured behavior
   - Forecast treatment effects for alternative childcare policies - test policy invariance of MTEs
3. Application 2: The effect of childbirth on female labor supply in the US
   - NLSY97 panel, 2001 and 2006 has data on expected fertility, realized fertility, and realized labor supply $\Rightarrow$ estimate MTEs

The Data Generating Process: The Generalized Roy Model

The Generalized Roy Model 1/2

- Let $D^i \in \{0,1\}$ indicate individual i treatment status
- $Y_0^i \in \mathbb{R}$ is an outcome in the untreated state, $Y_1^i \in \mathbb{R}$ is an outcome in the treated state

$$D^i = \begin{cases} 1 & \text{if } \mu^i \geq V^i \\ 0 & \text{if } \mu^i < V^i. \end{cases}$$

- $\mu^i \sim K_{\mu|X^i}$ stochastic selection component, distribution a function of variables $X \in \mathbb{R}^{d_x}$
  - Random μ captures uncertainty about selection into treatment
  - Note: the distribution is the same for all people with the same X
- $V^i \in \mathbb{R}$ is idiosyncratic selection component (i.e., a distaste for treatment)

The Generalized Roy Model 2/2

- $\mathcal{I}^i$ is individual i's information set at time t before treatment selection occurs
  - Note: This is when we will survey people
- $\mathcal{I}^i$ contains X^i, V^i, and $K_{\mu|X^i}$, but does not include μ^i, and needn't include Y_0^i, Y_1^i
- At the time of treatment selection μ^i is realized and treatment status is determined
- $H_D^i(Y_D^i)$ is individual i's subjective probability distribution over outcomes in each state D

$$H_D^i(Y_D^i) := P(Y_D^i|\mathcal{I}^i) \;\; \forall \; D.$$

  - The subjective probability of outcomes, $H_D^i(Y_D^i)$, along with other idiosyncratic components like preferences and costs, are potential determinants of V^i
- Let G^i denote individual i's subjective probability of being treated:

$$G^i := P(D^i = 1|\mathcal{I}^i) \tag{1}$$

Linking Roy Model to Marginal Treatment Effects

- $F_{V|X}$ is CDF of V conditional on X
- $M^i = F_{V|X^i}(\mu^i)$ (M^i is the fraction of X^i-people with $V < \mu^i$)
- $U^i = F_{V|X^i}(V^i)$. (U^i is i's order in the X^i-distribution of distaste for treatment)

Since $F_{V|X}$ is an increasing function, $M^i \geq U^i \iff \mu^i \geq V^i$.

$$D^i = \begin{cases} 1 & \text{if } M^i \geq U^i \\ 0 & \text{if } M^i < U^i. \end{cases}$$

MTE is the expected gain from treatment $(Y_1^i - Y_0^i)$ conditional on X and U:

$$MTE(u, x) := \mathbb{E}\left[Y_1^i - Y_0^i | U^i = u, X^i = x\right].$$

The *MTE* is the average treatment effect local to the population with similar values of X and U (distaste for treatment).

Estimation of MTEs using Survey Data

The Key Idea (see paper for precise assumptions/propositions)

- Subjective treatment probabilities, G^i, are sufficient to identify U^i for each individual
- Consider two individuals, i and j, who have the same observable characteristics X
- Assume people use $\tilde{K}$ to form subjective expectations of μ^i (not necessarily correct, but necessarily common)
- Then, G^i only differ because $V^i \neq V^j$
- Probability of receiving treatment is decreasing in V and V is in an individual's information set
- Thus, individual i will have a higher subjective probability of selection into treatment than individual j iff $V^i < V^j$

The Key Data

- G^i and H^i_D are both subjective probability distributions, so they can be measured directly via appropriately designed surveys
- Let $(\hat{G}^i, \hat{H}^i_D)$ denote the reported values of subjective beliefs
- Define $\hat{U}^i$ as one minus individual i's percentile in the distribution of $\hat{G}^i$, conditional on X:

$$\hat{U}^i := 1 - \frac{Percentile(\hat{G}^i|X)}{100}. \tag{2}$$

Reported Subjective Treatment Probabilities, $\hat{G}^i$, identify U_i

Assumption 2

Suppose that $\hat{G}^i$ is reported such that the order of individuals equals their order in the true subjective probabilities. That is, $G^i \leq G^j \iff \hat{G}^i \leq \hat{G}^j \;\; \forall\, i, j.$

Proposition 1

Let Assumption 2 hold. If there is a sufficiently large number of individuals within each set of conditioning variables X, then $\hat{U}^i \to_P U^i$.

- Loosely speaking, measuring the subjective probability of selection into treatment makes the unobserved latent propensity parameter "observed," so that we can proceed to estimate *MTEs* by directly conditioning on the now-observed latent propensity.

Estimating *MTEs* with Subjective Treatment Probs and Realized Outcomes

- Define the realized outcome for individual i as $Y^i = Y_0^i(1 - D^i) + Y_1^i D^i$

Proposition 2

Let Assumption 2 hold. Furthermore, let $\{Y^i\}, \{D^i\}, \{X^i\}$ be measured for a representative random sample of the population. Then the MTE function can be estimated by

$$\hat{MTE}(x, u) = \mathbb{E}\left[Y^i | D^i = 1, X^i = x, \hat{U}^i = u\right] - \mathbb{E}\left[Y^i | D^i = 0, X^i = x, \hat{U}^i = u\right]$$

- Compare the average outcome among treated individuals with the average outcome among non-treated individuals for those who have the same ex ante probability of being treated

- Useful because of increased measurement of subjective probabilities in many surveys

Estimating *MTEs* with Subjective Beliefs about Treatment and Outcomes

Can estimate MTEs without realized outcomes if we have expectations about outcomes.

- The ex ante ITE^i is the expected difference in treatment-contingent outcomes for individual i

$$ITE^i = \mathbb{E}_{H_1^i}\left[Y_1^i\right] - \mathbb{E}_{H_0^i}\left[Y_0^i\right]$$

- By nature, *ITEs* are impossible to measure using realized outcomes
- Note: It is the subjective beliefs about ITEs that enter into the selection equation
- Can measure subjective expected individual treatment effects using survey data:

$$I\hat{T}E^i = \mathbb{E}_{\hat{H}_1^i}\left[Y_1^i\right] - \mathbb{E}_{\hat{H}_0^i}\left[Y_0^i\right]$$

Subjective Expectations Data Identifies $MTE(x, u)$

Assumption 3

$$\mathbb{E}_{\hat{H}^i_D}\left[Y^i_D\right] = \mathbb{E}_{H^i_D}\left[Y^i_D\right] \;\forall\; D, i$$

- Assume the mean reported subjective state contingent outcome equals the mean subjective state contingent outcome for each state and individual

Proposition 3

Let Assumptions 2 and 3 hold. Then the MTE function can be estimated by

$$M\hat{T}E(x, u) = \mathbb{E}\left[I\hat{T}E^i | X^i = x, \hat{U}^i = u\right]$$

Summary of Method

- Nonrandom selection into treatment is a big concern for estimating externally-valid treatment effects
- One large concern is selection based on idiosyncratic preferences/costs
- In many contexts, individuals may have private information about their idiosyncratic preferences/costs
- Idea: use surveys to measure subjective beliefs of selection
- Show, under explicit conditions, that subjective beliefs identify key elements of Roy model
- Use subjective expectations to nonparameterically estimate MTEs for the whole population
- Tradeoff between stronger assumptions and stronger propositions. . . currently working on finding the right balance

Applications

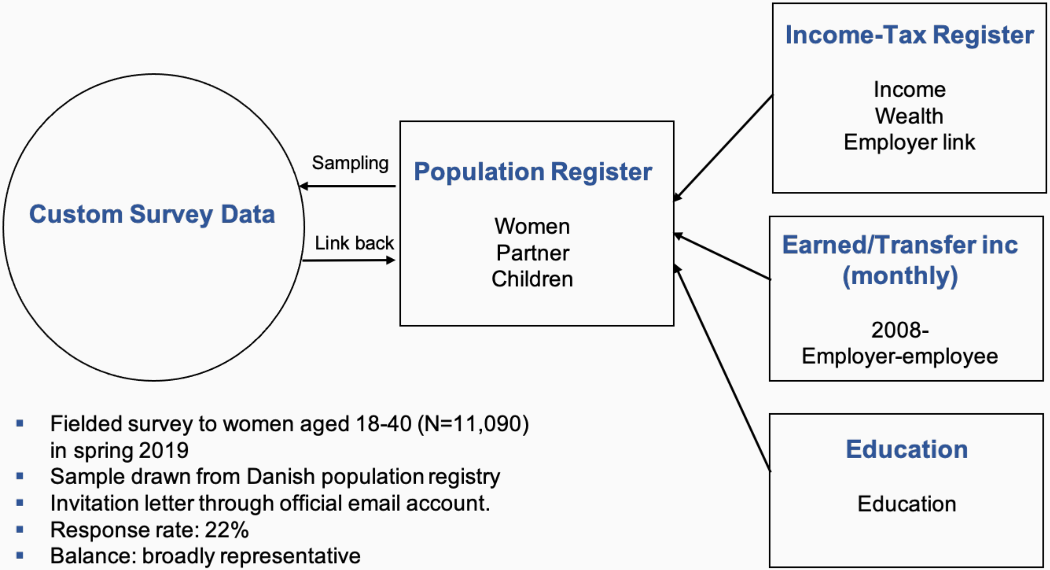
Outline

1. Roy model and marginal treatment effects with survey data
2. Application 1: The expected effect of childbirth on female labor supply in Denmark
    - Custom survey in Denmark: measures beliefs regarding fertility and labor supply $\Rightarrow$ estimate ex ante MTE
    - Merge survey data with registry data to validate survey responses
    - Forecast treatment effects for alternative childcare policies—test policy invariance of MTE
3. Application 2: The effect of childbirth on female labor supply in the US
    - NLSY97, 2001 and 2006 holds information about expected fertility and realized fertility and labor supply
    - Estimate MTE by comparing realized outcomes of treated and non-treated individuals who have the same ex ante probability of being treated.

Institutional Setting: High Quality Affordable Childcare and Job Protection

- Universal public child care for children aged 0-5 (83% of children attend).
- Opening hours for public day care are typically 6:30am to 5:00pm on weekdays
- Child care is of relatively high quality (regulation of quality and quantity of staff, physical surroundings, safety, and hygienic standards)
  - Nursery (0-2 years): three children per employee.
  - Kindergartens (3-5years): six children per employee
- Heavily subsidized: out of pocket expenditures about 1/3 of total costs
  - Nursery: 3,000 DKK/month (425 USD/Month)
  - Kindergarten: 1,700 DKK/month (245 USD/Month)
- Job protection by law
- 52 weeks of paid maternity/parental leave (leave benefits $\approx$ UI benefits)
- Child care and job protection system has been in place for 40+ years
  $\Rightarrow$ well-known and stable

Data and Sampling

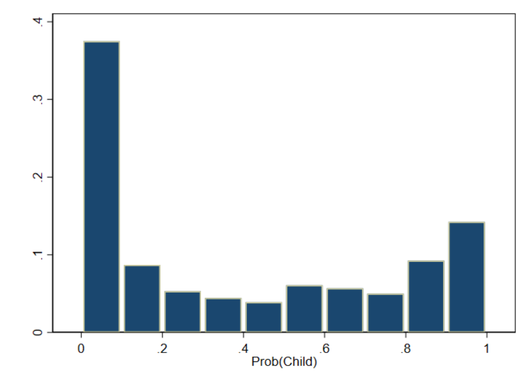


- Fielded survey to women aged 18-40 (N=11,090) in spring 2019
- Sample drawn from Danish population registry
- Invitation letter through official email account.
- Response rate: 22%
- Balance: broadly representative

Key Original Survey Data

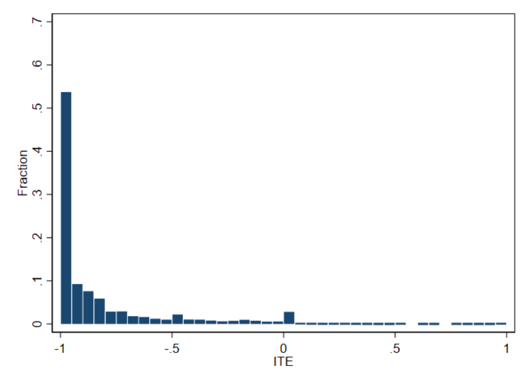
- Skipping details of survey design/fielding, sample details, balance, etc.
- Treatment is having a child
- Outcome is working t months around having a child, $t \in \{-3, 3, 9, 18, 36\}$ or working in a typical month for the not having a child state
- Probability of having a child in next four years $\hat{G}^i$
- Probability of working at t horizons if respondents do/do not have a child $\hat{H}_{D,t}(Y^i_{D,t})$
- Repeat questions for counterfactual policy environments
- Will now show you key data from survey

Reported Probability of Having a Child: $\hat{G}^i$



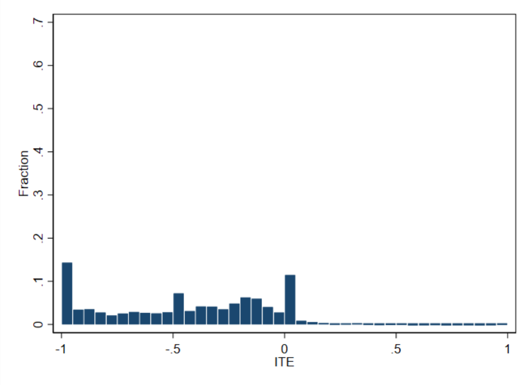
- Average Expectation: 39%
- Population Average, 2013-2018: 41%

ITE Density at Month 3: $\hat{ITE}^i = \mathbb{E}_{\hat{H}^i_1}\left[Y^i_1\right] - \mathbb{E}_{\hat{H}^i_0}\left[Y^i_0\right]$



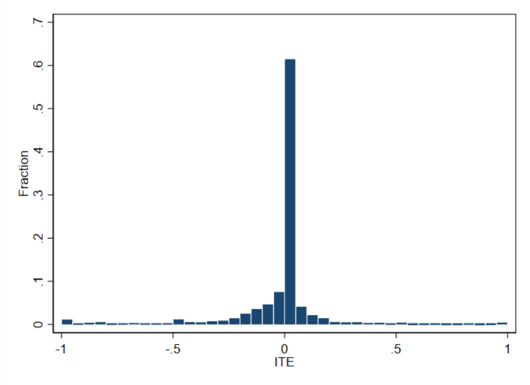
- Most women typically work and most are not working 3 months after childbirth ($ITE =$ -1)
- Some *ITE* heterogeneity $\rightarrow$ some scope for heterogeneity in *MTE*s

ITE Density at Month 9



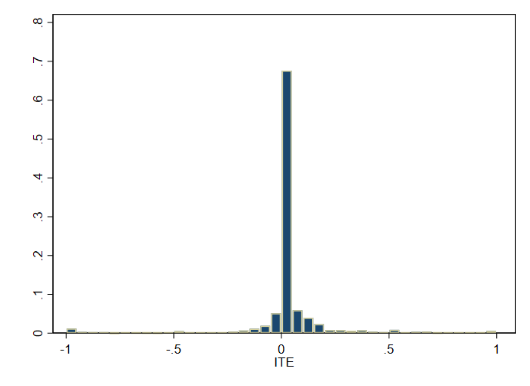
- Lots of *ITE* heterogeneity $\rightarrow$ scope for heterogeneity in *MTE*s

ITE Density at Month 18



- Limited *ITE* heterogeneity $\rightarrow$ will be limited heterogeneity in *MTE*s

ITE Density at Month 36



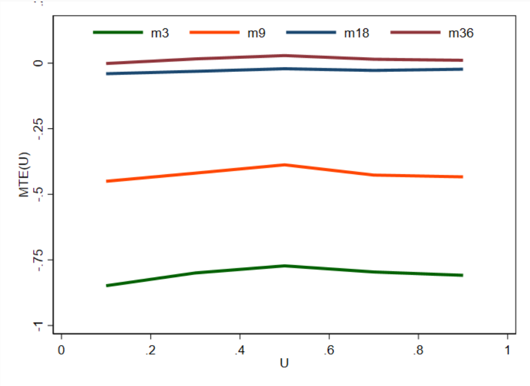
- Very limited *ITE* heterogeneity → will be limited heterogeneity in *MTE*s

Computing ex ante *MTEs*, by U and X bins

- Select some X variables (e.g., age and number of children)
- Create partition over X
  - E.g., (age $\leq$ 30, no child); (age $>$ 30, no child); (age $\leq$ 30, 1 child); etc.
- Within each subset of the X partition, order individuals in U. I.e., generate $\hat{U}^i$ according to their $\hat{G}^i$
- Partition the $\hat{U}^i$ set for each X subset
  - E.g., 4 quartiles
- For each subset of the $\hat{U} \times X$ partition, compute $MTE(u, x)$ as the average ITE for members of that subset
- We can compute ATE, LATE, ATT, ATU by appropriately weighting the MTE functions by $\hat{G}^i$, as outlined in Heckman and Vytlacil (2005)
  - E.g., ATE is average ITE. ATT is weighted-average of ITE, with more weight on people more likely to be treated $\left(\omega^i = \frac{\hat{G}^i}{\bar{\hat{G}}}\right)$

Ex ante *MTEs*, unconditional on X

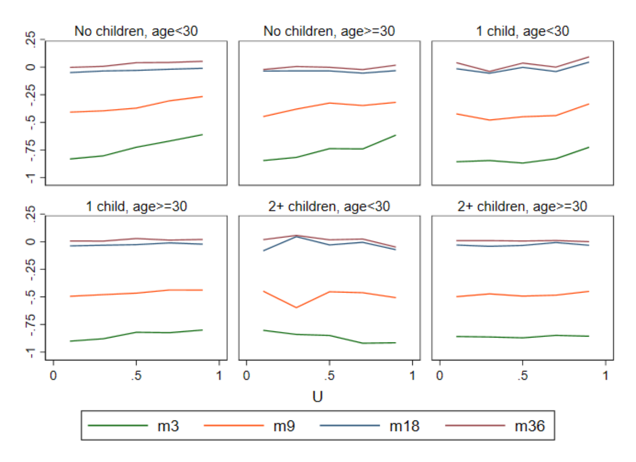
Ex ante *MTE*



- Very limited *MTE* heterogeneity ⇒ limited selection on unobserved factors
- $LATE \approx ATT \approx ATE$
- Could be hiding heterogeneity at disaggregated level (conditional on X)

Ex ante *MTEs*, by *U* and *X* bins

Ex ante *MTE*



- Main source of selection on unobservables is at 3 months in women with no other children

Comparison with Estimates Based on Historical Data on Realized Outcomes

- In 2 slides will show you ATT using our method on subjective expectations survey data
- Fundamentally it is impossible to validate expected fertility and labor supply until realizations are recorded (in 5-6 years time)[1]
- If environment is stable (no major shocks or policies affecting labor supply and fertility) then historical realizations may be informative
- Consider fertility in period 2013-2018 of women aged 20-40 in 2012
- Estimate labor supply response around child births happening 2010-2015 using event study design (i.e., estimate ATT with event study)
- Compare ATT from survey with ATT from event study

[1] In Application 2 we have both expected and realized fertility

Compare with Effect Estimated on Registry data - Event Study

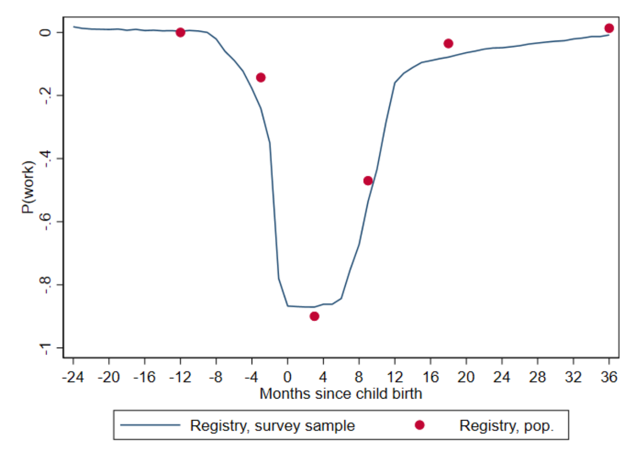
$$y_{it} = \alpha_t + \alpha_2 D_{it}^{Age} + \sum_{k=-B}^{A} D_{it}^{k} \delta_k + \alpha_3 X_{it} + u_{it}$$

- y_{it}: dummy for working
- α_t: time fixed effects
- D^{Age}: age fixed effects
- $D1_{it}^{k}$: dummy for child born k periods ago,
- δ_k: measures the effect k periods after birth
- $-B$: earliest period on the event axis
- A: latest period on the event axis for the first child
- X: vector of dummy variables controlling for timing of surrounding children

$\rightarrow$ Estimate on monthly registry data for survey sample (have child, 2010-2015)

$\rightarrow$ δ_k is ATT (no pretrend). Compare with ATT computed using survey methodology

Effect of Childbirth on Labor Supply, Survey and Event Study



Extra

Summary, comparison with historical data

Take away from comparison of survey data with registry data estimates

- Labor supply: Very close match for ATT on months -3, 3, 9, 18, and 36 ($<$5% deviation)
- Level of fertility is comparable to historical level and pattern of correlations with covariates is the same for fertility expectations and historical fertility (not reported in this presentation)

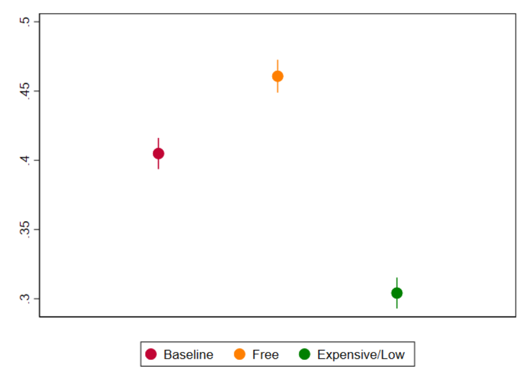
$\Rightarrow$ Overall, we take evidence to suggest that the survey data contain useful information about fertility and labor supply, loosely speaking to validity of assumptions

Policy Invariance of *MTE*

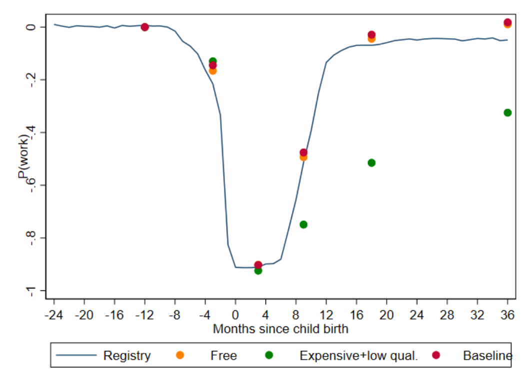
- Policy Relevant Treatment Effects (*PRTE*) can be constructed from baseline *MTEs* if policy affects the subjective treatment probability, G^i, but not state contingent outcomes $(\mathbb{E}_{H_1^i}[Y_1], \mathbb{E}_{H_0^i}[Y_0])$
- Test policy invariance by quantifying ex ante *MTEs* in alternative policy regimes.
- We forecast $(\mathbb{E}_{\hat{H}_1^i}[Y_1], \mathbb{E}_{\hat{H}_0^i}[Y_0])$ and $\hat{G}^i$ for two policy changes:
  1. Make childcare free
  2. Increase cost ($\times 3$), and reduce quality of childcare (cut staff by 50% + unskilled)

Counterfactual MTEs

Average Childbirth Expectations



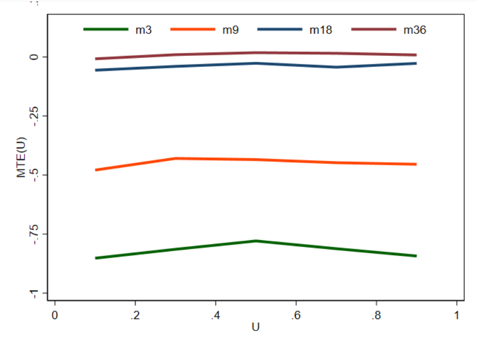
Labor Supply Conditional on Childbirth (ATT)



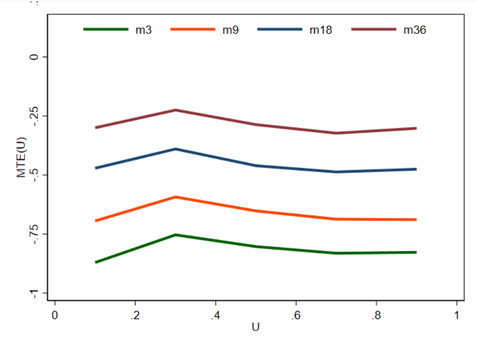
More

Counterfactual Fertility and Labor Supply

Free Childcare



Expensive/Low Quality Childcare



- *MTEs* in free childcare regime unchanged ⇒ *MTE* policy invariant
- *MTEs* in expensive/low quality childcare regime shift down, ⇒ *MTE not* policy invariant
- Advance information about who will respond to policy

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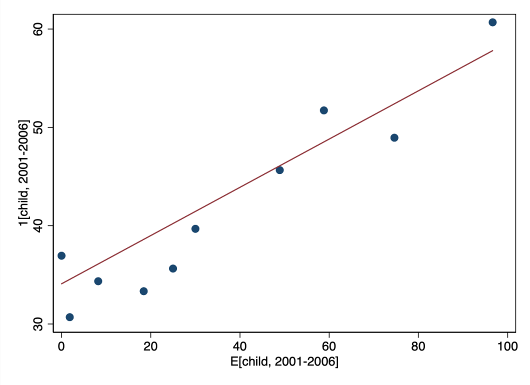
Application 2: The Effect of Childbirth on Female Labor Supply in US

- The NLSY97's 2001 wave contained an expectations module
  - Key question: What is the percent chance that you will have [a/another] child within the next five years?
- Match responses to this question to labor market outcomes in 2006
- Drawback: Sample is to small to estimate MTEs for very granular partition of X, U
  - Sample includes women aged 17-21 at time of expectation measurement (Dec. 2001)

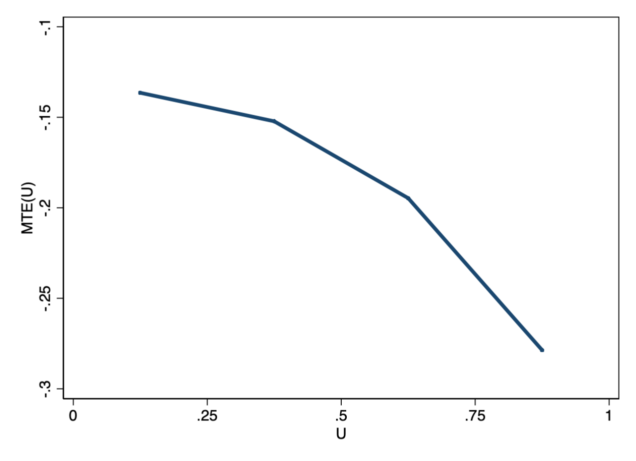
Expected vs Realized Fertility, NLSY



Notes. The figure shows a binned scatter plot of realized fertility during 2001-2006 by bins of expected 5-year fertility measured in 2001.

- Suggestive evidence that rank of $\hat{G}^i$ is preserved as is required in Assumption 2

MTE



- Women who are idiosyncratically more likely to have a child are also women who are more likely to work after having a child

Treatment Effects

Parameter	Expression	Estimate
Average Treatment Effect (ATE)	$\mathbb{E}[Y_1 - Y_0]$	-0.19
Average Treatment of the Untreated (ATU)	$\mathbb{E}[Y_1 - Y_0 \mid D = 0]$	-0.21
Average Treatment of the Treated (ATT)	$\mathbb{E}[Y_1 - Y_0 \mid D = 1]$	-0.14
Local Average Treatment Effect (LATE)	$\mathbb{E}[Y_1 - Y_0 \mid u \in (\underline{u}, \bar{u})]$	
	$(\underline{u} = 0,\ \bar{u} = .25)$	-0.14
	$(\underline{u} = .25,\ \bar{u} = .5)$	-0.15
	$(\underline{u} = .5,\ \bar{u} = .75)$	-0.19
	$(\underline{u} = .75,\ \bar{u} = 1)$	-0.28
Policy Relevant Treatment Effect (PRTE)	$\frac{\mathbb{E}[Y^\star]-\mathbb{E}[Y]}{\mathbb{E}[D^\star]-\mathbb{E}[D]}$	-0.13

Notes. *PRTE* summarizes the effect of a 5 percentage point increase in fertility. Average treatment effects are calculated as $(\frac{1}{B})\sum_b w_b \Delta Y_b$, where B is the number of bins defined over the support of U for which the parameter is calculated and where $w_b^{ATE} = 1$, $w_b^{ATT} = \hat{G}_b/\bar{\hat{G}}_b$ and $w_b^{ATU} = 1 - \hat{G}_b/1 - \bar{\hat{G}}_b$

Summary and Conclusion

- We introduce a data framework and set of assumptions that identify ex ante *MTEs* nonparametrically across the entire population
- The method relies on collecting subjective beliefs regarding the probability of treatment and either subjective beliefs about state contingent outcomes or realized outcomes
- The key assumption is that measured subjective beliefs concerning future treatment status are ordered similarly to true latent treatment propensity so that differences in measured subjective treatment probabilities reflect differences in latent treatment propensities
- Two applications about female labor supply around child birth:
  - Danish purpose built survey confronted with registry data
  - NLSY with both subjective probability of treatment and subsequent realizations
- Our method
  - is useful when individual is well-informed about treatment and potential outcomes
  - is useful when hard to obtain credible estimates from historical observations or for specific subpopulations of interest
  - can provide useful advance information on treatment effects of proposed new policies