Econ 237: Introduction

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Tolstoi on heterogeneous agent models

Happy families are all alike;
every unhappy family is unhappy in its own way.
Anna Karenina, p.1

Why heterogenous agent models?

- "Frictionless" models with competitive markets
  - aggregation theorems under some conditions
    - e.g. asset pricing (complete markets),
      Gorman aggregation of demand functions (linear Engel curves)
  - even if representative agent exists (literally or approximately)
    - heterogeneity still matters for some predictions, e.g. trading volume
    - welfare: e.g. who gains & loses from free.trade?
  - representative agent often assumed to simplify
    - appropriate abstraction for some questions
- Models with frictions
  - household borrowing limits & collateral requirements
  - boundaries of the firm: financing, organization
  - incomplete and/or illiquid markets
- Why care about frictions?
  - new predictions
  - effects on aggregates & cross section

Frictions & predictions

- Frictionless models & irrelevance theorems
  - Modigliani-Miller theorem: indeterminacy of shareholder payout & capital structure
  - Ricardian equivalence & irrelevance of money extensions
  - many asset structures complete the market
- Data: household, firm & investor patterns
  - payout & leverage procyclical & vary by size, profitability etc.
  - household wealth composition different over time & by age, income
  - financial institutions' leverage & portfolio composition in a crisis
  - frictionless models have little to say
- Data: market patterns beyond price & quantity
  - search activity: unemployment, vacancies, house listings, dealer inventory, search ranges by geography
  - duration: time on market, time to find a job, holding period
  - network effects: geography, trading relationships
  - even for P & Q: price dispersion & its dependence on trade quantity

Effects of frictions on aggregates & cross section

- Composition effects matter for aggregates
  - household responses to stimulus payments
    - consumption of constrained agents responds more
  - firm responses to banking crisis
    - activity of bank-dependent firms responds more
  - risky asset price responses to news
    - risk tolerant agents lose more $\longrightarrow$ amplification
- Redistribution shocks
  - inflation & interest rate changes
  - tax reform
- Learning from the from cross section (often short time series!)
  - match house prices & credit in aggregate & across regions / qualities
  - match jointly asset prices, trading volume & portfolio holdings

Organization of the course

- Daily life
  - lectures on tools + topics
  - weekly readings + 5 minute presentations
  - computer problem sets using tools: weeks 2-4, 6, 7
  - class project: quantitative modeling exercise
    - week 2: area of interest (1 paragraph)
    - week 4: project outline (1 page)
    - week 5: individual meetings $\longrightarrow$ executable plan
    - week 10: presentation of results
  - groups of up to three people allowed & encouraged
- Calendar
  - week 1: intro
  - weeks 2-4: lectures on optimal choice problems & equilibrium concepts
  - week 5: individual meetings on class problem
  - weeks 6-7 lectures on properties & computation of equilibrium
  - weeks 8-9: lectures on topics TBD
  - week 10: presentation of class projects

Numerical tools for problem sets & class project

- Matlab works well for many problem set questions
- For frontier research, invest in one faster language
  - now is a good time to make such an investment
  - the classic choice is C++; it remains a great option, but hard to learn
- Two strong newer languages:
  - Julia: domain specific language for scientific computing & stats
    - https://julialang.org
    - syntax similar to matlab, so works well for matlab users
  - Python: general purpose language
    - one neat distribution: https://www.anaconda.com
    - harder to learn; good at strings $\rightarrow$ popular in data science
  - very nice tutorials by Tom Sargent and John Stachurski https://lectures.quantecon.org
- Homework for next class
  - get set up to be able to write code in one faster language

Weeks 2-4 – three basic classes of models

1. Consumption & savings with labor income risk
   ("one tradable riskless asset, one nontradable asset")
   - data on household income & time series models
   - computation of consumption-savings problems
   - role of borrowing constraints
2. Portfolio choice
   ("many tradable assets")
   - data on asset returns & time series models
   - computation of portfolio problems
   - role of non-separable preferences
3. Consumption, savings & portfolio choice
   ("combining 1. and 2.")
   - data on household wealth positions, institutions' holdings
   - problems with multiple risks & various constraints
   - application: lifecycle portfolio choice with housing
   - matching models & data, presentation of results

Weeks 6-7: equilibrium

1. Equilibrium without aggregate risk
   - Setups with & without production
   - Stationary recursive rational expectations equilibrium
     - key simplification: constant distribution of endogenous state variables (e.g. wealth, internal funds)
   - Existence, uniqueness, computation & calibration
   - Transition dynamics
   - Welfare
2. Equilibrium with aggregate risk
   - Adding aggregate shocks
   - Approximations to stationary REE (Krusell-Smith, Reiter)
     - key challenge: moving distribution of endogenous state variables
   - Temporary equilibrium
   - Applications
     - business cycles with heterogeneous agents
     - asset pricing with frictions

Optimal choice models: a household example

- Many household index by i choose consumption, savings & portfolios
- Preferences: time separable power utility over consumption plans

$$E_0\left[\sum_{t=0}^{T}\beta_i^t\frac{\left(c_t^i\right)^{1-\gamma_i}-1}{1-\gamma_i}\right]$$

  - could allow random T; define I_t = set of agents alive at t
- Stochastic income process y_t^i (idiosyncratic & aggregate components)
- Assets
  - short safe bond: quantity b_t trades at price q_t, pays off one next period
  - infinitely lived stock: quantity s_t trades at price p_t, pays off stochastic dividend stream (d_t)
  - collateral constraint: borrow only up to ϕ (value of stocks)
- Constraints

$$c_t^i + q_t b_t^i + p_t s_t^i = b_{t-1}^i + (p_t + d_t)\, s_{t-1}^i + y_t^i$$
$$-b_t \leq \phi p_t \theta_t, \qquad c_t \geq 0; \qquad b_{-1}, \theta_{-1} \text{ given}$$

Optimal choice models: a household example

- Optimal choice problem

$$\max_{\left\{c_t^i,\theta_t^i,b_t^i\right\}_{t=0}^T} E_0\left[\sum_{t=0}^T \beta_i^t \frac{\left(c_t^i\right)^{1-\gamma_i}-1}{1-\gamma_i}\right]$$

$$c_t^i + q_t b_t^i + p_t s_t^i = b_{t-1}^i + (p_t + d_t)\, s_{t-1}^i + y_t^i = \text{cash on hand}$$
$$-b_t \leq \phi p_t \theta_t, \qquad c_t \geq 0; \qquad b_{-1}^i, s_{-1}^i \text{ given}$$

- Exogenous variables (stochastic processes)
  - individual specific: household income y_t^i
  - aggregate: bond price q_t, stock price p_t, dividend d_t
- Parameters
  - parameters describing distribution of $\left(q_t, p_t, d_t, (y_t^i)_{i\in I_t}\right)$
  - other individual specific parameters: β_i, γ_i
  - other common parameters: maximal LTV ratio ϕ
- Individual choices: consumption c_t^i, # bonds b_t^i, # stocks s_t^i
- Aggregates: consumption $C_t = \int_{I_t} c_t^i di$ etc

Optimal choice models: a household example

- Optimal choice problem

$$\max_{\left\{c_t^i, \theta_t^i, b_t^i\right\}_{t=0}^T} E_0 \left[\sum_{t=0}^T \beta_i^t \frac{\left(c_t^i\right)^{1-\gamma_i} - 1}{1-\gamma_i} \right]$$

$$c_t^i + q_t b_t^i + p_t s_t^i = b_{t-1}^i + (p_t + d_t) s_{t-1}^i + y_t^i = \text{cash on hand}$$
$$-b_t \leq \phi p_t \theta_t, \qquad c_t \geq 0; \qquad b_{-1}^i, s_{-1}^i \text{ given}$$

- Parameters
  - parameters describing distribution of $\left(q_t, p_t, d_t, (y_t^i)_{i \in I_t}\right)$
  - other individual specific parameters: β_i, γ_i
  - other common parameters: maximal LTV ratio ϕ

- **What changes with heterogenous beliefs?**

Optimal choice models: a firm example

- Many firms indexed by i choose inputs, payout & capital structure
- Technology: capital K_t^i & labor L_t^i make output $Z_t^i (K_t^i)^{\alpha_i} (L_t^i)^{1-\alpha_i}$
  - Z_t^i = total factor productivity shocks (again aggregate + idiosyncratic)
  - capital chosen one period ahead, rel price = 1, depreciation rate δ_i
- Firms maximize shareholder value $E_0 \left[\sum_{t=0}^{T} M_0^t D_t^i \right]$
  - D_t^i = payout to shareholders; M_0^t = state prices normalized by probs
- Financing
  - equity & nonnegative shareholder payout $D_t^i \geq 0$;
  - one period debt & borrowing constraint $B_t^i \leq \phi K_{t+1}^i$
  - firm cash flow

$$D_t^i = Z_t^i (K_t^i)^{\alpha_i} (L_t^i)^{1-\alpha_i} - w_t L_t^i + q_t B_t^i - B_{t-1}^i - I_t^i - \text{taxes}$$

  - taxes proportional to firm profits $\longrightarrow$ advantage for debt

$$\text{taxes} = \tau \left(Z_t^i (K_t^i)^{\alpha_i} (L_t^i)^{1-\alpha_i} - w_t L_t^i - I_t^i - (1 - q_{t-1}) B_{t-1}^i - \delta_i K_t^i \right)$$

Optimal choice models: a firm example

- Optimal choice problem

$$\max_{\left\{I_t^i, K_t^i, L_t^i, B_t^i, D_t^i\right\}_{t=0}^{T}} E_0\left[\sum_{t=0}^{T} M_0^t D_t^i\right]$$

$$D_t^i = Z_t^i (K_t^i)^{\alpha_i} (L_t^i)^{1-\alpha_i} - w_t L_t^i + q_t B_t^i - B_{t-1}^i - I_t^i - \text{taxes}$$
$$K_{t+1}^i = (1-\delta_i) K_t^i + I_t^i, \qquad D_t^i \geq 0, \qquad B_t^i \leq \phi K_{t+1}^i$$

- Parameters
  - parameters describing distribution of $\left((M_0^t), q_t, w_t, (Z_t^i)_{i \in I_t}\right)$
  - other individual specific parameters: deprec rate δ_i, capital share α_i
  - other common parameters: tax rate τ
- Individual choices: I_t^i, K_t^i, L_t^i, B_t^i, D_t^i
- Aggregates: investment $I_t = \int I_t^i di$ etc

Optimal choice models: a firm example

- Optimal choice problem

$$\max_{\left\{I_t^i, K_t^i, L_t^i, B_t^i, D_t^i\right\}_{t=0}^{T}} E_0\left[\sum_{t=0}^{T} M_0^t D_t^i\right]$$

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$$K_{t+1}^i = (1-\delta_i) K_t^i + I_t^i, \qquad D_t^i \geq 0, \qquad B_t^i \leq \phi K_{t+1}^i$$

- Parameters
  - parameters describing distribution of $\left((M_0^t), q_t, w_t, (Z_t^i)_{i \in I_t}\right)$
  - other individual specific parameters: deprec rate δ_i, capital share α_i
  - other common parameters: tax rate τ

- **What changes if we allow the option to default?**

Optimal choice models: general notation

- Discrete time, probability Pr in background, agents $i \in I_t$
- Exogenous variables $x_t = (x_t^0, (x_t^i)_{i \in I_t})$
  - individual specific $x_t^i \in X_t^i$: aggregate $x_t^0 \in X_t^0$
  - "exogenous" is context-specific – e.g. weather!
- Fixed parameters $\theta = (\theta^x, (\theta^i)_{i \in \cup_t I_t}, \theta^0) \in \Theta$
  - parameters θ^x describing distribution of x_t (incl. correlation!)
  - individual characteristics θ^i: risk aversion, beliefs, technology
  - common θ^0: maximal LTV ratio, bankruptcy cost, tax rates
- Individual choice over horizon T (may depend on i!)
  - histories $x^t = (x_0, .., x_t) \in \times_t X_t$
  - actions $a = (a_t)_{t=0}^T$, $a_t : X^t \to A_t$: C & savings; sales, I, leverage
  - constraints $a_t \in C_t^i \left(x^t, a^{t-1}; \theta\right)$: budget, borr. const., cap. accum.
- Objective function $U_t \left(a; x^t, a^{t-1}, \theta\right)$
  - describes ranking of plans from x^t on: utility, shareholder value,...
  - may reflect beliefs $\neq$ Pr; dependence on a^{t-1} from nonseparability
- Information structure: measurability restrictions on C_t^i, U_t^i

Optimal choice models & data

- Content of model

$$((x^0_\tau, x^i_\tau,)^t_{\tau=0}, \theta) \Rightarrow \quad \text{optimal choices } a^i_t$$

  - find mapping by solving agent optimization problems
- Quantitative analysis with micro data
  1. Inputs
     - parameters θ
     - initial conditions for x and past as (or sufficient state variables)
  2. Outputs
     - distribution of a given $x, \theta \longrightarrow$ joint distribution of x and a
     - optimal choice restricts joint distribution
- Comparison model vs data
  - form moments of x and a in cross section & time series
    - savings & portfolio composition by household over time
    - investment, leverage, payout by firm over time
- Once we have the mapping: counterfactuals
  - change θ, prediction of future x, a, compute welfare; e.g. tax reform

Optimal choice models: numbers

- What can be done before computing the model?
  - if xs observable, fit parametric distribution to x data $\longrightarrow$ part of θ^x
  - some other elements from literature or direct observation; e.g. factor shares, maximal LTV values
- Unobservable x
  - specify parametric distribution
- Finding other parameters in θ
  - select set of target moments from distribution of x and a
  - infer θ by comparing model & data moments
  - form distance function & minimize (calibration, estimation or mix)
- Identification of parameters
  - exploit variation in choices
- Measures of success
  - degree of data compression
  - plausibility of structure (interpret of data through lens of a model)

Optimal choice problems: numbers ctd.

- Extra assumption that deliver parsimony
  - rational expectations
    - tightly relates U_t and Pr
    - requires "probabilistically sophisticated" preferences
  - stationarity: x transitions & decision rules independent of calendar time $\longrightarrow$ stronger identification from time series variation
- Use of univariate linear regression
  - observe exogenous variable z_t orthogonal to other xs
    e.g. construct z by projection (IV)
  - identify derivative of optimal action function da_t / dz_t locally
  - provides information about model fit & sometimes directly delivers parameter of interest
  - info content depends on model structure
- Beware of nonlinearities (orange juice futures!)

General equilibrium: an example

- Asset pricing with heterogeneous agents
  - income & assets as in household problem before
  - assume infinite horizon $T = \infty$
  - competitive equilibrium
    - all agents maximize utility given prices + price & income expectations
    - market clearing: bonds $\int b_t^i di = 0$, stocks $\int s_t^i di = 1$, goods
- Rational expectations equilibrium without aggregate risk
  - suppose dividends deterministic, income shocks idiosyncratic
  - cash on hand is only endogenous state variable needed
  - steady state: interest rate, stock price + distribution for cash on hand
  - transition: moving from initial asset distribution to new steady state
  - e.g. unanticipated shock that tightens borrowing limits
- Rational expectations equilibrium with aggregate risk
  - suppose dividends stochastic and/or aggregate component in income
  - assume stationary processes for x, find stationary equilibrium process for x, prices & wealth distribution
  - time varying risk premia as wealth shifts between agents with diff. γ_is

General equilibrium models: general notation

- Additional endogenous variables y_t
  - prices, aggregates,...
  - in constraints $C_t^i\left(x^t, a^{t-1}, y^t; \theta\right)$: price in budget, collateral constraint
  - in objective function $U_t\left(a; x^t, a^{t-1}, y^t, \theta\right)$: externalities (habit, knowledge spillover)
  - objective function must detail expectations about future y_t
  - determined by agents' interaction $y_t = G\left((a_\tau^i)_{\tau=0}^t, x^t, y^{t-1}, \theta\right)$
- Flavors of interaction
  - competitive equilibrium
  - equilibrium of a game
  - hybrid (e.g. search models)
- Definition of equilibrium contains optimal choice given y!
- Content of model

$$(x_\tau^0)_{\tau=0}^t, (y_t)_{\tau=0}^{t-1}, \theta, \left\{(x_\tau^i,)_{\tau=0}^t, (a_\tau^i)_{\tau=0}^{t-1})\right\}_{i\in I_t} \Rightarrow \quad \text{choices } a_t^i \text{ \& } y_t$$

Example second year papers

- Tim Landvoigt (RFS 2016)
  - household behavior in boom: optimism or lax credit constraints?
  - optimal savings & portfolio choice with collateral constraints
  - allow for general beliefs about housing returns
  - fit to wealth holding in SCF 2001 and 2007
- Juliane Begenau & Juliana Salomao (RFS 2019)
  - firm financing by size over the business cycle
  - optimal investment, payout & capital structure with endogenous default
  - fit to Compustat data on quantities
- Edison Yu (JET 2018)
  - information aggregation in markets, volume & prices
  - dynamic GE model of asset trading; two agents of different sophistication
  - relate to data on institutional trading & returns

Stanford flyouts, winter 2020

- GSB Finance January 15: Geoffrey Zheng
  - overlapping generations of heterogenous households
  - role of incumbents vs new entrants in evolution of inequality?
- GBS Finance January 22: Chen Lian
  - investors who misperceive their tradable wealth
  - puzzles about response of consumption to income shocks?
- Econ January 28: Christian Wolf
  - GE effects of government transfers with heterogenous households or firms
  - in what economies can they be measured with reduced form tools?
- Econ February 4: Adrien Bilal
  - what explains regional unemployment & job losing rates?
  - heterogenous firms sort into heterogenous locations

Three families of agents, three literatures

1. Households
2. Firms
3. Investors

Common themes across literatures

- Inequality
  - long run distribution of wealth / size depends on shocks, opportunity sets
- Systematic life cycle patterns
  - cross section: consumption / payout, savings/ investment /financing
  - time series: slow moving trends (demographics, market structure)
- Distributional dynamics over the business cycle
  - time variation in volatility, skewness of income, employment etc.
- Financial frictions
  - misallocation (opportunities vs funding)
  - debt & propagation: originally firms, now investors, households
- Risk sharing
  - asset structure shapes gains from trade

Households

- Data
  - Panel Study of Income Dynamics, Survey of Consumer Finances
  - international data, e.g. European HFCS
- Key questions
  - sources of inequality (consumption, income, wealth)
  - household finance (participation & portfolio choice by group)
  - policy & risk sharing (social security, welfare progams)
- Tools
  - choice models & competitive models with frictions
- Gaps
  - making sense of risky assets with volatile prices
  - beyond "economic" characteristics
- Recent survey papers
  - risk sharing & inequality Krueger-Mitman-Perri 16
  - household finance: Campbell 12
  - aging: De Nardi 16
  - survey expectations: Manski 17

Housing & mortgage debt

- Data
  - CoreLogic transaction & assessor data
  - HMDA mortgage applications
- Key questions
  - determination of house prices
  - household debt overhang, business cycle & policy responses
  - mortgage design, securitization & government subsidies
- Tools
  - as above, but increasingly also search models
- Gaps
  - as above + supply side & volume
  - international evidence
- Recent survey papers
  - Piazzesi-Schneider 16, Han-Strange 15 (search), Campbell 16

Firms

- Data
  - Compustat
  - Census data (public use + restricted data from census data center)
- Key questions
  - size distribution
  - investment, payout & capital structure in the cross section
  - cross section of returns & firm characteristics
- Gaps
  - what is a firm: beyond production functions or entrepreneurs
  - mergers
- Recent survey papers
  - firm size distribution & misallocation: Hopenhayn 16
  - corporate finance: Strebulaev-Whited 2012

Investors

- Data
  - Prices: CRSP, FISD, Bloomberg...
  - Positions: Financial Accounts of the US, Call Reports, Dealscan, TAQ
- Key questions
  - excess volatility of asset prices & predictability of excess returns
  - connection prices & investor choice (households, institutions)
  - macroprodential regulation
  - monetary policy & asset prices
- Gaps
  - volume, prices & positions; networks
- Recent survey papers
  - asset pricing: Cochrane 12
  - institutions: active literature

More sources

- Elsevier handbooks
- Annual Reviews of Econ, Fin Econ
- Journal of Economic Perspectives
- Journal of Economic Literature
- google scholar

New themes?

- Financial literacy & endogenous information acquisition
- Belief formation & survey data
- Corporate governance
- Flavors of intermediation, segmentation & asset pricing
- Geography within countries (or cities)
- Climate & environment