Econ 237/Mgmt 617: Introduction

Martin Schneider & Chris Tonetti

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Why heterogenous agent models?

- Traditional macro: focus on aggregates
  - but built on microfoundations: household & firm behavior
  - aggregation theorems that justify representative household/firm hold only under restrictive conditions
  - representative agents often assumed to simplify (appropriate abstraction for some questions)
- Micro foundations & micro data
  - lots of data on households, firms, financial institutions...
  - $\longrightarrow$ understand mechanisms at micro level
    - identification from cross section, not only time series
    - especially useful when time series short, e.g. with structural change!
  - $\longrightarrow$ if composition effects matter for aggregates, aggregation misleading!
  - $\longrightarrow$ new moments
    - lots of trading within broad sectors of economy, classes of goods
    - details of trading beyond Walrasian P&Q
- Welfare
  - many events & policies affect different people differently!

Organization of the course

- Lectures on tools (weeks 3, 4, 6) & topics (weeks 8, 9)
- Class project: quantitative modeling exercise
  - week 1: area of interest (email us 3 slides as descried in syllabus)
  - week 2: project group meetings to map out initial modeling exercise
  - week 5: present results of initial exercise
  - week 7: project group meetings to map out final exercise
  - week 10: presentation of results
- Groups allowed & encouraged for all tasks!

Numerical tools for problem sets & class project

- Python, Julia or Matlab work for projects
- For frontier research, consider two useful investments
  - the second year is the time to make investments!

1. Parallelization
   - solving models often involves similar & independent tasks
     - value function iteration: optimization in different states
     - optimal choice for different agent types given prices
   - large gains in speed by doing those tasks in parallel, not sequentially
   - using the GPU: simpler set of instructions, but lots of processors
   - Stanford cluster has very powerful GPUs
2. Machine learning
   - already widely used for data analysis in industry & academia
   - use also for large nonlinear models = complicated functional equations
     - allows flexible parameterization of policy functions, laws of motion...
     - efficient algorithms for minimizing loss functions

- See notes by Jesus Fernandez-Villaverde (UPenn) for introductions

Weeks 3-4 – three classes of models

1. One tradable riskless asset, one nontradable risky asset
   - aka Aiyagari model: bonds + income shocks
   - data on household income & time series models
   - computation of consumption-savings problems & interest rates
   - role of borrowing constraints
2. Many tradable assets
   - data on asset returns & time series models
   - computation of portfolio problems & asset pricing
   - role of non-separable preferences
3. Combining 1. and 2.
   - data on wealth positions, institutions' holdings
   - setups with multiple risks & various constraints
   - application: OLG model with housing, stocks & bonds
   - matching models & data, presentation of results

Week 6: more on computing equilibria

1. Equilibrium without aggregate risk
   - Stationary recursive rational expectations equilibrium
     - key simplification: constant distribution of endogenous state variables (e.g. wealth, internal funds)
   - Transition dynamics & welfare calculations
2. Equilibrium with aggregate risk
   - Adding aggregate shocks
   - Approximations to stationary REE (Krusell-Smith, Reiter)
     - key challenge: moving distribution of endogenous state variables
   - Temporary equilibrium

A framework for thinking through models

- Physical environment
  - consumers' preferences, technology, information
  - what are feasible allocations?
  - no reference to markets, prices, firms etc
  - helpful for thinking about optimal allocations, alternative decentralizations
- Decentralization (= "markets & institutions")
  - more agents: firms & government
  - actions: taken to interact with others in markets
  - endogenous variables that coordinate action: prices...
  - equilibrium concept: agents' optimal choice + rules of interaction

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  - $n_t(\omega)$ available goods $\in N_t(\omega)$
  - agent i's consumption plans $c^i = \left(c_t^i\right)$; $c_t^i(\omega) \in \mathbb{R}^{n_t(\omega)}$
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  - endowments + production functions + resource constraints
  - endowment economy $F(c) = 0$, $c = \left(c^i\right)_{i \in I}$
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- Studying allocations
  - planner problem: rank feasible allocations
  - decentralization: allocation selected by markets & institutions

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- Equilibrium: optimal choices a_t^i given y_t + determination of y_t

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- How to model frictions?
  - any imperfection described by constraints on allocations only (incentive compatibility, participation, etc)
  - markets & institutions implement optimal allocations
  - strong traditions in public finance, banking (sometimes framed as "observed institutions must be optimal")
  - alternatively, impose contract structure (incomplete markets,...)

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- Defining "market failure"
  - discuss deviation from first best
  - constrained efficiency
    - define planner problem over (a, y)
    - planner respects constraints, price formation
    - equilibria may not be constrained efficient

Model vs data

- Exogenous variables x_t, parameters θ
- Content of model

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- Once we have the mapping: counterfactuals
  - change θ, prediction of future x, y, a, compute welfare

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- Identification of parameters
  - exploit variation in choices a^i & endogenous y given x
- Measures of success
  - degree of data compression
  - plausibility of structure (interpret of data through lens of a model)
  - targeted + untargeted moments vs one set of moments
    makes a difference if obvious tensions in fit: dark matter problem

Numbers ctd.

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- Use of univariate linear regression to define target moments
  - observe exogenous variable z_t orthogonal to other xs
    e.g. construct z by projection (IV)
  - identify derivative of optimal action function da_t / dz_t locally
  - provides information about model fit & sometimes directly delivers parameter of interest
  - info content depends on model structure

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- Evaluate moment conditions that connect endogenous observables
  - example: consumption Euler equation for asset prices
    more generally, any optimal choice in a model
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    - interpretation of counterfactuals: hold fixed production side

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- Sufficient statistics
  - care only about some function of endogenous observables, e.g. welfare
  - example: $d\theta =$ policy change, $w =$ agent wealth, $V =$ value function
    & envelope theorem says $dV/d\theta = V'(w)\, dw/d\theta = u'(c) dw/d\theta$
  - measure only $u'(c)$ & w change, not other features that go into V
  - large program evaluation literature in public finance