Exploration and Exploitation (Bandits)

Last Time

- What is Reinforcement Learning?

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- What are the main challenges in Reinforcement Learning?

Last Time

- What is Reinforcement Learning?
- What are the main challenges in Reinforcement Learning?
- How do we categorize RL approaches?

1. Exploration vs Exploitation
2. Credit Assignment
3. Generalization

Model-Free — Learn π^*, or Q

Model-Based — Learn T, R

On Policy — Exploration policy = Optimized policy

Off Policy

Batch

Tabular

Deep

Last Time

Last Time

First RL Algorithm:

Last Time

First RL Algorithm:

Tabular Maximum Likelihood Model-Based Reinforcement Learning

Last Time

First RL Algorithm:

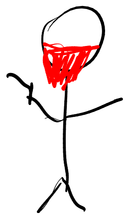
Tabular Maximum Likelihood Model-Based Reinforcement Learning

loop
    choose action a
    gain experience
    estimate T, R
    solve MDP with T, R

Guiding Questions

- What are the best ways to trade off Exploration and Exploitation?

Bandits



Bandits

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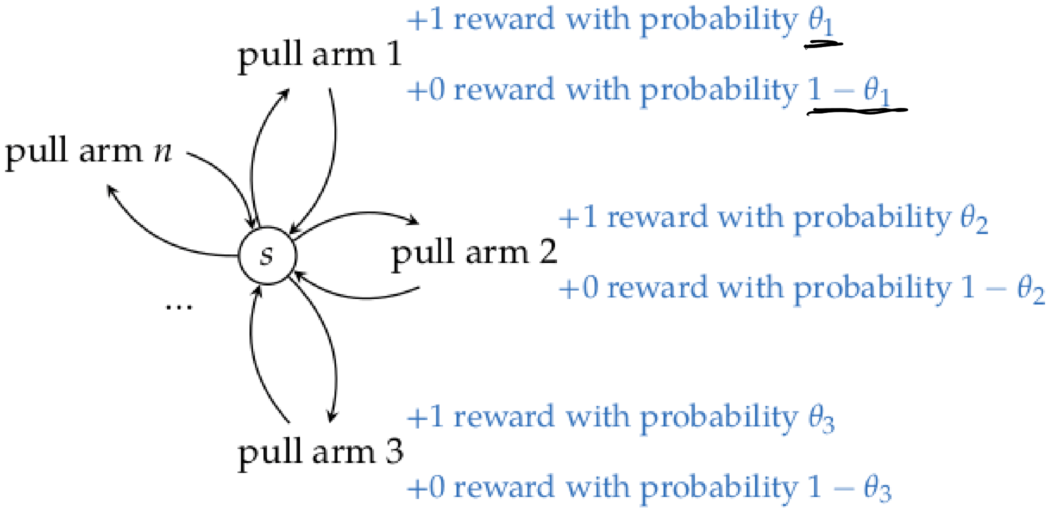
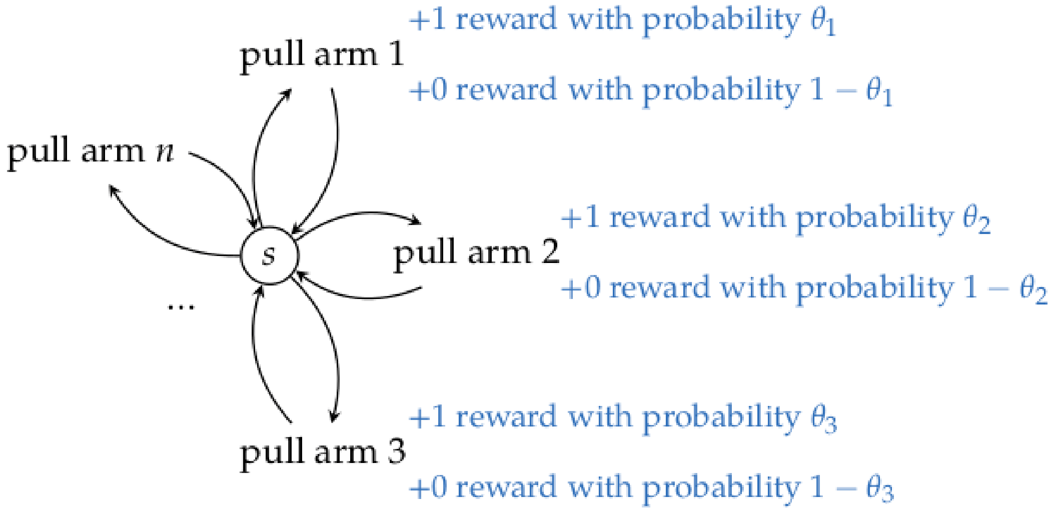


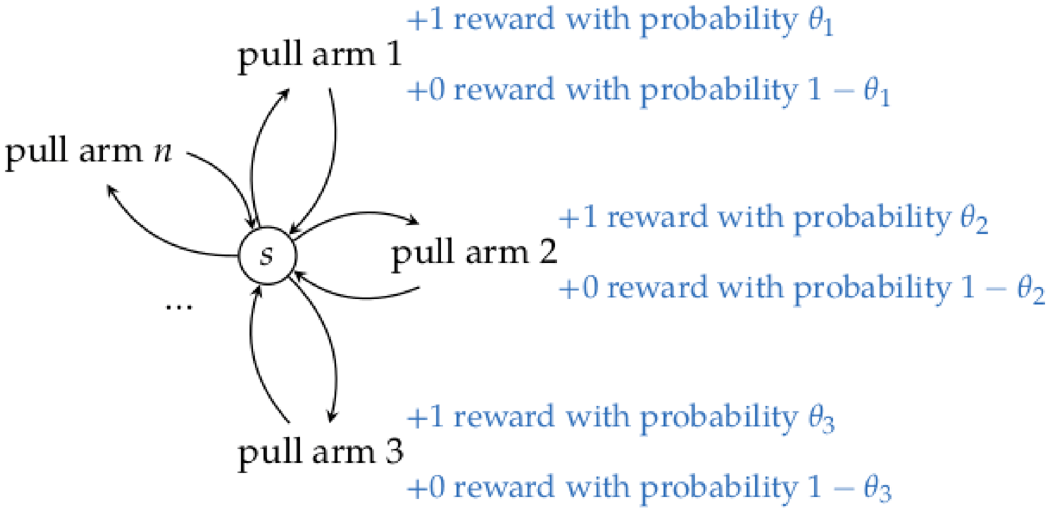
+1 reward with probability θ_1
pull arm 1
+0 reward with probability $1 - \theta_1$
pull arm n
+1 reward with probability θ_2
s
pull arm 2
+0 reward with probability $1 - \theta_2$
...
+1 reward with probability θ_3
pull arm 3
+0 reward with probability $1 - \theta_3$

Bandits



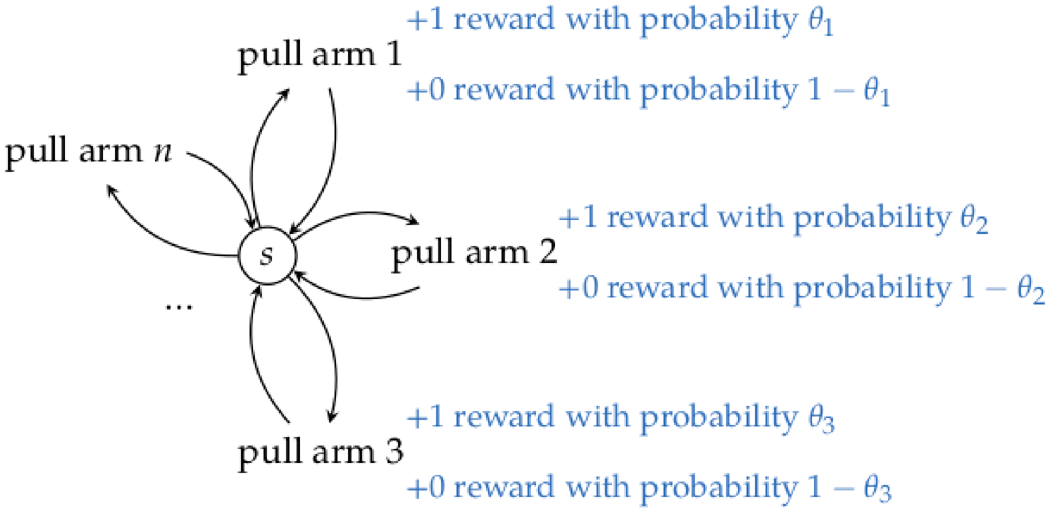
- Bernoulli Bandit with parameters θ

Bandits



- Bernoulli Bandit with parameters θ
- $\theta^* \equiv \max \theta$

Bandits



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"According to Peter Whittle, "efforts to solve [bandit problems] so sapped the energies and minds of Allied analysts that the suggestion was made that the problem be dropped over Germany as the ultimate instrument of intellectual sabotage.""

Greedy Strategy

$$\rho_a = \frac{\text{number of wins}+1}{\text{number of tries}+1}$$

Choose $\operatorname*{argmax}_a \rho_a$

Undirected Strategies

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- Explore then Commit
  Choose a randomly for k steps
  Then choose $\underset{a}{\operatorname{argmax}}\, \rho_a$

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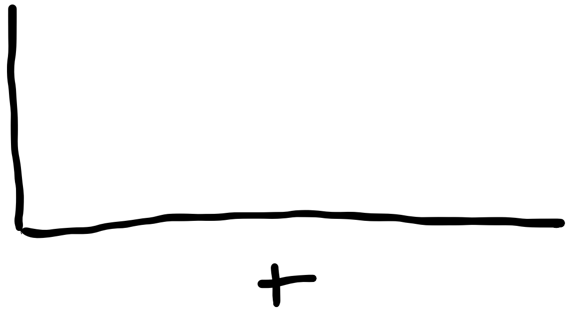

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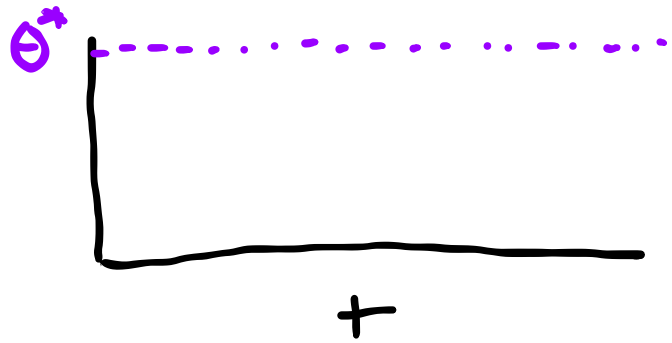

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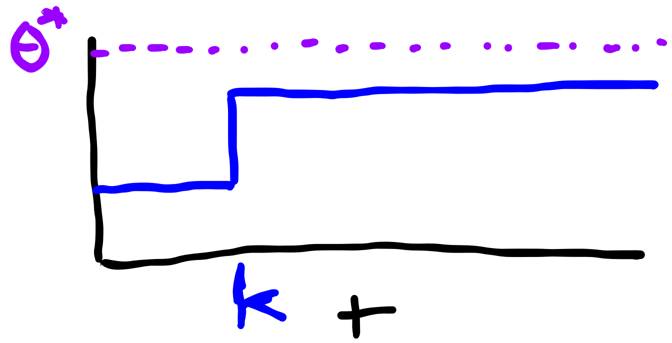

Undirected Strategies

- Explore then Commit
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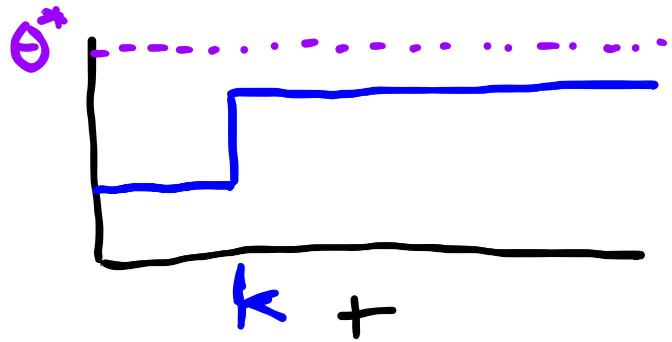
- ϵ - greedy
  With probability ϵ, choose randomly
  Otherwise choose $\operatorname*{argmax}_a \rho_a$



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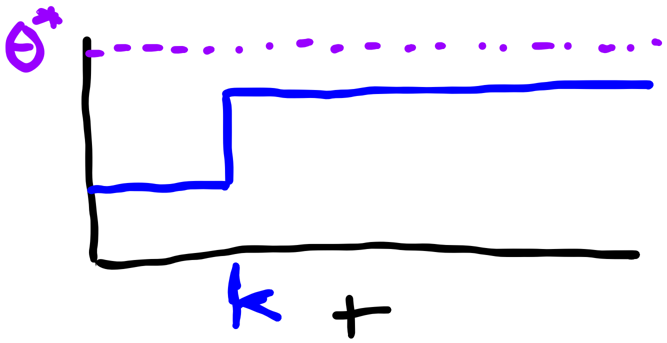

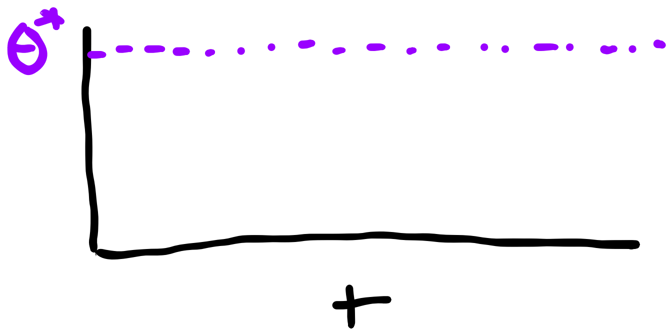

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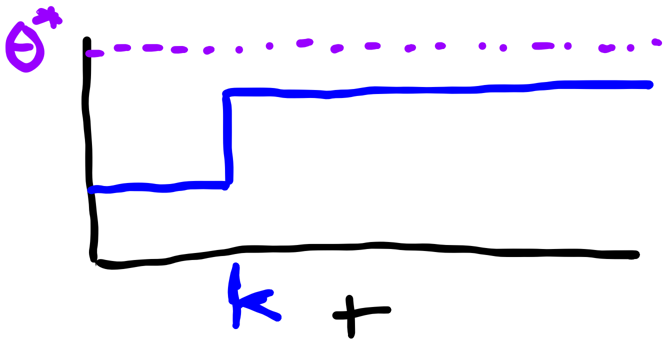

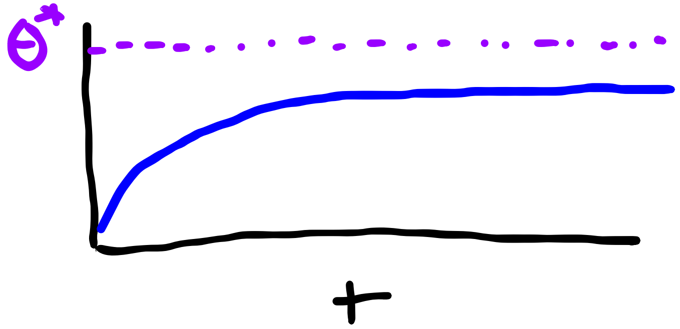

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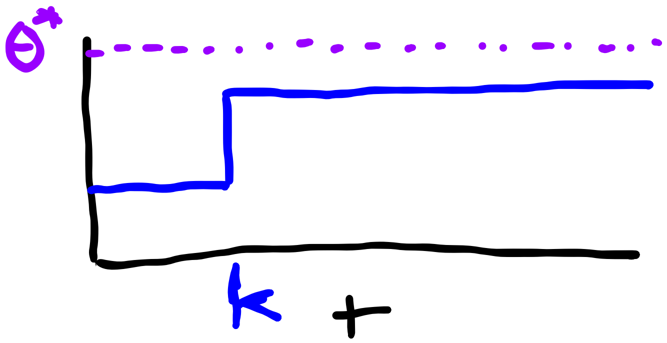

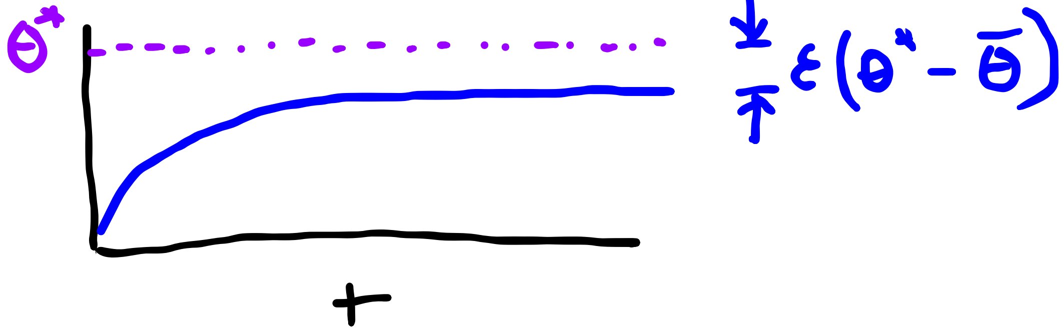

Directed Strategies

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  Choose a with probability
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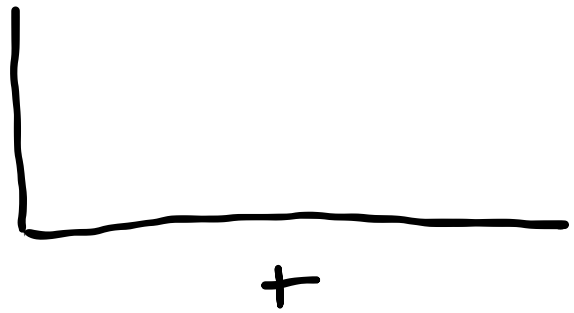

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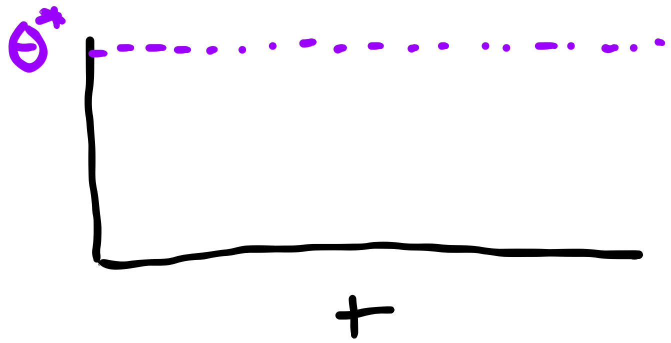

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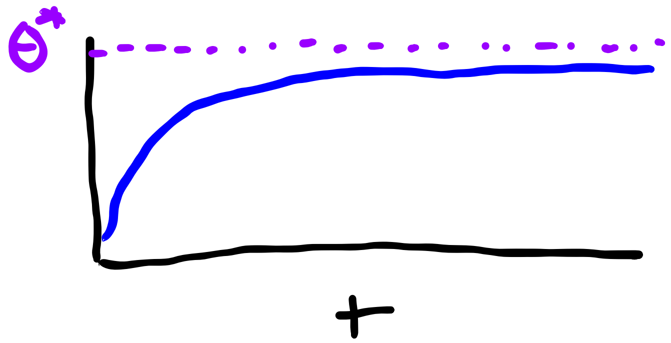

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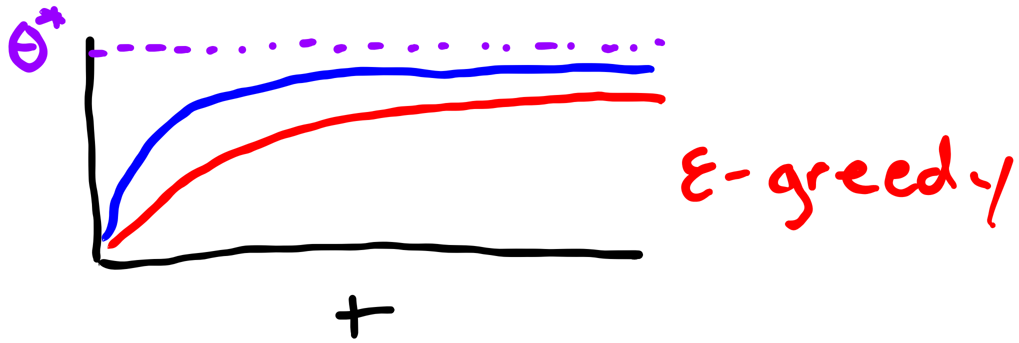

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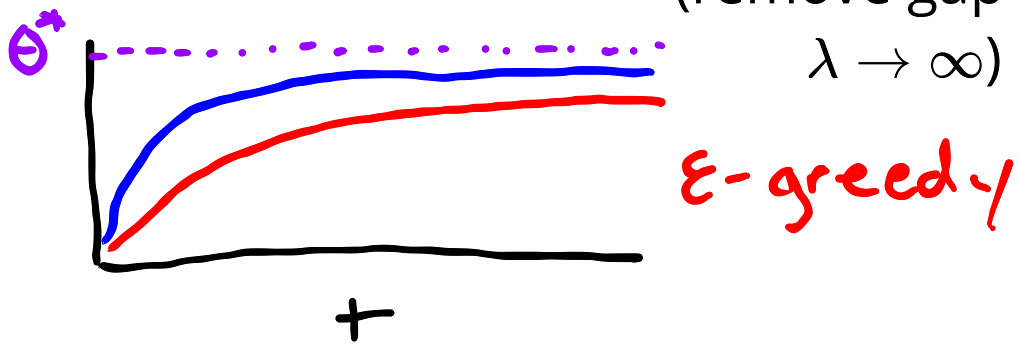
(remove gap with $\lambda \to \infty$)



Directed Strategies

- Softmax
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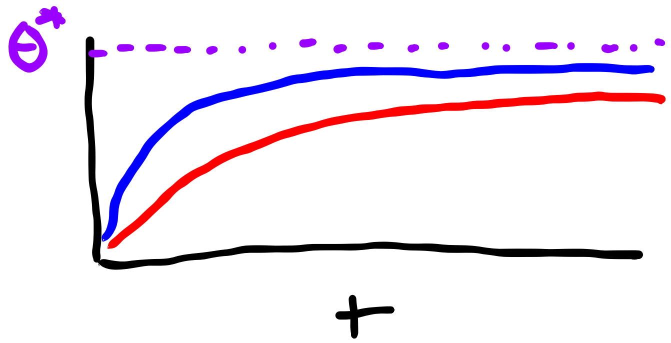
- Upper Confidence Bound (UCB)
  Choose $\underset{a}{\operatorname{argmax}}\, \rho_a + c\sqrt{\frac{\log N}{N(a)}}$



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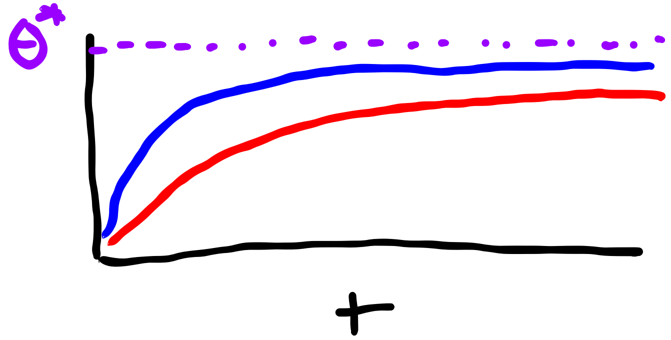
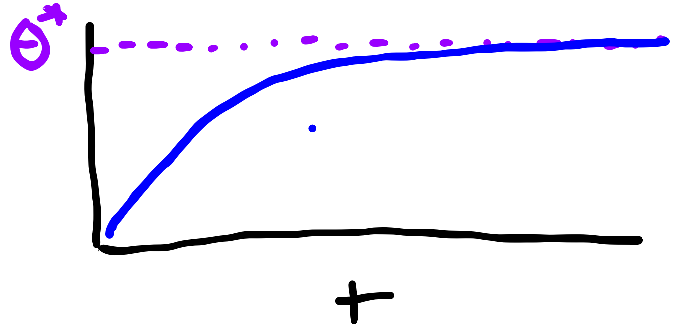
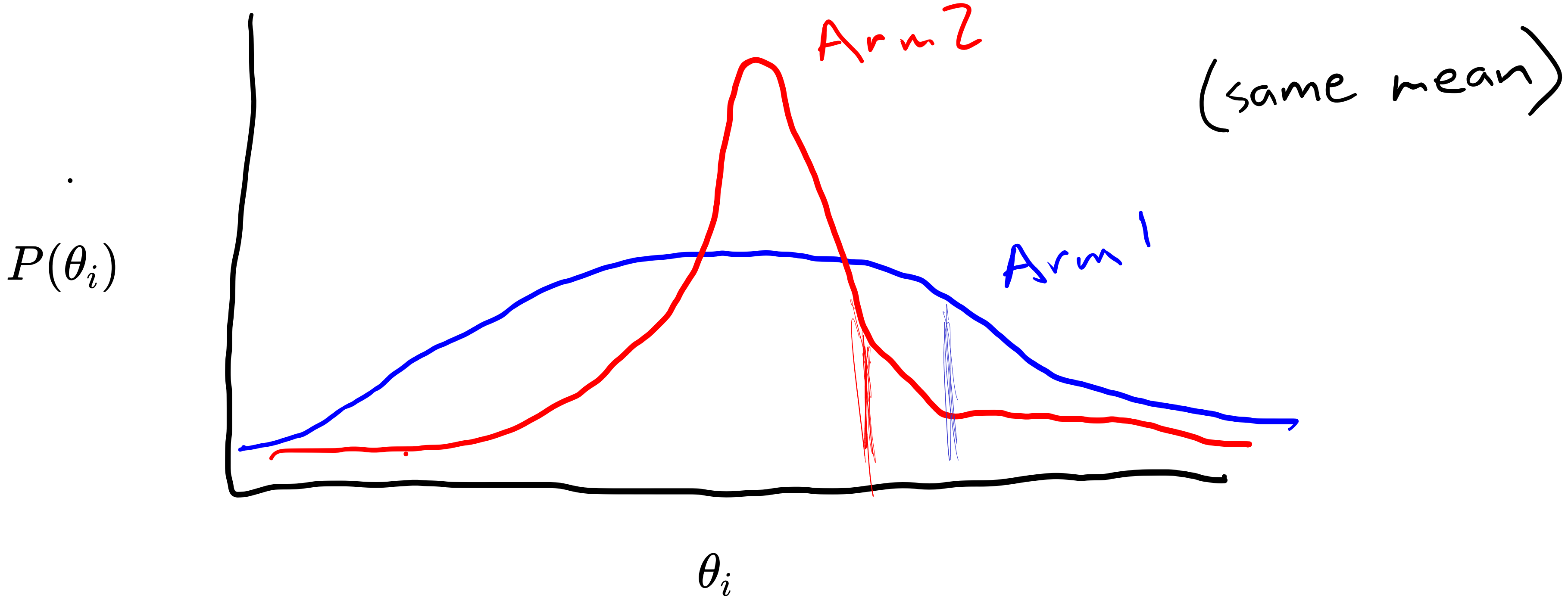


ε-greedy



Break

Discuss with your neighbor: Suppose you have the following *belief* about the parameters θ. Which arm should you choose to pull next?



Bayesian Estimation

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Bernoulli Distribution

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$$\text{Bernoulli}(\theta)$$

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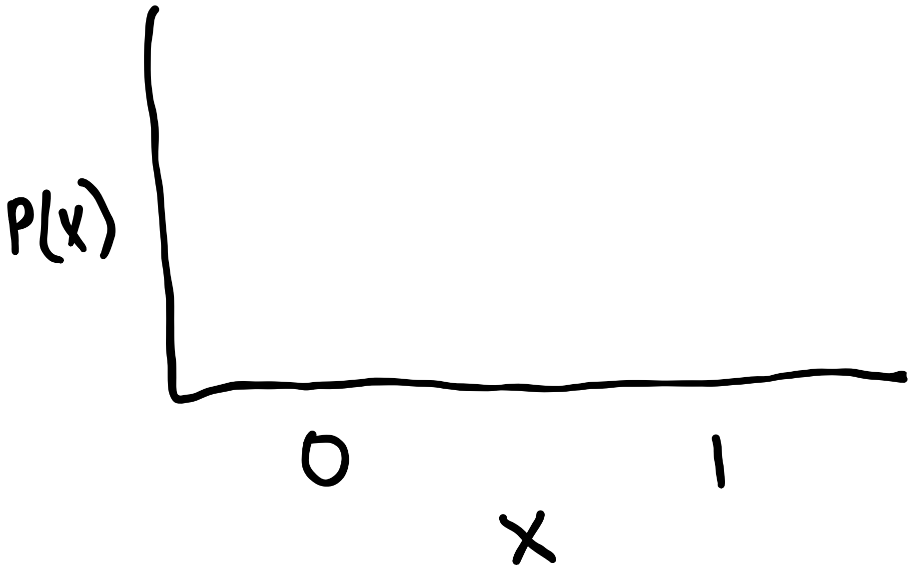

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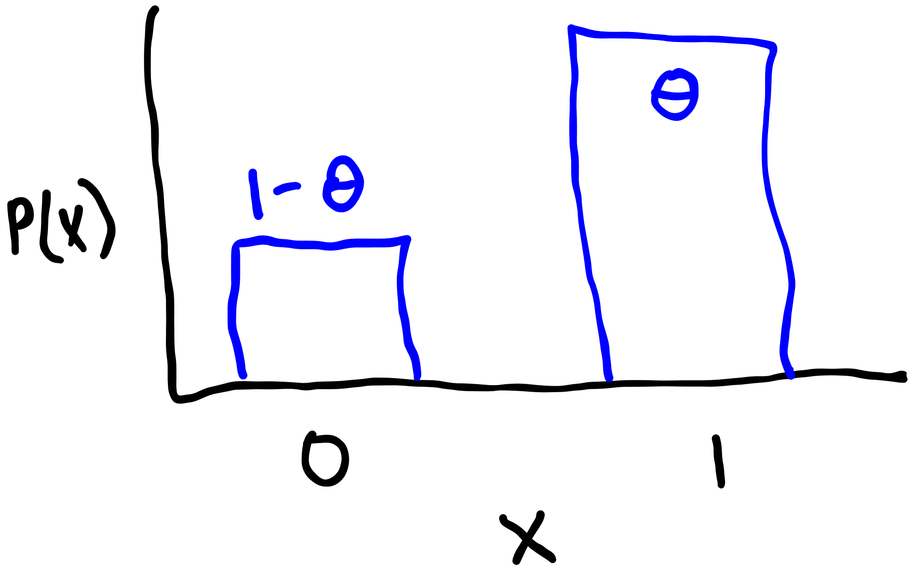

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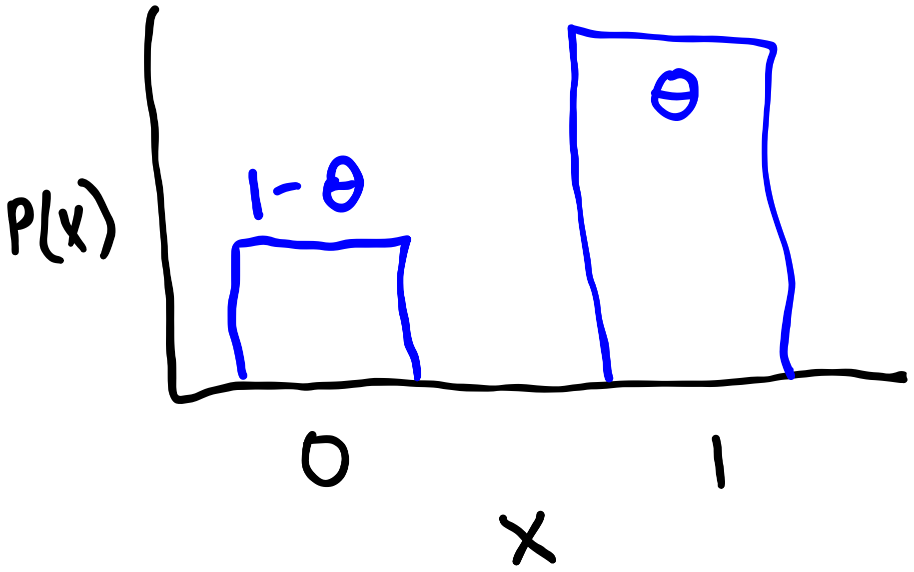

Discussion: Given that I have received w wins and l losses, what should my belief (probability distribution) about θ look like?

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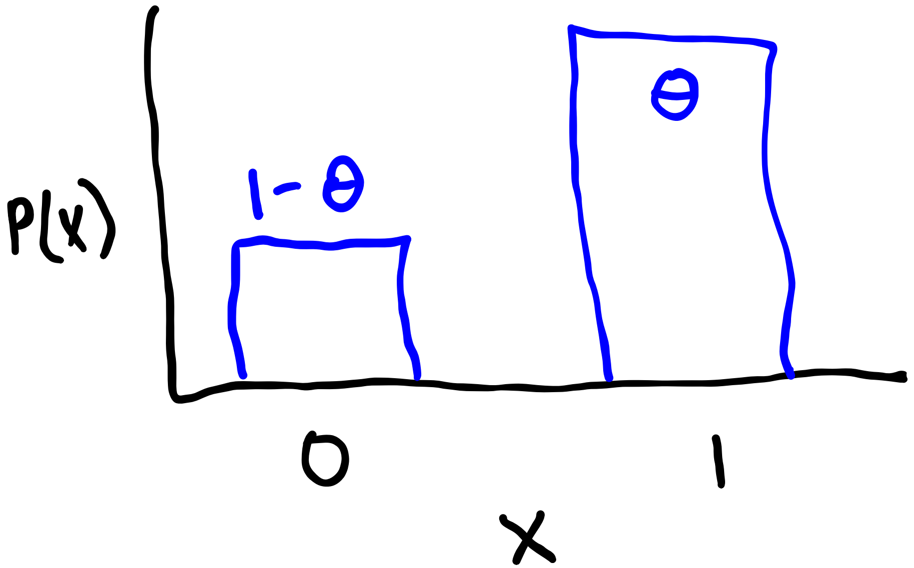

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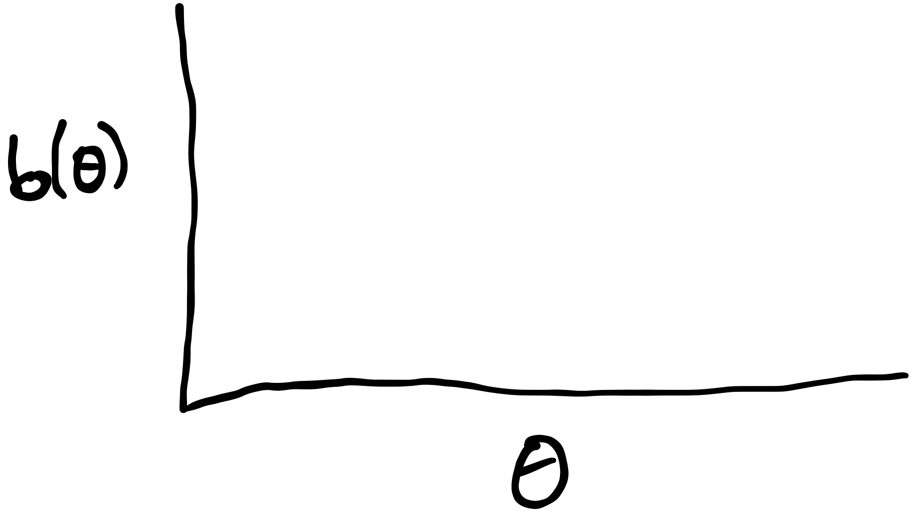

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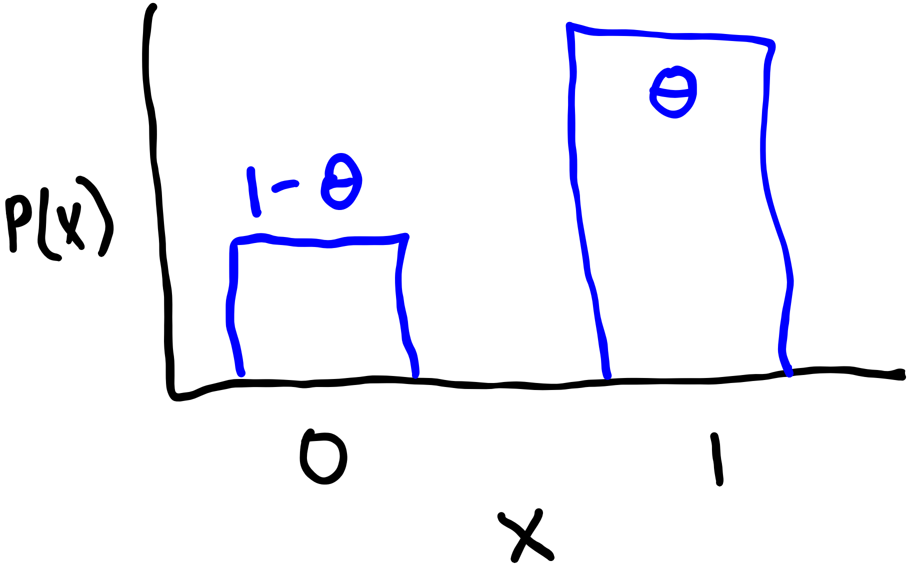

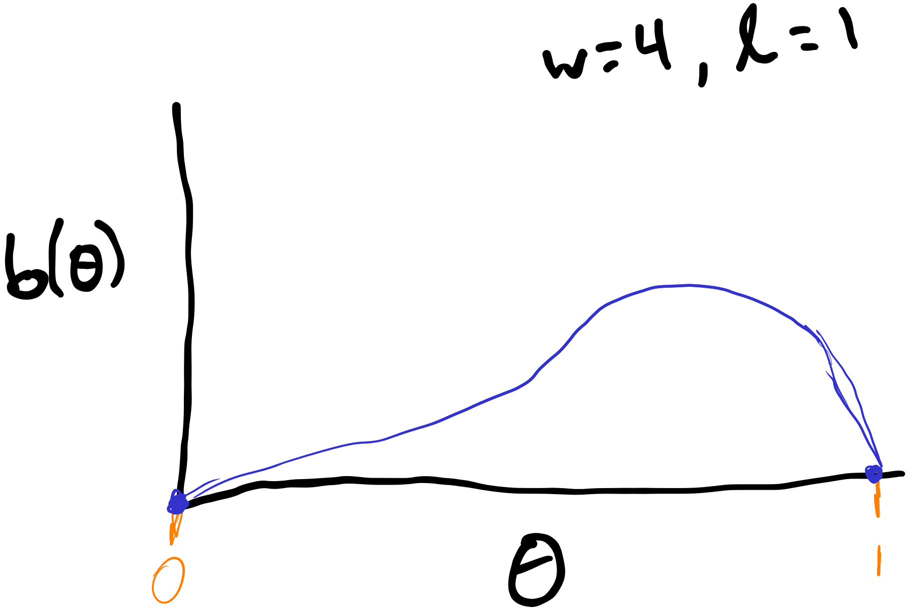

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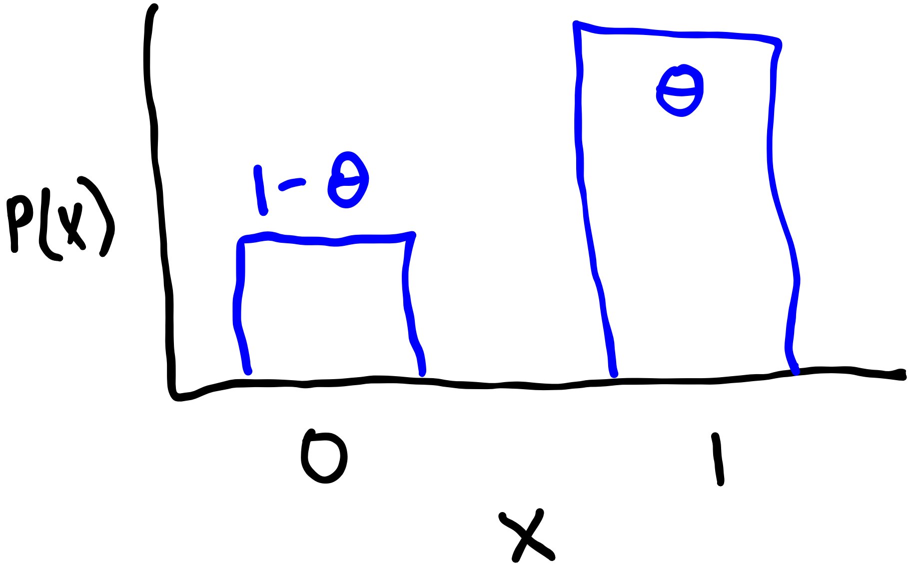

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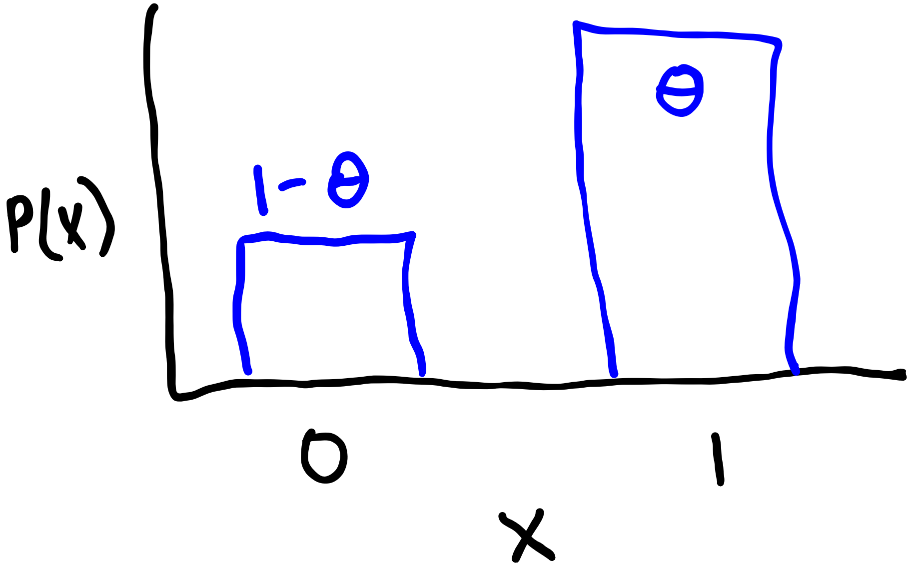

Beta Distribution

(distribution over Bernoulli distributions)

Bayesian Estimation

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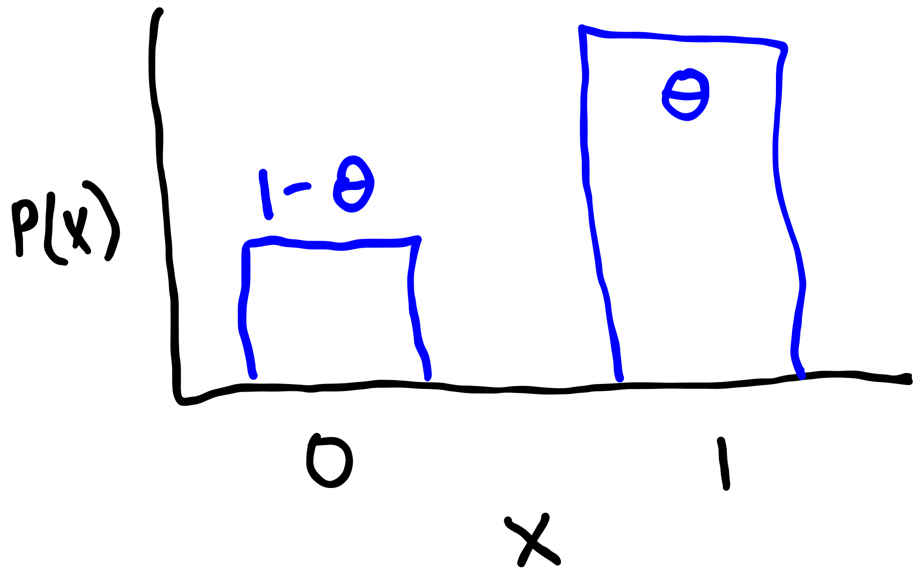

Beta Distribution

(distribution over Bernoulli distributions)

$$\mathrm{Beta}(\alpha, \beta)$$

Bayesian Estimation

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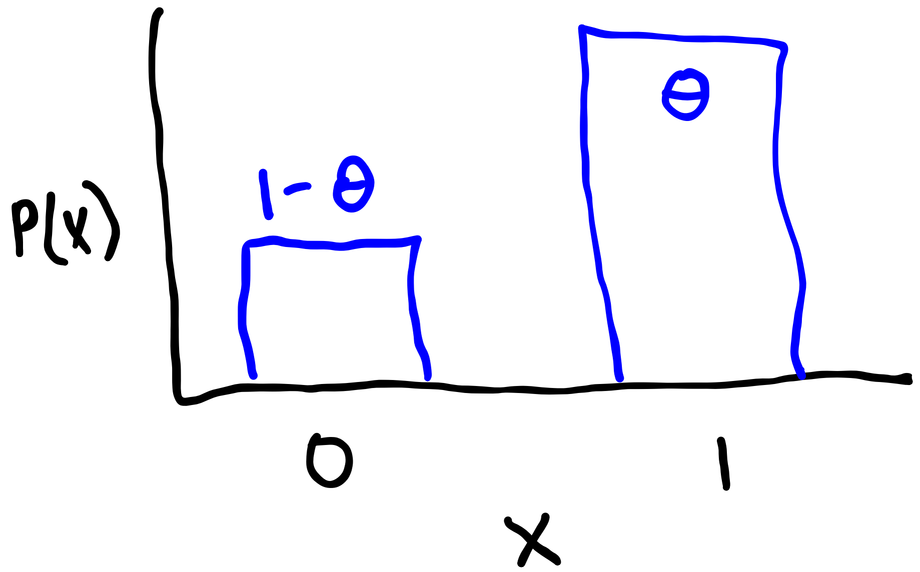

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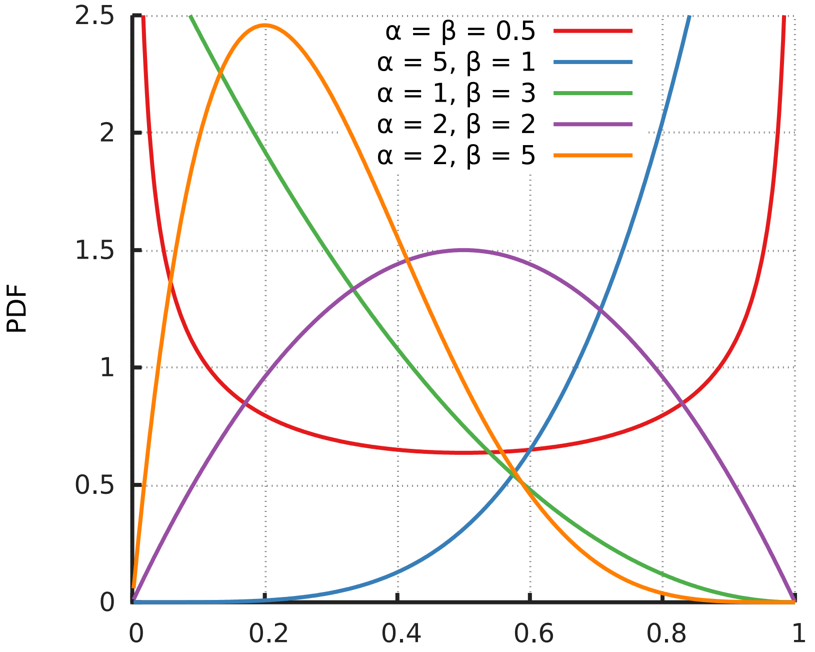

Bayesian Estimation

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Given a $\mathrm{Beta}(1, 1)$ prior distribution

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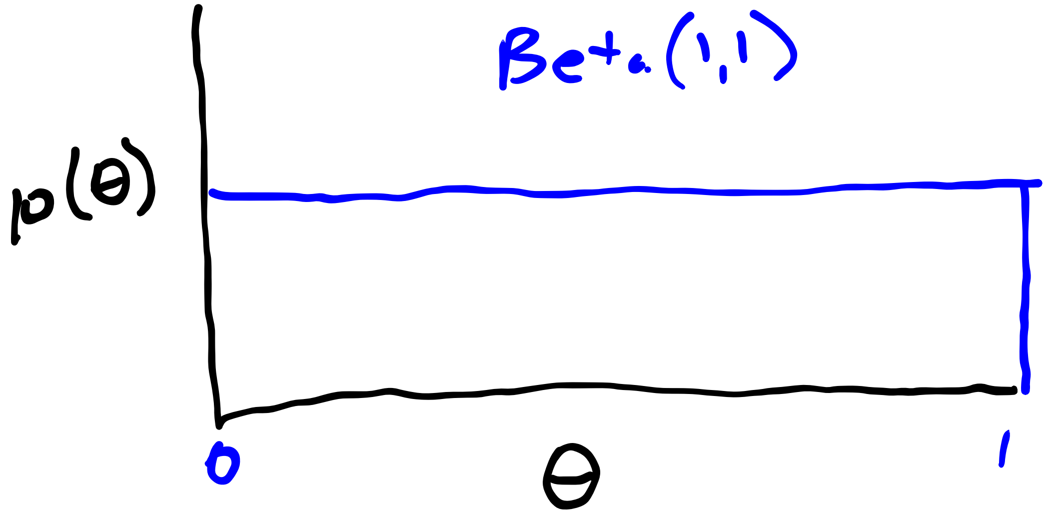

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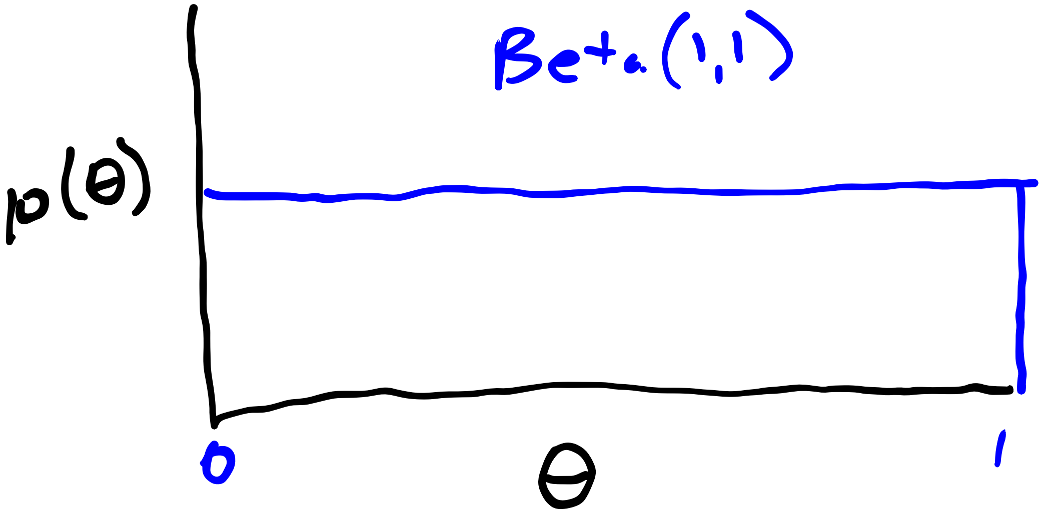

The posterior distribution of θ is

$\mathrm{Beta}(w + 1, l + 1)$

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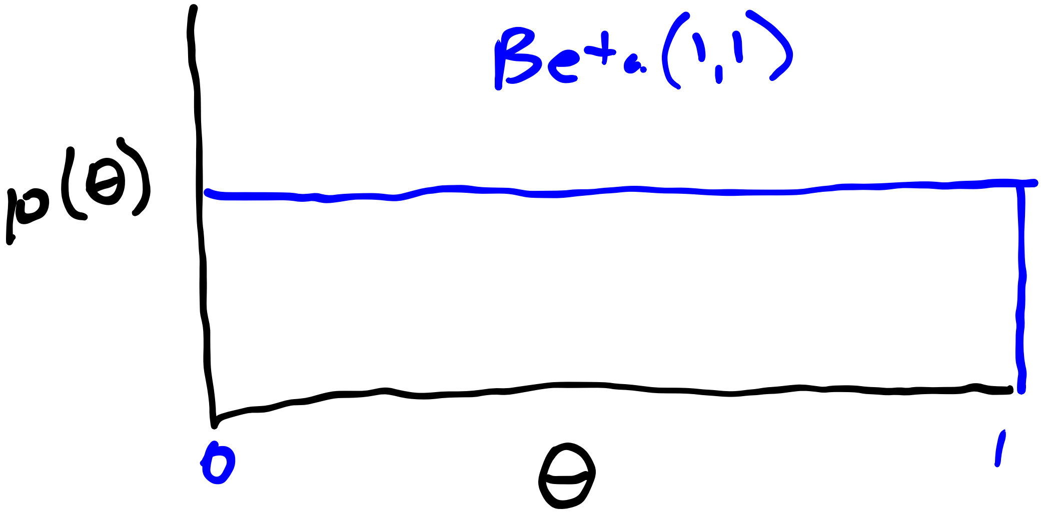

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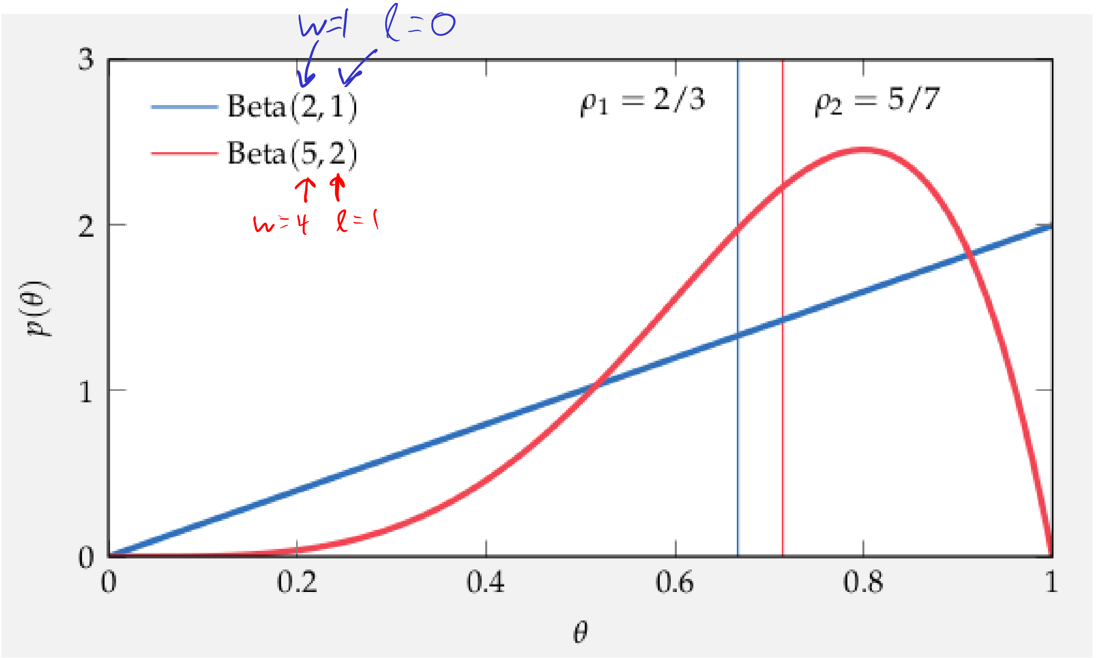

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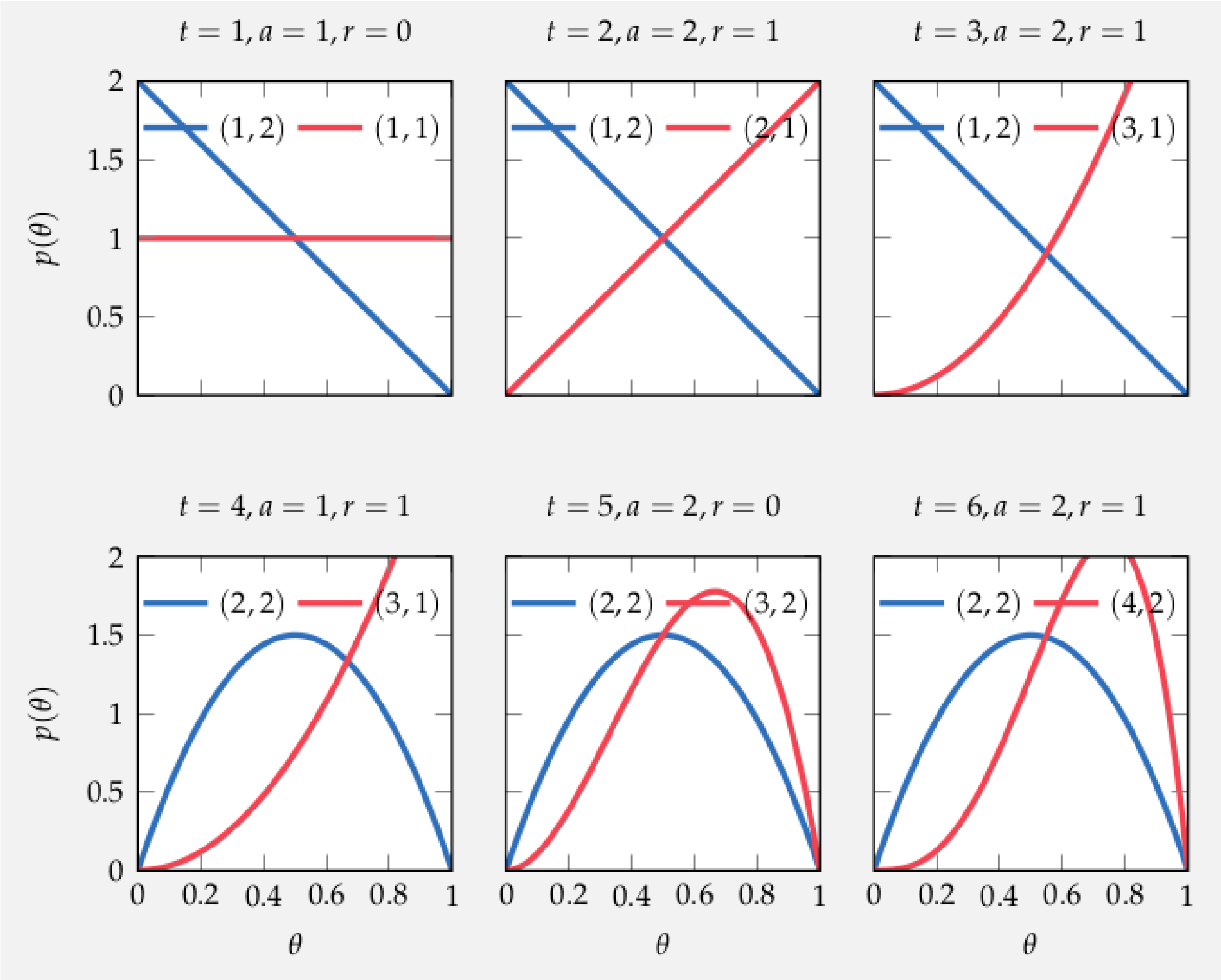

t = time

a = arm pulled

r = reward

Bayesian Bandit Algorithms

Bayesian Bandit Algorithms

- Quantile Selection
  Choose a for which the α quantile of $b(\theta)$ is highest

Bayesian Bandit Algorithms

$\alpha = 0.9$

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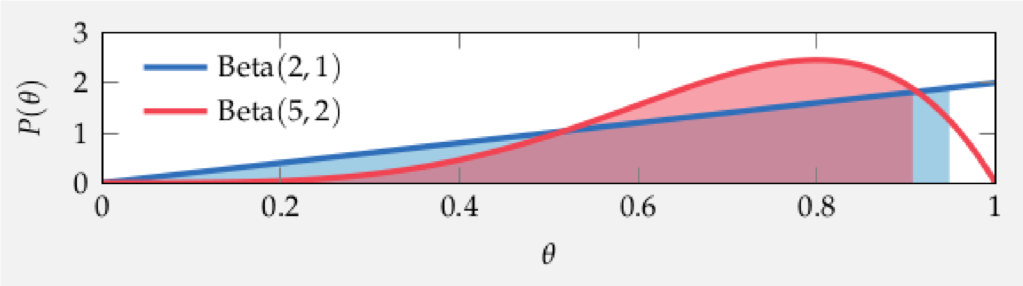
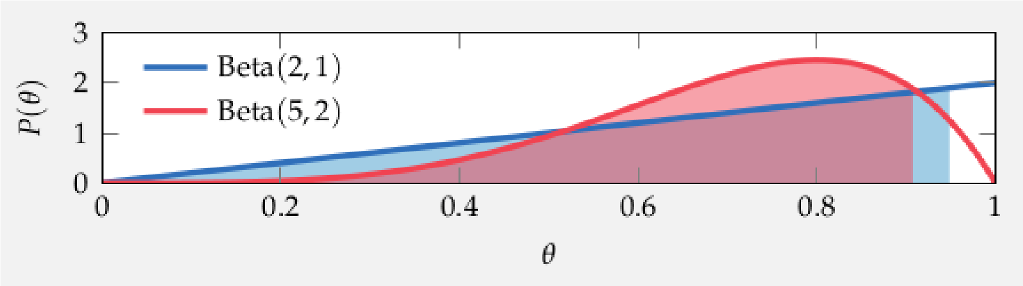


Bayesian Bandit Algorithms

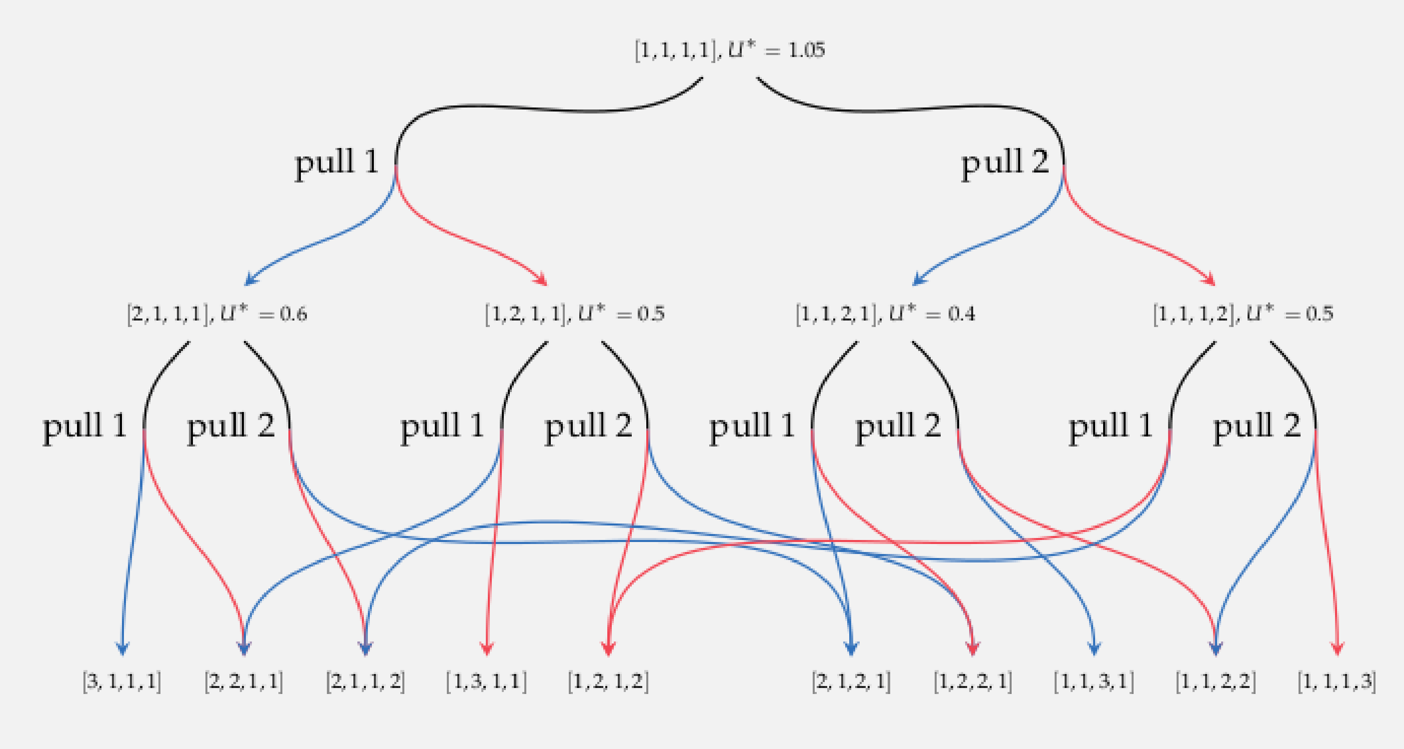
- Quantile Selection
  Choose a for which the α quantile of $b(\theta)$ is highest

- Thompson Sampling
  Sample $\hat{\theta}$
  Choose $\underset{a}{\operatorname{argmax}}\, \hat{\theta}_a$

$\alpha = 0.9$



Optimal Algorithm - Dynamic Programming



Regret Analysis

Roughly:

Regret Analysis

$$\text{Regret}(n) \equiv \theta^* n - \sum_{t=1}^{n} r_t$$

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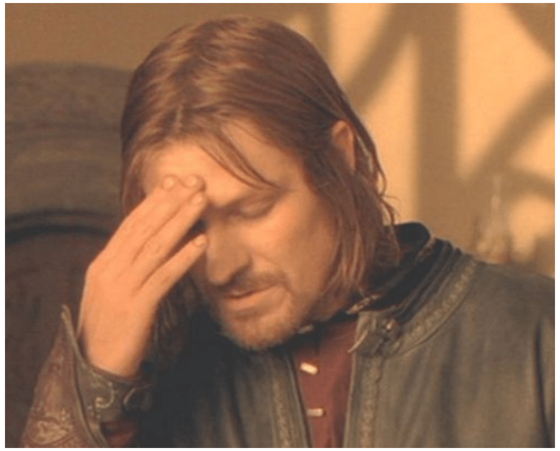
Regret Analysis



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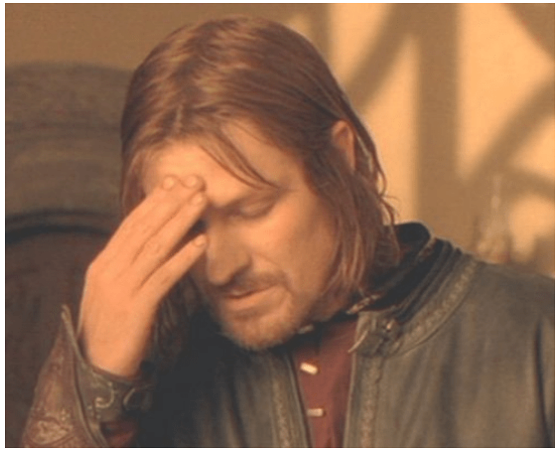
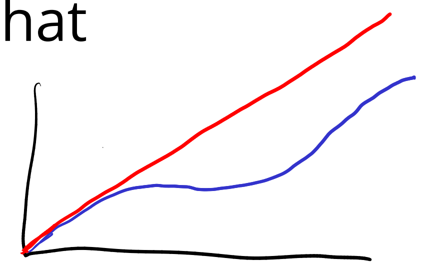


$$\text{Regret}(n) \equiv \theta^* n - \sum_{t=1}^{n} r_t$$

Recall: $f(n) = O(g(n))$ means that there exists a $C > 0$ and $N > 0$ such that $f(n) < C\, g(n)$ for all $n > N$.



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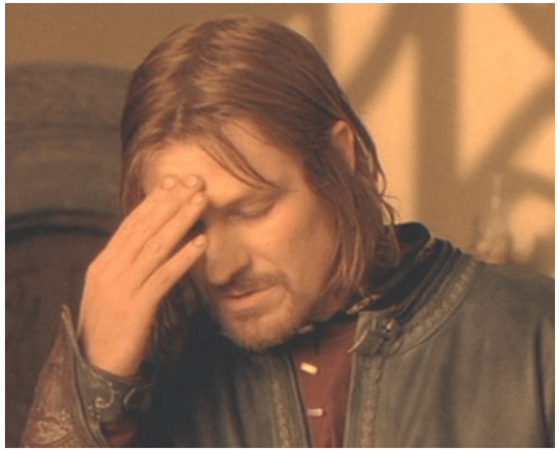

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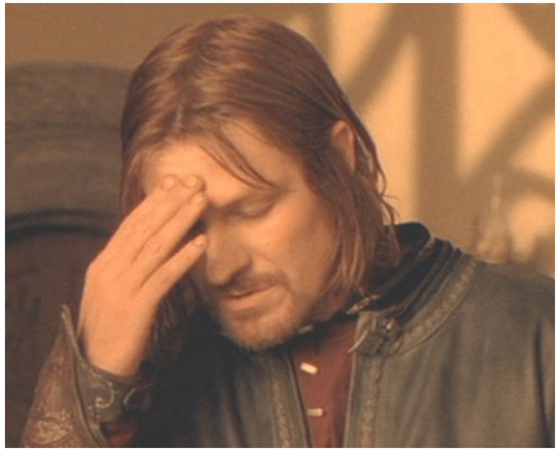

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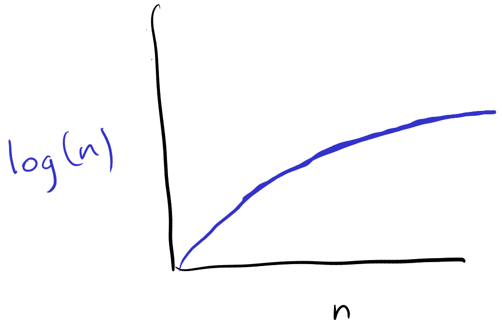
Roughly:

- $O(n)$ regret means you might keep picking the wrong arm forever
- $O(\log(n))$ regret means that you keep learning



Review

Easier to implement

	"Optimal in limit" (parameter)	Regret
undirected — Greedy	No	$O(n)$
undirected — ε-Greedy	$\varepsilon \to 0$	$O(n)$
undirected — explore then commit	$k \to \infty$	$O(n)$
- softmax $\propto e^{\lambda p_a}$	$\lambda \to \infty$	$O(n)$
- UCB $\arg\max_a p_a + c\sqrt{\frac{\log(n)}{n(a)}}$	✓	$O(\log(n))$
Bayesian — Quantile selection	✓	$O(\log(n))$
Bayesian — Thompson sampling	✓	$O(\log(n))$
- Dynamic Programming	✓	(only for finite horizon)

$O(\log(n))$

$c > 0$ #arms

$\varepsilon_n = \min\left(1, \frac{cK}{d^2 n}\right)$

$0 < d \le \min_i(\theta^* - \theta_i)$

Less Regret

Guiding Questions

- What are the best ways to trade off Exploration and Exploitation