Probability and Random Variables

Concepts

1. Utility and Probability
2. Random Variables
3. Relationships between Random Variables

Utility and Probability

Full story: https://projecteuclid.org/journals/statistical-science/volume-1/issue-3/The-Axioms-of-Subjective-Probability/10.1214/ss/1177013611.full

Utility and Probability

Consider events A and B:

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Utility and Probability

Consider events A and B:

Utility indicates preference

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Utility and Probability

Consider events A and B:

Utility indicates preference

Probability indicates plausibility

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Utility and Probability

Consider events A and B:

Utility indicates preference

$U(A) > U(B)$ Indicates A is *preferable* to B

$U(A) = U(B)$ Indicates *indifference* between A and B

Probability indicates plausibility

Full story: https://projecteuclid.org/journals/statistical-science/volume-1/issue-3/The-Axioms-of-Subjective-Probability/10.1214/ss/1177013611.full

Utility and Probability

Consider events A and B:

<u>Utility indicates preference</u>

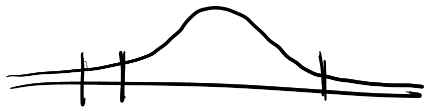
$U(A) > U(B)$	Indicates A is *preferable* to B
$U(A) = U(B)$	Indicates *indifference* between A and B

<u>Probability indicates plausibility</u>

$P(A) > P(B)$	Indicates A is *more plausible* (or likely) than B
$P(A) = P(B)$	Indicates A is *equally as plausible* (or equally likely) as B

Full story: https://projecteuclid.org/journals/statistical-science/volume-1/issue-3/The-Axioms-of-Subjective-Probability/10.1214/ss/1177013611.full

What is a Random Variable?



R.V. X

- Variable can take on multiple possible values
  ↖ finite
- Each possible value has an associated probability

$$P(X=1) = 0.6$$
$$P(X=0) = 0.4$$

- Variable
  - Discrete
  - Continuous
- Relationships between R.V.s

$$P(X|Y)$$

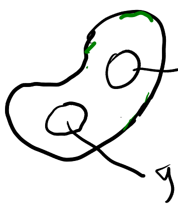
(Ω, Σ, μ)

sample space

σ-algebra

Probability measure assigns prob. to every element of Σ

$$X : \Omega \longrightarrow E$$



Vocabulary/Notation

Vocabulary/Notation

Term	Definition	Coinflip Example

Vocabulary/Notation

Term	Definition	Coinflip Example
support(X)		

Vocabulary/Notation

Term	Definition	Coinflip Example
support(X)	All the values that X can take	

Vocabulary/Notation

Term	Definition	Coinflip Example
support(X) $x \in X$	All the values that X can take	

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Term	Definition	Coinflip Example
support(X) $x \in X$	All the values that X can take	$\{h, t\}$ or $\{0, 1\}$ "Binary random variable"

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Distribution		

Vocabulary/Notation

Term	Definition	Coinflip Example
support(X) $x \in X$	All the values that X can take	$\{h, t\}$ or $\{0, 1\}$ "Binary random variable"
Distribution	Maps each value in the support to a real number indicating its probability	

Vocabulary/Notation

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support(X) $x \in X$	All the values that X can take	$\{h, t\}$ or $\{0, 1\}$ "Binary random variable"
Distribution	Maps each value in the support to a real number indicating its probability	$\text{Bernoulli}(\cancel{0.6})$ 0.5 $P(X = 1) = \cancel{0.6}$ 0.5 $P(X = 0) = \cancel{0.4}$ 0.5

Vocabulary/Notation

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$P(X)$ is a table

X	P(X)
0	0.4
1	0.6

Vocabulary/Notation

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Expectation		

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Expectation	First moment of the random variable, "mean"	

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Expectation $E[X]$	First moment of the random variable, "mean"		

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Expectation $E[X]$	First moment of the random variable, "mean"	$E[X] = \sum_{x \in X} x P(x)$

$P(X)$ is a table

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Vocabulary/Notation

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Distribution	Maps each value in the support to a real number indicating its probability	$\text{Bernoulli}(0.6)$ $P(X = 1) = 0.6$ $P(X = 0) = 0.4$
Expectation $E[X]$	First moment of the random variable, "mean"	$E[X] = \sum_{x \in X} xP(x)$ $= 0.5$ for fair coin

$P(X)$ is a table

X	P(X)
0	0.4
1	0.6

Distributions of related R.V.s

Distributions of related R.V.s

Joint Distribution

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.08
0	0	1	0.31
0	1	0	0.09
0	1	1	0.37
1	0	0	0.01
1	0	1	0.05
1	1	0	0.02
1	1	1	0.07

Distributions of related R.V.s

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Conditional Distribution

Distributions of related R.V.s

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Conditional Distribution

$$P(X \mid Y, Z)$$

Distributions of related R.V.s

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1	1	1	0.07

Conditional Distribution

$$P(X \mid Y, Z)$$

(Distribution - valued function)

X	$P(X \mid Y=1, Z=1)$
0	0.84
1	0.16

Distributions of related R.V.s

Joint Distribution

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0	0	0	0.08
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Marginal Distribution

Distributions of related R.V.s

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0	1	0	0.09
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Marginal Distribution

$$P(X)\ P(Y)\ P(Z)$$

Distributions of related R.V.s

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Conditional Distribution

$$P(X \mid Y, Z)$$

(Distribution - valued function)

X	$P(X \mid Y{=}1, Z{=}1)$
0	0.84
1	0.16

Marginal Distribution

$$P(X)\ P(Y)\ P(Z)$$

X	$P(X)$
0	0.85
1	0.15

Y	$P(Y)$
0	0.45
1	0.55

Z	$P(Z)$
0	0.20
1	0.80

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

Conditional Distribution

$$P(X \mid Y, Z)$$

Marginal Distribution

$$P(X)\ P(Y)\ P(Z)$$

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

Conditional Distribution

$$P(X \mid Y, Z)$$

Marginal Distribution

$$P(X)\ P(Y)\ P(Z)$$

(Burrito-level)

3 Rules

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

Conditional Distribution

$$P(X \mid Y, Z)$$

Marginal Distribution

$$P(X)\, P(Y)\, P(Z)$$

3 Rules

(Burrito-level)

(Gourmet Level: Axioms of Probability)

not on exam

AXIOM 1. STRUCTURE OF UNKNOWN REAL NUMBERS AND PLAUSIBLE VALUE. *We assume a set T of unknown numbers is a partially ordered commutative algebra over $\mathbb{R}$ with identity, 1.*

We assume in addition a given sub-Boolean algebra E of $E(T)$ with $0, 1 \in E$ and denote by E_0 the set of non-zero members of E. We assume that the partial ordering in $E(T)$ as a Boolean algebra coincides with the ordering that $E(T)$ inherits from the algebra T. Finally, we assume a function $PV : T \times E_0 \to \mathbb{R}$, called **PLAUSIBLE VALUE**, *whose value on the pair (x, e) is denoted $PV(x|e)$.*

AXIOM 2. STRONG RESCALING FOR PLAUSIBLE VALUE. *If a, b belong to $\mathbb{R}$, if x belongs to T, and if e belongs to E_0, then*

$$PV(ax + b|e) = aPV(x|e) + b. \tag{2}$$

AXIOM 3. ORDER CONSISTENCY FOR PLAUSIBLE VALUE. *If $x, y \in T$ and if $e \in E_0$, implies that $x \leq y$, then $PV(x|e) \leq PV(y|e)$.*

Notice that if $e \in E(T)$, then $0 \leq e \leq 1$, in T, as it is true in the lattice ordering of $E(T)$.

AXIOM 4. THE COX AXIOM FOR PLAUSIBLE VALUE: *If e, c are fixed in E, with $ec \in E_0$, if x_1, x_2 are in T, if $PV(x_1|ec) = PV(x_2|ec)$, then $PV(x_1 e|c) = PV(x_2 e|c)$. That is, we assume that as a function of x, the plausible value $PV(xe|c)$ depends only on $PV(x|ec)$.*

AXIOM 5. RESTRICTED ADDITIVITY OF PLAUSIBLE VALUE. *For each fixed $y \in T$ and $e \in E_0$, the plausible value $PV(x + y|e)$ as a function of $x \in T$ depends only on $PV(x|e)$, which is to say that if $x_1, x_2 \in T$ and $PV(x_1|e) = PV(x_2|e)$, then $PV(x_1 + y|e) = PV(x_2 + y|e)$.*

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

(Burrito-level)

1)

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\, P(Y)\, P(Z)$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

   b) $\sum_{x \in X} P(x \mid Y) = 1$

Y = making it to class

X = passing quiz

$P(X=1 \mid Y=0) = 0$

$P(X=1 \mid Y=1) = 0.$

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

   b) $\sum_{x \in X} P(x \mid Y) = 1$

2) "Law of total probability"

$$P(X) = \sum_{y \in Y} P(X, y)$$

$$P(X) = \sum_{y \in \{0,1\}} P(X, y)$$

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

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Joint $\rightarrow$ Marginal

"Marginalization"

Distributions of related R.V.s

Joint Distribution

$P(X,Y,Z)$

Conditional Distribution

$P(X \mid Y,Z)$

Marginal Distribution

$P(X)\,P(Y)\,P(Z)$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

   b) $\sum_{x\in X} P(x \mid Y) = 1$

2) "Law of total probability"

$$P(X) = \sum_{y\in Y} P(X,y)$$

Joint $\rightarrow$ Marginal

3) Definition of Conditional Probability

$$P(X \mid Y) = \frac{P(X,Y)}{P(Y)}$$

Distributions of related R.V.s

Joint Distribution

$$P(X, Y, Z)$$

Conditional Distribution

$$P(X \mid Y, Z)$$

Marginal Distribution

$$P(X)\ P(Y)\ P(Z)$$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

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3) Definition of Conditional Probability

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Joint $\rightarrow$ Marginal

Joint + Marginal $\rightarrow$ Conditional

Distributions of related R.V.s

Joint Distribution

$P(X, Y, Z)$

Conditional Distribution

$P(X \mid Y, Z)$

Marginal Distribution

$P(X)\ P(Y)\ P(Z)$

3 Rules

(Burrito-level)

1) a) $0 \leq P(X \mid Y) \leq 1$

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2) "Law of total probability"

$$P(X) = \sum_{y \in Y} P(X, y)$$

3) Definition of Conditional Probability

$$P(X \mid Y) = \frac{P(X, Y)}{P(Y)}$$

Marginal Probability of passing Quiz

Combine 2 and 3

$$P(X=1) = \sum_{y \in \{0,1\}} P(X=1, Y=y)$$

$$= \sum_{y \in \{0,1\}} P(X=1 \mid Y=y)\, P(Y=y)$$

Joint $\rightarrow$ Marginal

Joint + Marginal $\rightarrow$ Conditional

Marginal + Conditional $\rightarrow$ Joint

$$P(X, Y) = P(X|Y)\, P(Y)$$

Distributions of related R.V.s

Joint Distribution

$P(X,Y,Z)$

Conditional Distribution

$P(X \mid Y,Z)$

Marginal Distribution

$P(X)\;P(Y)\;P(Z)$

3 Rules

1) a) $0 \le P(X \mid Y) \le 1$

   b) $\sum_{x\in X} P(x \mid Y) = 1$

2) "Law of total probability"

$$P(X) = \sum_{y\in Y} P(X,y)$$

3) Definition of Conditional Probability

$$P(X \mid Y) = \frac{P(X,Y)}{P(Y)}$$

X	Y	Z	$P(X,Y,Z)$
0	0	0	0.08
0	0	1	0.31
0	1	0	0.09
0	1	1	0.37
1	0	0	0.01
1	0	1	0.05
1	1	0	0.02
1	1	1	0.07

$P(X)$

X	P(X)
0	0.85
1	0.15

$= \sum_{y\in Y, z\in Z} P(X=0, Y=y, Z=z)$

Joint → Marginal

$P(X=0 \mid Y=1, Z=1) = \frac{P(X=0, Y=1, Z=1)}{P(Y=1, Z=1)} = \frac{0.37}{0.44} = 0.84$

Joint + Marginal → Conditional

$= P(X=0, Y=1, Z=1) + P(X=1, Y=1, Z=1) = 0.37 + 0.07$

Marginal + Conditional → Joint

$$P(X,Y) = P(X|Y)\,P(Y)$$

Naive Inference

(Book introduces unnormalized "factors", but process is the same.)

Naive Inference

Three Random Variables: A, B, C (Works for any number)

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Want to find $P(\underbrace{A = a}_{\text{query}} \mid \underbrace{B = b}_{\text{evidence}})$

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Want to find $P(\underbrace{A = a}_{\text{query}} \mid \underbrace{B = b}_{\text{evidence}})$

C is a "hidden variable"

1. Determine the joint distribution $P(A, B, C)$.

(Book introduces unnormalized "factors", but process is the same.)

Naive Inference

Three Random Variables: A, B, C (Works for any number)

Want to find $P(\underbrace{A = a}_{\text{query}} \mid \underbrace{B = b}_{\text{evidence}})$

C is a "hidden variable"

1. Determine the joint distribution $P(A, B, C)$.
2. Marginalize over hidden and query variables to get

$$P(A = a, B = b) = \sum_c P(A = a, B = b, C = c)$$

and

$$P(B = b) = \sum_{a,c} P(A = a, B = b, C = c)$$

(Book introduces unnormalized "factors", but process is the same.)

Naive Inference

Three Random Variables: A, B, C (Works for any number)

Want to find $P(\underbrace{A = a}_{\text{query}} \mid \underbrace{B = b}_{\text{evidence}})$

C is a "hidden variable"

1. Determine the joint distribution $P(A, B, C)$.
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$$P(A = a, B = b) = \sum_c P(A = a, B = b, C = c)$$

and

$$P(B = b) = \sum_{a,c} P(A = a, B = b, C = c)$$

3. $P(A = a \mid B = b) = \frac{P(A=a, B=b)}{P(B=b)}$

(Book introduces unnormalized "factors", but process is the same.)

1) a) $0 \le P(X \mid Y) \le 1$

b) $\sum_{x \in X} P(x \mid Y) = 1$

2) $P(X) = \sum_{y \in Y} P(X, y)$

3) $P(X \mid Y) = \frac{P(X,Y)}{P(Y)}$

$\boxed{P(X,Y) = P(X|Y)\,P(Y)}$

Break

Joint: "And"

$P(C=1) = P(C=1, P=1) + P(C=1, P=0)$

0.8 0.1 0.7

P	C	P(P, C)
0	0	0.1
0	1	0.7
1	0	0.1
1	1	0.1

$P(C=0|P=1)\,P(P=1)$

$\underbrace{P(C=1|P=1)P(P=1)}_{0.5}$

- $P \in \{0, 1\}$: Powder Day
- $C \in \{0, 1\}$: Pass Clear
- 1 in 5 days is a powder day
- The pass is clear 8 in 10 days
- If it is a powder day, there is a 50% chance the pass is blocked

$P(P=1) = 0.2$

$P(C=1) = 0.8$

$P(C=0 \mid P=1) = 0.5$

$\underbrace{P(C=0 \mid P=1)}_{0.5} + \underbrace{P(C=1 \mid P=1)}_{0.5} = 1$

- Write out the joint probability distribution for P and C.
- Suppose it is a non-powder day, what is the probability that the pass is blocked?

$P(\underbrace{C=0}_{\text{query}} \mid \underbrace{P=0}_{\text{evidence}}) = \frac{P(C=0, P=0)}{P(P=0)} = \frac{0.1}{0.8} = \boxed{0.125}$

Bayes Rule

- Know: $P(B \mid A)$, $P(A)$, $P(B)$
- Want: $P(A \mid B)$

$$P(A|B) = \frac{P(A,B)}{P(B)}$$

$$P(B|A) = \frac{P(A,B)}{P(A)}$$

$$P(A|B)P(B) = P(A,B) = P(B|A)P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$

Definitions: Conditional Expectation and Independence

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Definition: The conditional expectation of X given Y is

$$E[X \mid Y] = \sum_x x \, P(X = x \mid Y)$$

(function from values of Y to expectations of X)

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Definition: X and Y are *independent* iff $P(X, Y) = P(X)\, P(Y)$

Definitions: Conditional Expectation and Independence

Definition: The conditional expectation of X given Y is

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(function from values of Y to expectations of X)

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Concepts

1. Utility and Probability
2. Random Variables
3. Relationships between Random Variables