Continuous Space MDPs

Last Time

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- What are the differences between online and offline solutions?
- Are there solution techniques that are *independent* of the state space size?

Guiding Questions

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- What tools do we have to solve MDPs with continuous S and A?

Current Tool-Belt

Notation: Continuous Random Variables

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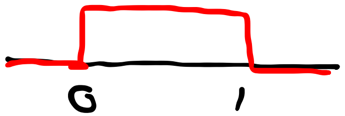 </td></tr>
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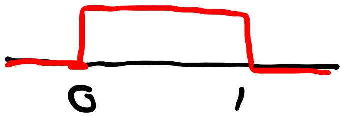
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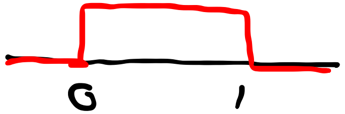
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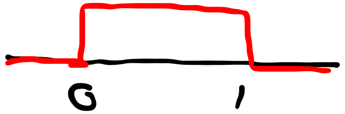
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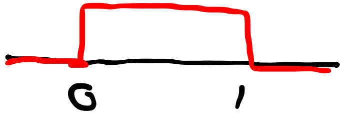

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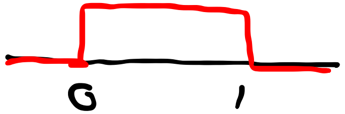
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1) a) $0 \leq P(X \mid Y) \leq 1$

   b) $\sum_{x \in X} P(x \mid Y) = 1$

2) $P(X) = \sum_{y \in Y} P(X, y)$

3) $P(X \mid Y) = \frac{P(X,Y)}{P(Y)}$

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Multivariate Gaussian Distribution

Joint Distribution

Conditional Distribution

Marginal Distribution

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The old rules still work!

Today: Four Tools

1. Linear Dynamics, Quadratic Reward

$S = \mathbb{R}^n$, $A = \mathbb{R}^m$ $\quad$ $T(s' \mid s, a) = \mathcal{N}(T_s s + T_a a, \Sigma)$ $\quad$ $R(s, a) = s^\top R_s s + a^\top R_a a$

$s' = T_s s + T_a a + w \quad w \sim \mathcal{N}(0, \Sigma)$ (Also works with other zero-mean w.)

Finite Horizon: $U_h^*(s) = \max_\pi \left[\sum_{t=0}^{h} R(s_t, a_t)\right]$ $\quad$ π_h^* is "optimal h-step policy"

We will show that $U_h^*(s) = s^\top V_h s + q_h$ and $\pi_h^*(s) = -K_h s$

1. Linear Dynamics, Quadratic Reward

We will show that $U_h^*(s) = s^\top V_h s + q_h$ and $\pi_h^*(s) = -K_h s$ by induction.

Base: $U_1^*(s) = \max_a \left(s^\top R_s s + a^\top R_a a\right) = s^\top R_s s$

Inductive step: show that if $U_t^* = s^\top V_t s + q_t$, then $U_{t+1}^* = s^\top V_{t+1} s + q_{t+1}$.

$$\begin{aligned}
U_{t+1}^*(s) &= \max_a \left(R(s,a) + \gamma E\left[U_t^*(s)\right]\right) \\
&= \max_a \left(s^\top R_s s + a^\top R_a a + \int p(w) U_t(T_s s + T_a a + w)\, dw\right) \\
&= s^\top R_s s + \max_a \left(a^\top R_a a + \int p(w)(T_s s + T_a a + w)^\top V_t (T_s s + T_a a + w) + q_t\right) dw \\
&= s^\top R_s s + s^\top T_s^\top V_t T_s s + \max_a \left(a^\top R_a a + 2 s^\top T_s^\top V_t T_a a + a^\top T_a^\top V_t T_a a\right) + \int p(w) w^\top V_t w\, dw + q_t
\end{aligned}$$

a^* is where $\nabla_a(\text{max term}) = 0$

$$0 = 2R_a a^* + 2T_a^\top V_t T_s s + 2T_a^\top V_t T_a a^*$$

$$a^* = -\underbrace{(R_a + T_a^\top V_t T_a)^{-1} T_a^\top V_t T_s}_{K_t} s$$

$$U_{t+1}^*(s) = s^\top \underbrace{\left(R_s + T_s^\top V_t T_s - (T_a^\top V_t T_s)^\top (R_a + T_a^\top V_t T_a)^{-1} (T_a^\top V_t T_s)\right)}_{V_{t+1}} s + \underbrace{\int p(w) w^\top V_t w\, dw + q_t}_{q_{t+1}}$$

$$U_{t+1}^*(s) = s^\top V_{t+1} s + q_{t+1} \qquad \square$$

1. Linear Dynamics, Quadratic Reward

As $h \to \infty$

$$V_\infty = T_s^\top \left(V_\infty - V_\infty T_a \left(T_a^\top V_\infty T_a + R_a\right)^{-1} T_a^\top V_\infty\right) T_s + R_s$$

$$K_\infty = \left(T_a^\top V_\infty T_a + R_a\right)^{-1} T_a^\top V_\infty T_s$$

$$\pi_\infty^*(s) = -K_\infty s$$

(K_∞ has no dependence on Σ)

Certainty-Equivalence Principle: For Linear-Quadratic problems, the optimal policy with noise is the same as the optimal policy without noise!

Practical Implication: If a continuous problem has roughly linear dynamics, a convex cost function, and roughly zero-mean additive noise, you can use *certainty-equivalent control*, i.e. control as if there is no noise.

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while not converged
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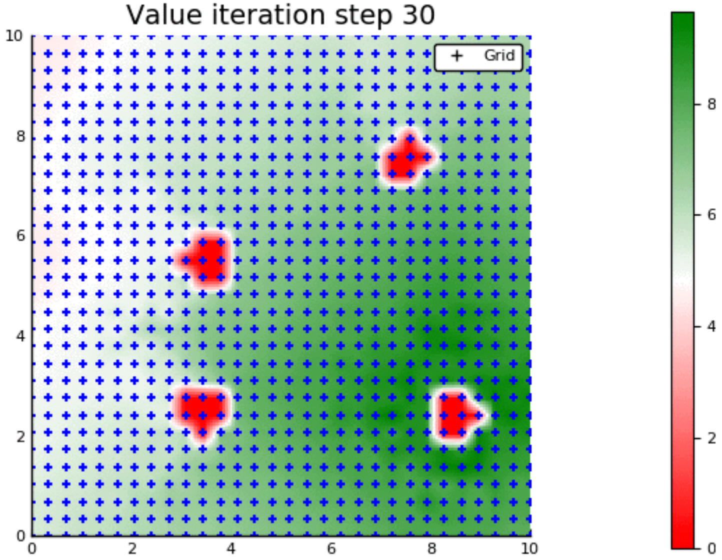

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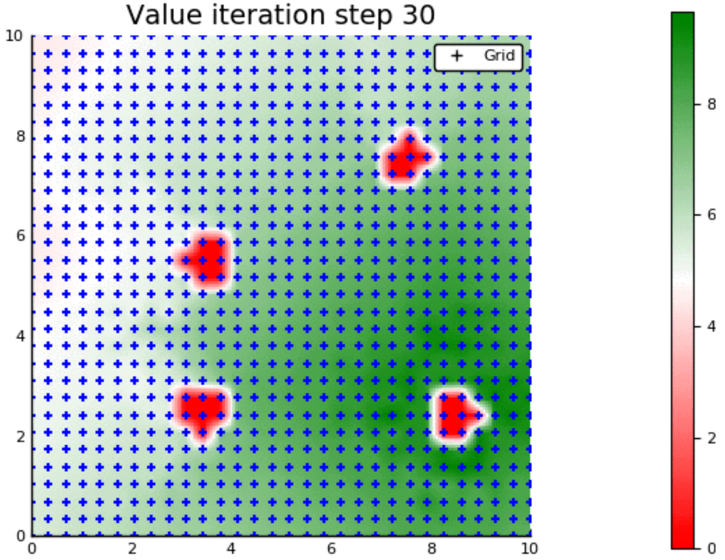
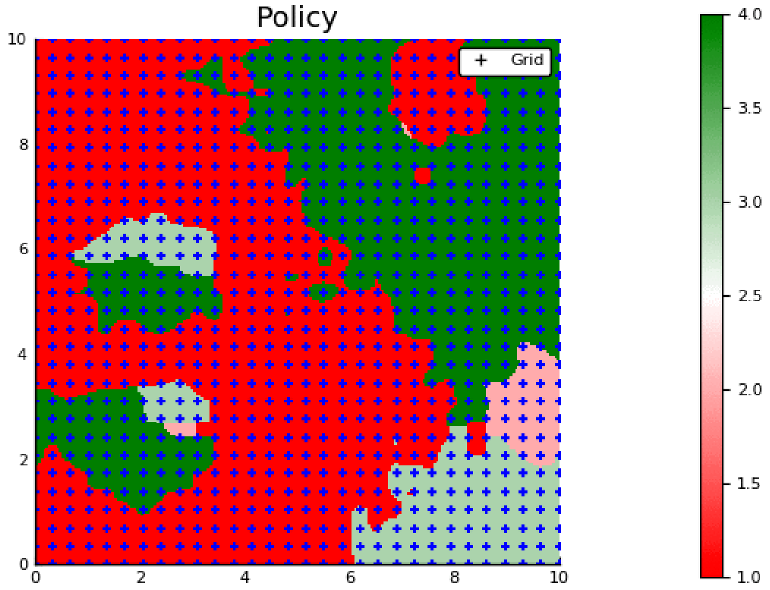
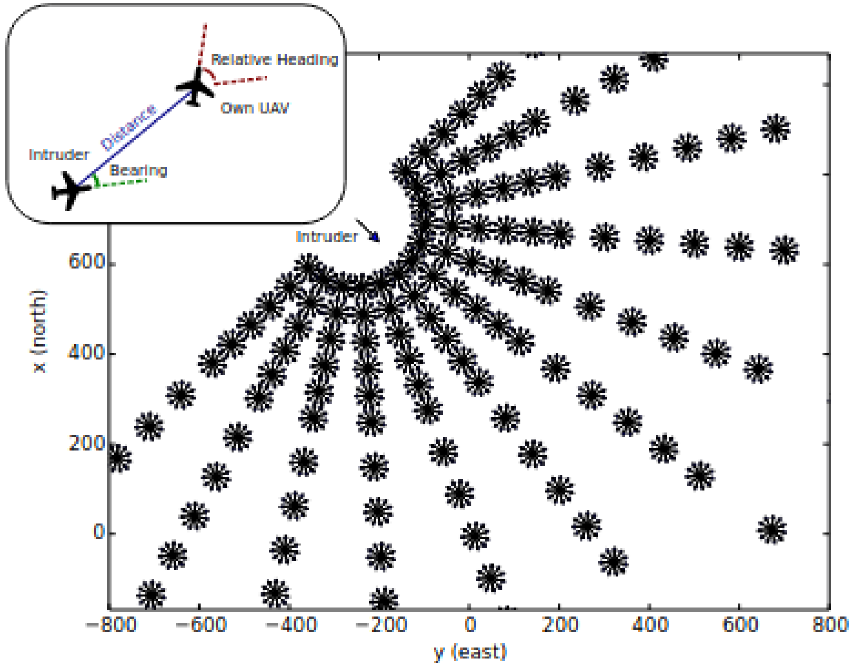
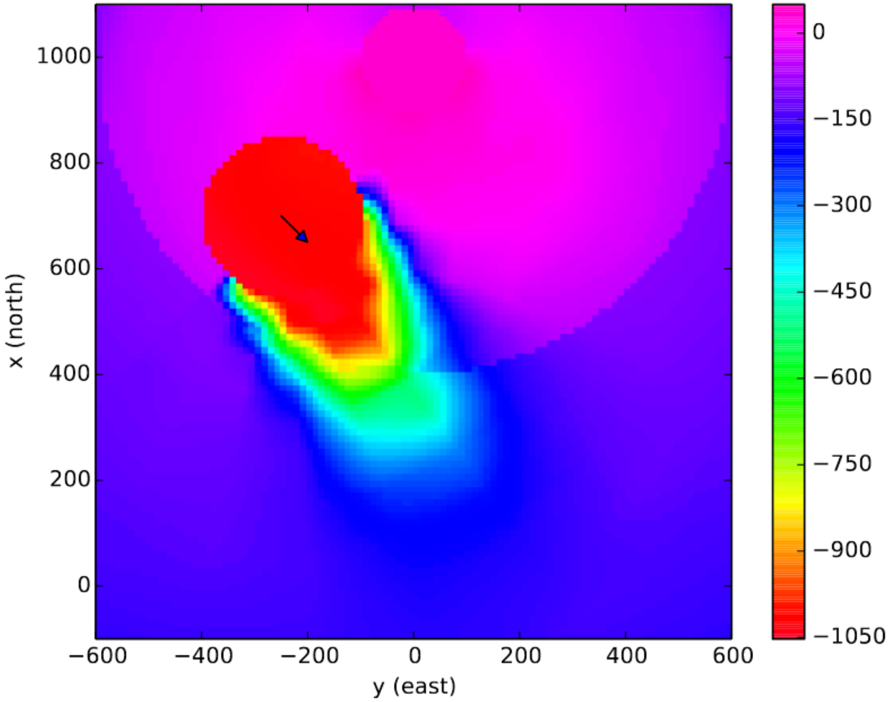




$$B_{\text{MC}(N)}[V_\theta](s) = \max_a \left(R(s,a) + \gamma \sum_{i=1}^{N} V_\theta(G(s,a,w_i)) \right)$$

2. Value Function Approximation





Break

What will a Monte Carlo Tree Search tree look like if run on a problem with continuous spaces?

3. Sparse Tree Search/Progressive Widening

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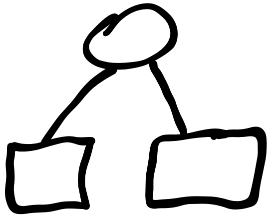

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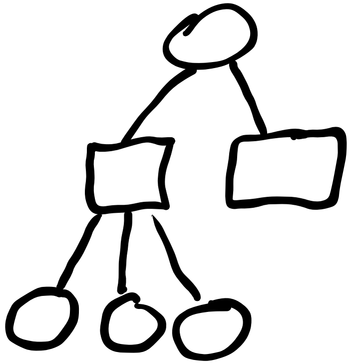

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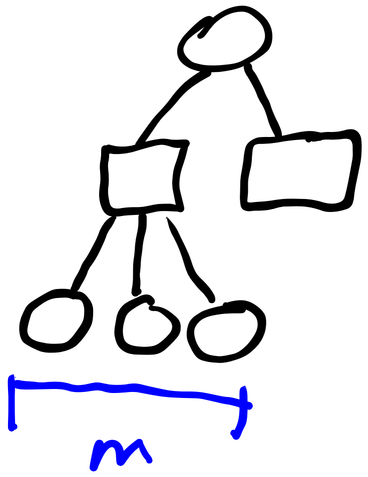

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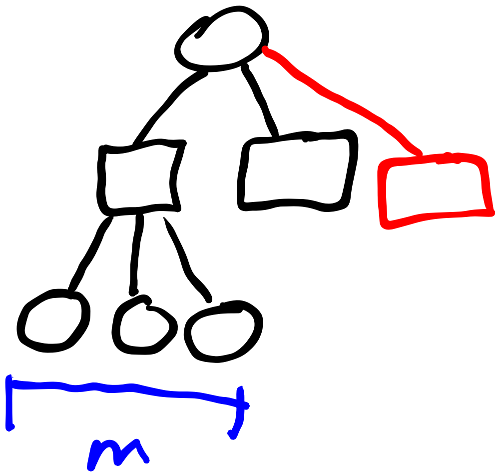

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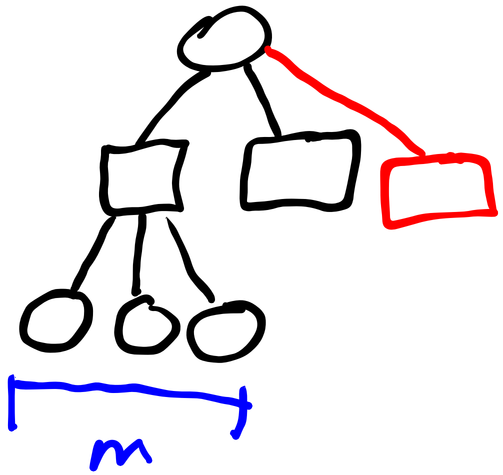

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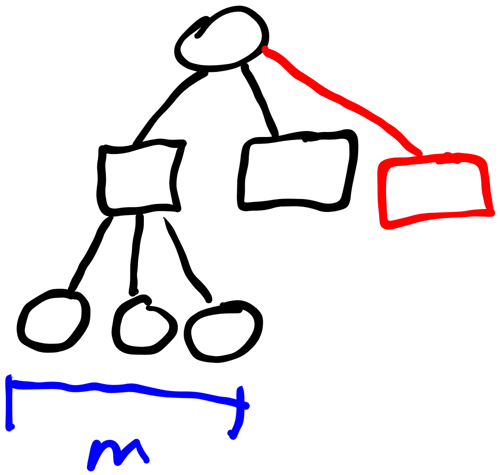

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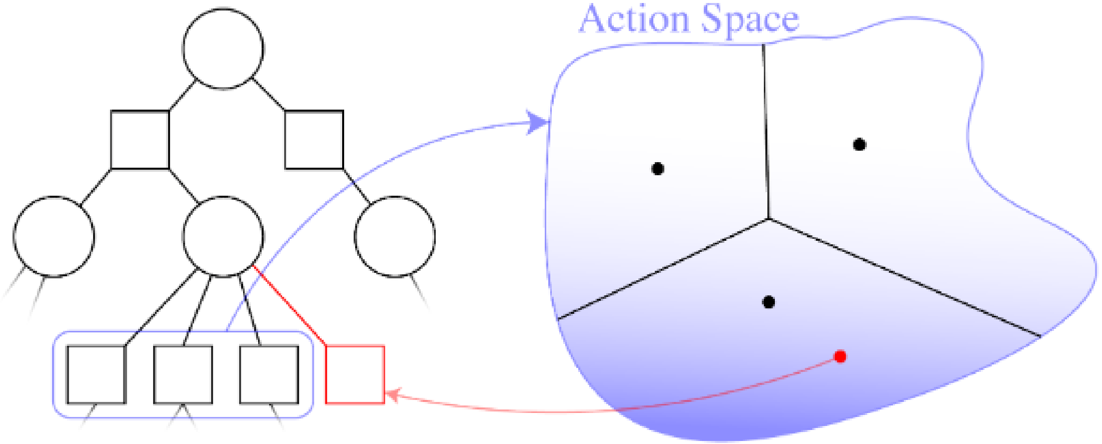

Online Tree Search Planner

Voronoi Progressive Widening

4. Model Predictive Control

(Use off-the-shelf optimization software, e.g. Ipopt)

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Certainty-Equivalent

$$\begin{aligned} \underset{a_{1:d}, s_{1:d}}{\text{maximize}} \quad & \sum_{t=1}^{d} \gamma^t R(s_t, a_t) \\ \text{subject to} \quad & s_{t+1} = \text{E}[T(s_t, a_t)] \quad \forall t \end{aligned}$$

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Open-Loop

$$\begin{aligned} \underset{a_{1:d}, s_{1:d}^{(1:m)}}{\text{maximize}} \quad & \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{d} \gamma^t R(s_t^{(i)}, a_t) \\ \text{subject to} \quad & s_{t+1} = G(s_t^{(i)}, a_t, w_t^{(i)}) \quad \forall t, i \end{aligned}$$

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Hindsight Optimization

$$\begin{aligned} \underset{a_{1:d}^{(1:m)}, s_{1:d}^{(1:m)}}{\text{maximize}} \quad & \frac{1}{m} \sum_{i=1}^{m} \sum_{t=1}^{d} \gamma^t R(s_t^{(i)}, a_t^{(i)}) \\ \text{subject to} \quad & s_{t+1} = G(s_t^{(i)}, a_t^{(i)}, w_t^{(i)}) \quad \forall t, i \\ & a_1^{(i)} = a_1^{(j)} \quad \forall i, j \end{aligned}$$

Guiding Questions

- What tools do we have to solve MDPs with continuous S and A?