Stochastic Processes and Simple Decisions

Review

Guiding Question

- What does "Markov" mean in "Markov Decision Process"?

Stochastic Process

- A stochastic process is a collection of R.V.s indexed by time.
- $\{X_0, X_1, X_2, \ldots\}$ or $\{X_t\}_{t=0}^{\infty}$ or just $\{X_t\}$

Example: Positive, Uniform Random Walk

$$X_0 = 0 \qquad\qquad X_{t+1} = X_t + V_t \qquad\qquad P(X_{t+1} \mid X_{0:t}) = \begin{cases} 0.5 & \text{if } X_{t+1} = X_t \\ 0.5 & \text{if } X_{t+1} = X_t + 1 \\ 0 & \text{otherwise} \end{cases}$$

$$V_t \sim \text{Bernoulli}(0.5) \text{ (i.i.d.)}$$

In a *stationary* stochastic process (all in this class), this

relationship does not change with time

Bayes Net   Trajectories

Simulating a Stochastic Process

030-Stochastic-Processes.ipynb

A More Complex Example

Markov Process

- A stochastic process $\{S_t\}$ is *Markov* if
$$P(S_{t+1} \mid S_{0:t}) = P(S_{t+1} \mid S_t)$$
$$S_{t+1} \perp S_{t-\tau} \mid S_t \quad \forall \tau \in 1:t$$
- S_t is called the "state" of the process

Dynamic Bayesian Networks

Break

Suppose you want to create a Markov process model that describes how many new COVID cases will start in a particular week. **What information should be in the state of the model?**

Assume:

- The population mixes thoroughly (i.e. there are no geographic considerations).
- COVID patients may be contagious up to 2 weeks after they contract the disease.
- Researchers have determined a probabilistic model for the number of new cases given the number of people in the first week of the disease and the number of people in the second week of the disease.

Simple Decisions

Simple Decisions

Outcomes
$S_1 \ldots S_n$ or A, B, C

Probabilities
$p_1 \ldots p_n$

Lottery
$[S_1 : p_1; \ldots; S_n : p_n]$

- Completeness: Exactly one holds: $A \succ B$, $B \succ A$, $A \sim B$
- Transitivity: If $A \succeq B$ and $B \succeq C$, then $A \succeq C$
- Continuity: If $A \succeq C \succeq B$, then there exists a probability p such that $[A : p; B : 1 - p] \sim C$
- Independence: If $A \succ B$, then for any C and probability p, $[A : p; C : 1 - p] \succeq [B : p; C : 1 - p]$

von Neumann - Morgenstern Axioms

These constraints imply a utility function U with the properties:

- $U(A) > U(B)$ iff $A \succ B$
- $U(A) = U(B)$ iff $A \sim B$
- $U([S_1 : p_1; \ldots; S_n : p_n]) = \sum_{i=1}^{n} p_i \, U(S_i)$

Decision Networks

Maximizing Expected Utility

Value of Information

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