Causal Bayesian Networks

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Today:

- Causal Bayesian Networks
- How do we reason about independence in Bayesian Networks?

Review: Distributions of Discrete R.V.s

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Joint	$P(X = x, Y = y)$ Shorthand: $P(x, y)$	Single number	"Probability that $X = x$ *and* $Y = y$"
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Conditional	$P(X = x \mid Y = y)$ Shorthand: $P(x \mid y)$	Single number	"Probability that $X = x$ *if* $Y = y$"
	$P(X \mid Y)$	A collection of tables for each y	"Conditional distribution of X given Y"

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Marginal	$P(X=x)$ Shorthand: $P(x)$	Single number	"Probability that $X=x$"
	$P(X)$	A table	"Marginal distribution of X"

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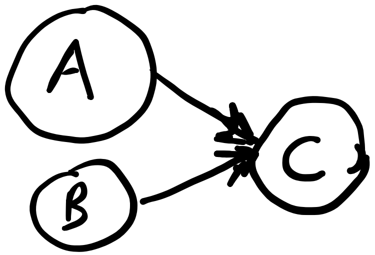

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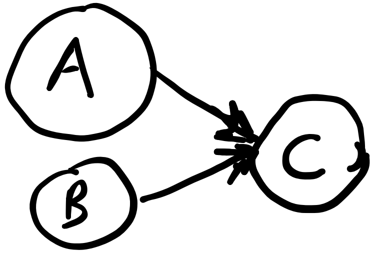

At each node, $P(X \mid pa(X))$

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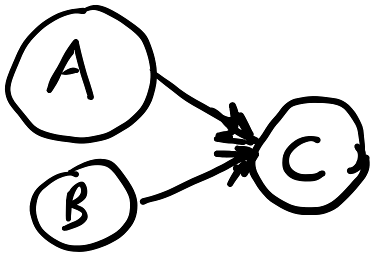

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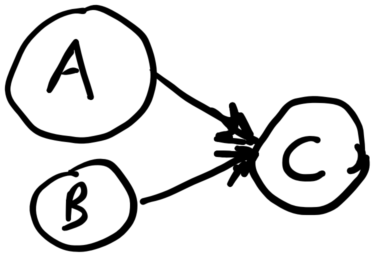

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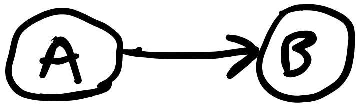

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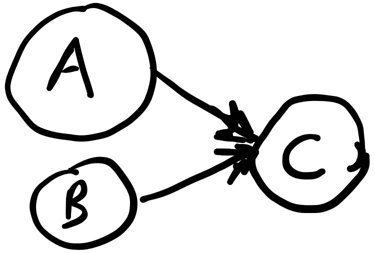
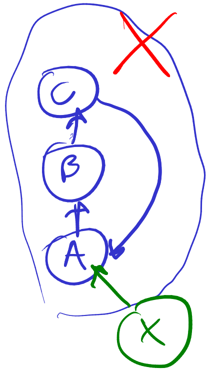




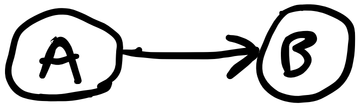
At each node, $P(X \mid pa(X))$

$P(C \mid A, B)$

$P(A)$

$P(B)$

In a *Causal Bayesian Network*, arrows denote causation.



B is a result of A (and some aleatory uncertainty)

$P(B \mid A)$ "dynamics of system"

$P(A)$

$P(X_{k+1} \mid X_k, U_k)$

$X_{k+1} = f(X_k, U_k, W_k)$

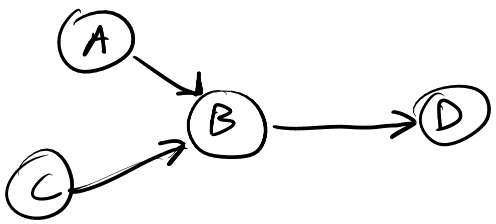
Chain rule for Bayesian Networks

Chain rule for Bayesian Networks

$P(X_1, X_2 \ldots X_n)$

$$P(X_{1:n}) = \prod_{i=1}^{n} P(X_i \mid \mathrm{pa}(X_i))$$

Joint



$$P(A, B, C, D) = \underline{P(A)\, P(B \mid A, C)\, P(C)\, P(D \mid B)}$$

Simple Causal Bayes Net Example

$S \rightarrow T$

snow traffic

$$\left.\begin{aligned} P(S=1) &= 0.1 \\ P(T=1 \mid S=0) &= 0.2 \\ P(T=1 \mid S=1) &= 0.7 \end{aligned}\right\} \text{3 parametes}$$

observe snow, traffic? $\Longrightarrow P(T=1 \mid S=1) = 0.7$

$S=1$ $\quad P(T=1 \mid S=1)$

observe traffic, snow? $\Longrightarrow P(S=1 \mid T=1) = \dfrac{P(S=1, T=1)}{P(T=1)} = \dfrac{P(T=1 \mid S=1)\,P(S=1)}{\sum_s (T=1 \mid S=s)\,P(S=s)}$

$T=1$ $\quad P(S=1 \mid T=1)$

$$= \frac{0.07}{0.2 \cdot 0.9 + 0.7 \cdot 0.1}$$

$$= 0.28$$

"Information can flow up or down edges"

Naive Inference on Bayes Nets

(Book introduces unnormalized "factors", but process is the same.)

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Bayes Net with 3 Random Variables: $A \to C \to B$

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1. $P(A, B = b, C) = P(B = b \mid C)\, P(C \mid A)\, P(A).$

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1. $P(A, B = b, C) = P(B = b \mid C)\, P(C \mid A)\, P(A)$.
2. Marginalize over hidden and query variables to get

$$P(A = a, B = b) = \sum_{c} P(A = a, B = b, C = c)$$

and

$$P(B = b) = \sum_{a,c} P(A = a, B = b, C = c)$$

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1. $P(A, B = b, C) = P(B = b \mid C)\, P(C \mid A)\, P(A)$. ← Chain Rule
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3. $P(A = a \mid B = b) = \frac{P(A=a, B=b)}{P(B=b)}$

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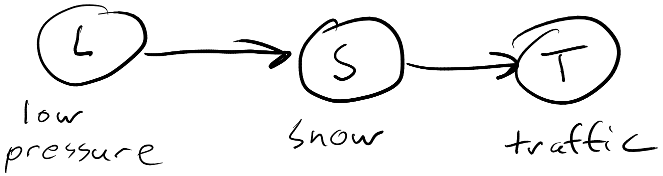
Conditional Independence in Bayes Nets

"Chain"



$T \perp L \mid S$?

Yes

Definition of cond. indep.

$$P(T, L \mid S) = P(T \mid S)\, P(L \mid S)$$

$$P(T \mid S) = P(T \mid S, L)$$

$$P(T \mid S) \stackrel{?}{=} P(T \mid S, L)$$

$$= \frac{P(T, S, L)}{P(S, L)}$$

$$= \frac{P(T \mid S)\, \cancel{P(S \mid L)}\, \cancel{P(L)}}{\underbrace{\sum_t P(T = t \mid S)}_{1}\, \cancel{P(S \mid L)}\, \cancel{P(L)}}$$

$$= P(T \mid S)$$

Conditional Independence: Fork

https://kunalmenda.com/2019/02/21/causation-and-correlation/

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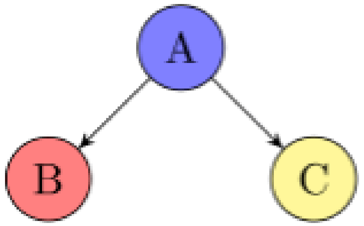

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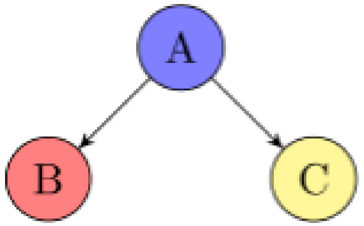

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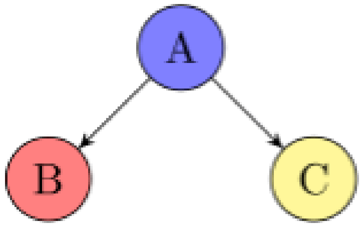

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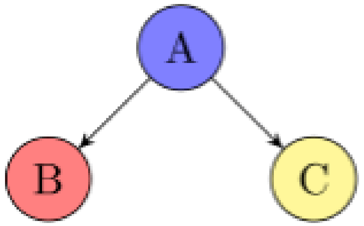

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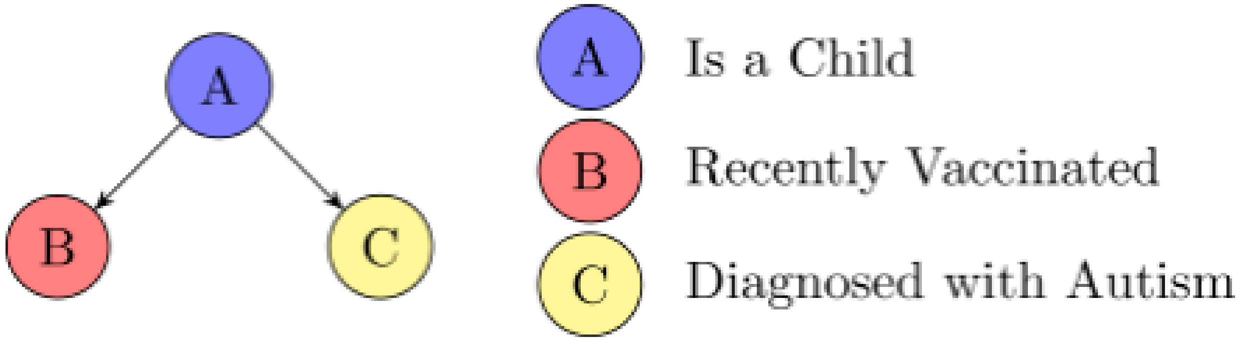

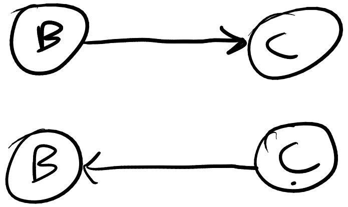

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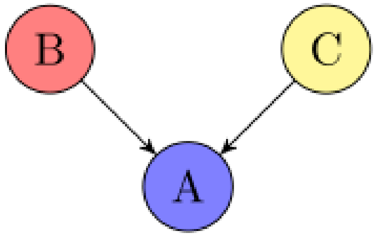

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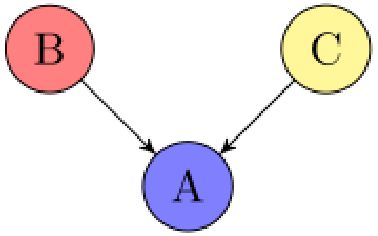
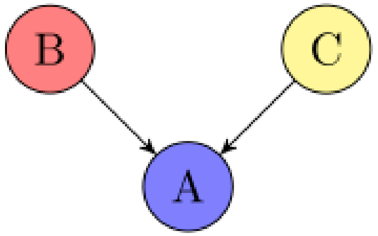


$$B \perp C \mid A \ ?$$

Inconclusive
based on structure

https://kunalmenda.com/2019/02/21/causation-and-correlation/

Conditional Independence: Inverted Fork



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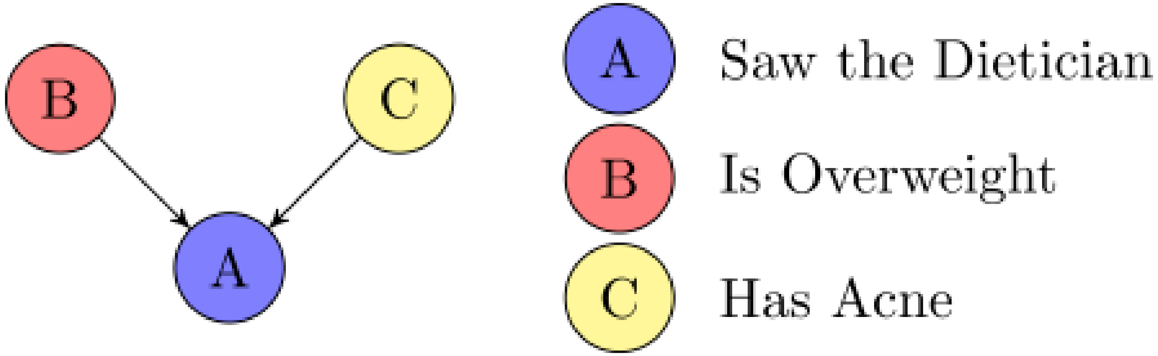
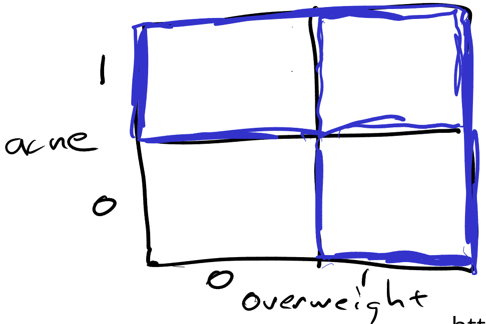


Assume $B \perp C$

Does not imply that $B \perp C \mid A$



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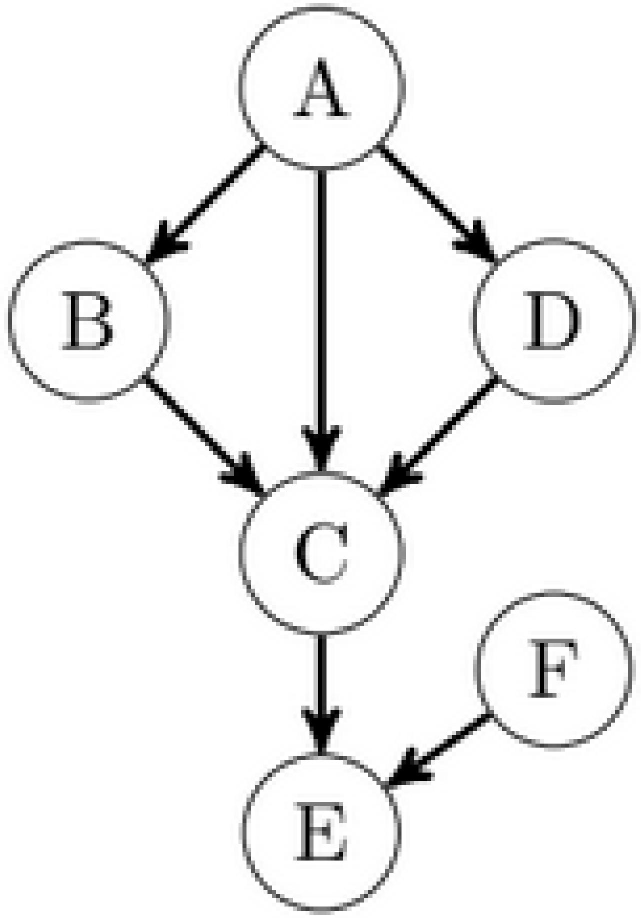

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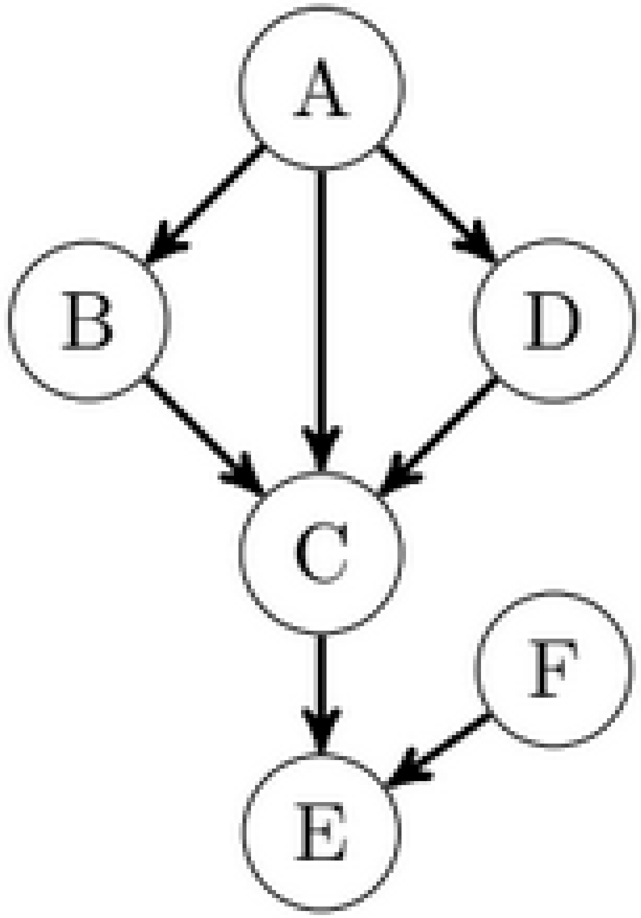

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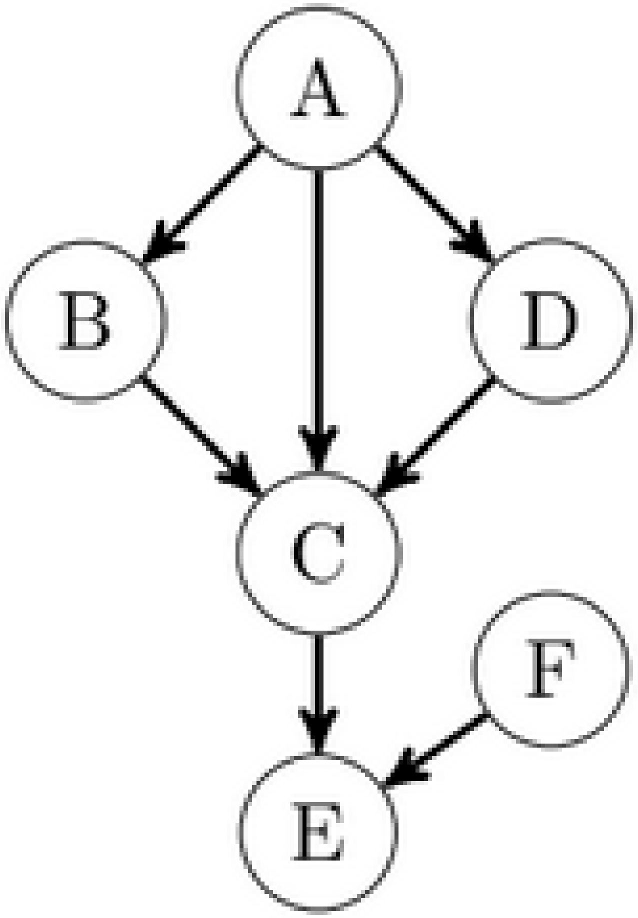

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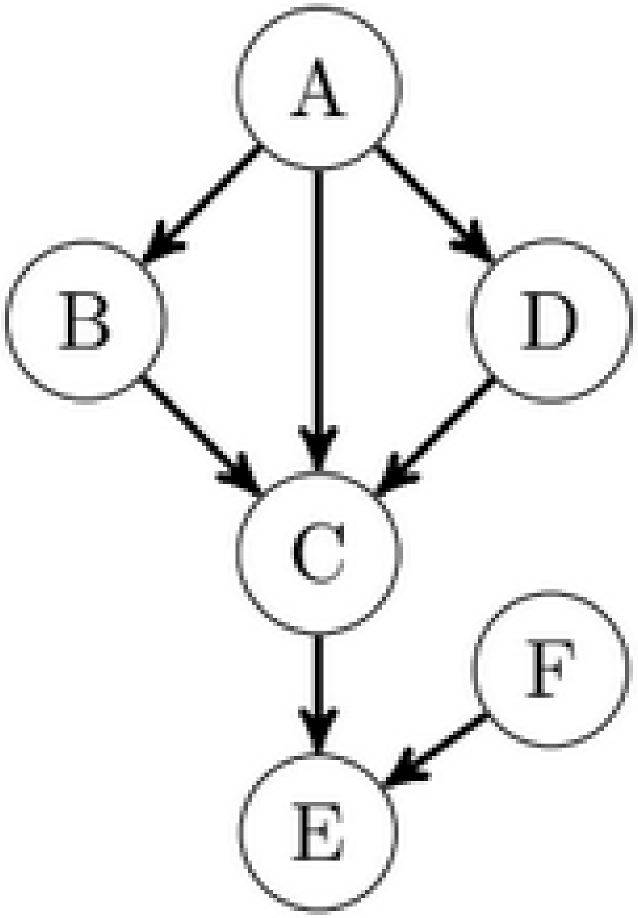

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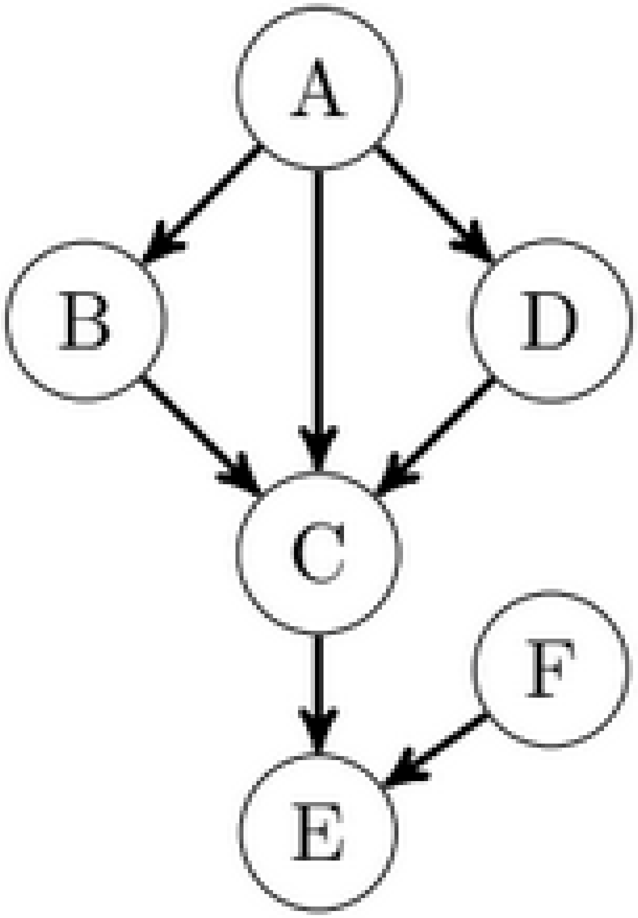

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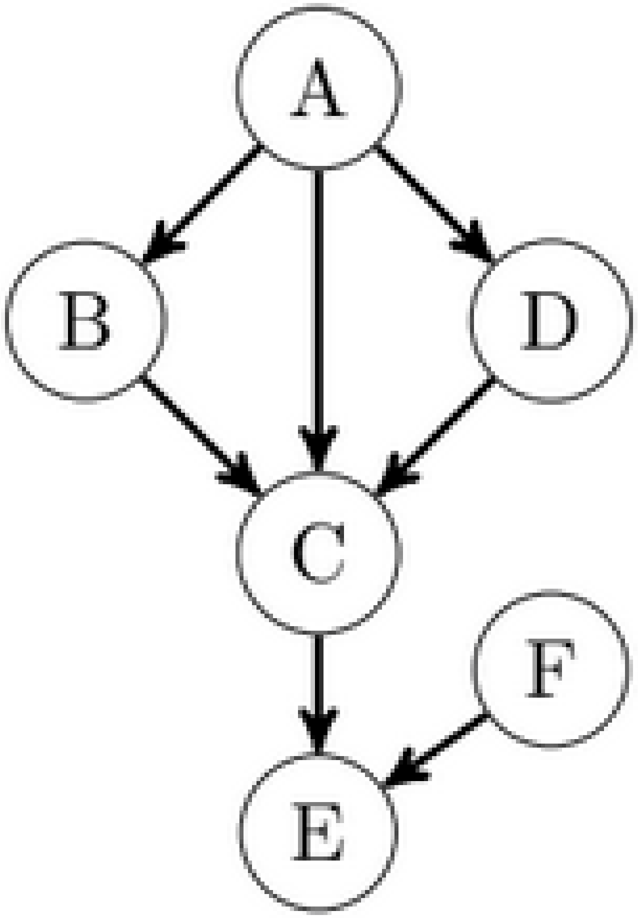

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Why is this relevant to decision making?

d-Separation

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Also:

- d-separated = "inactive"
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$$B \leftarrow \overset{A}{A} \to \overset{A}{C} \leftarrow D$$ → active / not d-separated

$$B \leftarrow \overset{I}{C} \to \overset{I}{A} \leftarrow D$$ → not active

Are these paths d-separated by $\mathcal{C} = \{C\}$?

d-Separation for Bayes Nets

*short for "directionally separated"

d-Separation for Bayes Nets

We say that A and B are *d-separated* by $\mathcal{C}$ if all acyclic paths between A and B are d-separated by $\mathcal{C}$.

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If A and B are d-separated by $\mathcal{C}$ then $A \perp B \mid \mathcal{C}$

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d-Separation for Bayes Nets

We say that A and B are *d-separated* by $\mathcal{C}$ if all undirected acyclic paths between A and B are d-separated by $\mathcal{C}$.

If A and B are d-separated by $\mathcal{C}$ then $A \perp B \mid \mathcal{C}$

In other words, if there is any active path w.r.t. $\mathcal{C}$ between A and B, we *cannot* conclude that $A \perp B \mid \mathcal{C}$ based on the structure alone.

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Proving Conditional Independence

Separators

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3. If all paths d-separated, then CE

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Proving Conditional Independence

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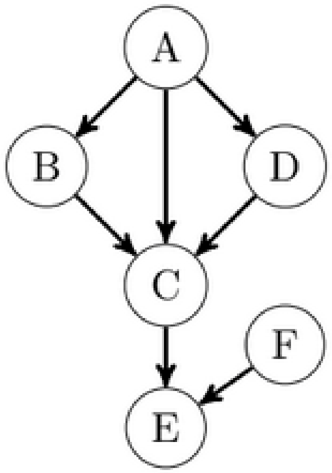
2. Check all paths for d-separation
3. If all paths d-separated, then CE



Separators

1. **Chain:** $X \rightarrow Y \rightarrow Z$ such that $Y \in \mathcal{C}$
2. **Fork:** $X \leftarrow Y \rightarrow Z$ such that $Y \in \mathcal{C}$
3. **Inverted Fork (v-structure):** $X \rightarrow Y \leftarrow Z$ such that $Y \notin \mathcal{C}$ and no descendant of Y is in $\mathcal{C}$.

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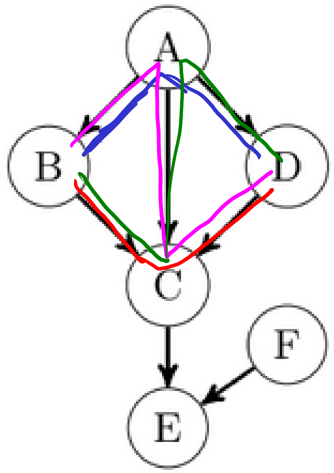

Example: $(B \perp D \mid \underbrace{C, E}_{\mathcal{C}})$?

$B \leftarrow A \rightarrow D$

$B \rightarrow C \leftarrow D$

$B \leftarrow A \rightarrow C \leftarrow D$

$B \rightarrow C \leftarrow A \rightarrow D$

active $\longrightarrow$ inconclusive

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Markov Blanket

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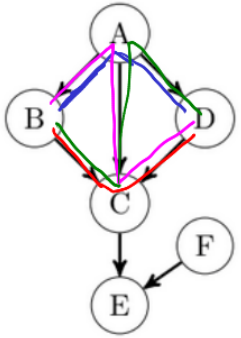
If $\mathcal{B}$ is the Markov blanket of $\mathcal{X}$, you can treat analyze $\mathcal{B} \cup \mathcal{X}$ alone, and ignore any other nodes.



What is the Markov Blanket of $\{E, F\}$?

C ✓

Exercise

1. The path contains a *chain* $X \rightarrow Y \rightarrow Z$ such that $Y \in \mathcal{C}$
2. The path contains a *fork* $X \leftarrow Y \rightarrow Z$ such that $Y \in \mathcal{C}$
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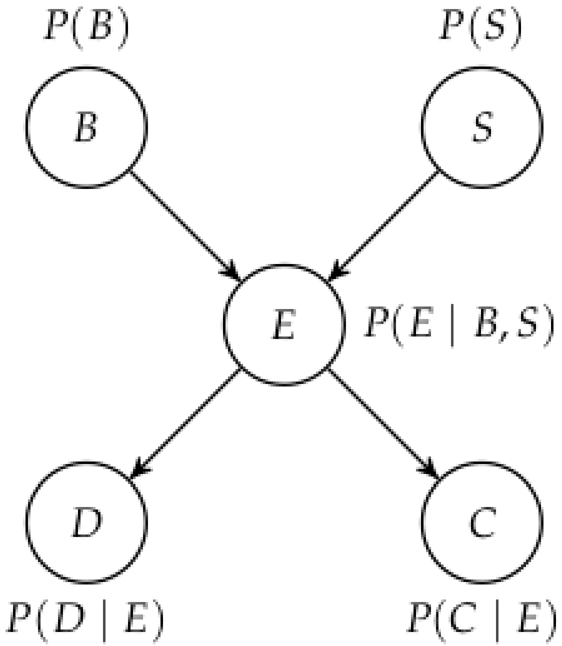
Exercise



B battery failure
S solar panel failure
E electrical system failure
D trajectory deviation
C communication loss

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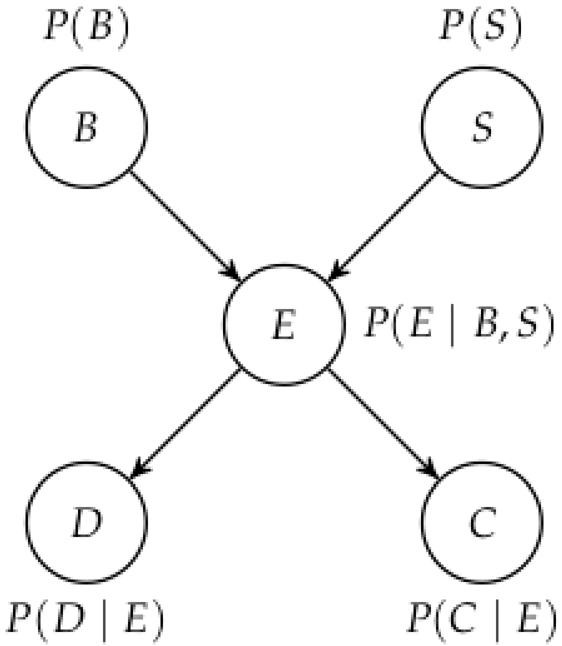

Exercise

$$D \perp C \mid B\,?$$

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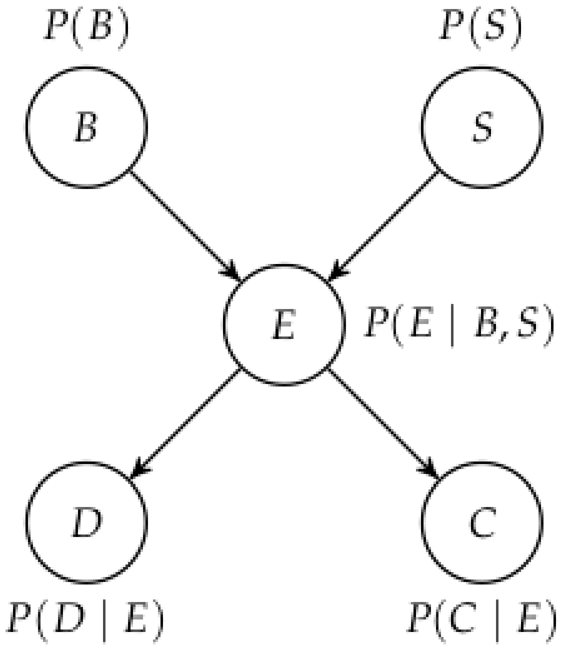

Exercise

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$$D \perp C \mid B \,?$$

$$D \perp C \mid E \,?$$

1. The path contains a *chain* $X \rightarrow Y \rightarrow Z$ such that $Y \in \mathcal{C}$
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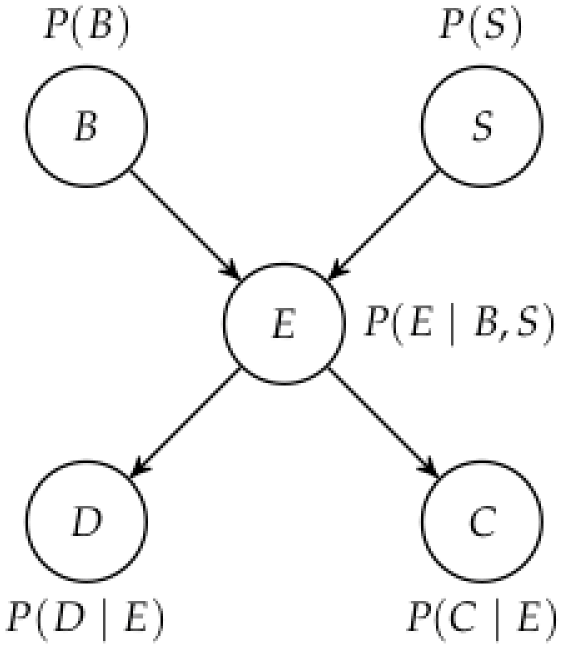
Approximate Inference

Approximate Inference: Direct Sampling



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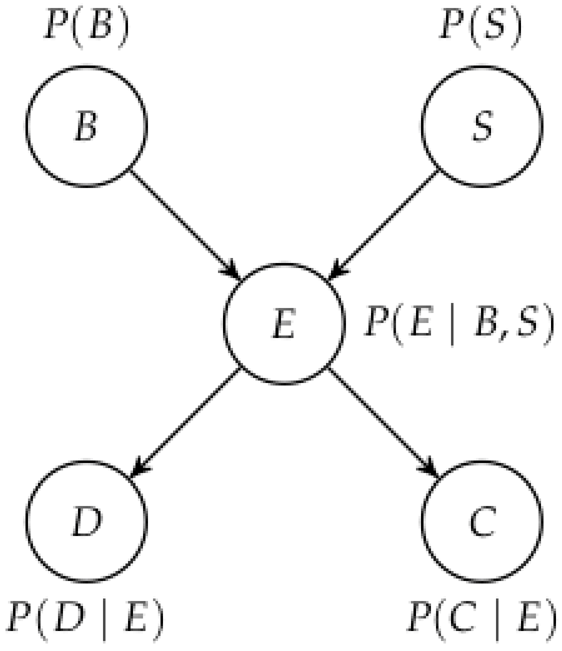

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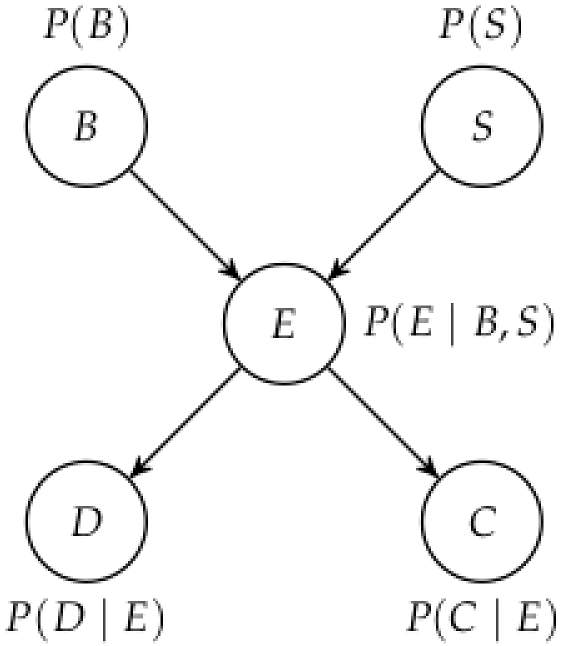
$$P(b^1 \mid d^1, c^1) \approx \frac{\sum_i (b^{(i)} = 1 \land d^{(i)} = 1 \land c^{(i)} = 1)}{\sum_i (d^{(i)} = 1 \land c^{(i)} = 1)}$$

Approximate Inference: Direct Sampling



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B	S	E	D	C	
0	0	1	1	0	
0	0	0	0	0	
1	0	1	0	0	
1	0	1	1	1	←
0	0	0	0	0	
0	0	0	1	0	
0	0	0	0	1	
0	1	1	1	1	←
0	0	0	0	0	
0	0	0	1	0	

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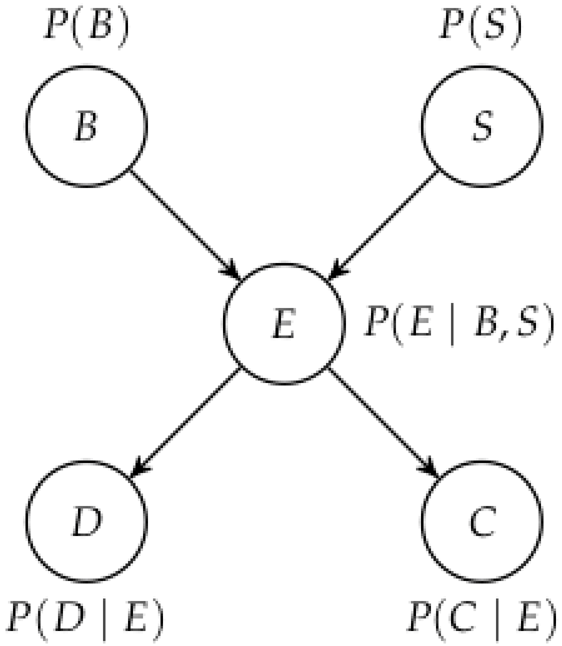

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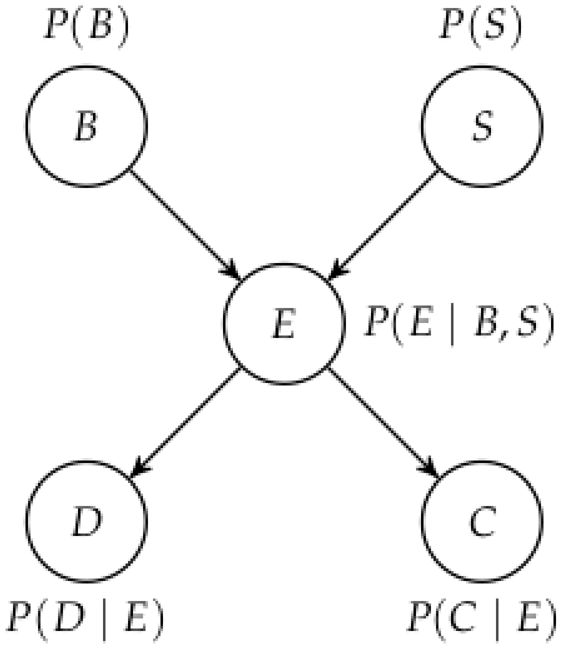
B	S	E	D	C
0	0	1	1	0
0	0	0	0	0
1	0	1	0	0
1	0	1	1	1
0	0	0	0	0
0	0	0	1	0
0	0	0	0	1
0	1	1	1	1
0	0	0	0	0
0	0	0	1	0

Analogous to

Approximate Inference: Direct Sampling



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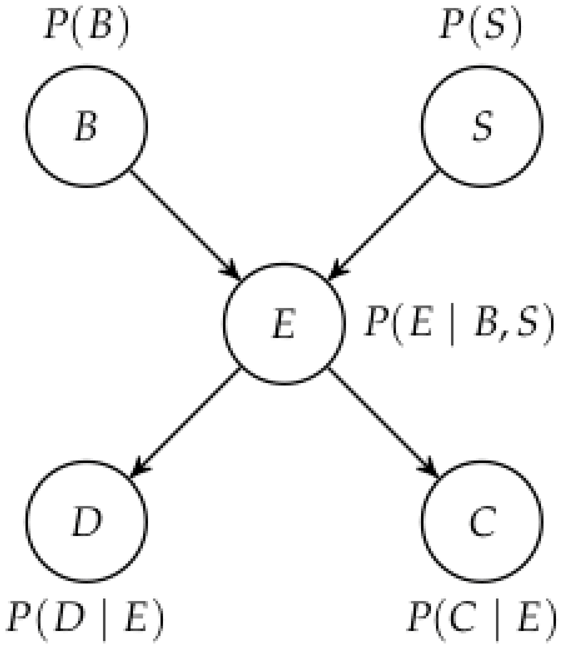
B	S	E	D	C
0	0	1	1	0
0	0	0	0	0
1	0	1	0	0
1	0	1	1	1
0	0	0	0	0
0	0	0	1	0
0	0	0	0	1
0	1	1	1	1
0	0	0	0	0
0	0	0	1	0

Analogous to **unweighted particle filtering**

Approximate Inference: Weighted Sampling



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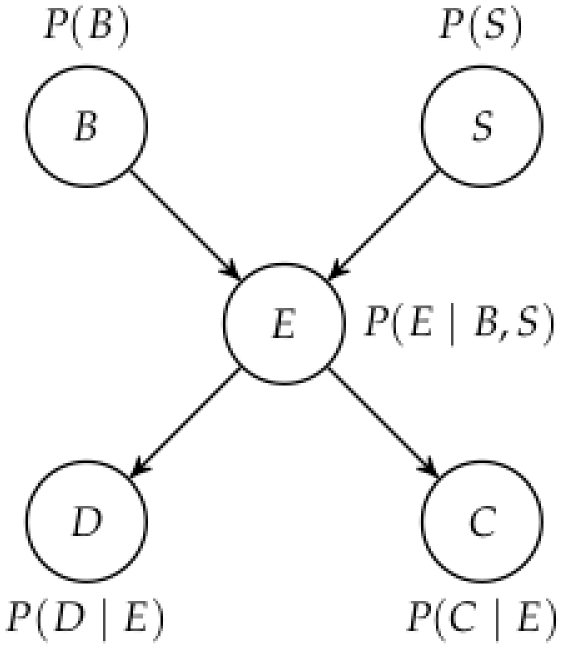
Approximate Inference: Weighted Sampling



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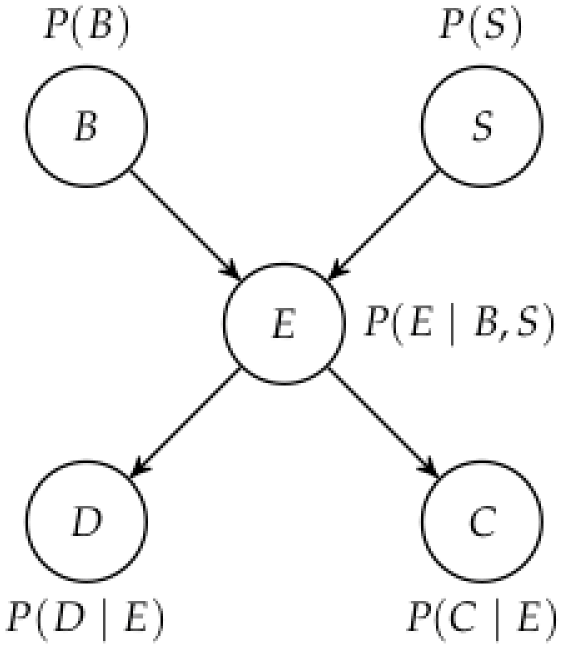
$$P(b^1 \mid d^1, c^1) \approx \frac{\sum_i w_i(b^{(i)} = 1 \land d^{(i)} = 1 \land c^{(i)} = 1)}{\sum_i w_i(d^{(i)} = 1 \land c^{(i)} = 1)} = \frac{\sum_i w_i(b^{(i)} = 1)}{\sum_i w_i}$$

Approximate Inference: Weighted Sampling



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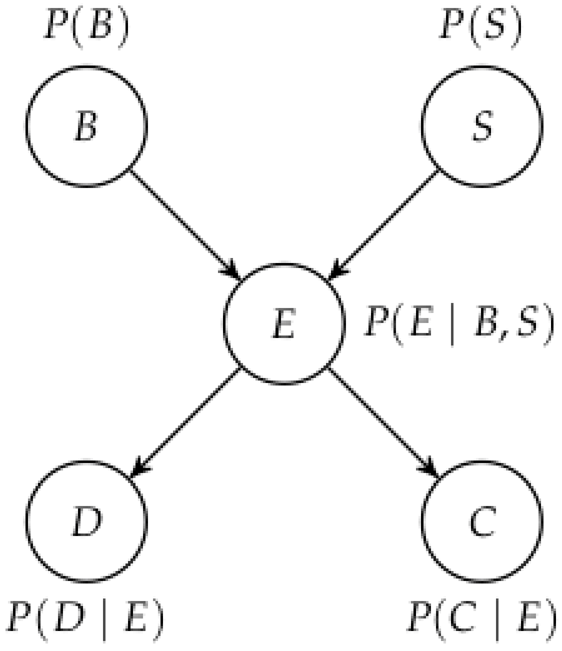
$$
\begin{aligned}
P(b^1 \mid d^1, c^1) &\approx \frac{\sum_i w_i(b^{(i)} = 1 \wedge d^{(i)} = 1 \wedge c^{(i)} = 1)}{\sum_i w_i(d^{(i)} = 1 \wedge c^{(i)} = 1)} \\
&= \frac{\sum_i w_i(b^{(i)} = 1)}{\sum_i w_i}
\end{aligned}
$$

B	S	E	D	C	Weight
1	0	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$
0	1	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$
0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
0	0	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$

Approximate Inference: Weighted Sampling



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$$= \frac{\sum_i w_i(b^{(i)} = 1)}{\sum_i w_i}$$

B	S	E	D	C	Weight
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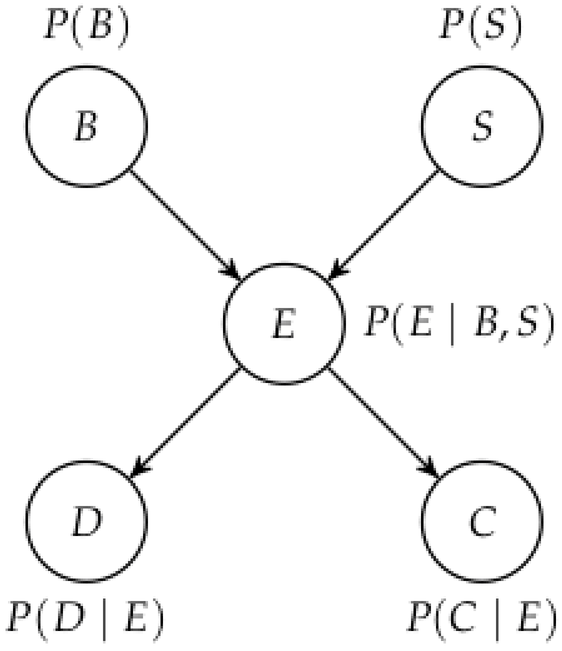
Analogous to

Approximate Inference: Weighted Sampling



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0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
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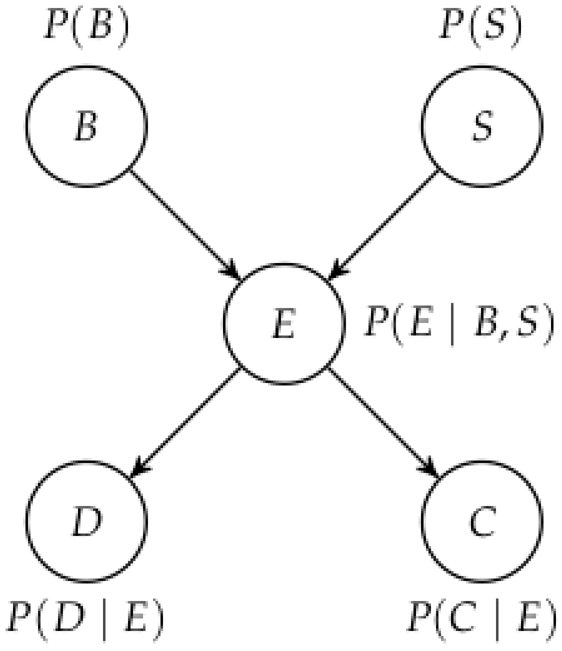
Analogous to **weighted particle filtering**

Approximate Inference: Gibbs Sampling



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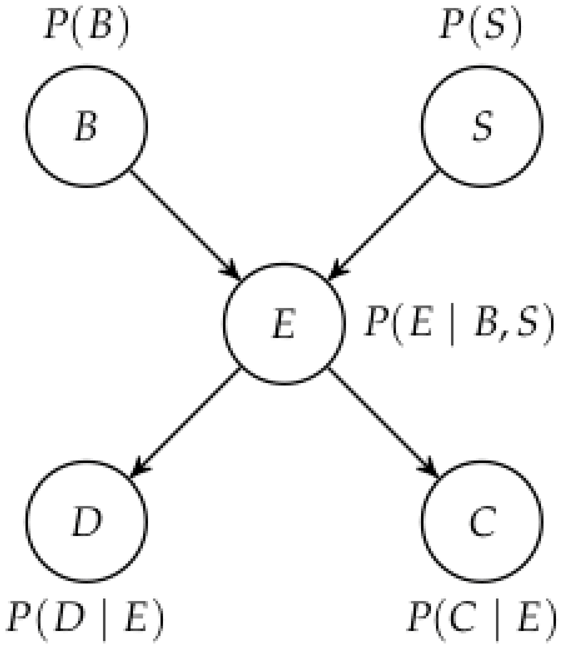

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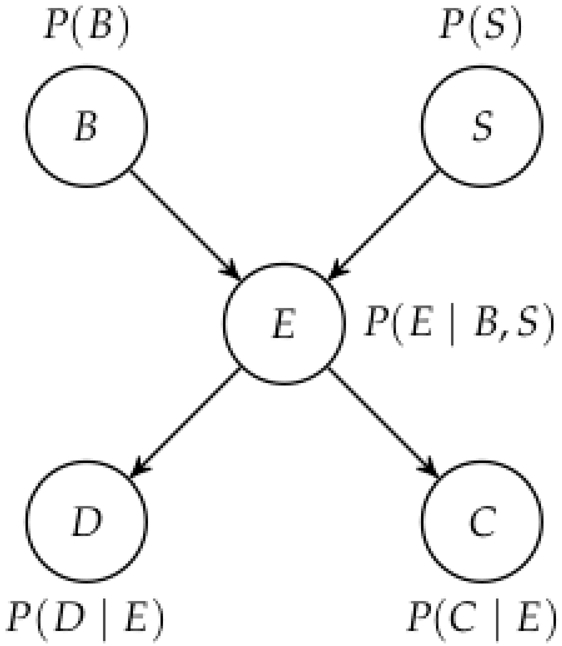
Markov Chain Monte Carlo (MCMC)

Approximate Inference: Gibbs Sampling



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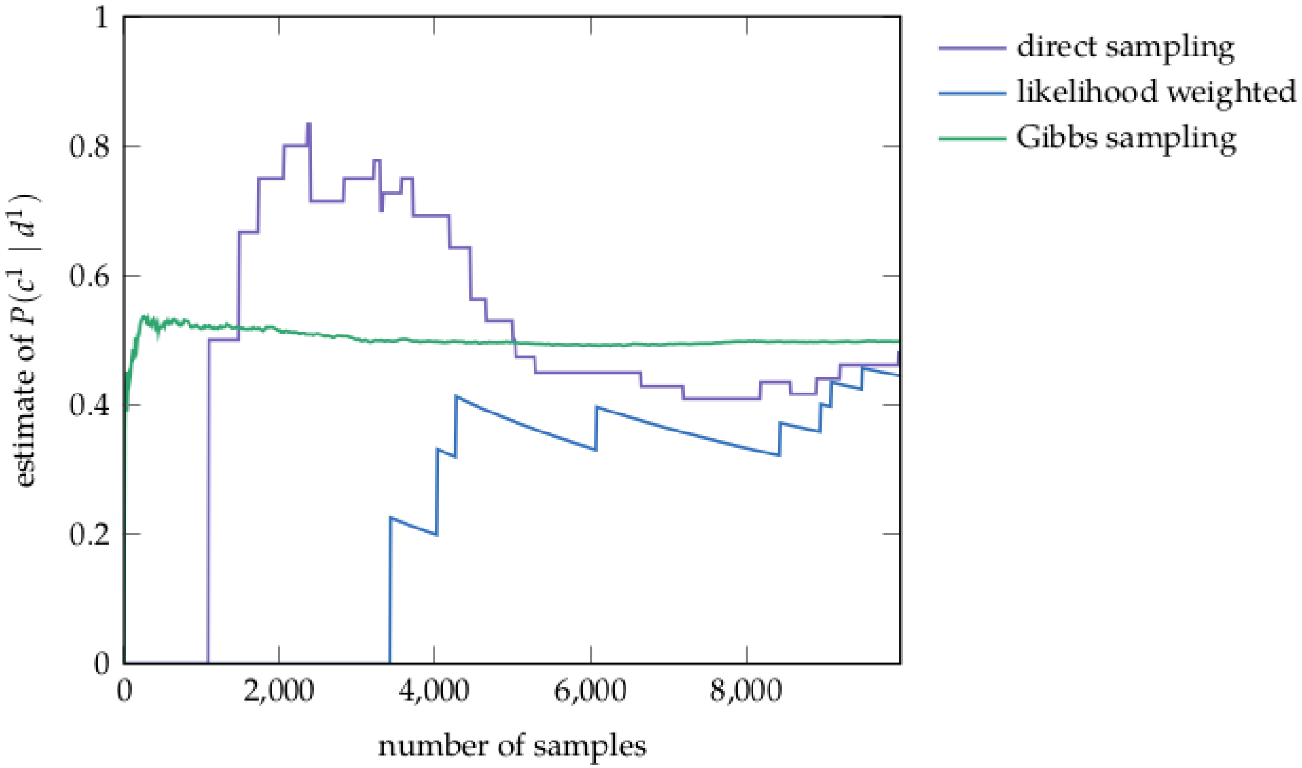

Recap