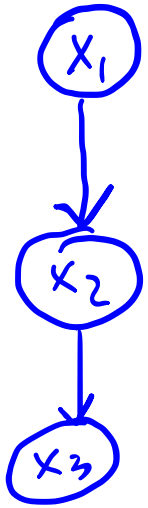
Bayesian Networks

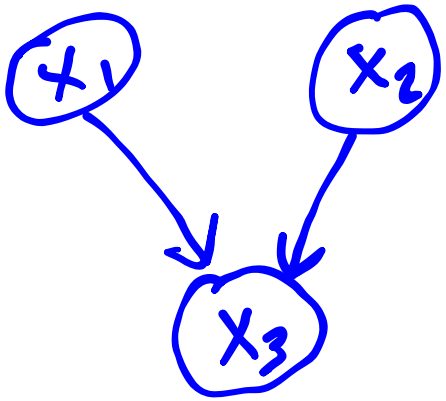
Today:

- Bayesian Networks
- How do we reason about independence in Bayesian Networks?
- How do we sample from Bayesian Networks?

Review of Definitions

Bayesian Network: Directed Acyclic Graph (DAG) that represents a **joint probability distribution**





- Node: Random Variable
- Edges encode:

$$P(x_{1:n}) = \prod_{i=1}^{n} P(x_i \mid \text{pa}(x_i))$$

Independence

$$P(X, Y) = P(X)\,P(Y)$$

$X \perp Y$

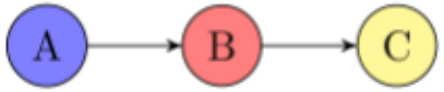
Conditional Independence

$$P(X, Y \mid Z) = P(X \mid Z)\,P(Y \mid Z)$$

$(X \perp Y \mid Z)$ $\Rightarrow$ $P(X|Z) = P(X|Y,Z)$

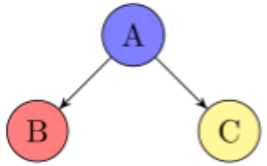
What does conditional independence mean?

$X \perp Y \mid Z \quad \Longrightarrow$ All of X's dependence on Y comes through Z



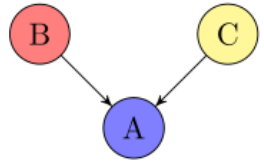
$A \perp C \mid B$? Yes

Mediator



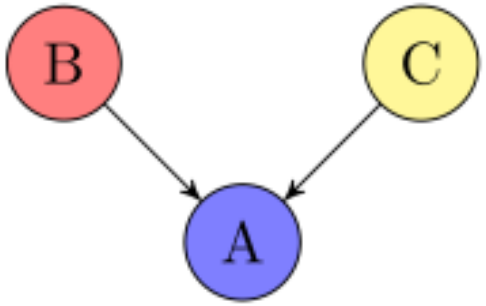
$B \perp C \mid A$? Yes

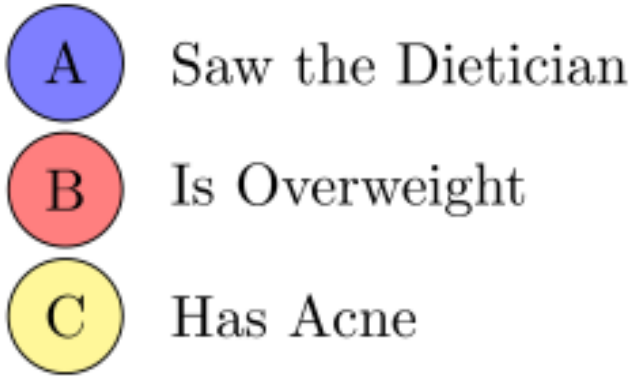
Confounder



$B \perp C \mid A$? No

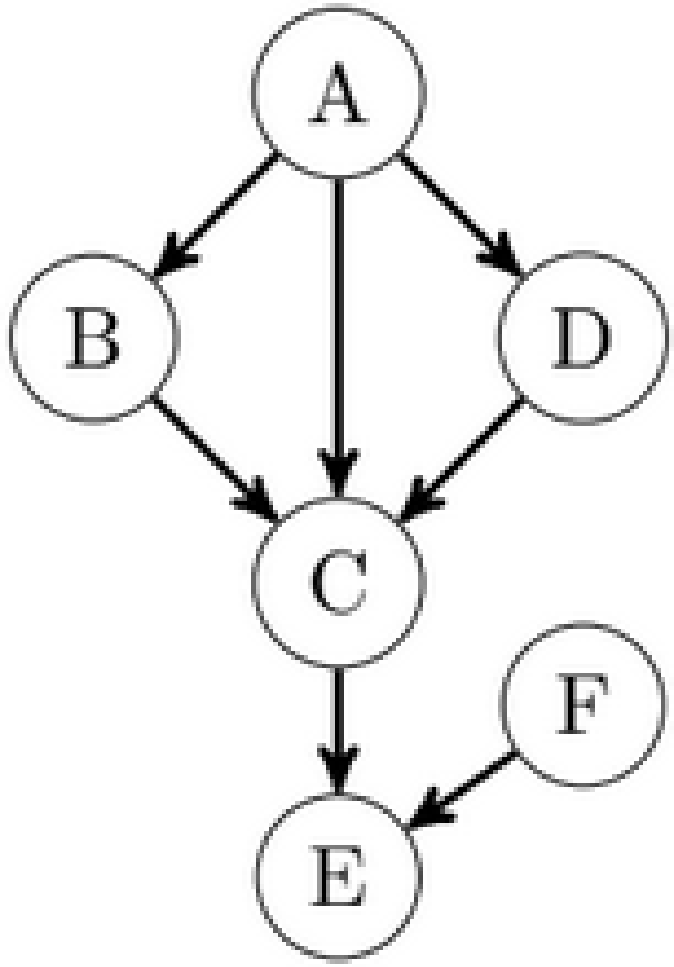
Collider





https://kunalmenda.com/2019/02/21/causation-and-correlation/

More Complex Example



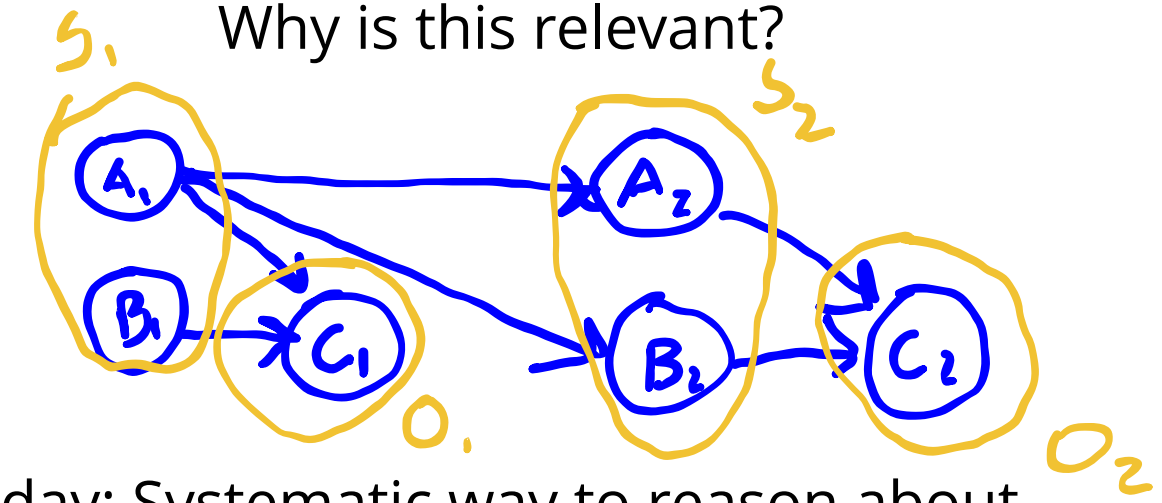
$(B \perp D \mid A)$?

Yes!

$(B \perp D \mid E)$?

No

Why is this relevant?



Today: Systematic way to reason about conditional independence

d-Separation

Let $\mathcal{C}$ be a set of random variables.

A *path* between A and B is *d-separated* by $\mathcal{C}$ if any of the following are true

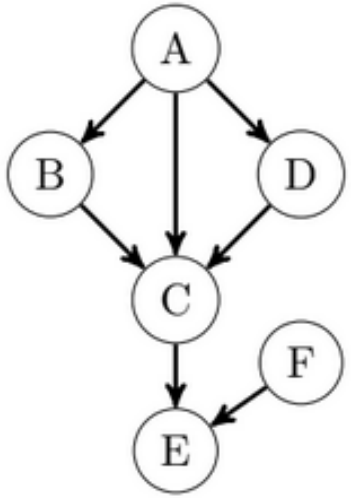
1. The path contains a *chain* $X \to Y \to Z$ such that $Y \in \mathcal{C}$
2. The path contains a *fork* $X \leftarrow Y \to Z$ such that $Y \in \mathcal{C}$
3. The path contains an *inverted fork* (v-structure) $X \to Y \leftarrow Z$ such that $Y \notin \mathcal{C}$

We say that A and B are *d-separated* by $\mathcal{C}$ if all paths between A and B are d-separated by $\mathcal{C}$.

If A and B are d-separated by $\mathcal{C}$ then $A \perp B \mid \mathcal{C}$

Proving Conditional Independence

1. Enumerate all paths between nodes in question
2. Check all paths for d-separation
3. If all paths d-separated, then CE



Example: $(B \perp D \mid C, E)$?

1. The path contains a *chain* $X \rightarrow Y \rightarrow Z$ such that $Y \in \mathcal{C}$
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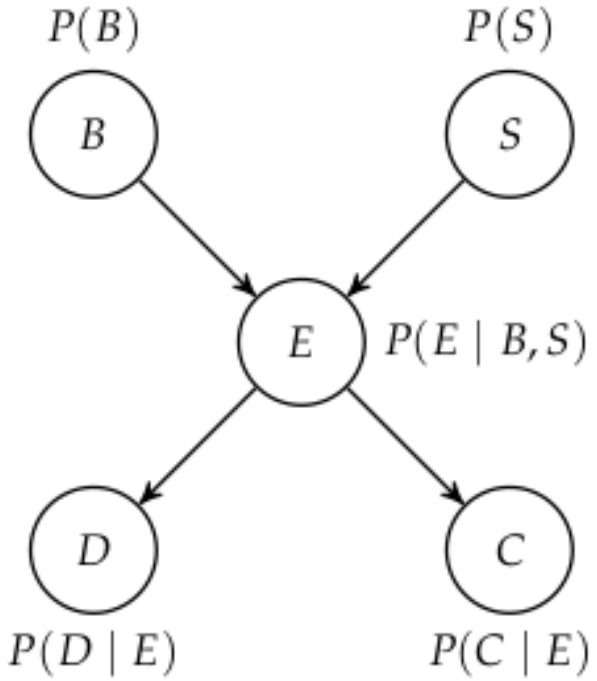

Exercise

$D \perp C \mid B$?

$D \perp C \mid E$?

B battery failure
S solar panel failure
E electrical system failure
D trajectory deviation
C communication loss

1. The path contains a *chain* $X \to Y \to Z$ such that $Y \in \mathcal{C}$
2. The path contains a *fork* $X \leftarrow Y \to Z$ such that $Y \in \mathcal{C}$
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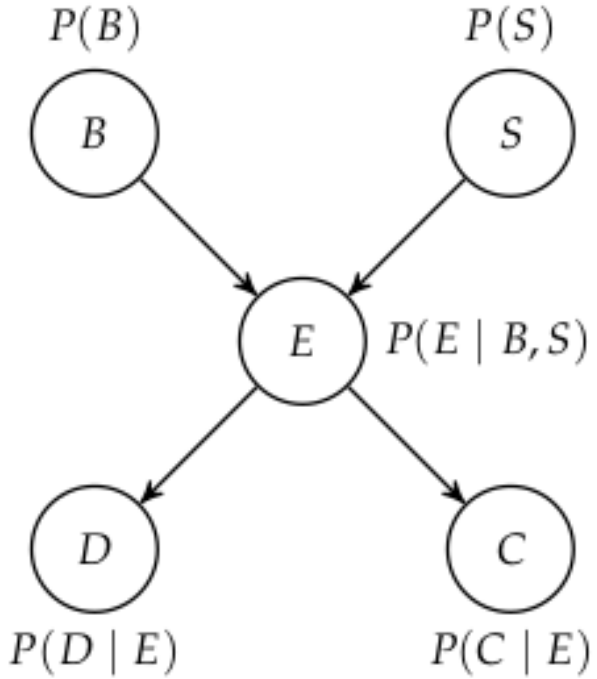
Sampling from a Bayesian Network

Given a Bayesian network, how do we sample from the joint distribution it defines?

1. Topoligical Sort (If there is an edge $A \rightarrow B$, then A comes before B)
2. Sample from conditional distributions in order of the topological sort



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Analogous to **Simulating** a (PO)MDP

Recap