

Exploration and Exploitation (Bandits)

Last Time

- What is Reinforcement Learning?
- What are the main challenges in Reinforcement Learning?

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- What are the main challenges in Reinforcement Learning?
- How do we categorize RL approaches?

Last Time

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First RL Algorithm:

Last Time

First RL Algorithm:

Tabular Maximum Likelihood Model-Based Reinforcement Learning

Last Time

First RL Algorithm:

Tabular Maximum Likelihood Model-Based Reinforcement Learning

loop
 choose action a
 gain experience
 estimate T, R
 solve MDP with T, R

Guiding Questions

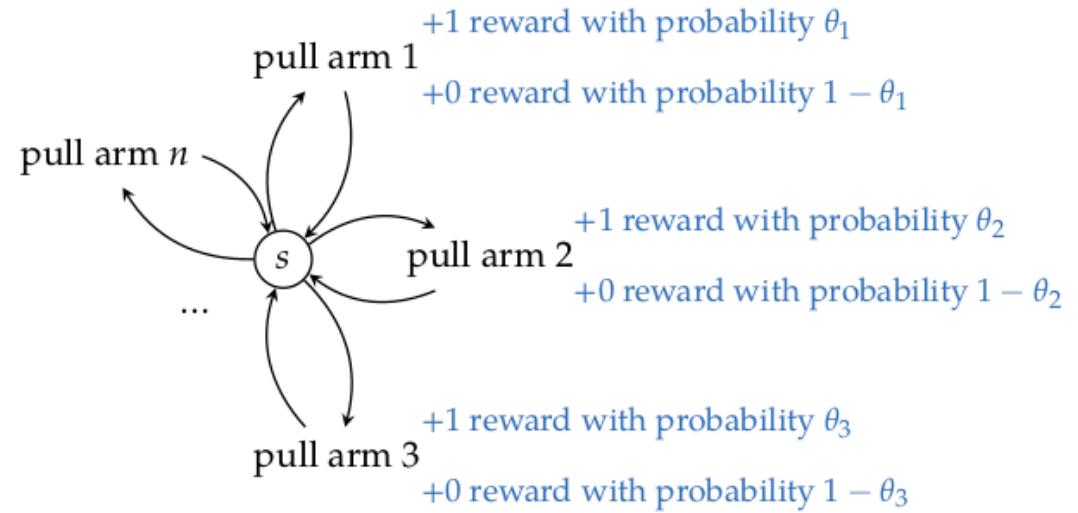
- What are the best ways to trade off Exploration and Exploitation?

Bandits

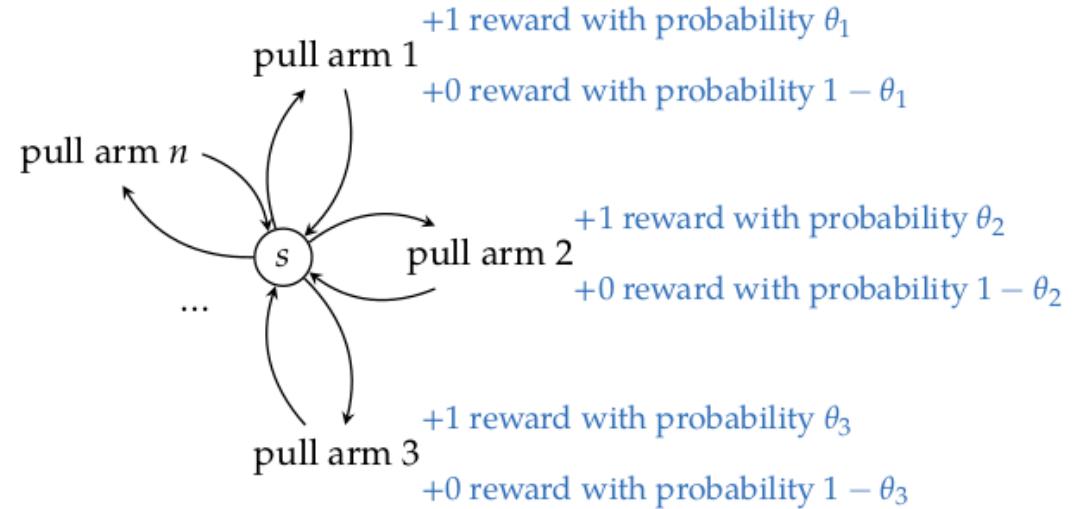
Bandits



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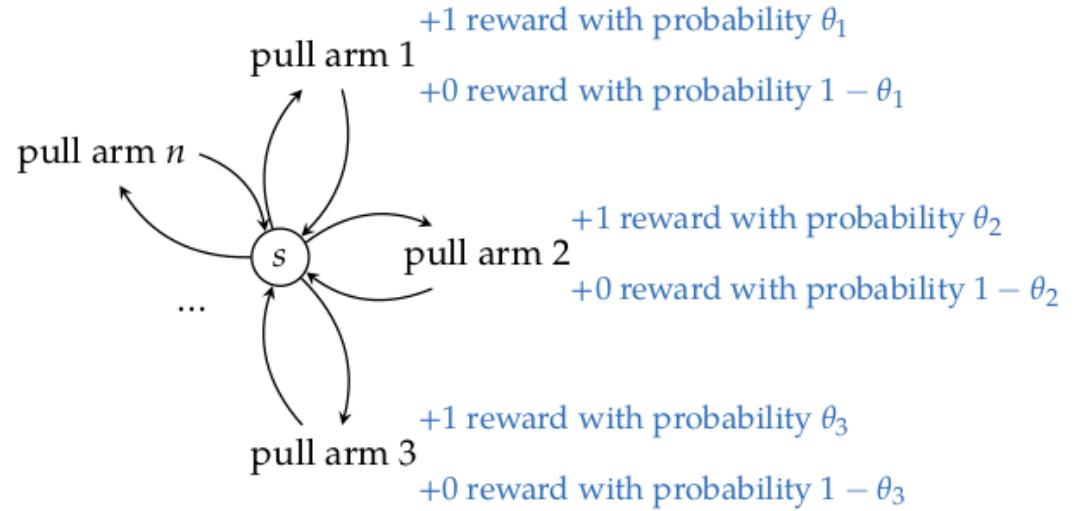


Bandits



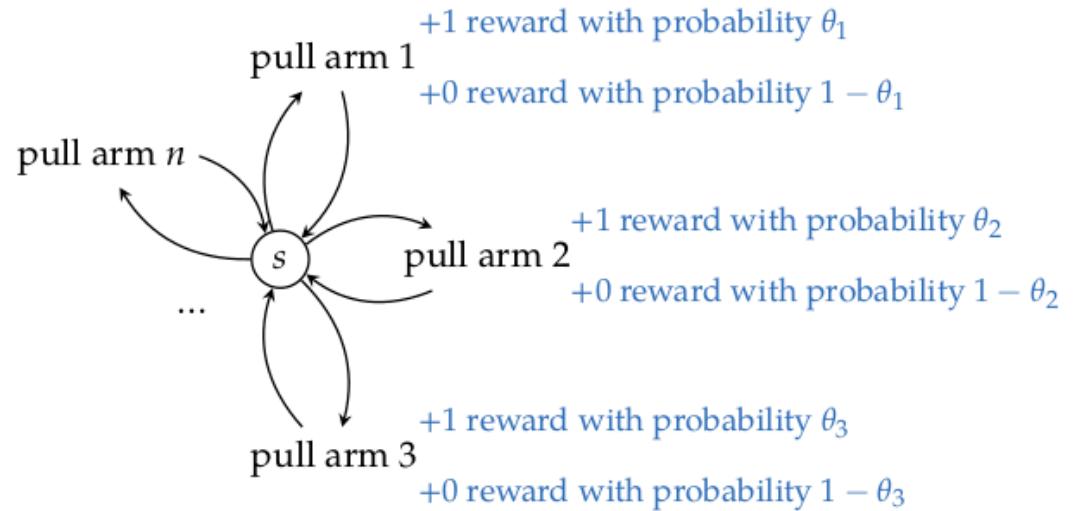
- Bernoulli Bandit with parameters θ

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- $\theta^* \equiv \max \theta$

Bandits



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“According to Peter Whittle, “efforts to solve [bandit problems] so sapped the energies and minds of Allied analysts that the suggestion was made that the problem be dropped over Germany as the ultimate instrument of intellectual sabotage.”

Greedy Strategy

$$\rho_a = \frac{\text{number of wins}+1}{\text{number of tries}+1}$$

Choose $\operatorname{argmax}_a \rho_a$

Undirected Strategies

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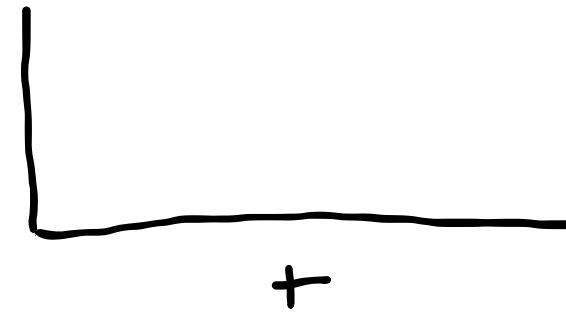
- Explore then Commit

Choose a randomly for k steps

Then choose $\operatorname{argmax}_a \rho_a$

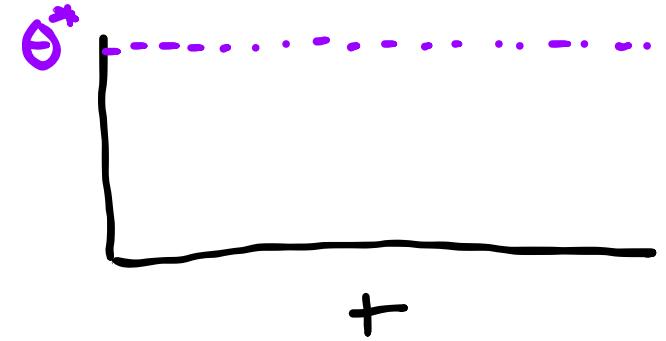
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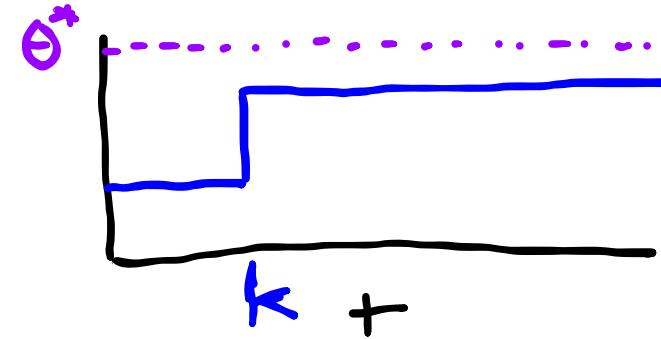
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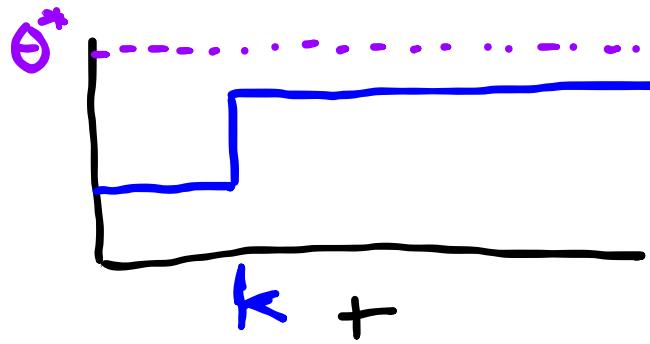
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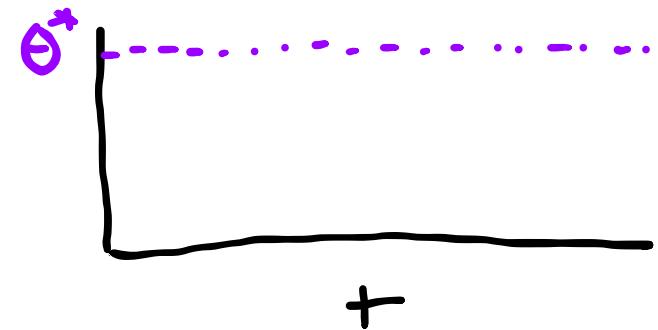
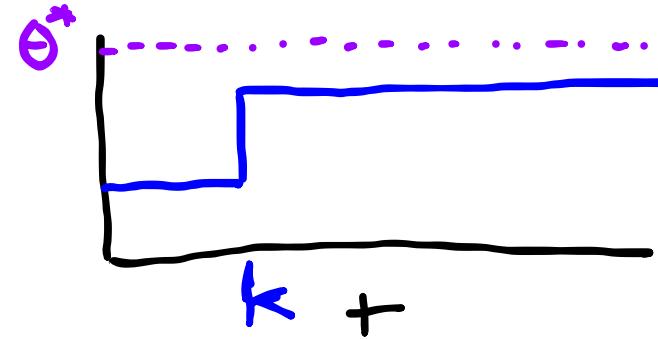
Undirected Strategies

- Explore then Commit
Choose a randomly for k steps
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- ϵ - greedy
With probability ϵ , choose randomly
Otherwise choose $\operatorname{argmax}_a \rho_a$



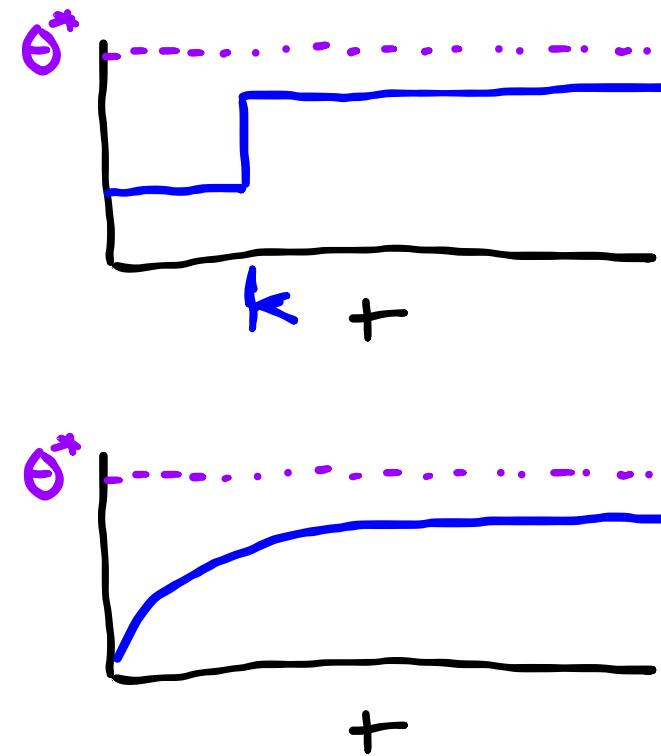
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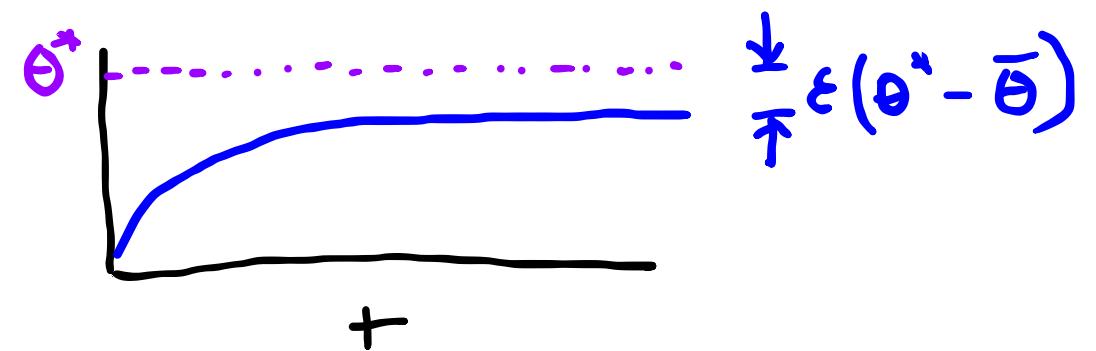
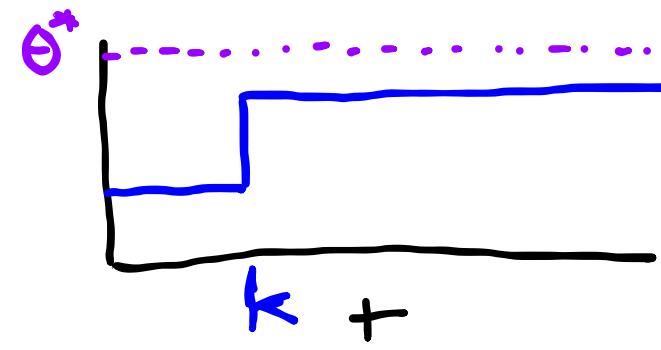
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Directed Strategies



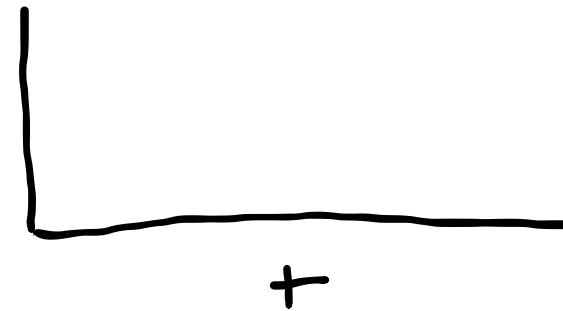
Directed Strategies

- Softmax
Choose a with probability
proportional to $e^{\lambda \rho_a}$



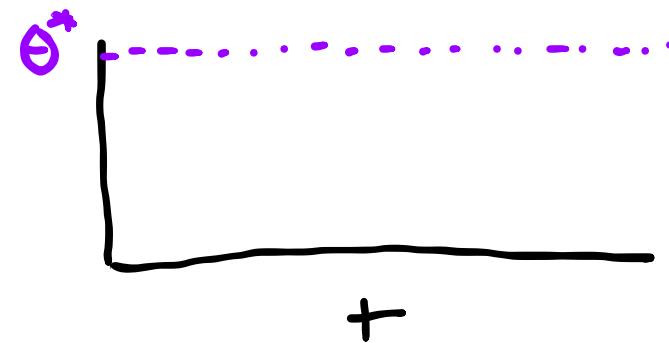
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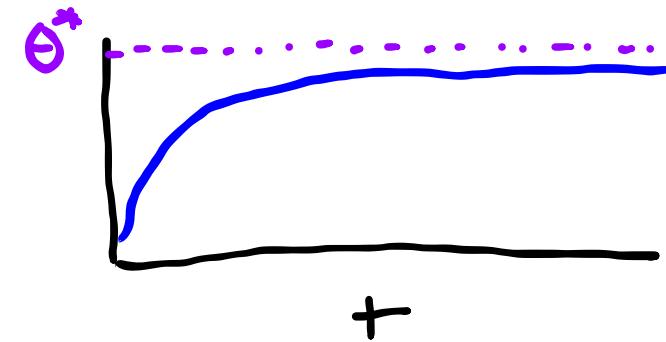
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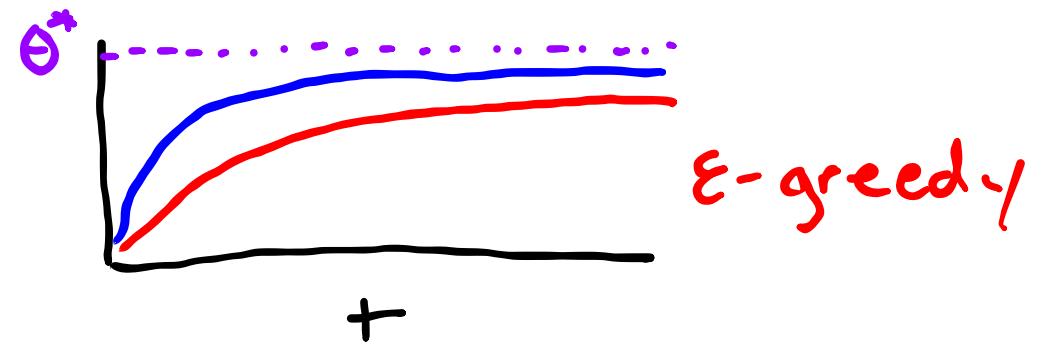
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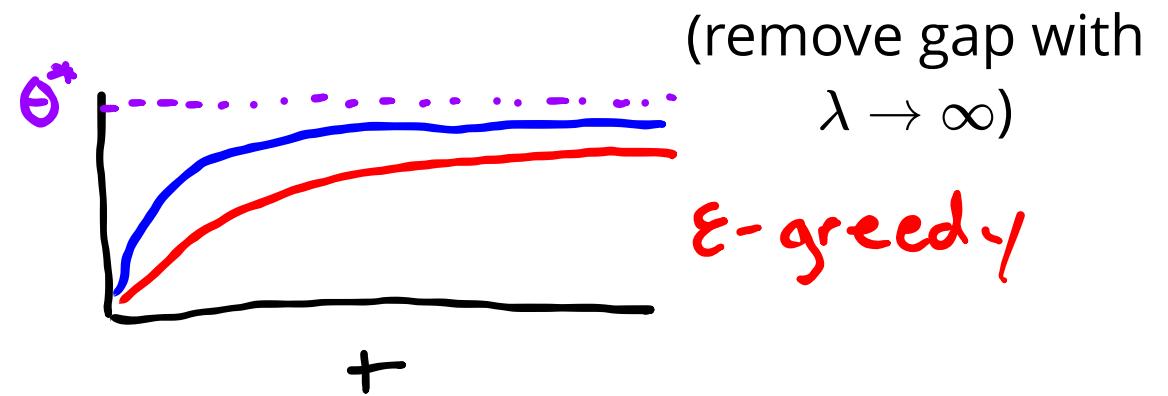
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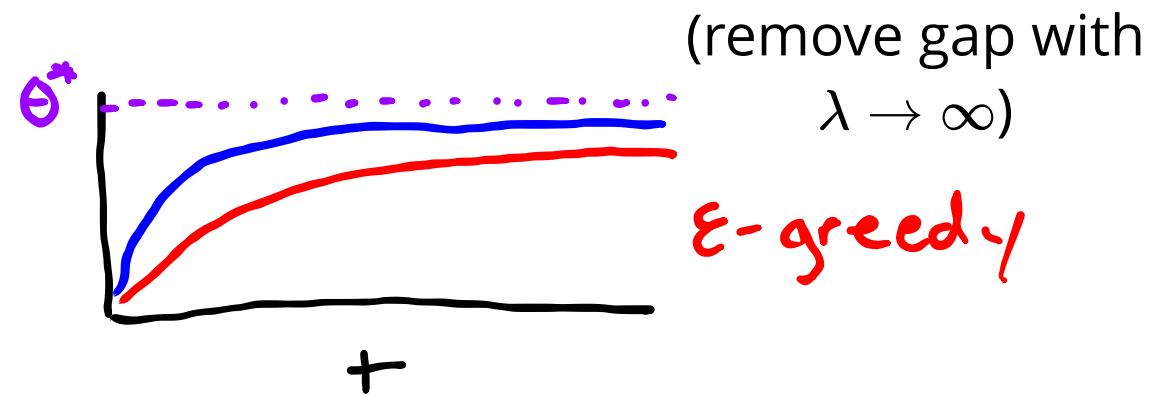
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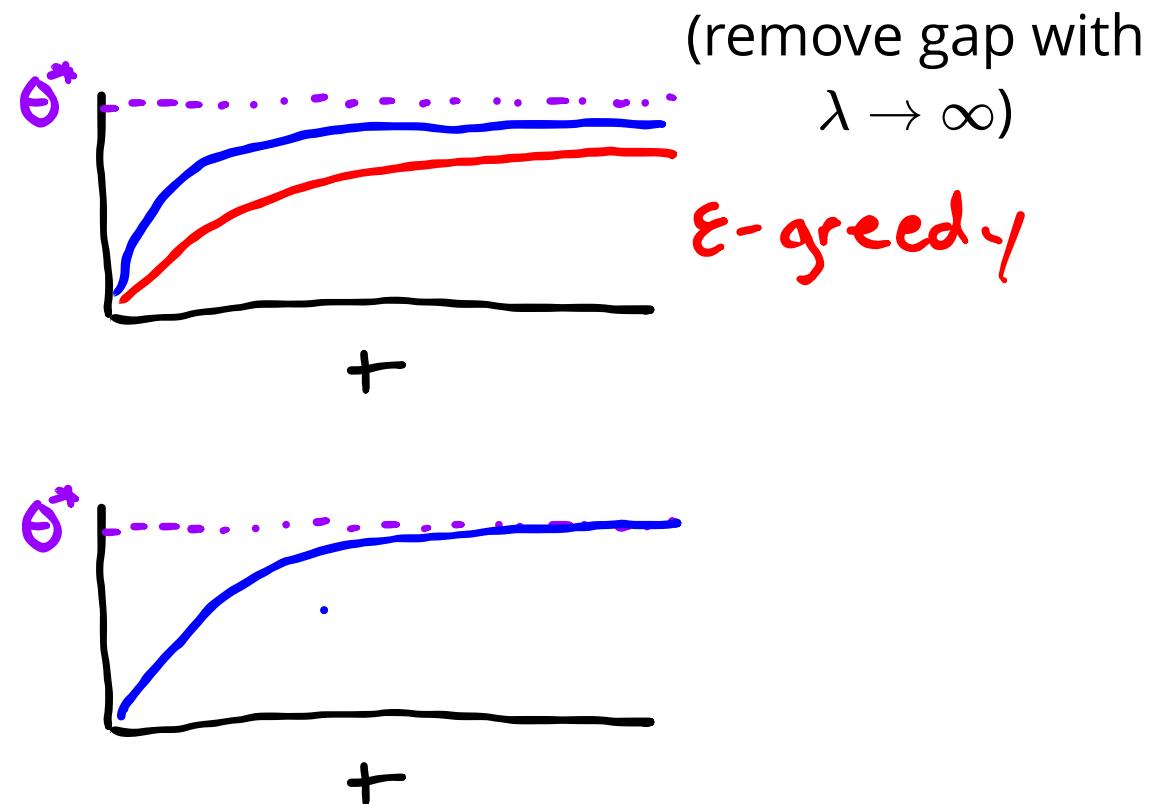
Directed Strategies

- Softmax
Choose a with probability proportional to $e^{\lambda \rho_a}$
- Upper Confidence Bound (UCB)
Choose $\underset{a}{\operatorname{argmax}} \rho_a + c \sqrt{\frac{\log N}{N(a)}}$



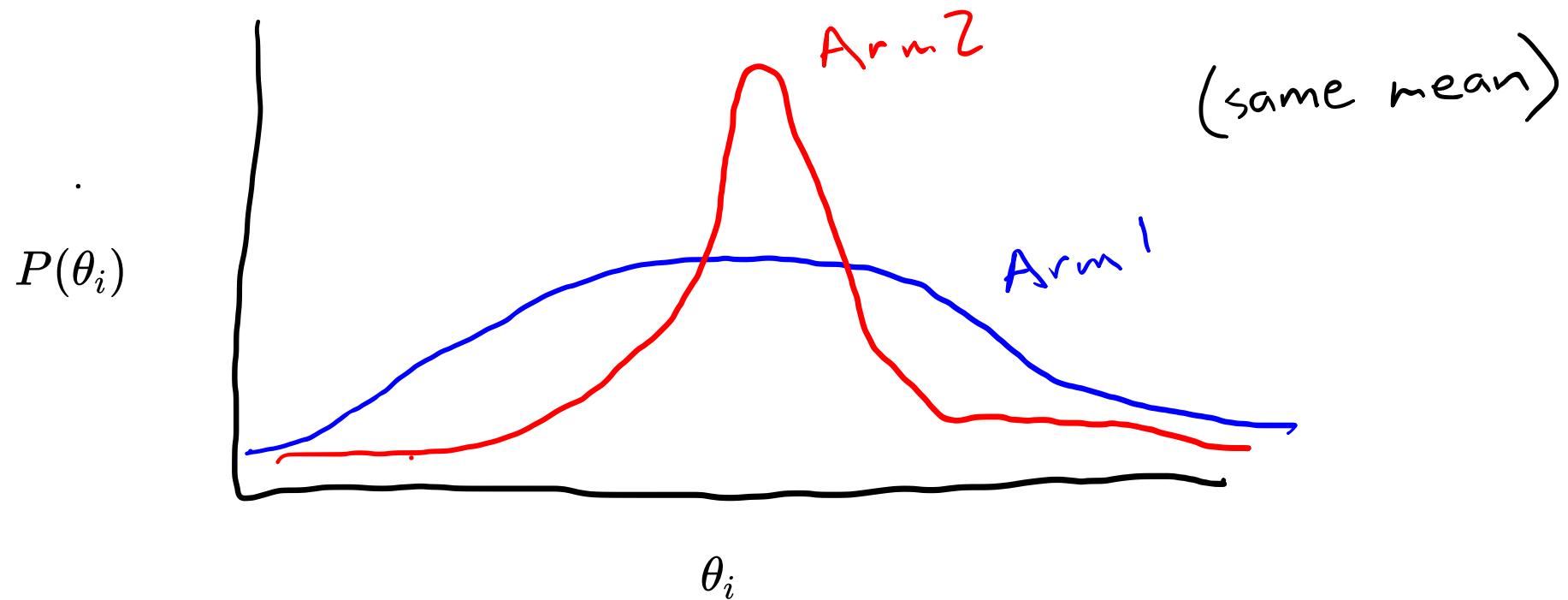
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Break

Discuss with your neighbor: Suppose you have the following *belief* about the parameters θ . Which arm should you choose to pull next?



Bayesian Estimation

Bayesian Estimation

Bernoulli Distribution

Bayesian Estimation

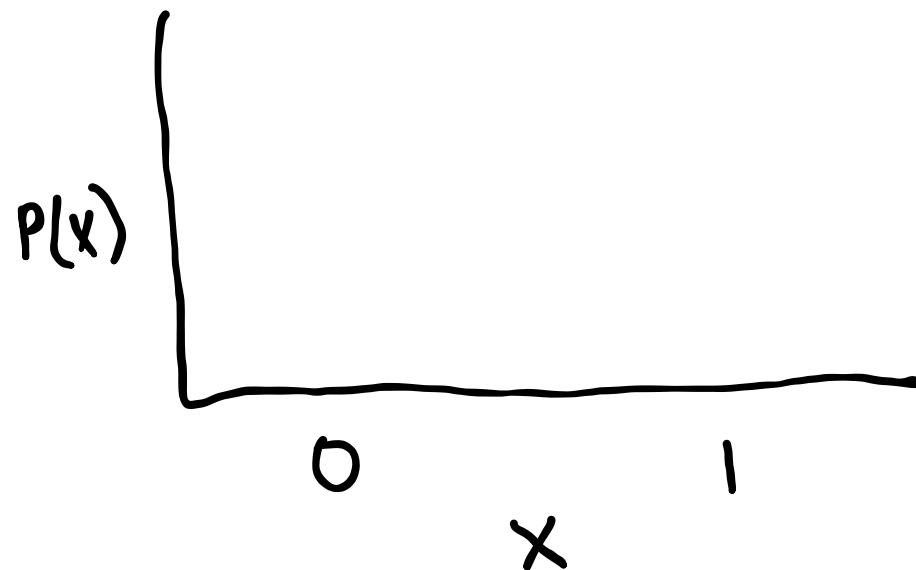
Bernoulli Distribution

$$\text{Bernoulli}(\theta)$$

Bayesian Estimation

Bernoulli Distribution

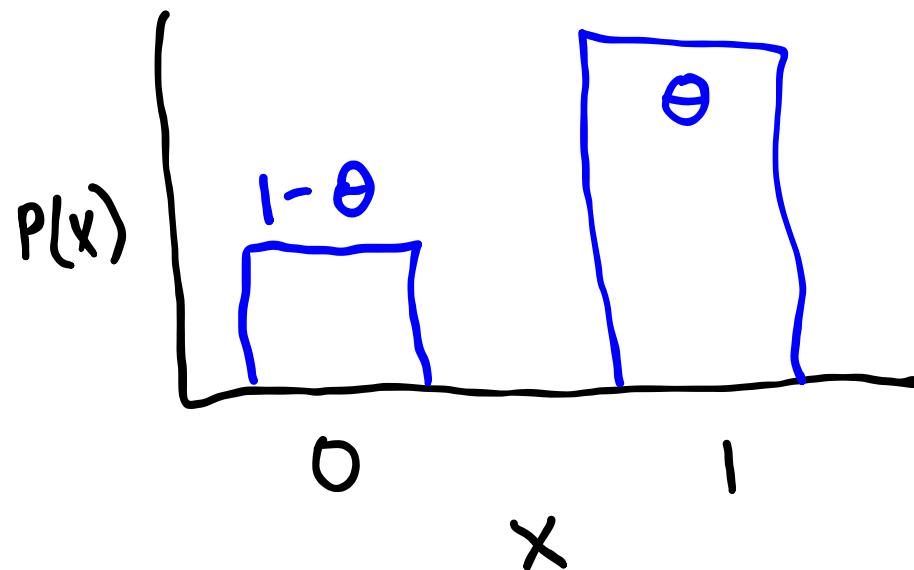
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Bayesian Estimation

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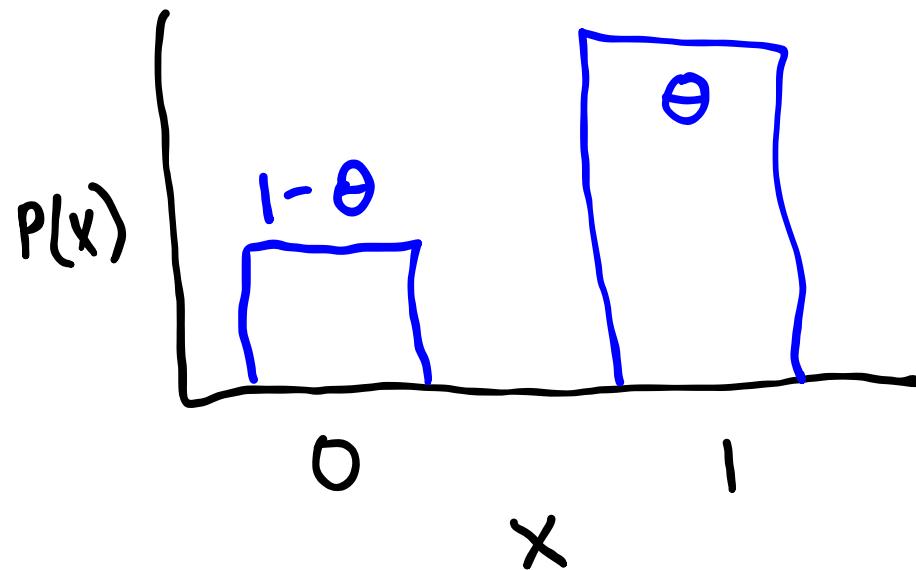


Bayesian Estimation

Bernoulli Distribution

$\text{Bernoulli}(\theta)$

Discussion: Given that I have received w wins and l losses, what should my belief (probability distribution) about θ look like?

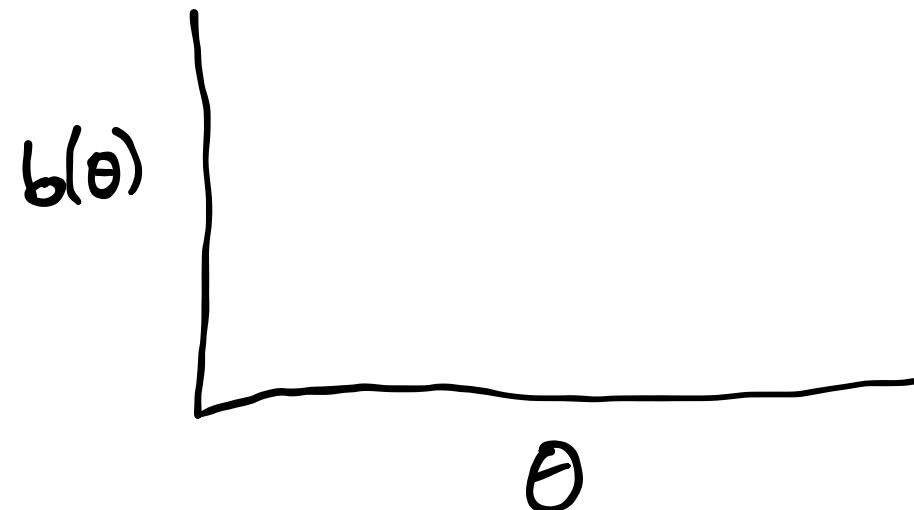
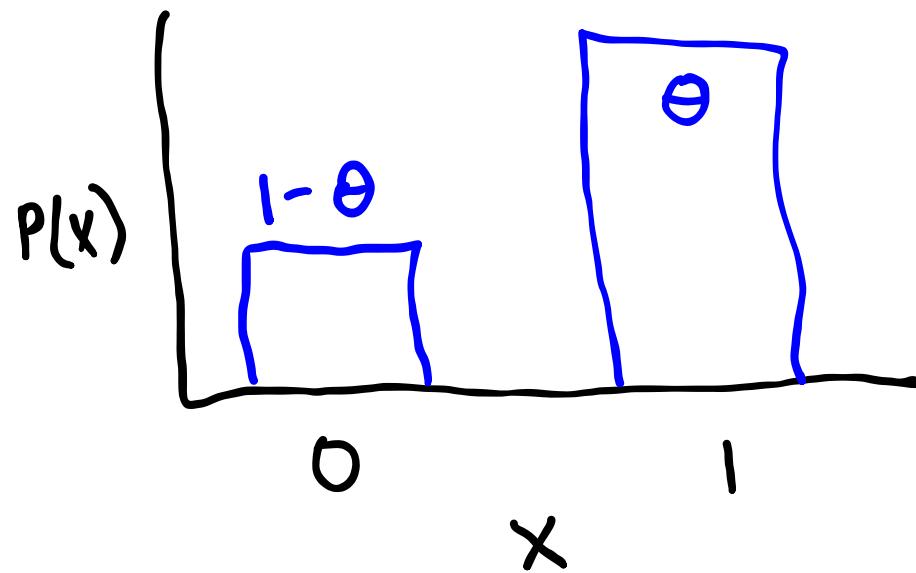


Bayesian Estimation

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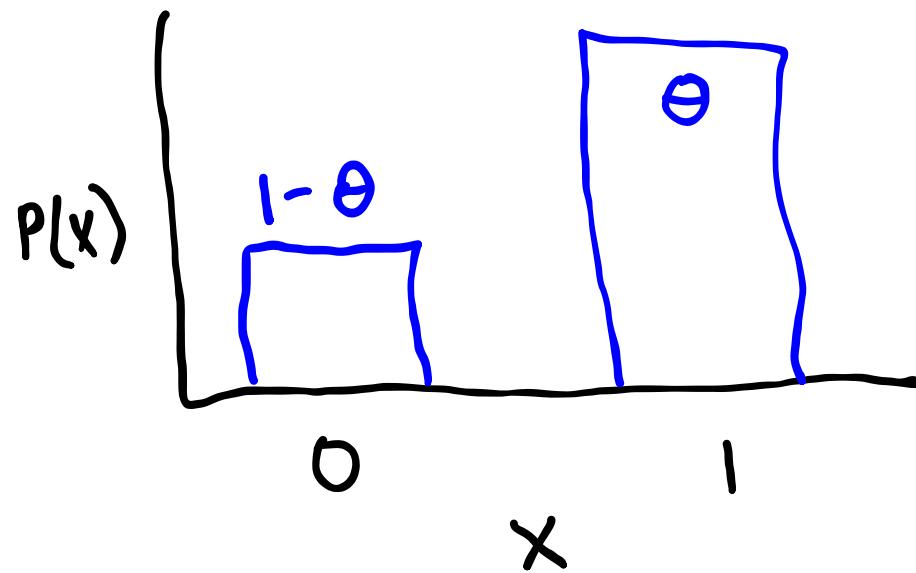


Bayesian Estimation

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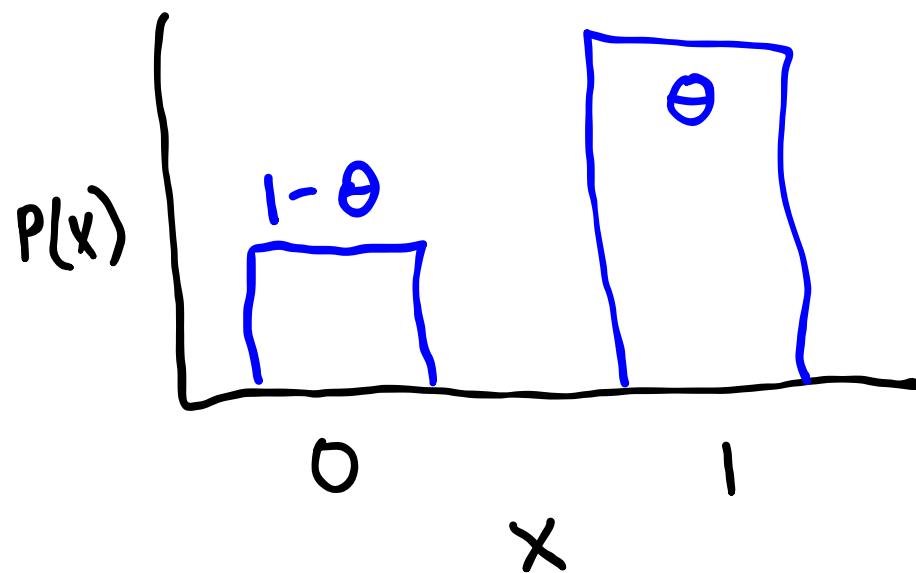
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Bayesian Estimation

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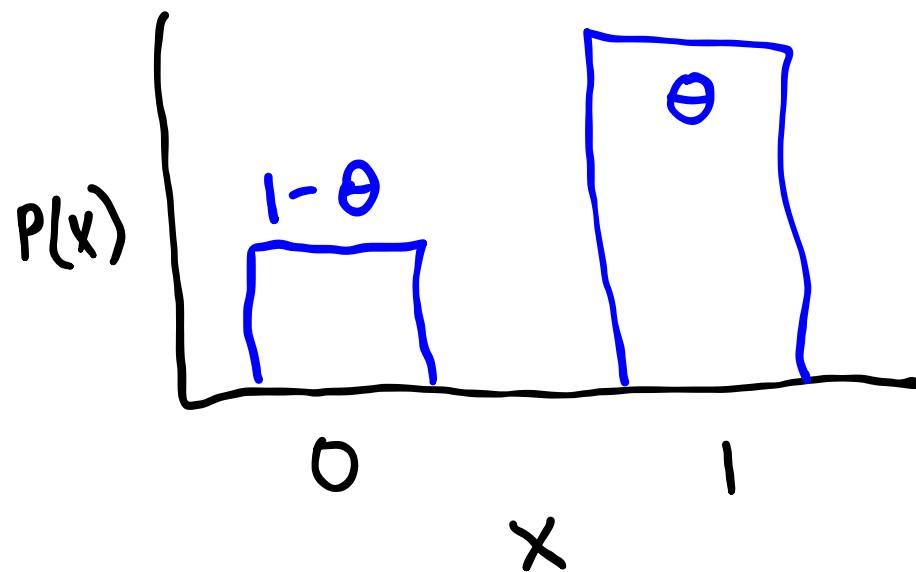
Bayesian Estimation

Bernoulli Distribution

$\text{Bernoulli}(\theta)$

Beta Distribution

(distribution over Bernoulli distributions)



Bayesian Estimation

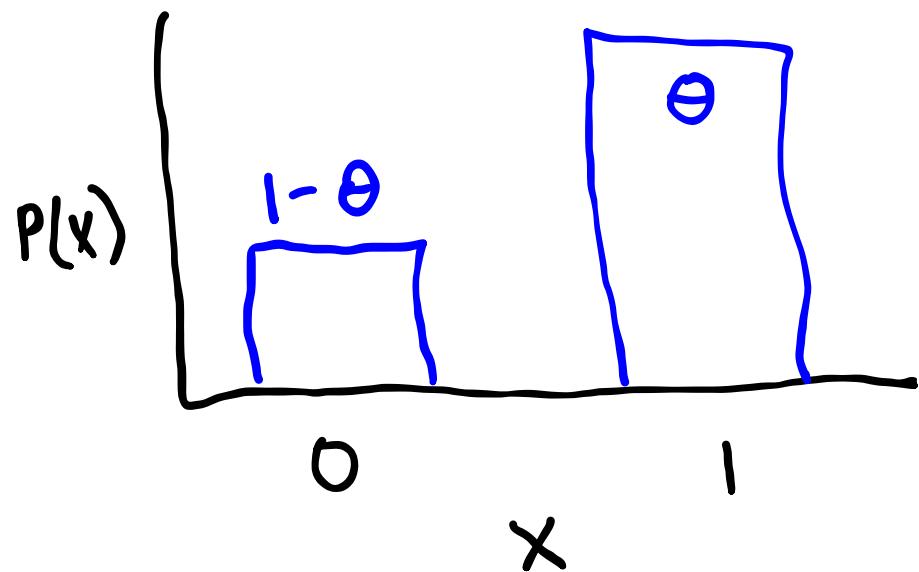
Bernoulli Distribution

$$\text{Bernoulli}(\theta)$$

Beta Distribution

(distribution over Bernoulli distributions)

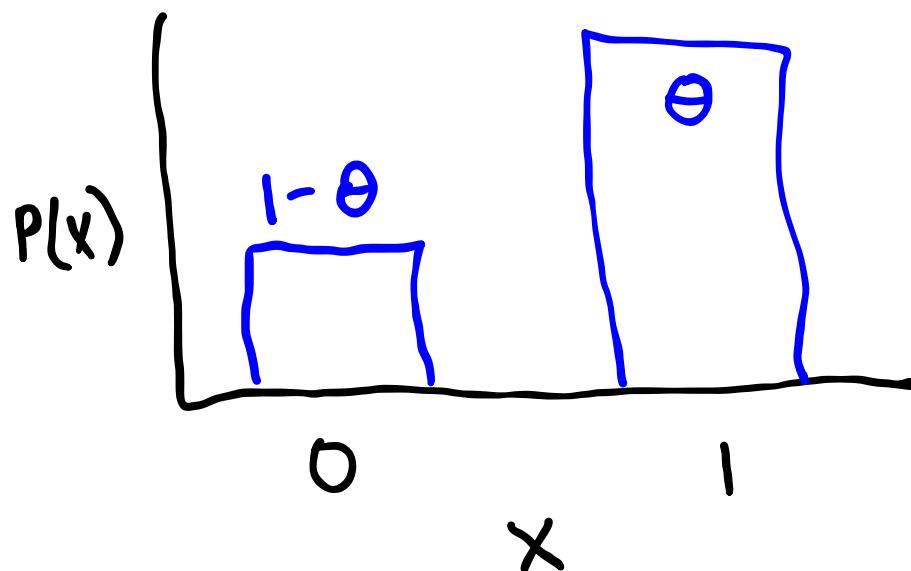
$$\text{Beta}(\alpha, \beta)$$



Bayesian Estimation

Bernoulli Distribution

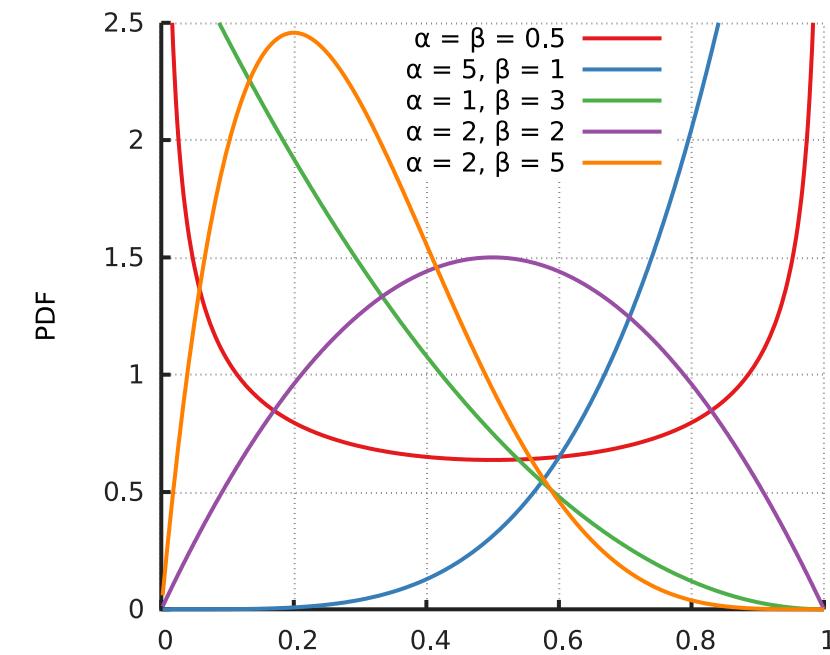
$\text{Bernoulli}(\theta)$



Beta Distribution

(distribution over Bernoulli distributions)

$\text{Beta}(\alpha, \beta)$



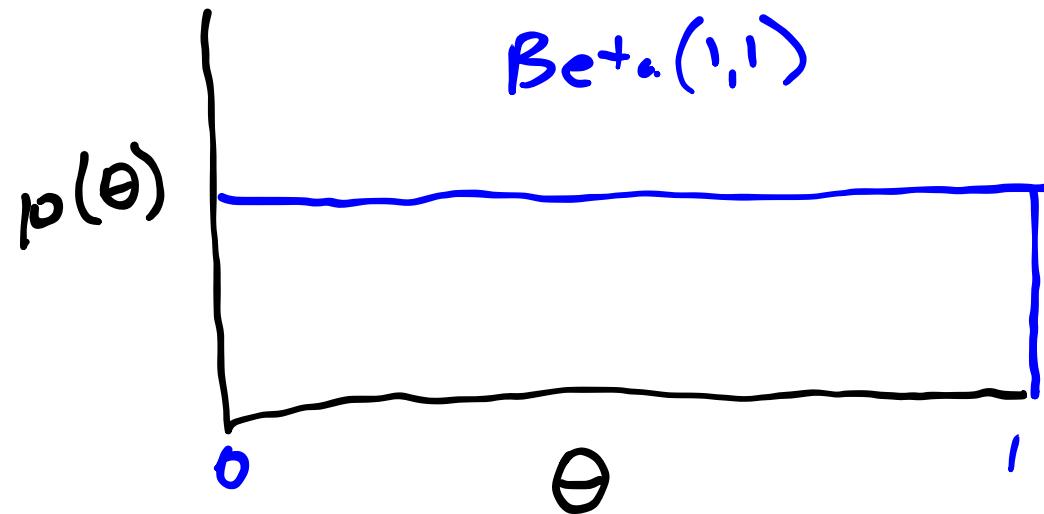
Bayesian Estimation

Bayesian Estimation

Given a $\text{Beta}(1, 1)$ prior distribution

Bayesian Estimation

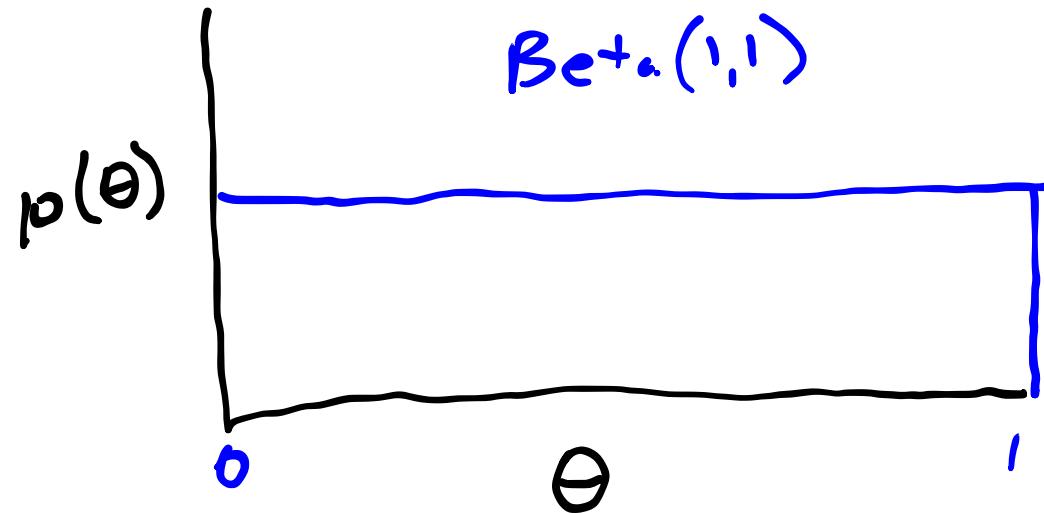
Given a Beta(1, 1) prior distribution



Bayesian Estimation

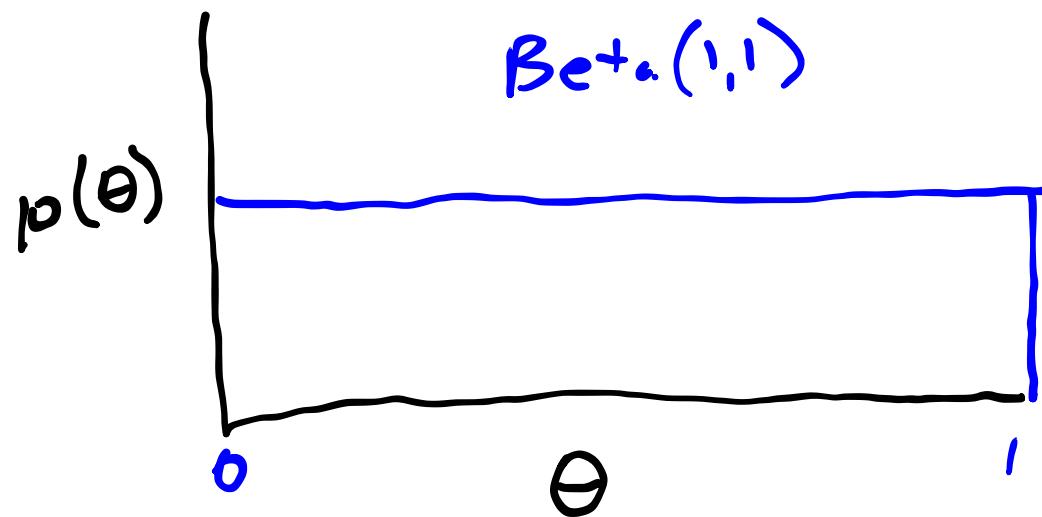
Given a $\text{Beta}(1, 1)$ prior distribution

The posterior distribution of θ is
 $\text{Beta}(w + 1, l + 1)$

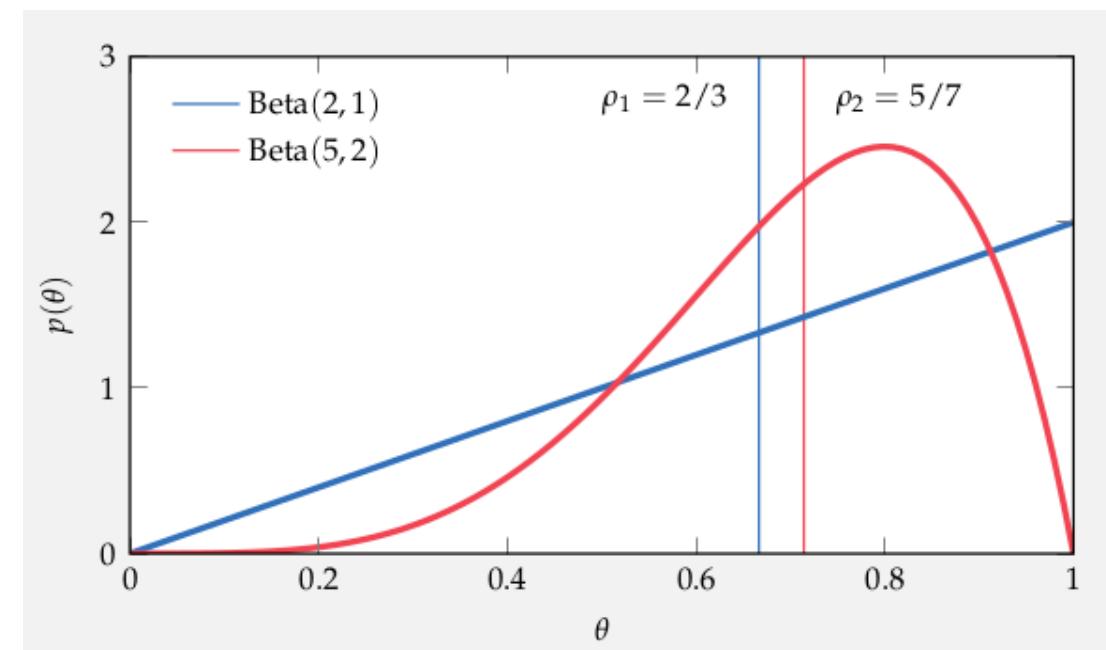


Bayesian Estimation

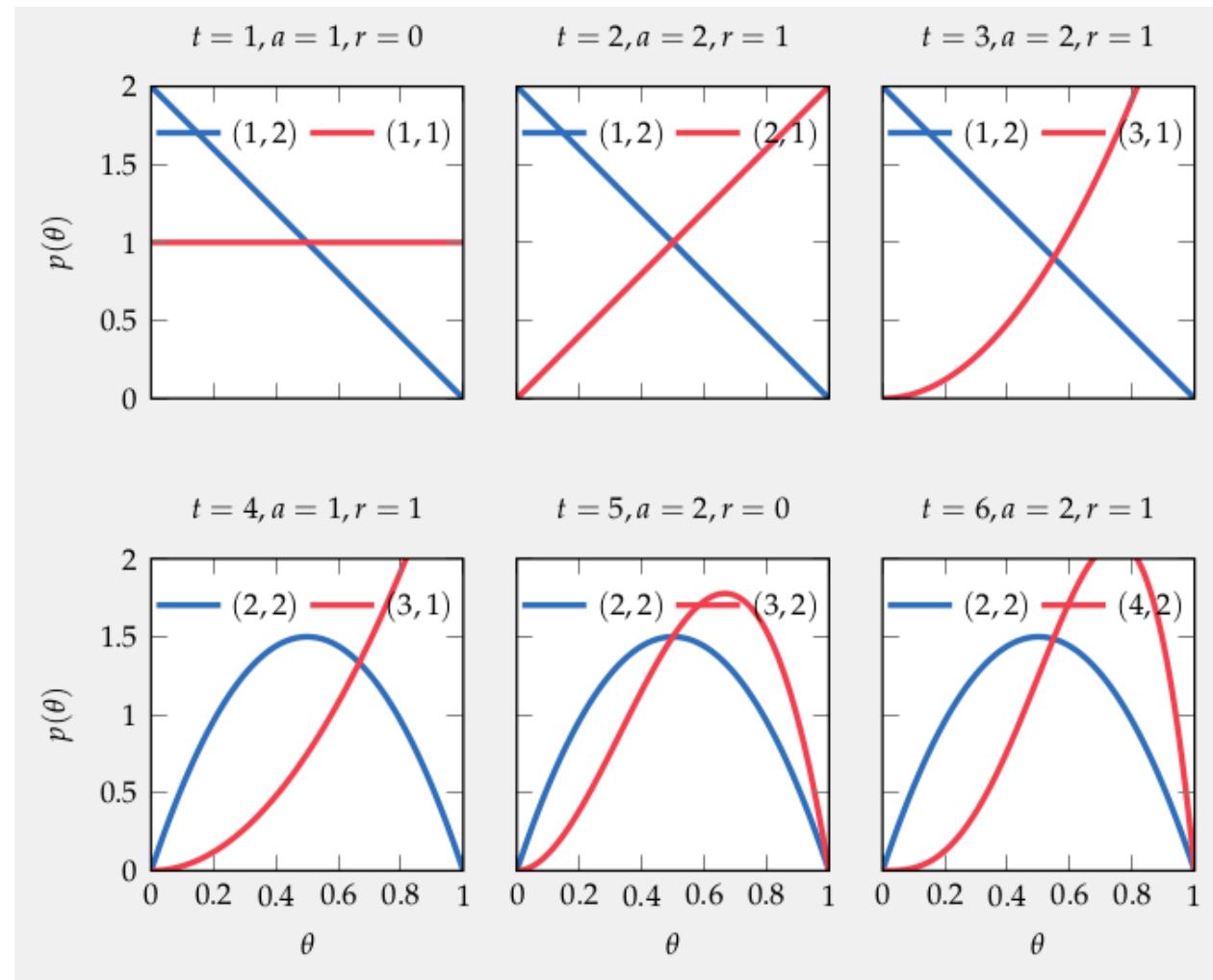
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Bayesian Estimation



t = time

a = arm pulled

r = reward

Bayesian Bandit Algorithms

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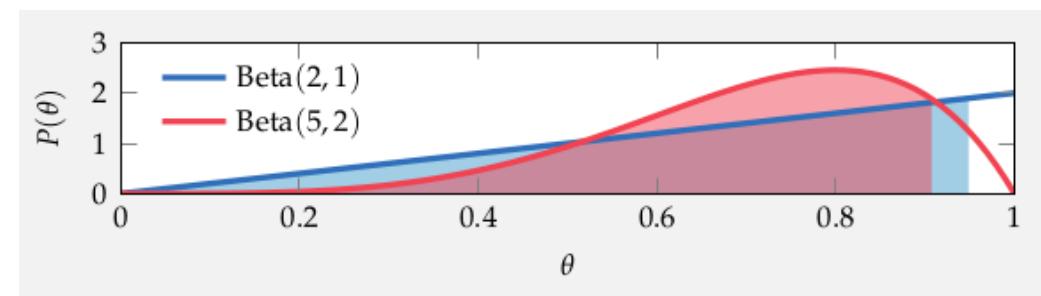
- Quantile Selection

Choose a for which the α quantile of
 $b(\theta)$ is highest

Bayesian Bandit Algorithms

- Quantile Selection
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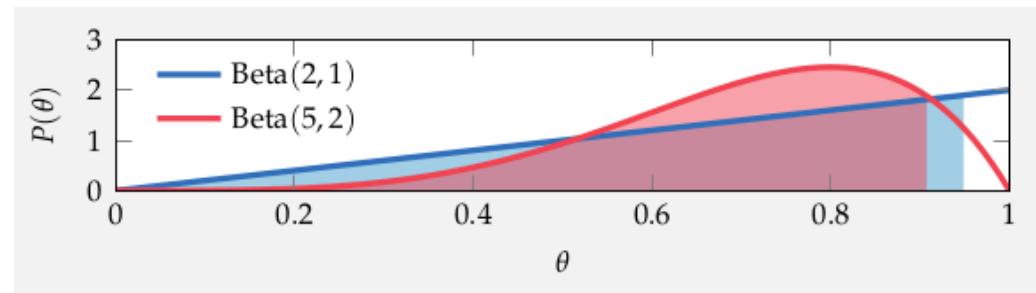
$$\alpha = 0.9$$



Bayesian Bandit Algorithms

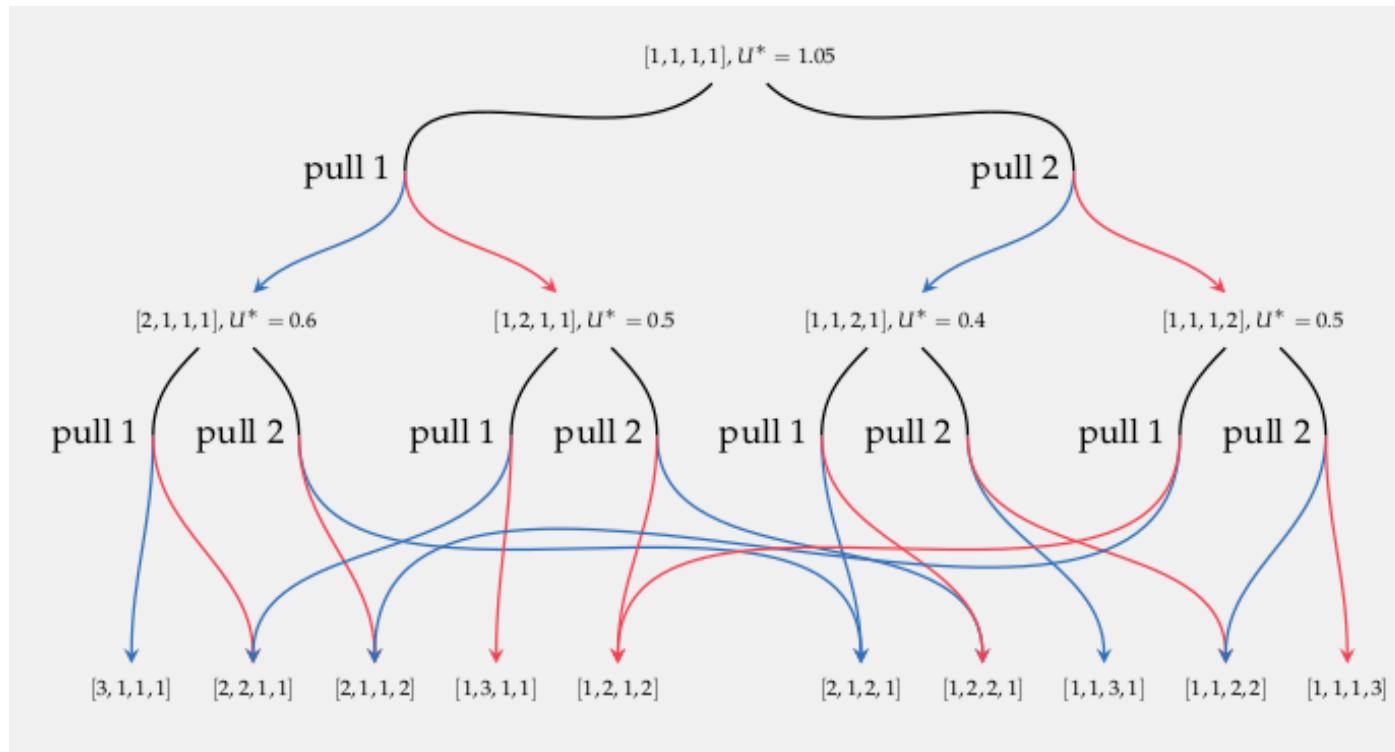
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- Thompson Sampling
Sample $\hat{\theta}$
Choose $\operatorname{argmax}_a \hat{\theta}_a$

Optimal Algorithm - Dynamic Programming



Review

Guiding Questions

- What are the best ways to trade off Exploration and Exploitation