Recap

DMU

- Probabilistic Models
- MDPs
- Reinforcement Learning
- POMDPs
- Games

Probabilistic Models

$P(A)$
$P(A,B)$
$P(A|B)$

3 Rules

1. $0 \leq P(X \mid Y) \leq 1$
   $\sum_{x \in X} P(x \mid Y) = 1$

2. $P(X) = \sum_{y \in Y} P(X, y)$

3. $P(X \mid Y) \ = \frac{P(X,Y)}{P(Y)}$

Bayes Rule

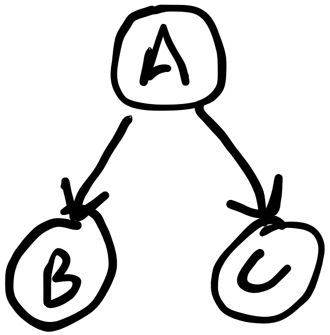
$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Independence

$$A \perp B \iff P(A, B) = P(A)P(B)$$

$$A \perp B \mid C \iff P(A, B \mid C) = P(A \mid C)P(B \mid C)$$

Bayesian Networks



Chain Rule

$$P(X_{1:n}) = \prod_i P(X_i \mid Pa(X_i))$$

Sampling

Topological sort, then sample from each node

Conditional Independence

$X \perp Y \mid \mathcal{C}$ if all paths between X and Y are d-separated by $\mathcal{C}$

Inference

- Input: BN, evidence values
- Output: Distribution of query variables

Exact: NP-Hard

Approximate via sampling: Direct, Likelihood Weighted, Gibbs

Learning

- Input: Data
- Output: BN structure and parameters

Markov Decision Processes

$$(S, A, R, T, \gamma)$$

Examples: $S = \{1, 2, 3\}$ or $S = \mathbb{R}^2$

$$s = (x, \dot{x}) \in S = \mathbb{R}^2$$

$$\underset{\pi}{\text{maximize}} \; E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t)\right]$$

$$Q^\pi(s, a) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \,\middle|\, s_0 = s, a_0 = a, a_t = \pi(s_t)\right]$$

$$V^\pi(s) = Q^\pi(s, \pi(s))$$

$V^\pi(s) = R(s, a) + \gamma E[V^\pi(s')]$ Policy Evaluation

$V^*(s) = \max_a \{R(s, a) + \gamma E[V^*(s')]\}$ Bellman's Equation: Certificate of Optimality

$B[V](s) = \max_a \{R(s, a) + \gamma E[V(s')]\}$ Bellman's Operator

Offline MDP Algorithms

Policy Iteration

loop

    Evaluate Policy

    Improve Policy

Converges because policy always improves and there are a finite number of policies

Value Iteration

loop

    $V \leftarrow B[V]$

Converges because B is a contraction mapping

Online MDP Planning

Monte Carlo Tree Search

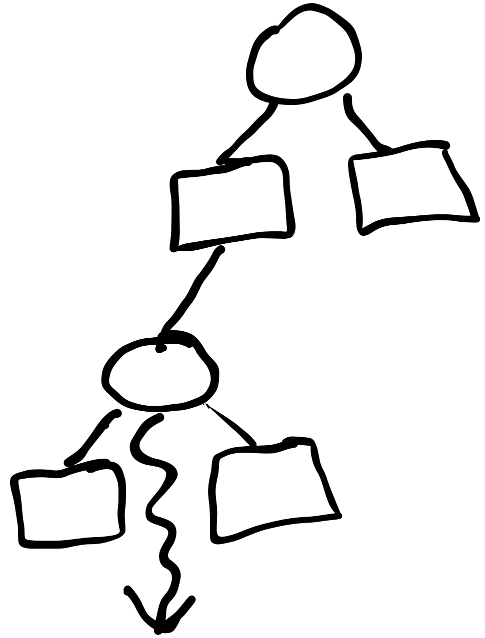
Search

Expand

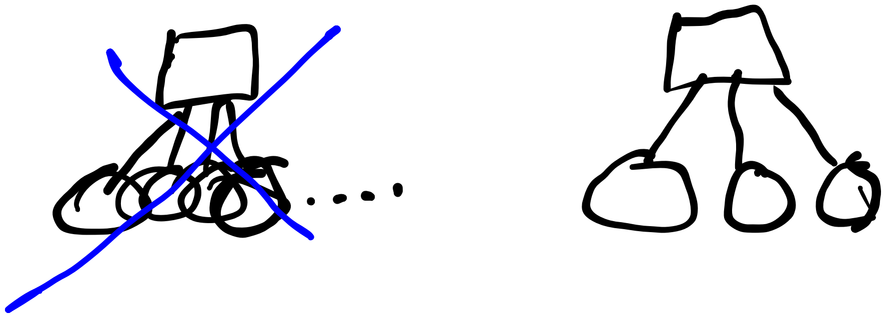
Rollout

Backup

$$Q(s,a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$$

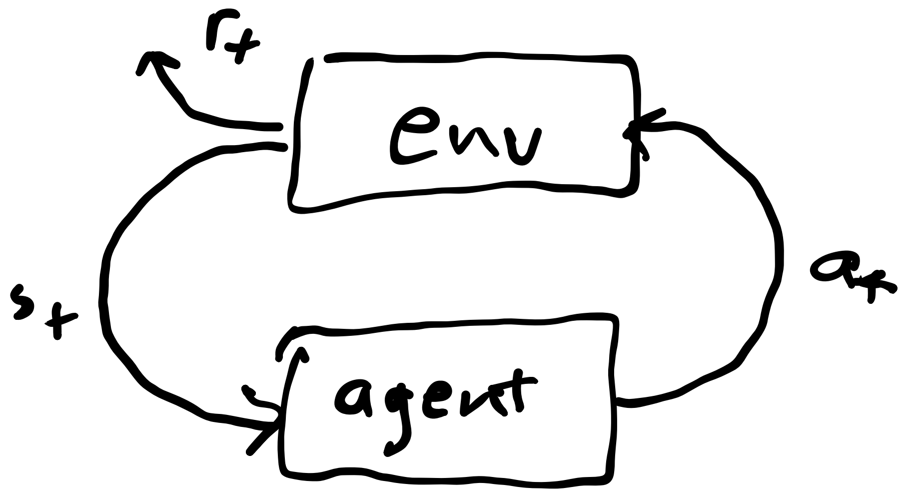


Sparse Sampling



Guarantees *independent* of $|S|$!!

Reinforcement Learning



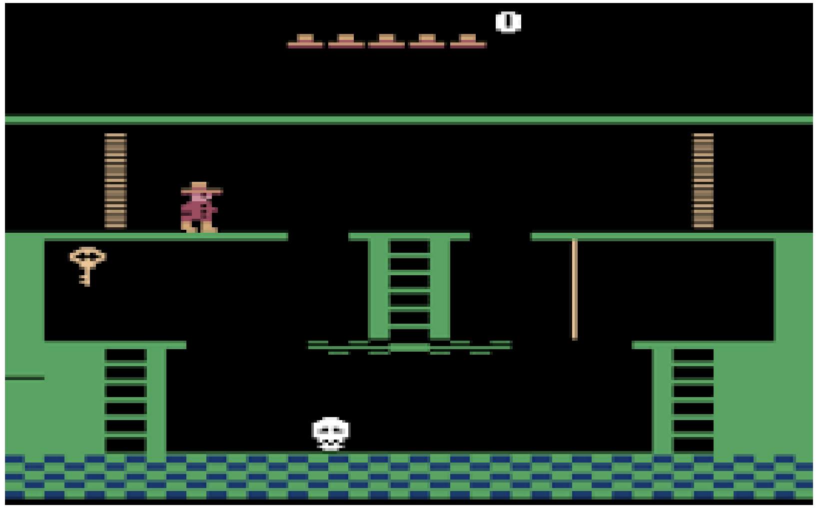
Challenges:

1. Exploration and Exploitation
2. Credit Assignment
3. Generalization

Exploration

Bandits

- ϵ-greedy
- softmax
- UCB
- Thompson Sampling
- Optimal DP Solution (solving a POMDP!)



Montezuma's Revenge!

- Pseudocounts
- Curiosity: extra reward for bad prediction
- Random network distillation

RL Algorithms

Model Based

Model Free

learn Q

SARSA

Q-learning

learn π

Policy Gradient

On Policy

Off Policy

MLMBRL
(learn T,R)

BAMDP
(POMDP $\tilde{s}=(s,T,R)$)

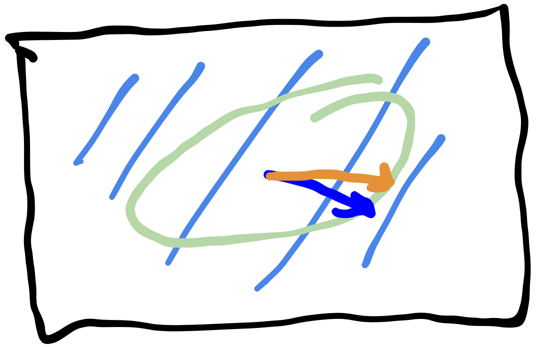
Policy Gradient

- Likelihood ratio trick
- Causality
- Baseline Subtraction

$$\nabla_\theta p_\theta(\tau) = p_\theta(\tau) \nabla_\theta \log p_\theta(\tau)$$

$$\nabla U(\theta) = \mathbb{E}_\tau \left[\sum_{k=1}^{d} \nabla_\theta \log \pi_\theta(a^{(k)} \mid s^{(k)}) \gamma^{k-1} \left(r_{\text{to-go}}^{(k)} - r_{\text{base}}(s^{(k)}) \right) \right]$$

- Natural Gradient



KL div.
Bound

Q-Learning

SARSA

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_t + \gamma Q(s',a') - Q(s,a))$$

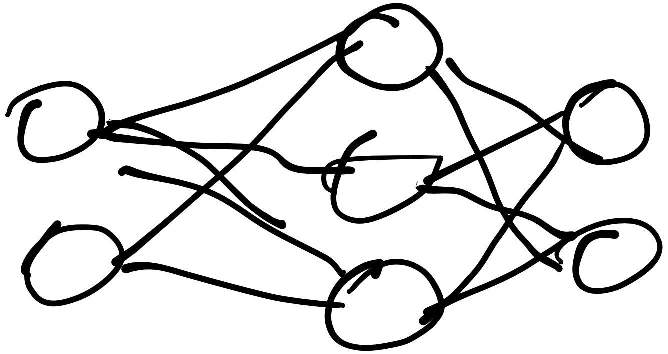
Eligibility Traces

Q-learning

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_t + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

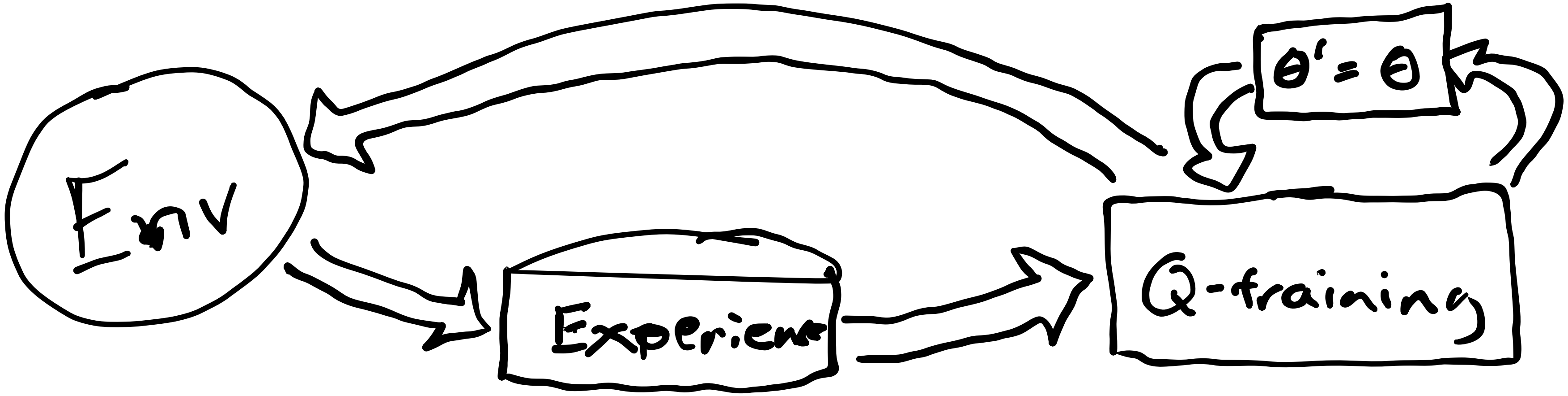
Double Q Learning

Neural Networks and DQN



$$f_\theta(x) = \sigma(W_2\sigma(W_1 x + b_1) + b_2)$$

Backprop



Actor-Critic

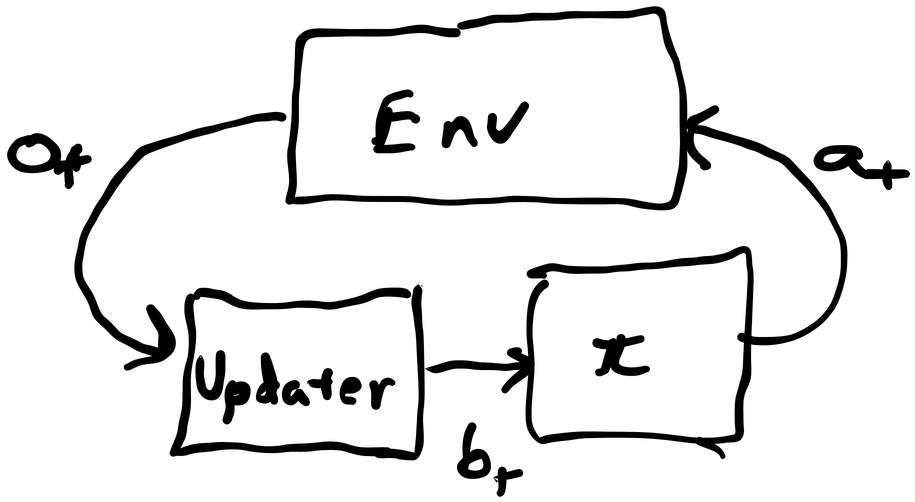
- Actor: π_θ
- Critic: Q_ϕ

Soft Actor Critic

$$J(\pi) = E\left[\sum_{t=0}^{\infty} \gamma^t \left(r_t + \alpha \mathcal{H}(\pi(\cdot \mid s_t))\right)\right]$$

POMDPs

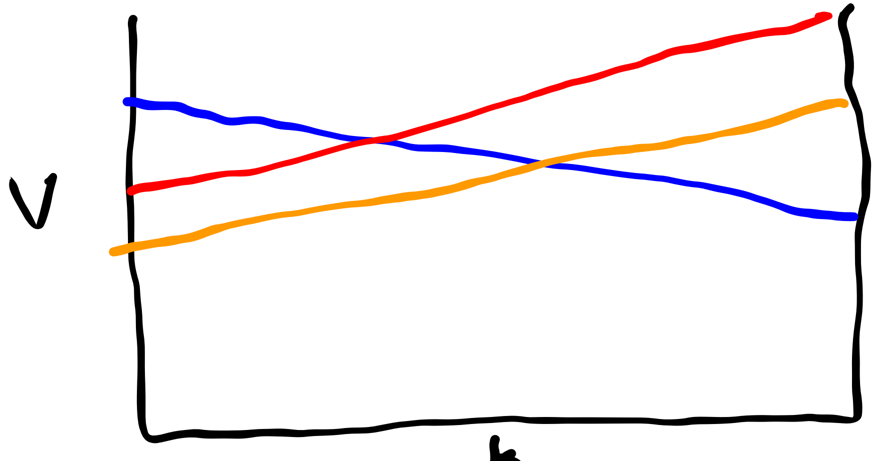
$$(S, A, T, R, O, Z, \gamma)$$



Belief Updates

- Discrete Bayesian Filter
- Particle Filter

Alpha Vectors



- Each alpha vector corresponds to a conditional plan
- You can prune alpha vectors by solving an LP

POMDP Approximations

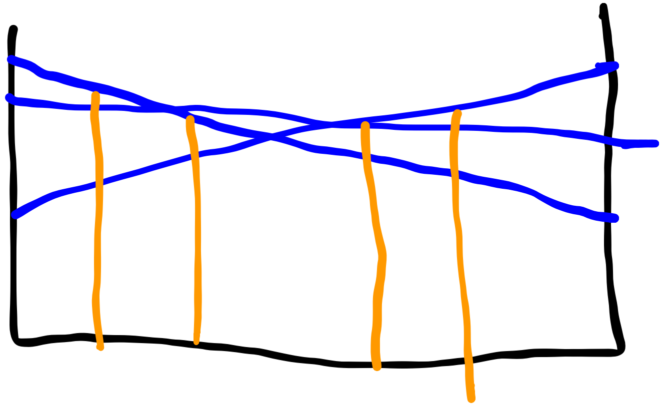
Formulation

- Certainty Equivalence
- QMDP

Numerical

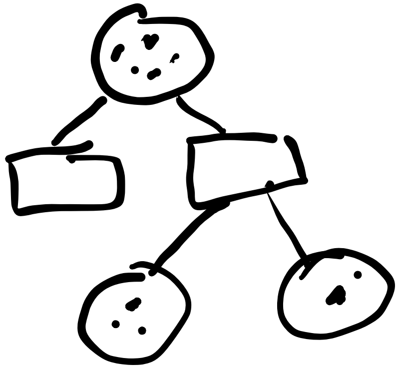
Offline

- Point-Based Value Iteration
- SARSOP



Online

- POMCP
- DESPOT



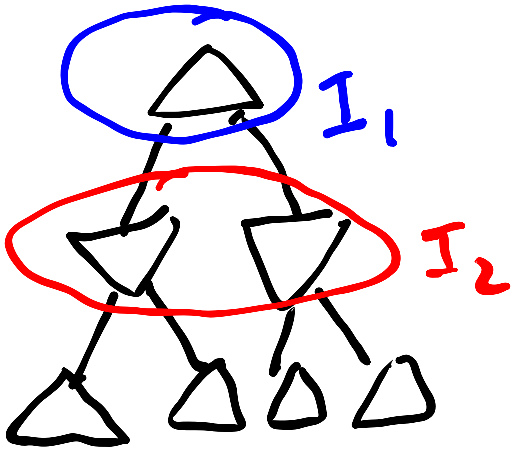
Simple Games

- ~~Optimal Solutions~~ No!
- Equilibria (e.g. Nash Equilibria)

-1,-1	-3,0
0,-3	-2,-2

- Every finite game has at least 1 Nash Equilibrium
- Might be pure or mixed
- Algorithms like fictitious play converge in special cases

Turn Taking Games

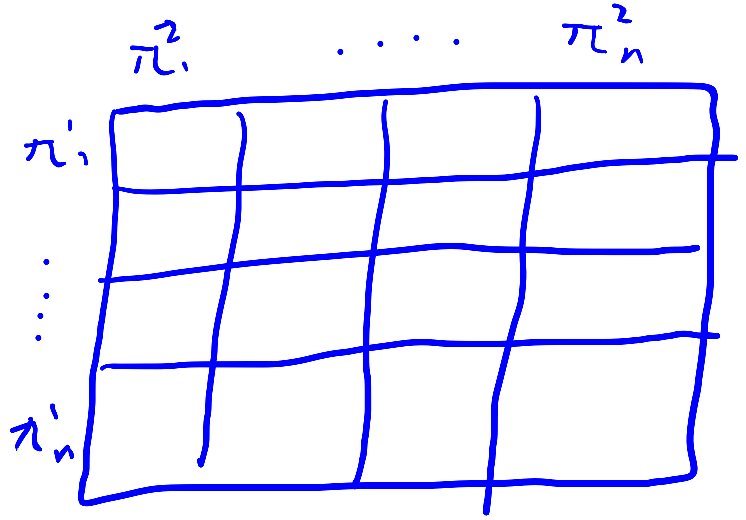


- Value Function Backup
- $\alpha\beta$ Pruning
- Incomplete Information Extensive Form

Markov Games and POMGS

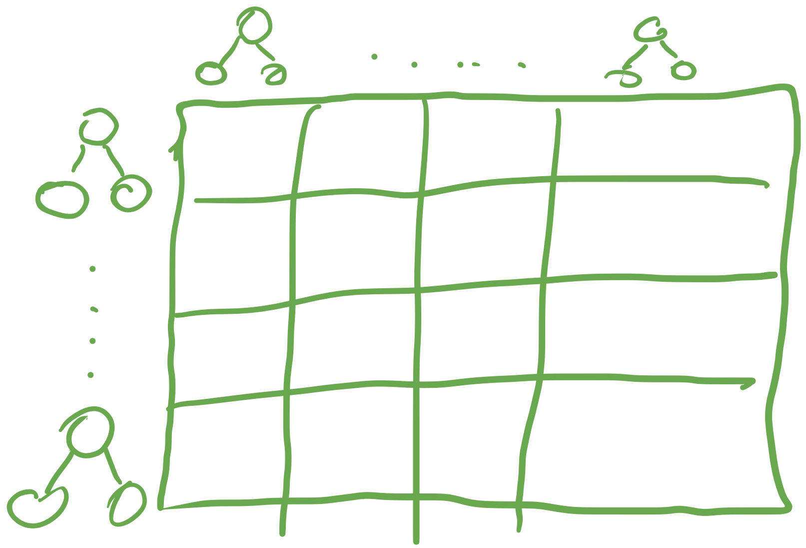
Markov Games

- All players play simultaneously
- Transitions are stochastic
- Best response involves solving an MDP
- Can be reduced to a simple game with policies as actions



Partially Observable Markov Games

- Each player receives a noisy observation at each step
- Beliefs not practical to compute
- Can be reduced to simple game with policies as actions



Recap

After DMU you have basic tools to deal with 4 Big Problems:

1. Immediate and Future Rewards
2. Unknown Models
3. Partial Observability
4. Other Agents