ASEN 5264 Decision Making under Uncertainty
Homework 1: Probabilistic Models

January 15, 2024

1 Questions

Question 1. (20 pts) Consider the following joint distribution of three binary-valued random variables, A, B, and C:

A	B	C	$P(A, B, C)$
0	0	1	0.15
0	1	0	0.05
0	1	1	0.01
1	0	0	0.14
1	0	1	0.18
1	1	0	0.29
1	1	1	0.06

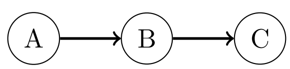
a) What is the probability of the outcome $A = 0$, $B = 0$, $C = 0$?

b) What is the marginal distribution of A?

c) What is the conditional distribution of A given $B = 0$ and $C = 1$?

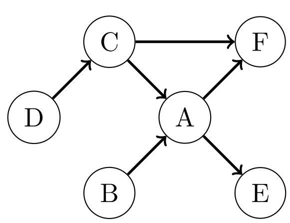
Question 2. (20 pts) 2% of women at age forty who participate in routine screening have breast cancer. 86% of those with breast cancer will get positive mammograms. 8% of those without breast cancer will also get positive mammograms. A woman in this age group had a positive mammogram in a routine screening. What is the probability that she actually has breast cancer?

Question 3. (20 pts) Suppose that A, B, and C are binary random variables with $P(A = 1) = 0.5$, $P(B = 1|A = 1) = 0.8$, $P(B = 1|A = 0) = 0.2$, $P(C = 1|A = 1) = 0.9$, $P(C = 1|A = 0) = 0.3$, and the following Bayesian network structure:



If we observe that $C = 1$, what is the probability that $B = 1$?

Question 4. (20 pts) Consider the following Bayesian network structure:



a) Is it possible to conclude from the structure that $B \perp F \mid C$? Justify your answer.
b) Is it possible to conclude from the structure that $B \perp F \mid A$? Justify your answer.
c) Is it possible to conclude from the structure that $B \perp E \mid A$? Justify your answer.

(assignment continues on next page)

2 Auto-graded Programming

Question 5. (20 pts autograder + 5 pts code) In this exercise, you will write and test a Julia function to ensure that you can get Julia and the course-specific code running and help you learn how to do a task that sometimes trips students up in homework 2. Your function should take two arguments:

- `a`: a matrix, and
- `bs`: a non-empty vector of vectors.

The function should multiply all of the vectors in `bs` by `a` and then return a vector where the ith element is the maximum of the ith elements of all of the resulting vectors, that is, the *elementwise* maximum of the resulting vectors.

Example: if

$$\texttt{a} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \texttt{bs} \text{ has the vectors } \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \tag{1}$$

then, after multiplication, the resulting vectors are

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \text{ and the elementwise maximum that should be returned is } \begin{bmatrix} 4 \\ 2 \end{bmatrix}. \tag{2}$$

In order to get full-credit, the function must be completely “type-stable” (see the “Performance Tips” section of the Julia manual). Your function should always return a vector with the same element type as `a`. You can assume the vectors in `bs` will have the same element type as `a`, but you should be able to handle `a` with any numeric element type.

Evaluate this function with `DMUStudent.HW1.evaluate` and submit the resulting json file *along with a listing of the code*. A score of 1 will receive full credit.