Exploration and Exploitation (Bandits)

Last Time

- What is Reinforcement Learning?

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- What are the main challenges in Reinforcement Learning?

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- What are the main challenges in Reinforcement Learning?
- How do we categorize RL approaches?

Last Time

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First RL Algorithm:

Last Time

First RL Algorithm:

Tabular Maximum Likelihood Model-Based Reinforcement Learning

Last Time

First RL Algorithm:

Tabular Maximum Likelihood Model-Based Reinforcement Learning

loop
- choose action a
- gain experience
- estimate T, R
- solve MDP with T, R

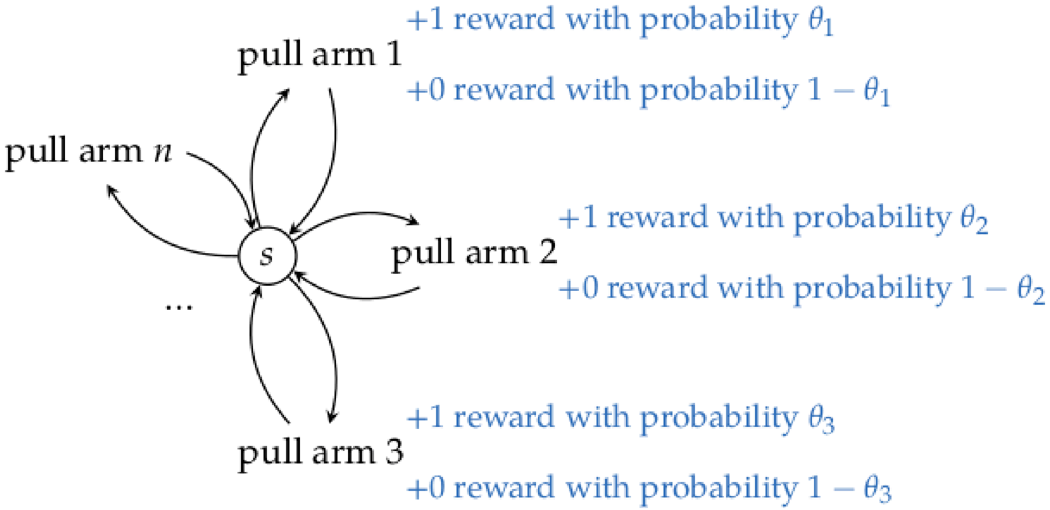
Guiding Questions

- What are the best ways to trade off Exploration and Exploitation?

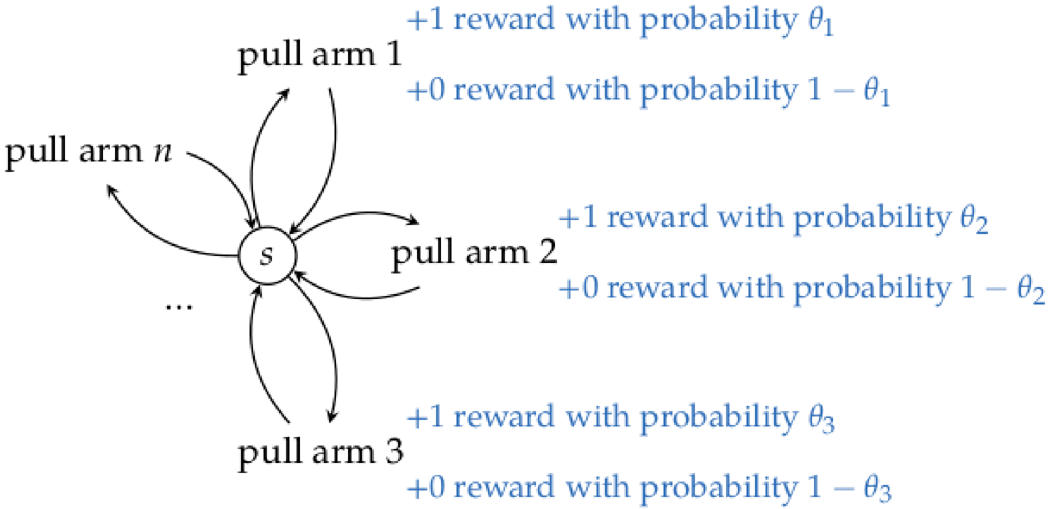
Bandits

Bandits

Bandits

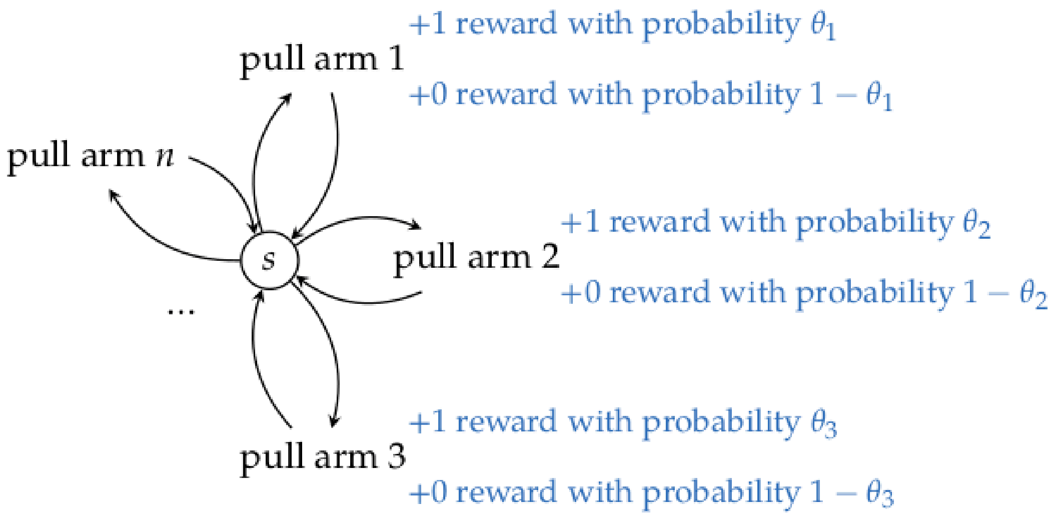

+1 reward with probability θ_1
pull arm 1
+0 reward with probability $1-\theta_1$
pull arm n
s
pull arm 2
+1 reward with probability θ_2
+0 reward with probability $1-\theta_2$
...
+1 reward with probability θ_3
pull arm 3
+0 reward with probability $1-\theta_3$

Bandits



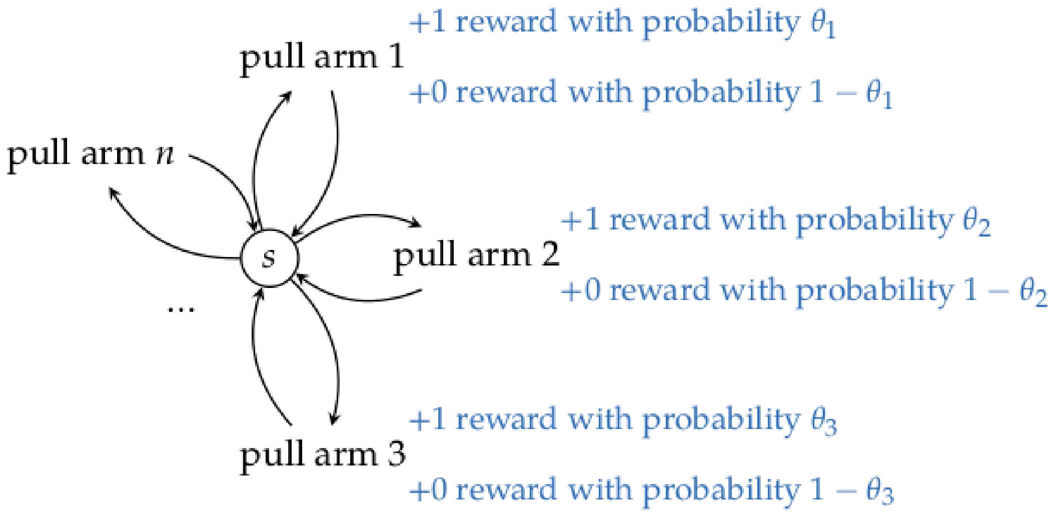
- Bernoulli Bandit with parameters θ

Bandits



- Bernoulli Bandit with parameters θ
- $\theta^* \equiv \max \theta$

Bandits



- Bernoulli Bandit with parameters θ
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"According to Peter Whittle, "efforts to solve [bandit problems] so sapped the energies and minds of Allied analysts that the suggestion was made that the problem be dropped over Germany as the ultimate instrument of intellectual sabotage.""

Greedy Strategy

$$\rho_a = \frac{\text{number of wins} + 1}{\text{number of tries} + 1}$$

Choose $\underset{a}{\operatorname{argmax}}\, \rho_a$

Undirected Strategies

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- Explore then Commit
  Choose a randomly for k steps
  Then choose $\underset{a}{\operatorname{argmax}}\, \rho_a$

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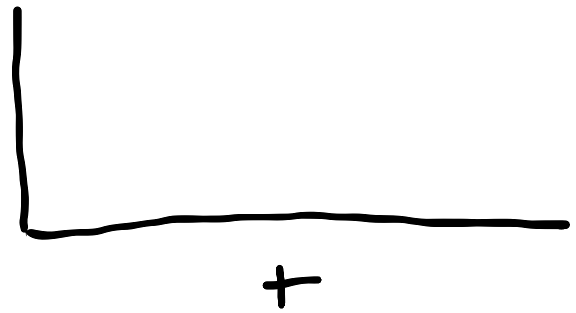

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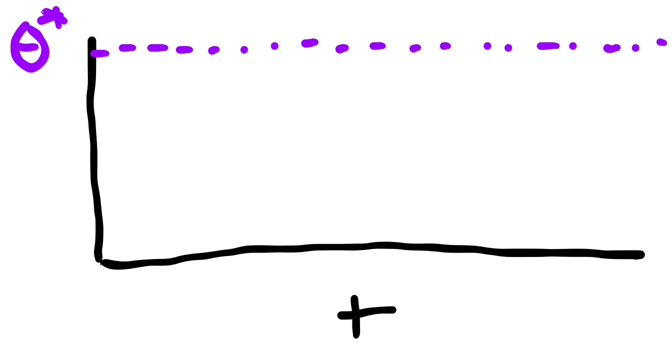

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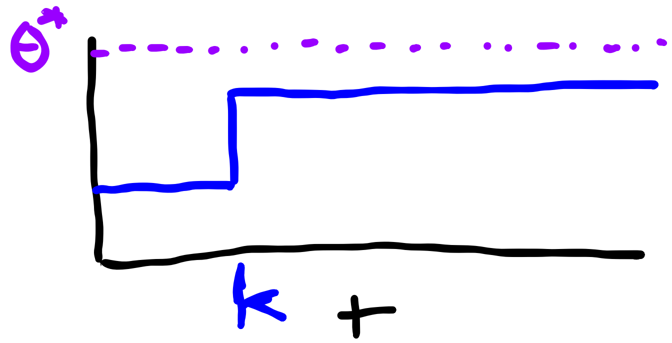

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- Explore then Commit
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- ϵ - greedy
  With probability ϵ, choose randomly
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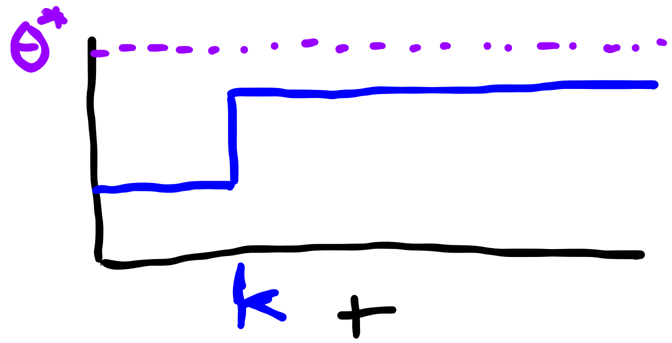

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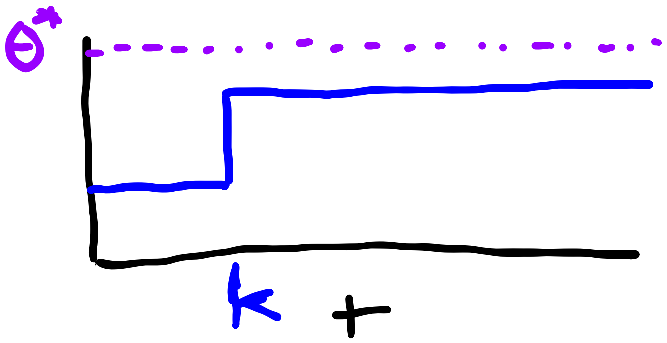

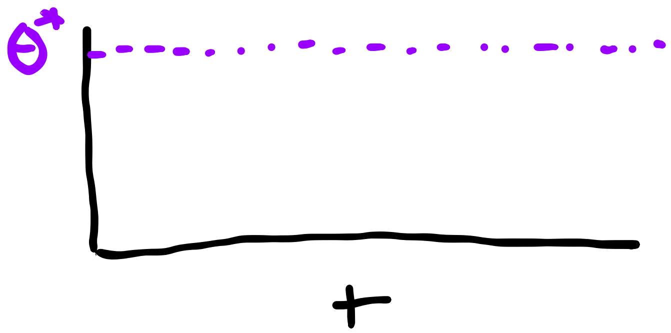

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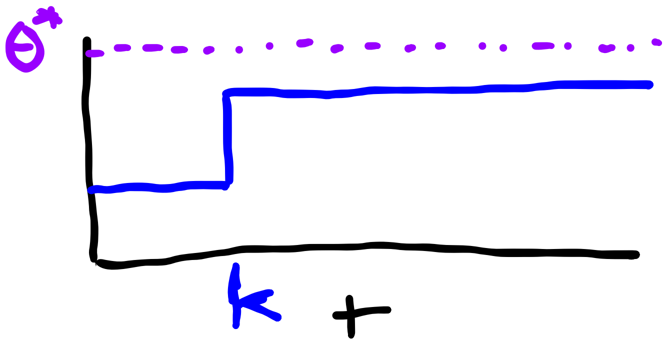

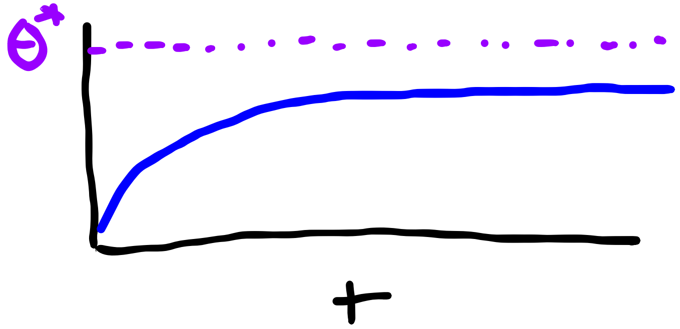

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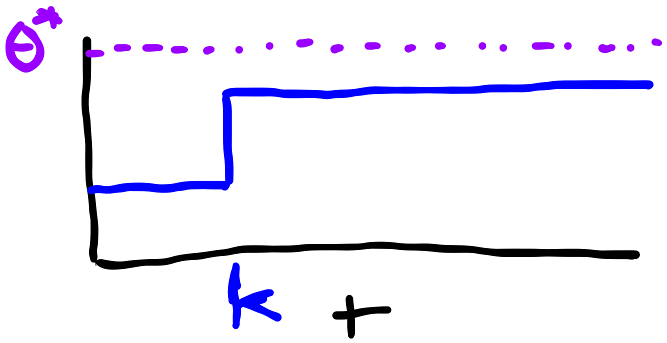

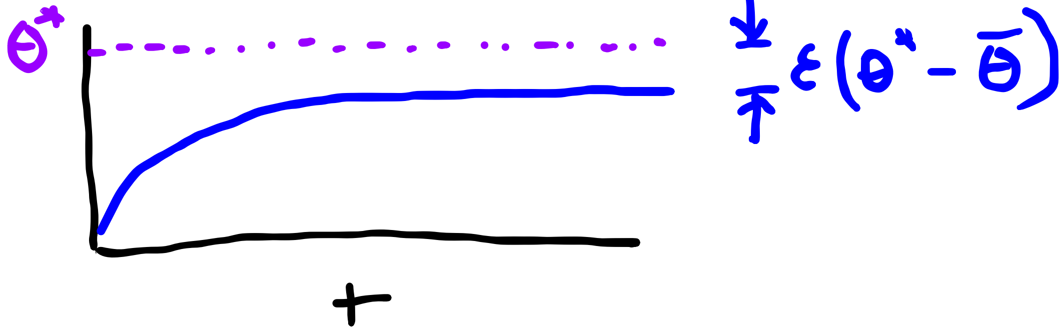

Directed Strategies

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- Softmax
  Choose a with probability proportional to $e^{\lambda \rho_a}$

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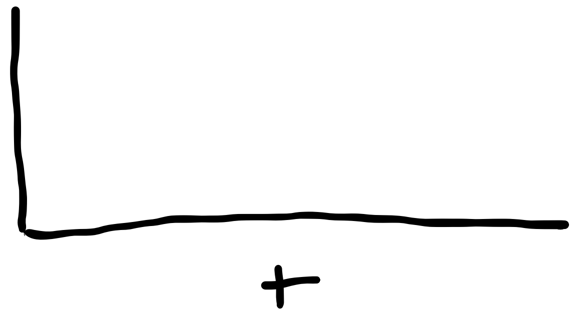

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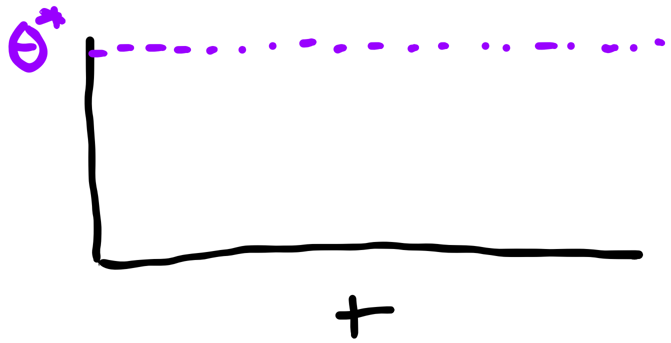

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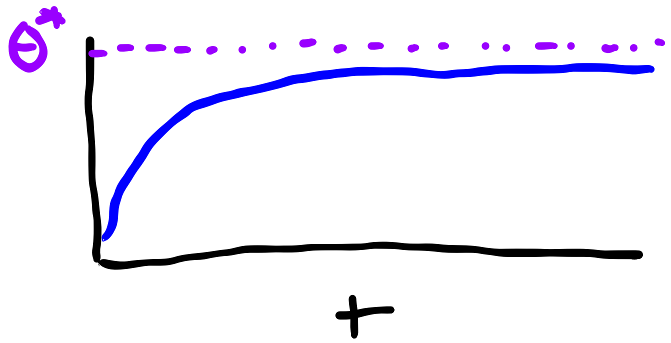

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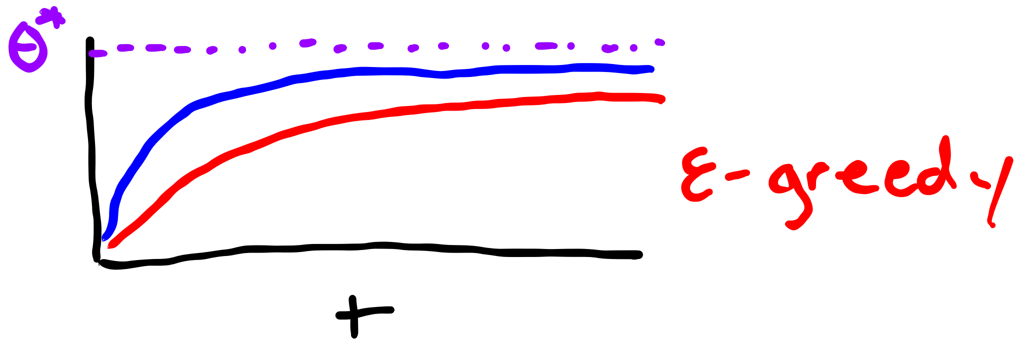

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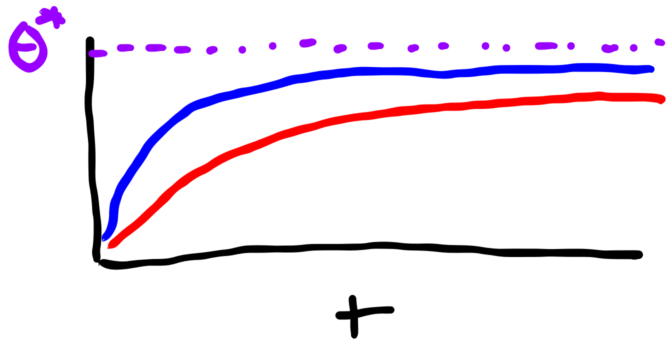


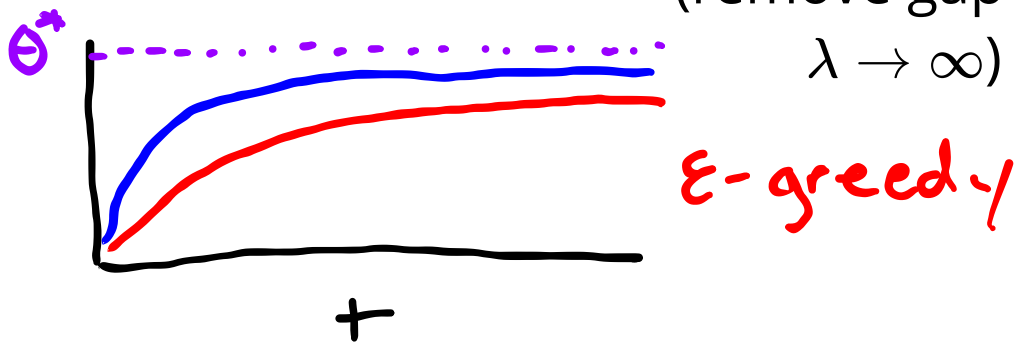
(remove gap with $\lambda \to \infty$)

ε-greedy

Directed Strategies

- Softmax
  Choose a with probability proportional to $e^{\lambda \rho_a}$

- Upper Confidence Bound (UCB)
  Choose $\underset{a}{\operatorname{argmax}}\, \rho_a + c\sqrt{\frac{\log N}{N(a)}}$



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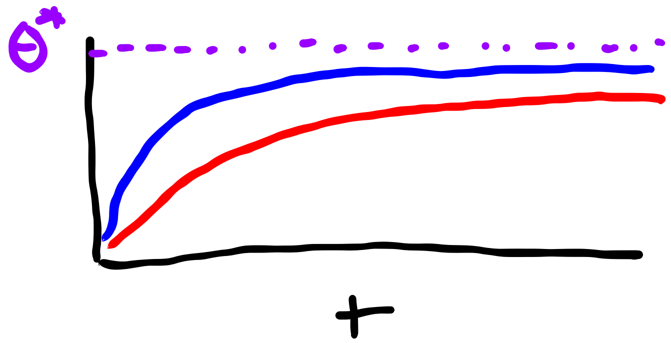

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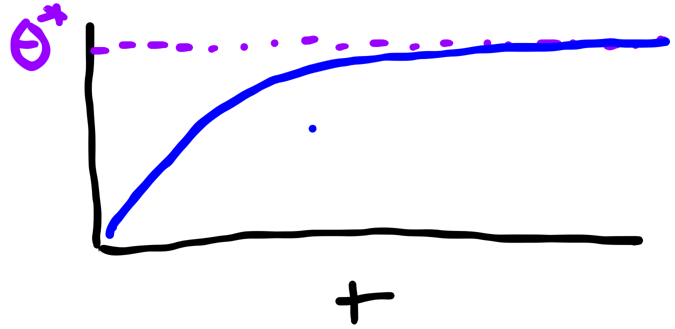
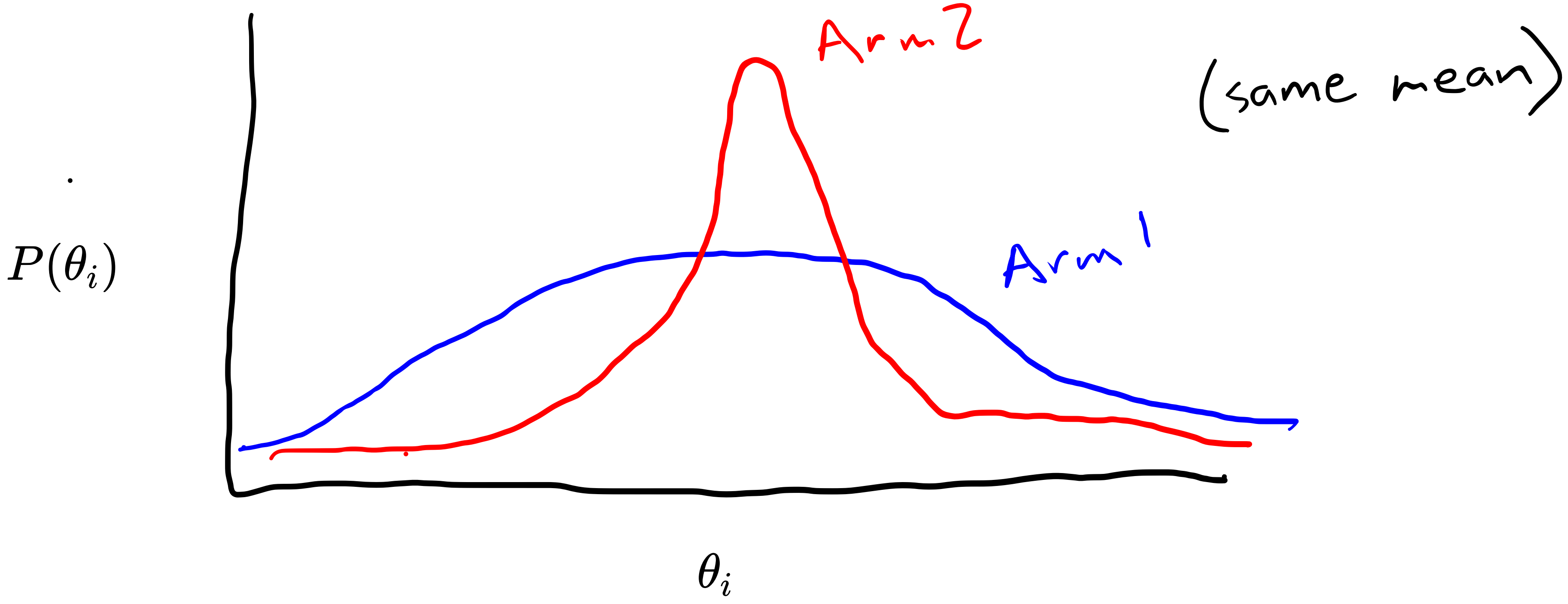


Break

Discuss with your neighbor: Suppose you have the following *belief* about the parameters θ. Which arm should you choose to pull next?



Bayesian Estimation

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Bernoulli Distribution

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$$\text{Bernoulli}(\theta)$$

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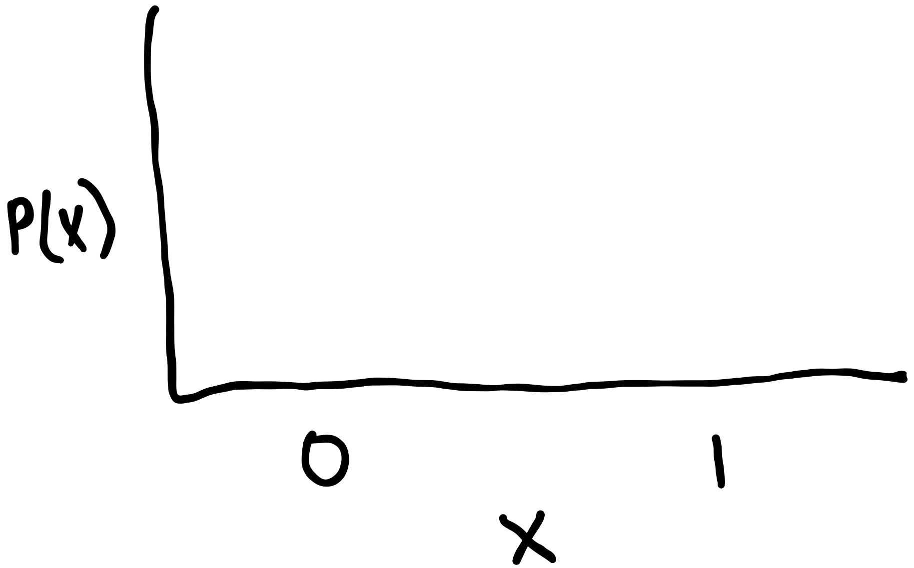

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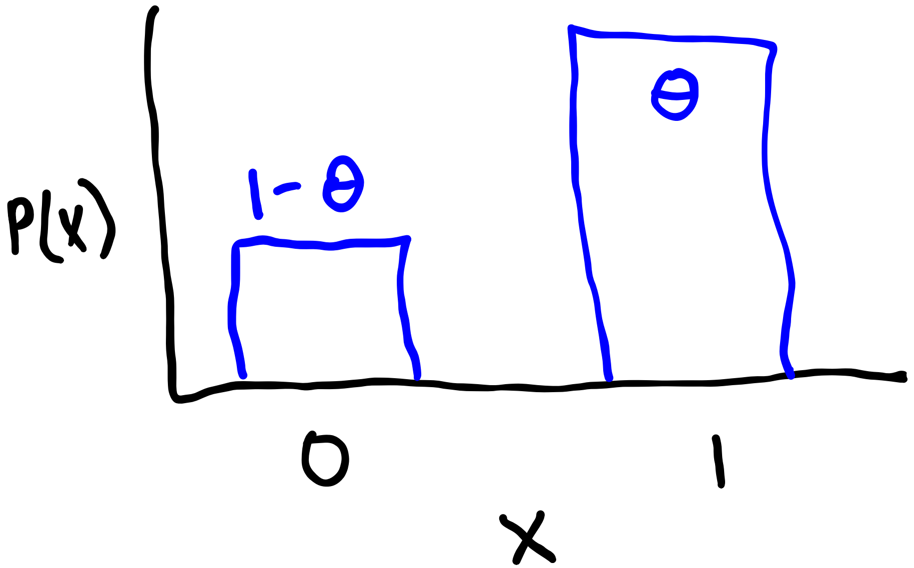

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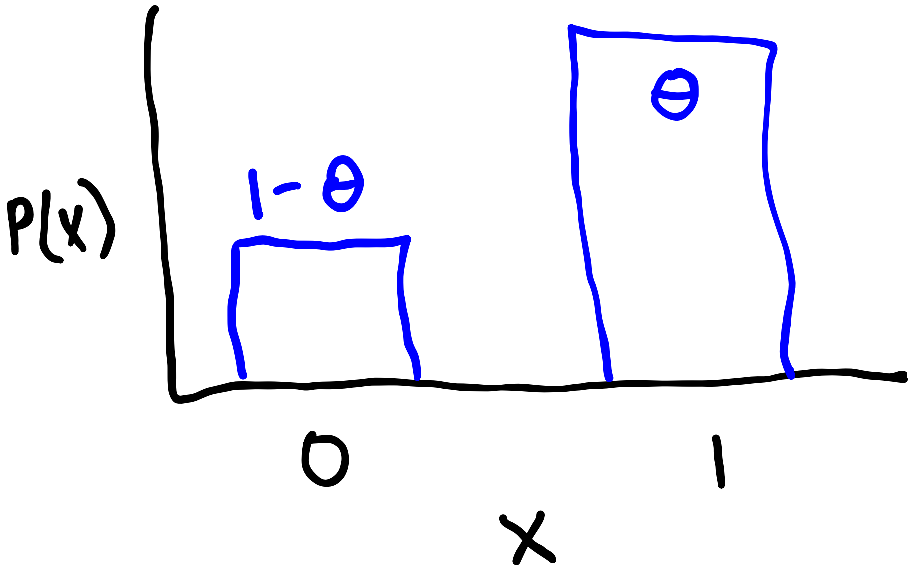

Discussion: Given that I have received w wins and l losses, what should my belief (probability distribution) about θ look like?

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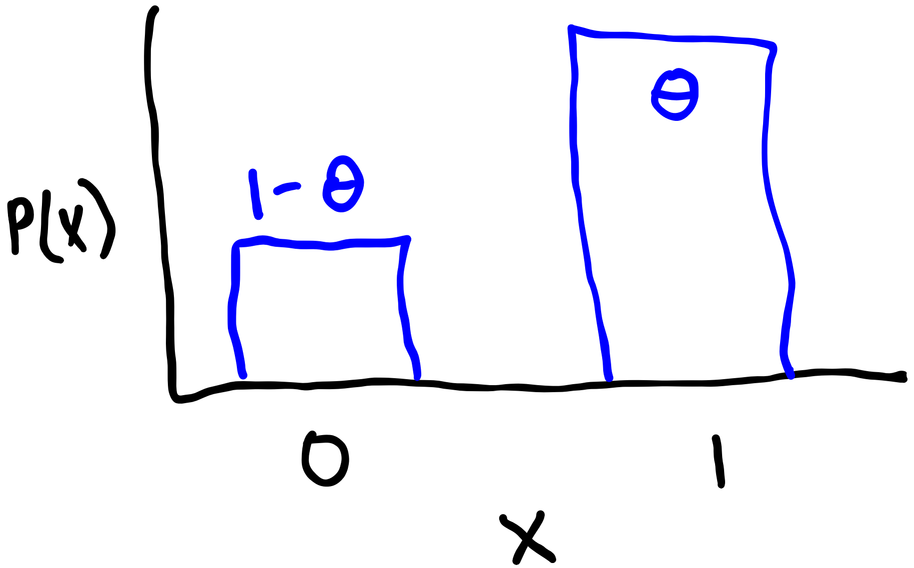

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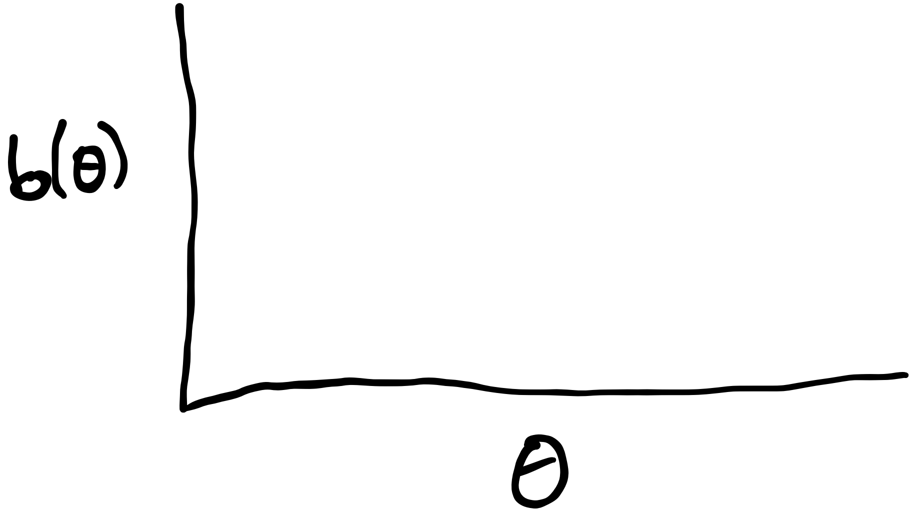

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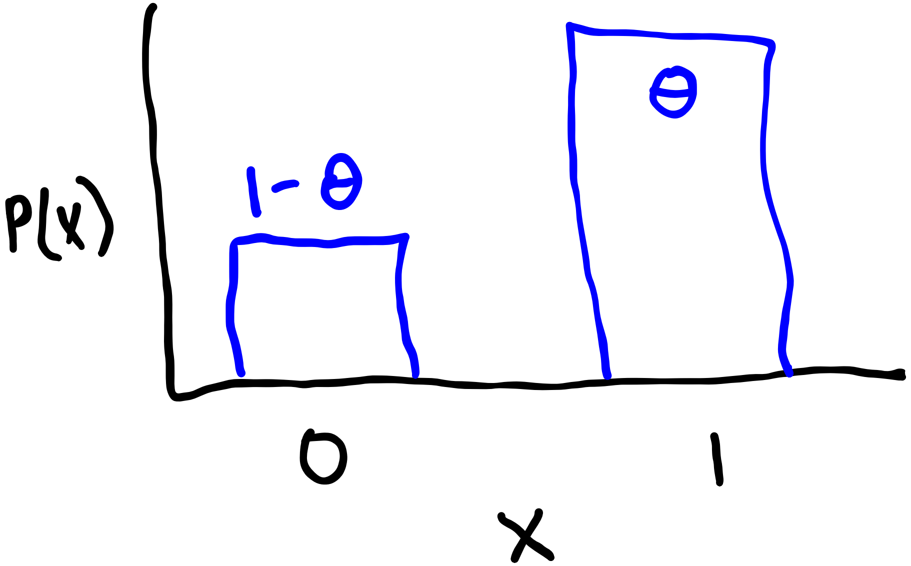

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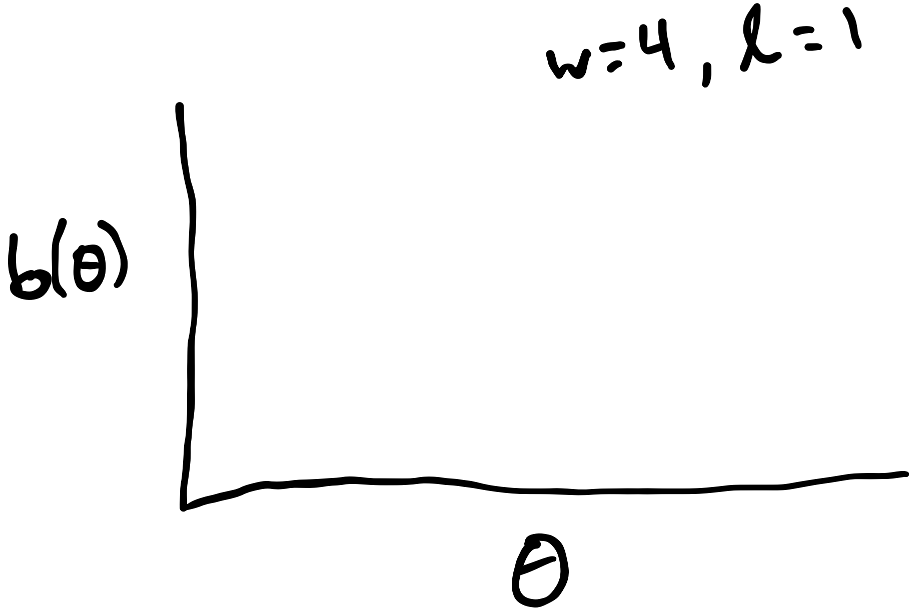

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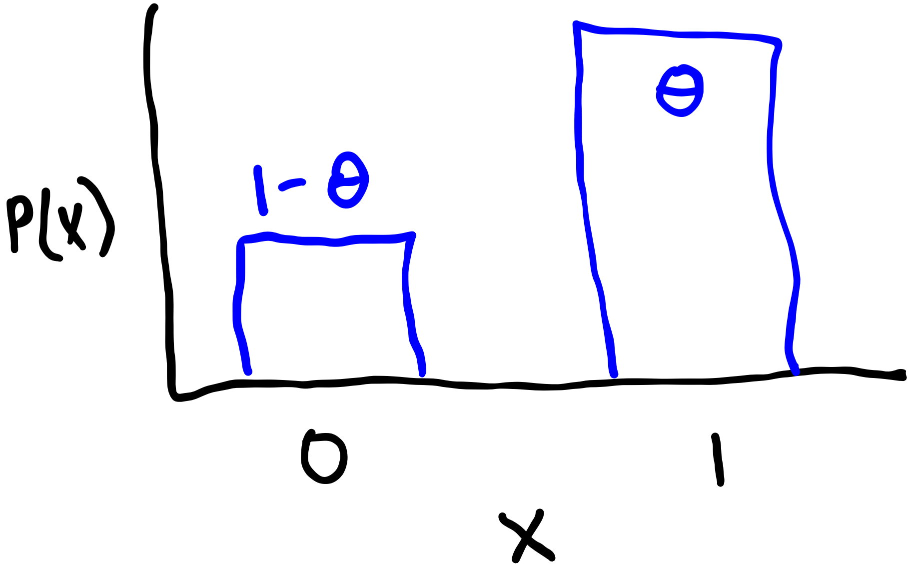

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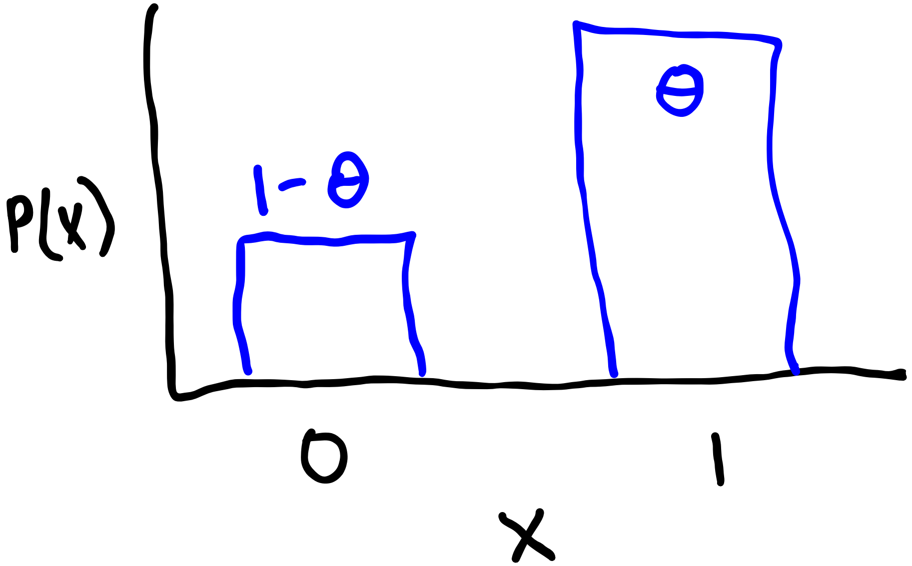

Beta Distribution

(distribution over Bernoulli distributions)

Bayesian Estimation

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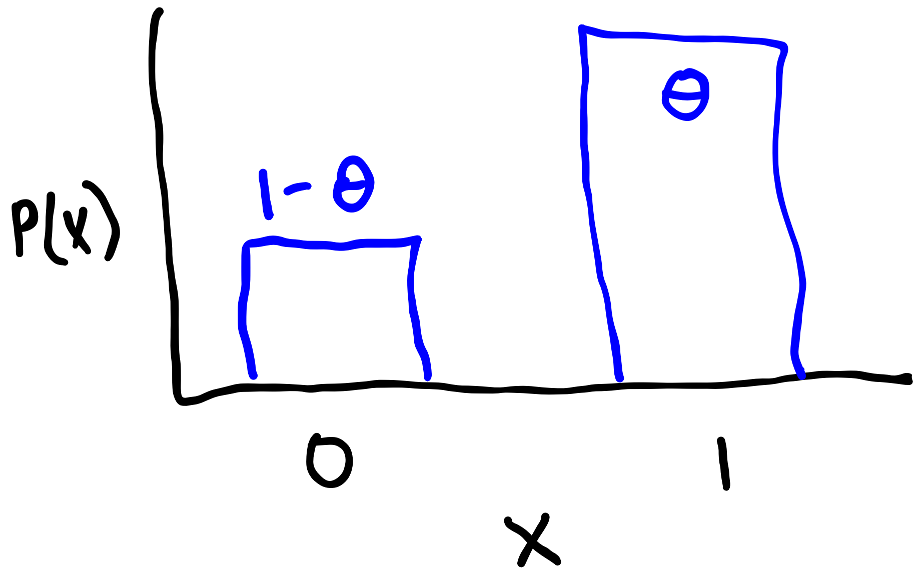

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$$\text{Beta}(\alpha, \beta)$$

Bayesian Estimation

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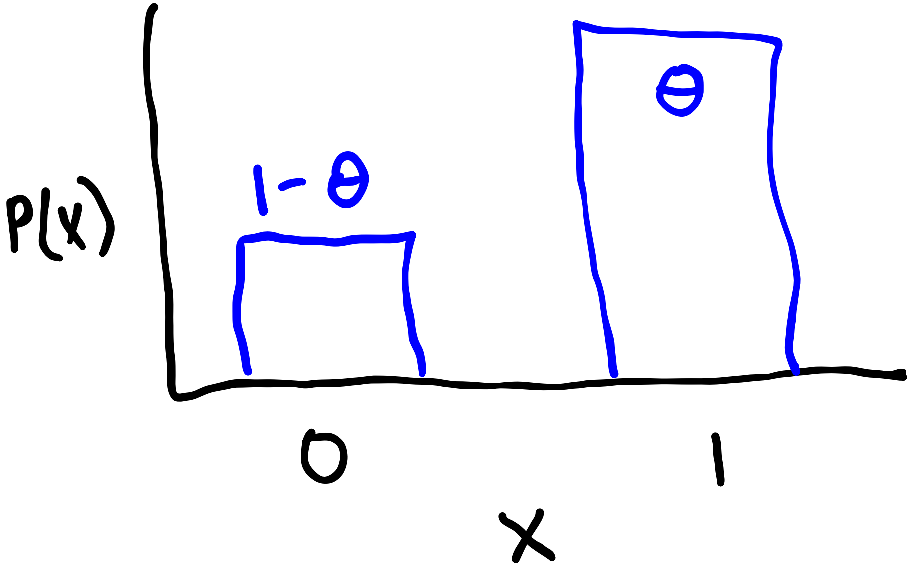

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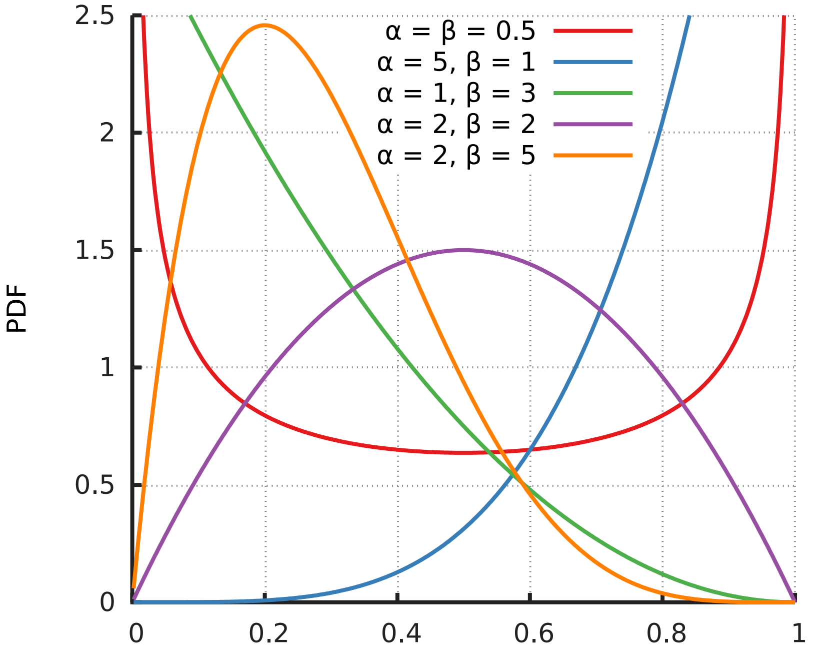

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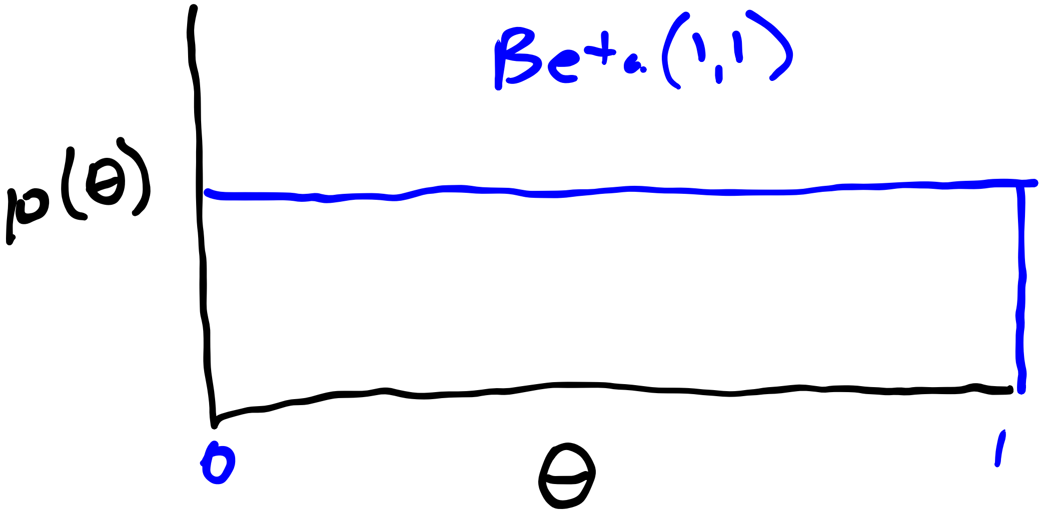

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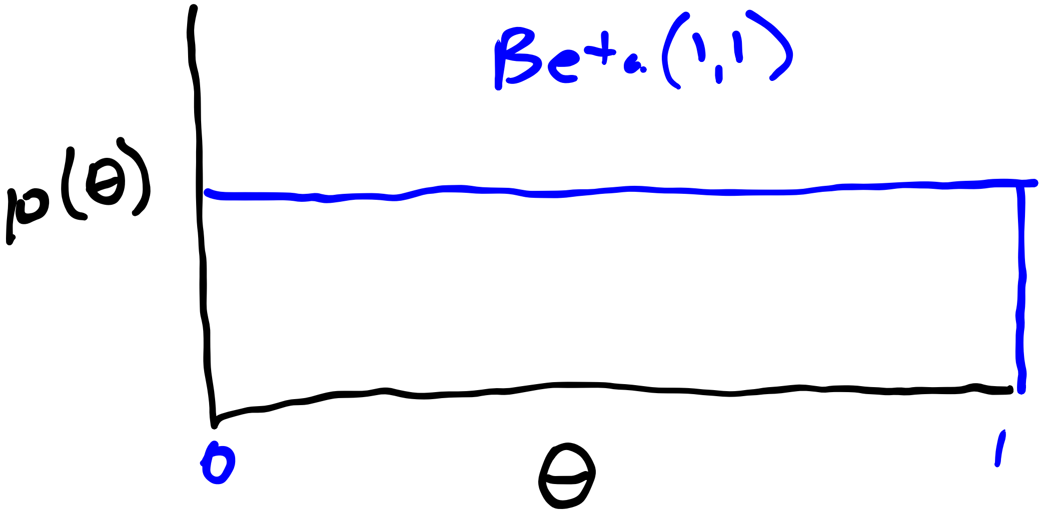

The posterior distribution of θ is

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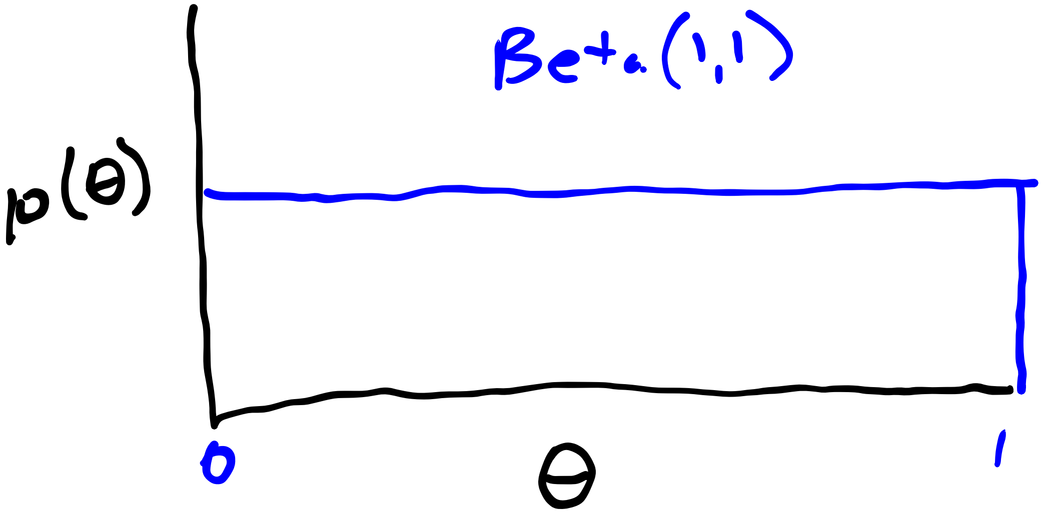

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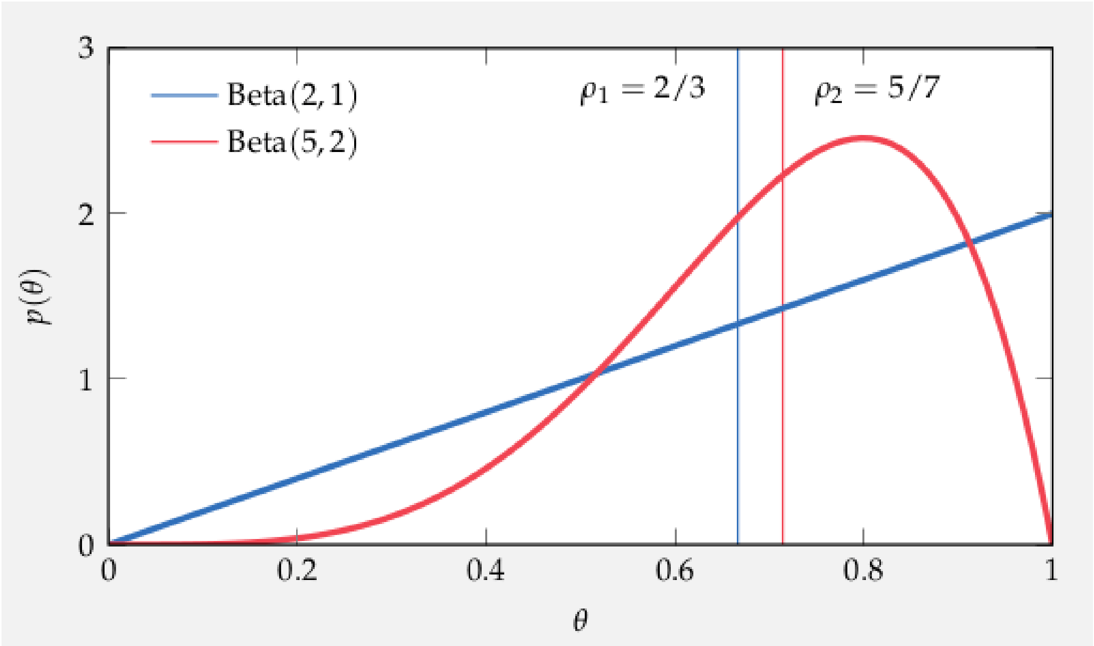

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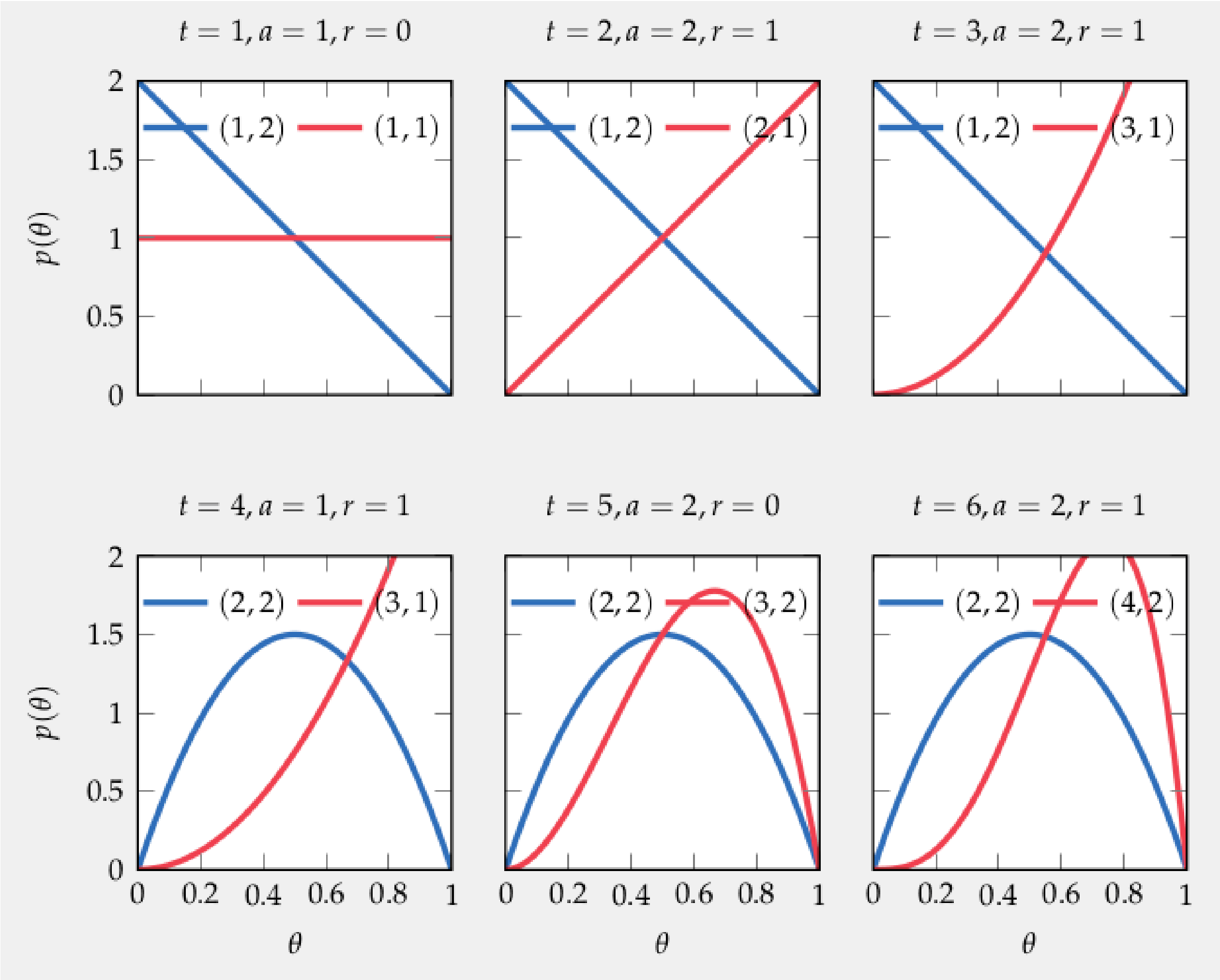

t = time

a = arm pulled

r = reward

Bayesian Bandit Algorithms

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- Quantile Selection
  Choose a for which the α quantile of
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$\alpha = 0.9$

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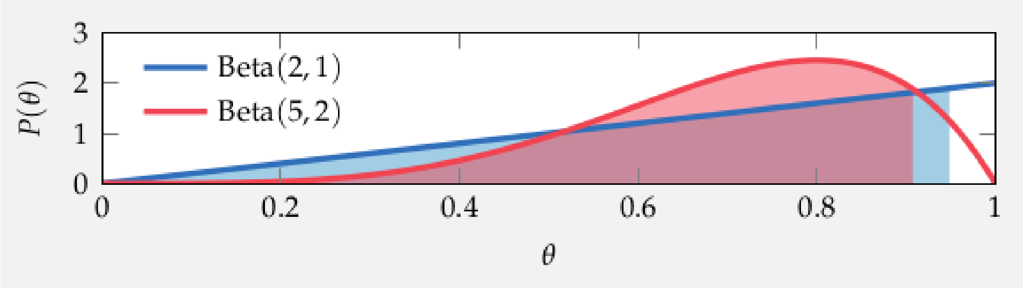

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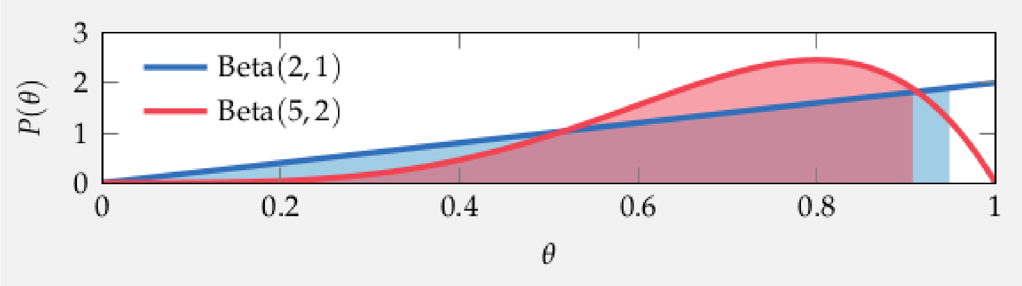

- Quantile Selection
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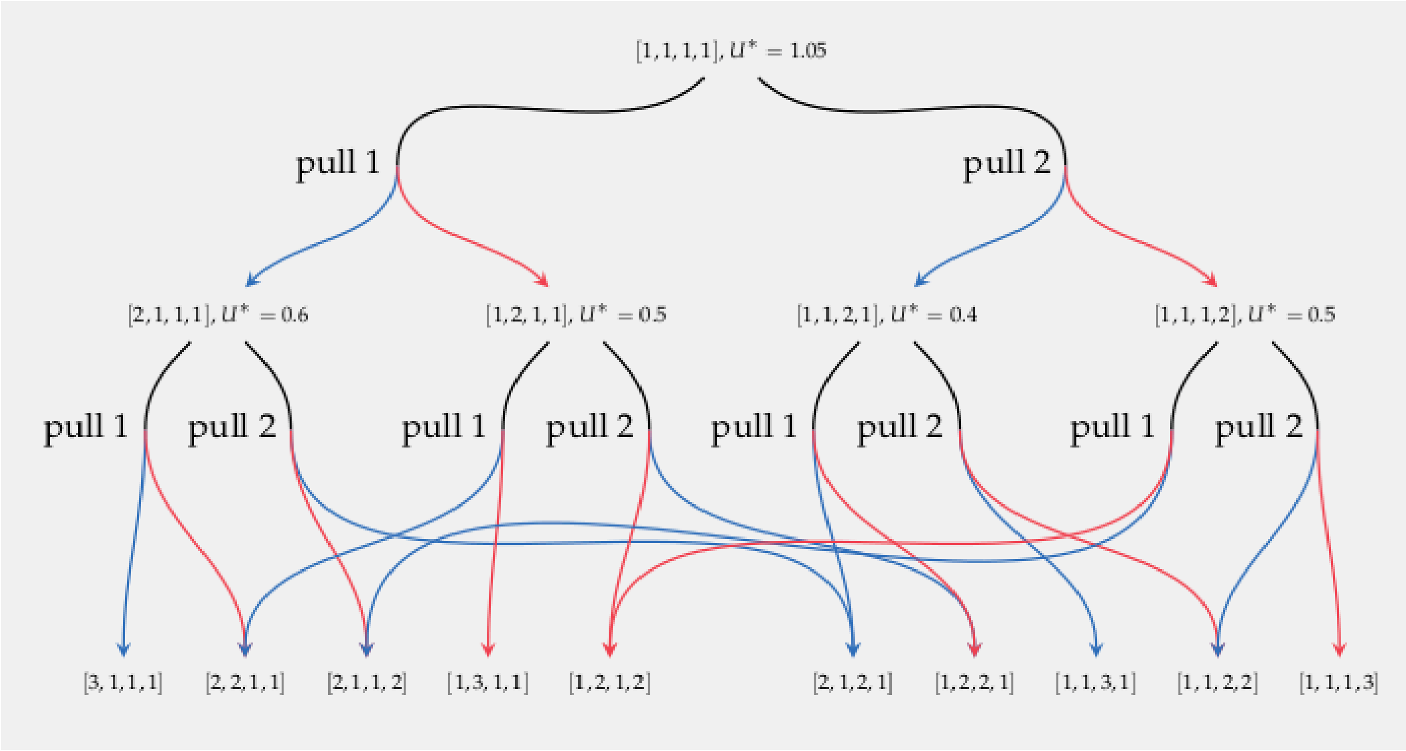
- Thompson Sampling
  Sample $\hat{\theta}$
  Choose $\arg\max_a \hat{\theta}_a$

Optimal Algorithm - Dynamic Programming



Regret Analysis

Roughly:

Regret Analysis

$$\text{Regret}(n) \equiv \theta^* n - \sum_{t=1}^{n} r_t$$

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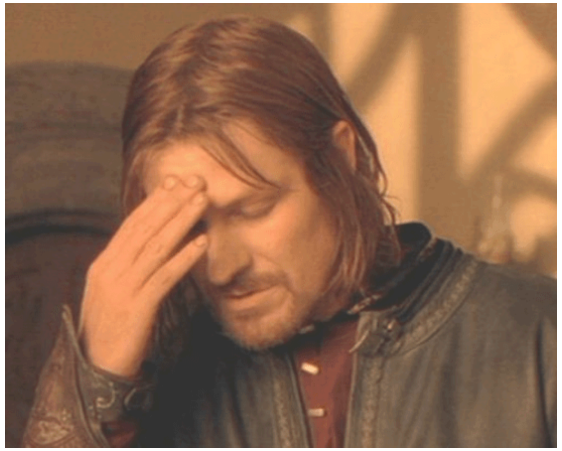

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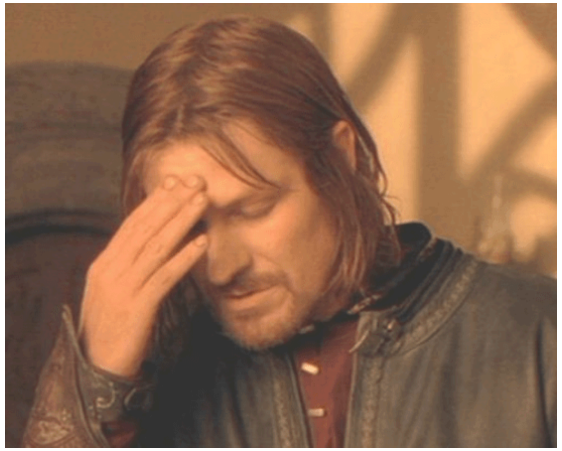
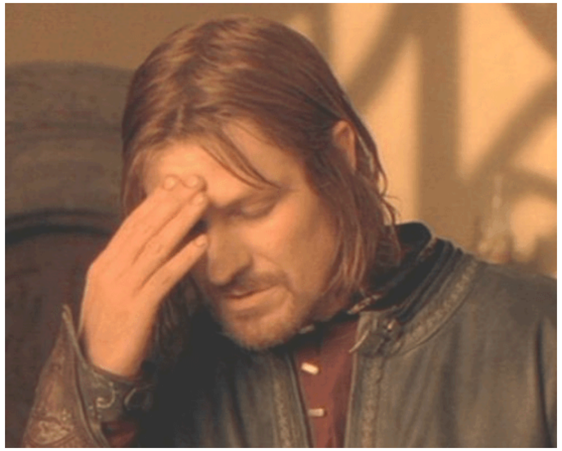


$$\text{Regret}(n) \equiv \theta^* n - \sum_{t=1}^{n} r_t$$

Recall: $f(n) = O(g(n))$ means that there exists a $C > 0$ and $N > 0$ such that $f(n) < C\, g(n)$ for all $n > N$.

Roughly:

Regret Analysis



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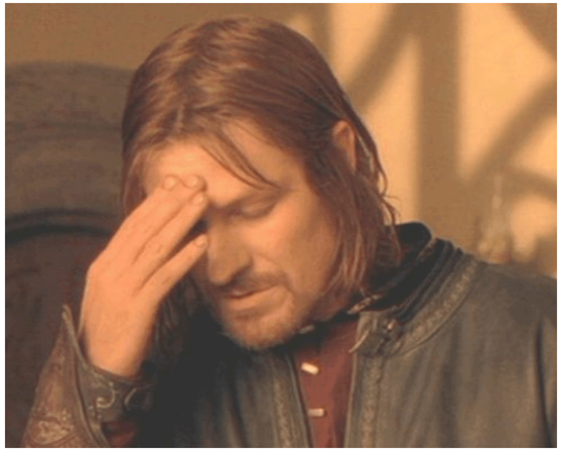

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Roughly:

- $O(n)$ regret means you might keep picking the wrong arm forever
- $O(\log(n))$ regret means that you keep learning

Review

Guiding Questions

- What are the best ways to trade off Exploration and Exploitation