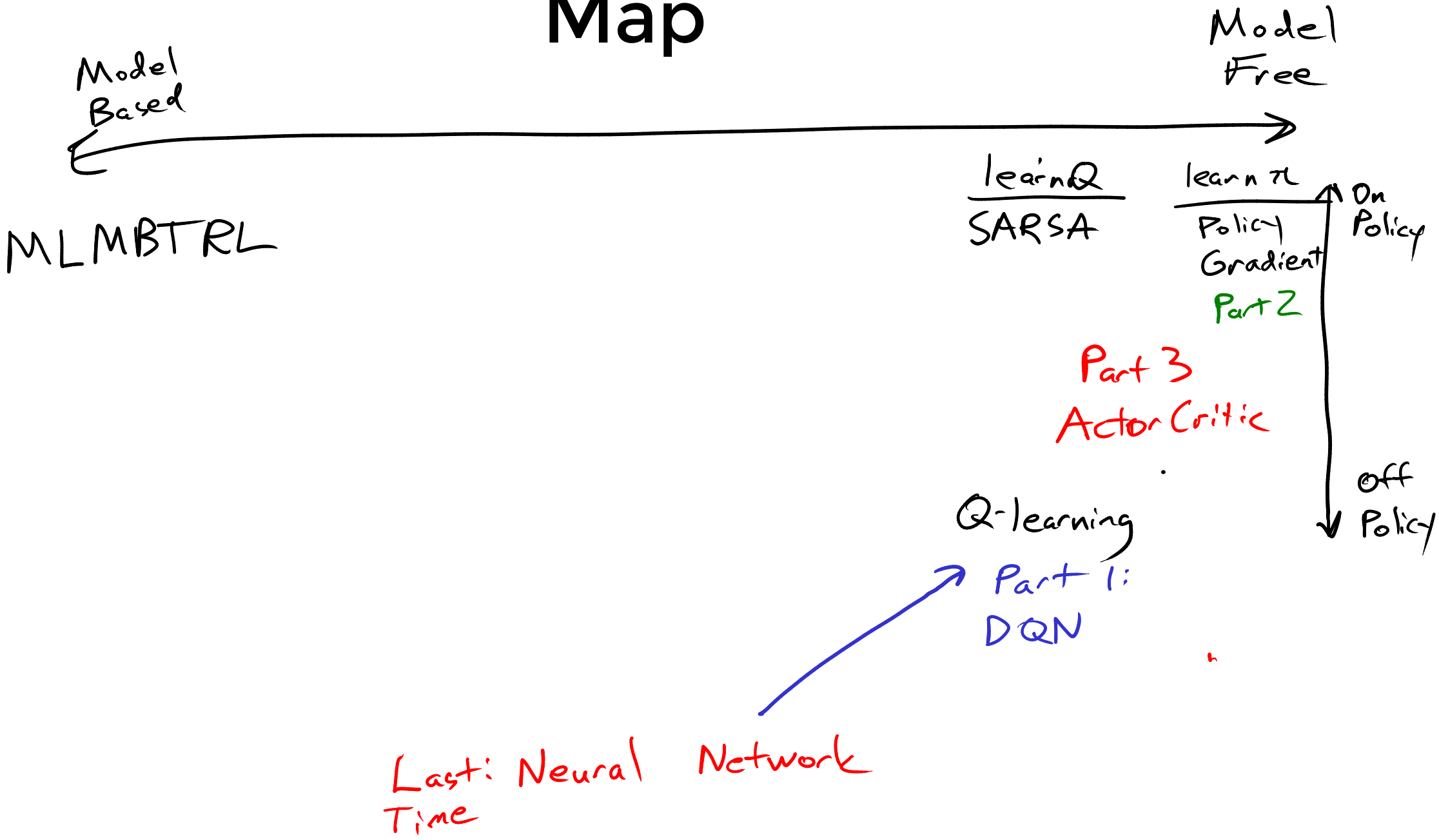
Map


Model Based
Model Free
MLMBTRL
learnQ
SARSA
learn π
Policy Gradient
Part 2
On Policy
Part 3
Actor Critic
off Policy
Q-learning
Part 1:
DQN
Last: Neural Network
Time

Part I
DQN

Q-Learning with Neural Networks

Q-Learning with Neural Networks

Q-Learning:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \left(r + \gamma \max_{a'} Q(s',a') - Q(s,a)\right)$$

Q-Learning with Neural Networks

Q-Learning:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \left(r + \gamma \max_{a'} Q(s',a') - Q(s,a)\right)$$

Neural Networks

$$\theta^* = \arg\min_\theta \sum_{(x,y)\in\mathcal{D}} l(f_\theta(x), y)$$

$$\theta \leftarrow \theta - \alpha \nabla_\theta \, l(f_\theta(x), y)$$

Q-Learning with Neural Networks

Q-Learning:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \ (r + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

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Deep Q learning:

- Approximate Q with Q_θ

Q-Learning with Neural Networks

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- What should (x, y) be?

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$$\theta^* = \arg\min_\theta \sum_{(x,y)\in\mathcal{D}} l(f_\theta(x), y)$$

$$\theta \leftarrow \theta - \alpha \nabla_\theta l(f_\theta(x), y)$$

Deep Q learning:

- Approximate Q with Q_θ
- What should (x, y) be?
- What should l be?

$$l(s,a,r,s') = \left(r + \gamma \max_{a'} Q(s',a') - Q(s,a)\right)^2$$

Q-Learning with Neural Networks

Q-Learning:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

Neural Networks

$$\theta^* = \arg\min_\theta \sum_{(x,y)\in\mathcal{D}} l(f_\theta(x), y)$$

$$\theta \leftarrow \theta - \alpha \nabla_\theta\, l(f_\theta(x), y)$$

Deep Q learning:

- Approximate Q with Q_θ
- What should (x, y) be?
- What should l be?

Candidate Algorithm:

loop

$$a \leftarrow \text{argmax}\, Q(s,a) \text{ w.p. } 1-\epsilon, \quad \text{rand}(A) \text{ o.w.}$$

$$r \leftarrow \text{act!}(\text{env}, a)$$

$$s' \leftarrow \text{observe}(\text{env})$$

$$\theta \leftarrow \theta - \alpha \nabla_\theta \left(r + \gamma \max_{a'} Q_\theta(s',a') - Q_\theta(s,a)\right)^2$$

$$s \leftarrow s'$$

DQN: The Atari Benchmark

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DQN: The Atari Benchmark

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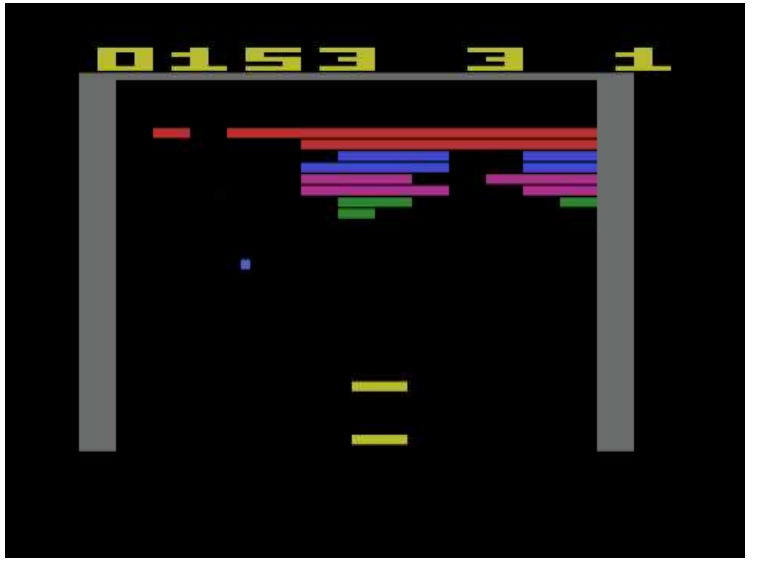

DQN: Problems with Naive Approach

DQN: Problems with Naive Approach

Candidate Algorithm:

loop

$a \leftarrow \text{argmax}\, Q(s,a) \text{ w.p. } 1-\epsilon, \quad \text{rand}(A) \text{ o.w.}$

$r \leftarrow \text{act!}(\text{env}, a)$

$s' \leftarrow \text{observe}(\text{env})$

$\theta \leftarrow \theta - \alpha \nabla_\theta \left(r + \gamma \max_{a'} Q_\theta(s',a') - Q_\theta(s,a)\right)^2$

$s \leftarrow s'$

DQN: Problems with Naive Approach

s, a, r, s'

s' a' r' s''

Candidate Algorithm:

loop

$$a \leftarrow \text{argmax}\, Q(s,a) \text{ w.p. } 1-\epsilon, \quad \text{rand}(A) \text{ o.w.}$$

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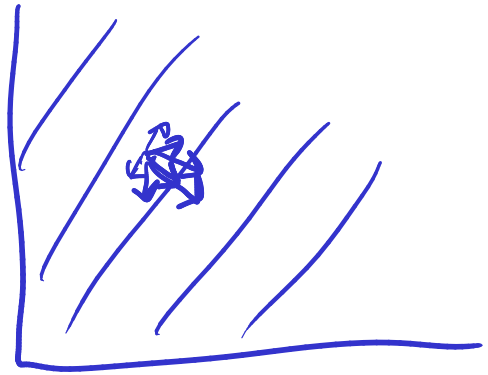
$$s \leftarrow s'$$

Problems:

1. Samples Highly Correlated

DQN: Problems with Naive Approach



Candidate Algorithm:

loop

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$$s \leftarrow s'$$

Problems:

1. Samples Highly Correlated
2. Size-1 batches

DQN: Problems with Naive Approach

Candidate Algorithm:

loop

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$$r \leftarrow \text{act!}(\text{env}, a)$$

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Problems:

1. Samples Highly Correlated
2. Size-1 batches
3. Moving target

DQN: Problems with Naive Approach

Candidate Algorithm:

loop

$$a \leftarrow \text{argmax}\, Q(s,a) \text{ w.p. } 1-\epsilon, \quad \text{rand}(A) \text{ o.w.}$$

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$$\theta \leftarrow \theta - \alpha \nabla_\theta \left(r + \gamma \max_{a'} Q_{\theta'}(s',a') - Q_\theta(s,a)\right)^2$$

$$s \leftarrow s'$$

Problems:

1. Samples Highly Correlated
2. Size-1 batches
3. Moving target

} data buffer / experience replay

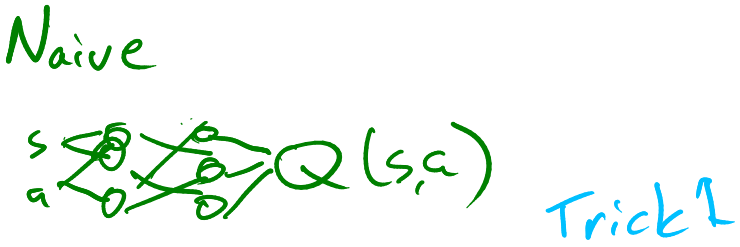
} periodically freeze target

DQN

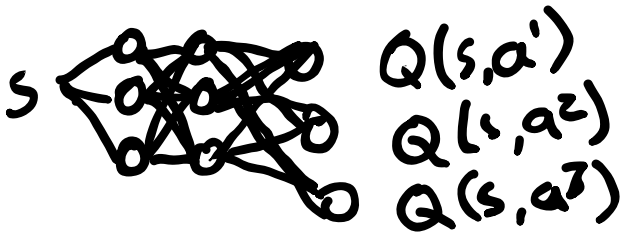
DQN

Q Network Structure:

DQN

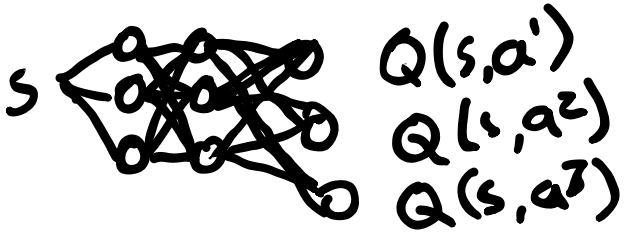


Q Network Structure:



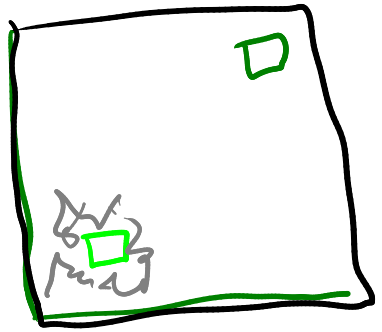
DQN

Q Network Structure:

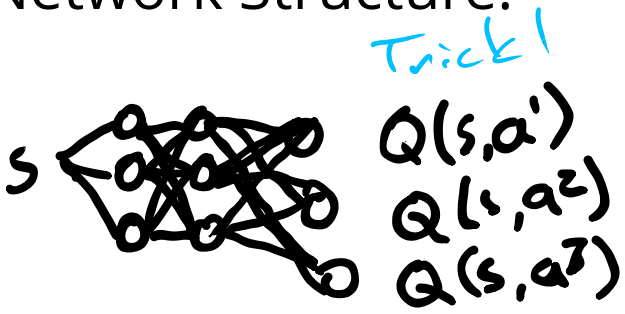


Experience Tuple: (s, a, r, s')

DQN



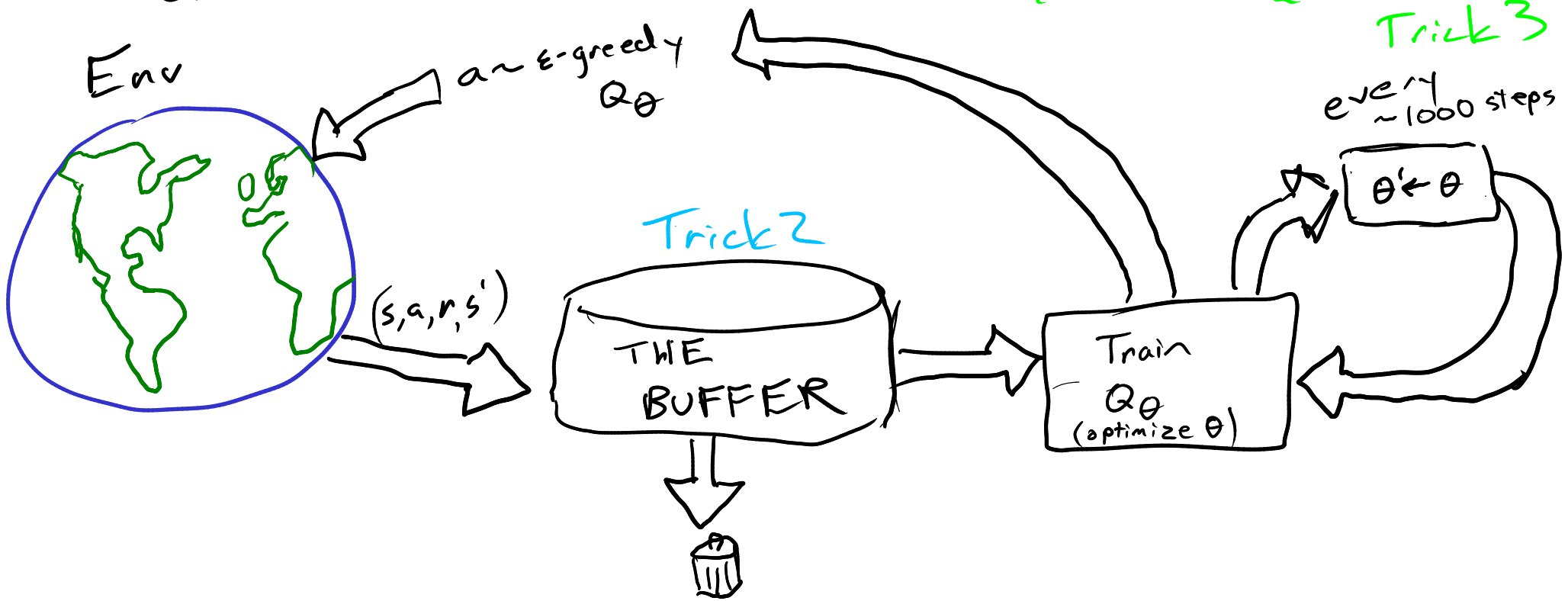
Q Network Structure:



Experience Tuple: (s, a, r, s')

Loss:

$$l(s, a, r, s') = \left(\overbrace{r + \gamma \max_{a'} Q_{\theta'}(s', a') - Q_{\theta}(s, a)}\right)^2$$

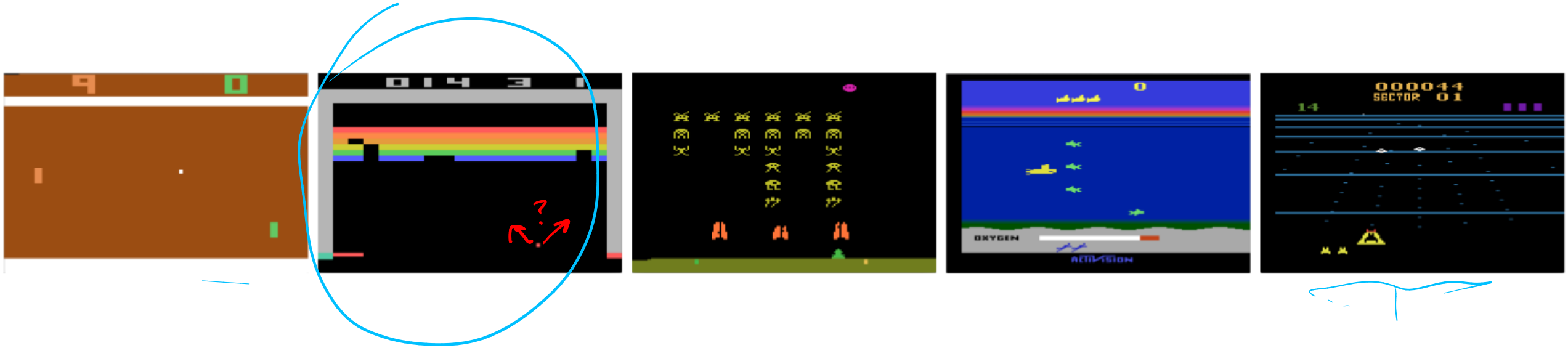


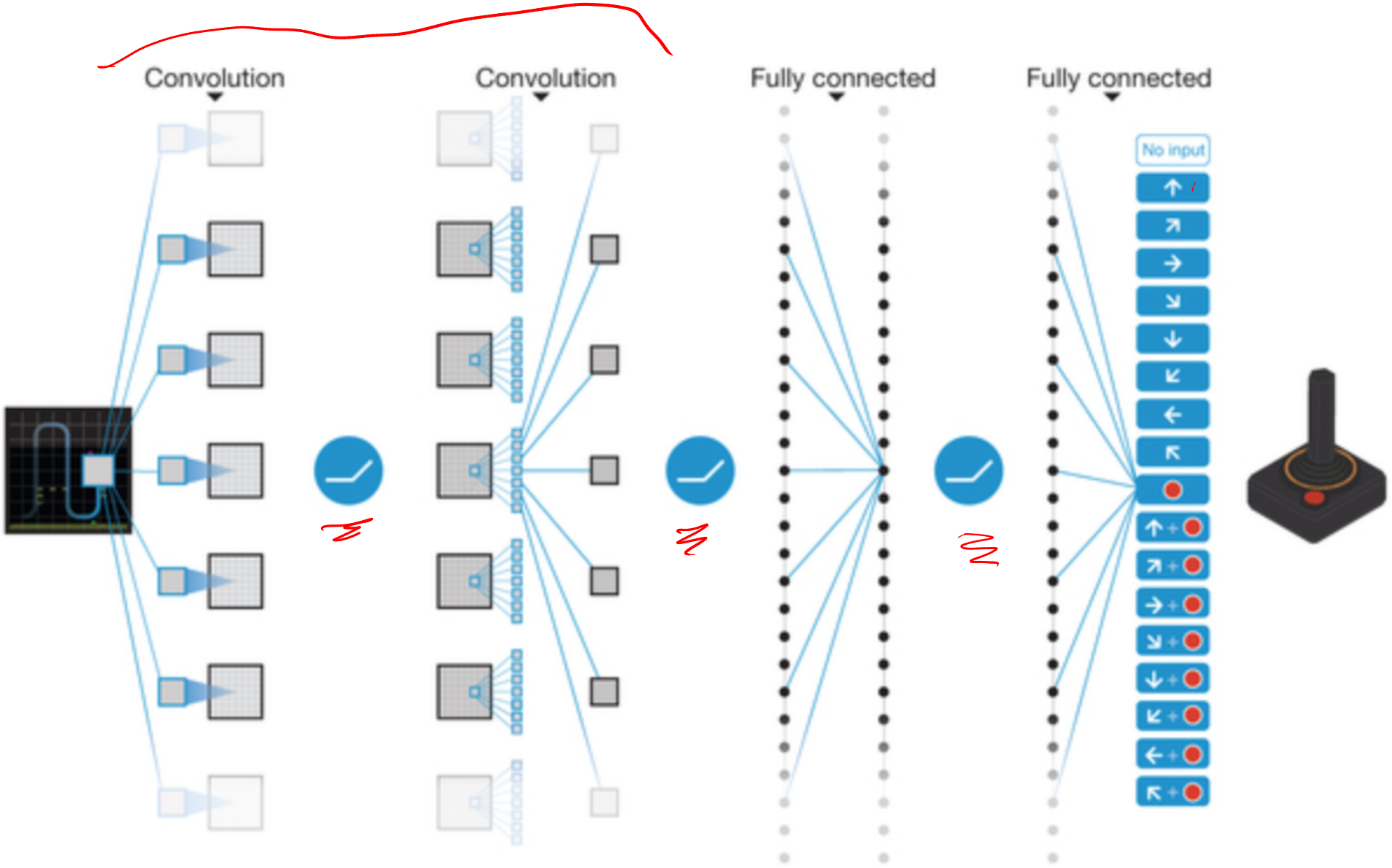
Playing Atari with Deep Reinforcement Learning

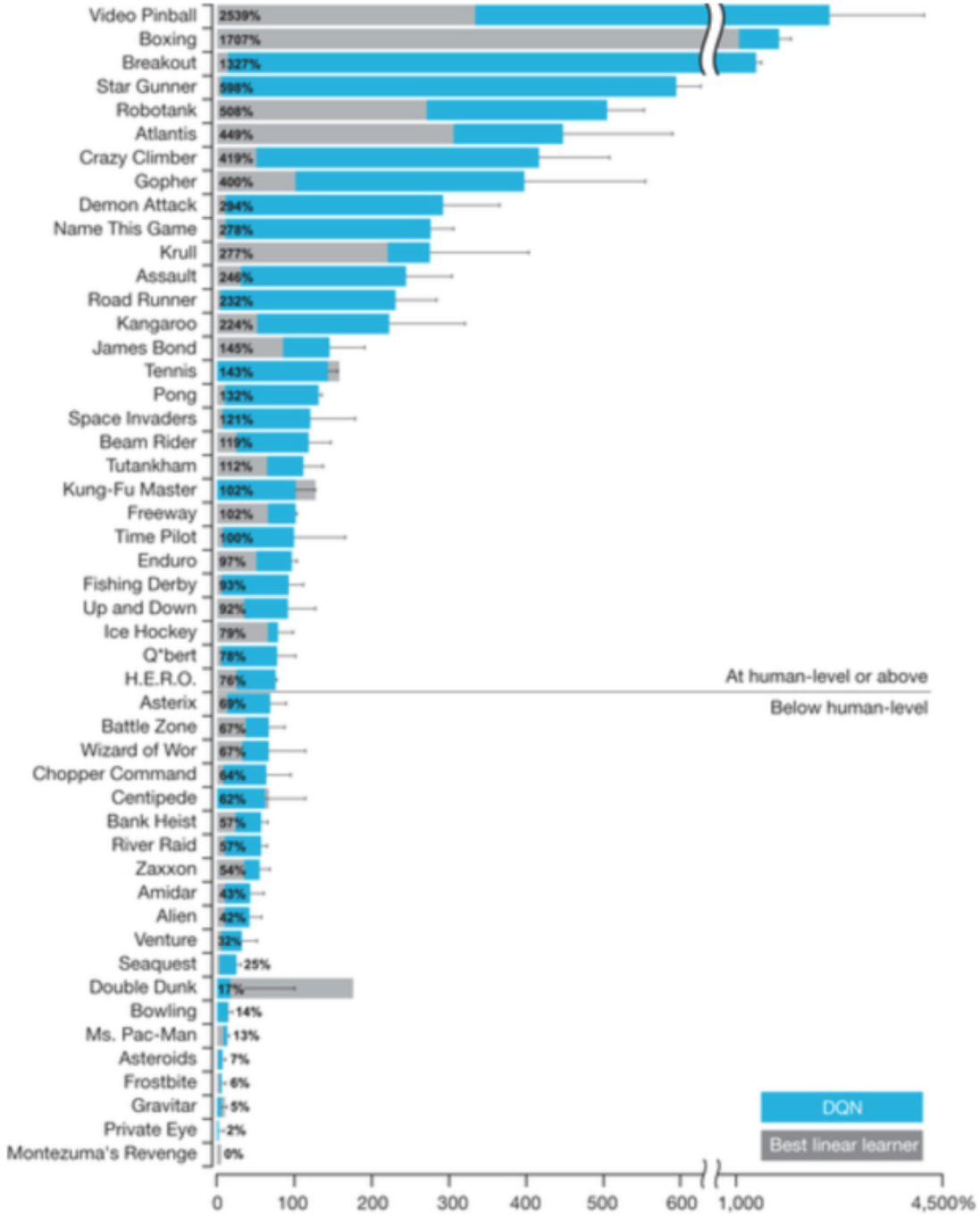
Volodymyr Mnih **Koray Kavukcuoglu** **David Silver** **Alex Graves** **Ioannis Antonoglou**

Daan Wierstra **Martin Riedmiller**

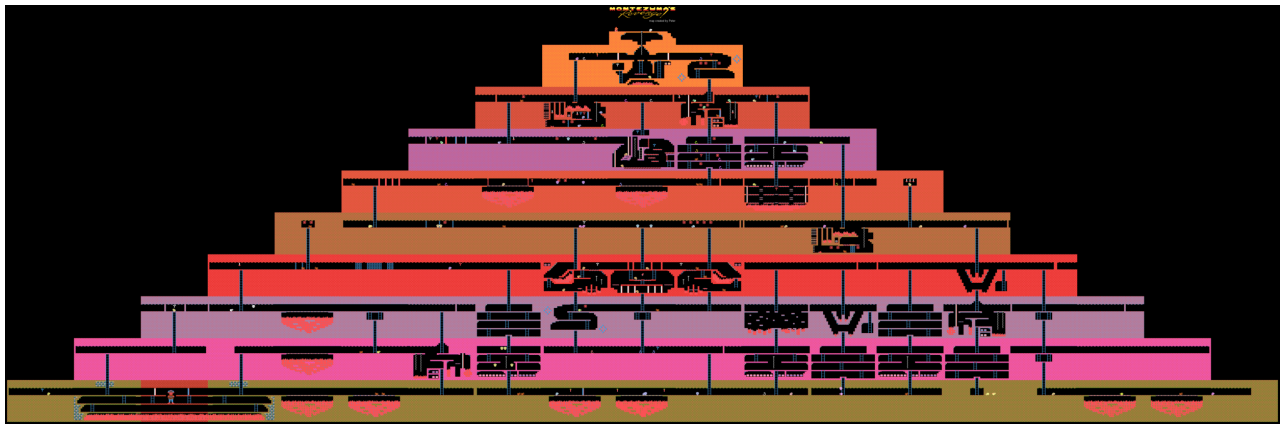
DeepMind Technologies




Convolution
Convolution
Fully connected
Fully connected
No input



https://www.youtube.com/watch?v=SuZVyOlgVek



Rainbow

Rainbow

- Double Q Learning

$$r + \gamma Q_{\theta_1'}\left(s', \operatorname{argmax}_{a'} Q_{\theta_2}(s', a')\right) - Q_{\theta_2}(s, a)$$

Rainbow

- Double Q Learning
- Prioritized Replay
  (priority proportional to last TD error)

Rainbow

- Double Q Learning
- Prioritized Replay
  (priority proportional to last TD error)
- Dueling networks
  Value network + advantage network
  $Q(s, a) = V(s) + A(s, a)$

Rainbow

- Double Q Learning
- Prioritized Replay
  (priority proportional to last TD error)
- Dueling networks
  Value network + advantage network
  $Q(s,a) = V(s) + A(s,a)$
- Multi-step learning
  $(r_t + \gamma r_{t+1} + \ldots + \gamma^{n-1} r_{t+n-1} + \gamma \max Q_\theta(s_{t+n}, a') - Q_\theta(s_t, a_t))^2$

Rainbow

- Double Q Learning
- Prioritized Replay
  (priority proportional to last TD error)
- Dueling networks
  Value network + advantage network
  $Q(s,a) = V(s) + A(s,a)$
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- Distributional RL
  predict an entire distribution of values instead of just Q

Rainbow

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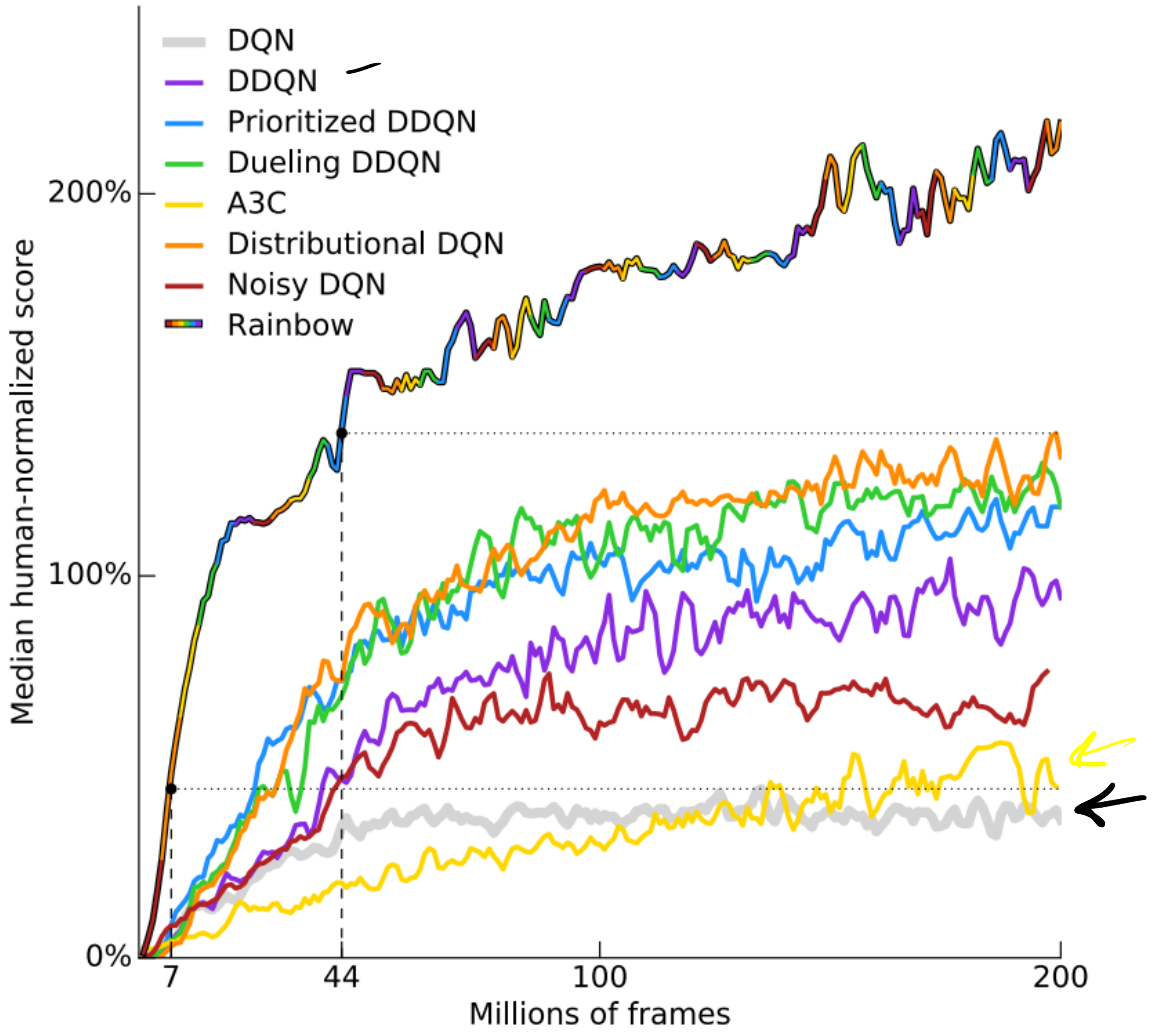
- Distributional RL
  predict an entire distribution of values instead of just Q
- Noisy Nets

Rainbow



- Double Q Learning
- Prioritized Replay
  (priority proportional to last TD error)
- Dueling networks
  Value network + advantage network
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- Distributional RL
  predict an entire distribution of values instead of just Q
- Noisy Nets

Part II
Improved Policy Gradients

Restricted Gradient Update

Restricted Gradient Update

$$\widehat{\nabla U}(\theta) = \sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) \gamma^k \left(r_{k,\text{to-go}} - r_{\text{base}}(s_k)\right)$$

Restricted Gradient Update

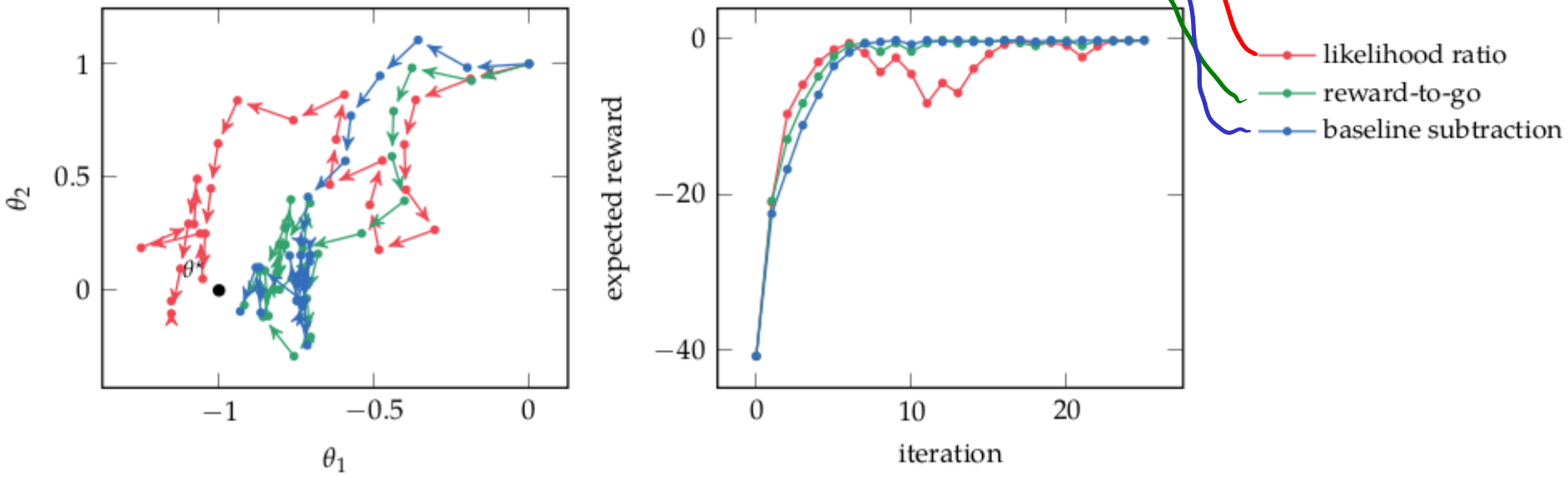
$$\widehat{\nabla U}(\theta) = \sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) \gamma^k (r_{k,\text{to-go}} - r_{\text{base}}(s_k))$$

$$\theta' = \theta + \alpha \widehat{\nabla U}(\theta)$$

Restricted Gradient Update

$$\widehat{\nabla U}(\theta) = \sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) \gamma^k (r_{k,\text{to-go}} - r_{\text{base}}(s_k))$$

$$\theta' = \theta + \alpha \widehat{\nabla U}(\theta)$$



Restricted Gradient Update

$$\widehat{\nabla U}(\theta) = \sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) \gamma^k (r_{k,\text{to-go}} - r_{\text{base}}(s_k))$$

$$\theta' = \theta + \alpha \widehat{\nabla U}(\theta)$$

1-step

$$\underset{\theta'}{\text{maximize}} \quad U(\theta') \approx U(\theta) + \nabla U(\theta)^T (\theta' - \theta)$$

$$\text{subject to} \quad g(\theta, \theta') \leq \varepsilon$$

$$g(\theta, \theta') = \| \theta - \theta' \|_2^2 = \frac{1}{2} (\theta' - \theta)^T (\theta' - \theta)$$

$$\theta' = \theta + u \sqrt{\frac{2\varepsilon}{u^T u}} = \theta + \sqrt{2\varepsilon} \frac{u}{\|u\|}$$

$$u = \nabla U(\theta)$$

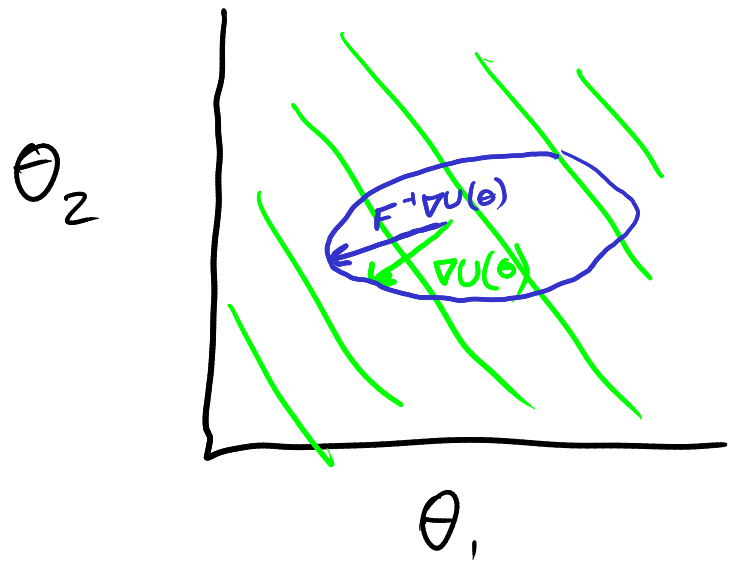
Natural Gradient

$$g(\theta,\theta') = D_{KL}\left(p(\tau|\theta) \,\|\, p(\tau|\theta')\right) \le \varepsilon$$

$$\underset{\theta'}{\text{maximize}} \quad U(\theta') \approx U(\theta) + \nabla U(\theta)^T(\theta' - \theta)$$

$$\text{s.t.} \quad g(\theta,\theta') = \frac{1}{2}(\theta' - \theta)^T F_\theta (\theta' - \theta) \le \varepsilon$$

$$\theta' = \theta + u\sqrt{\frac{2\varepsilon}{\nabla U(\theta)^T u}} \qquad u = F_\theta^{-1}\nabla U(\theta)$$

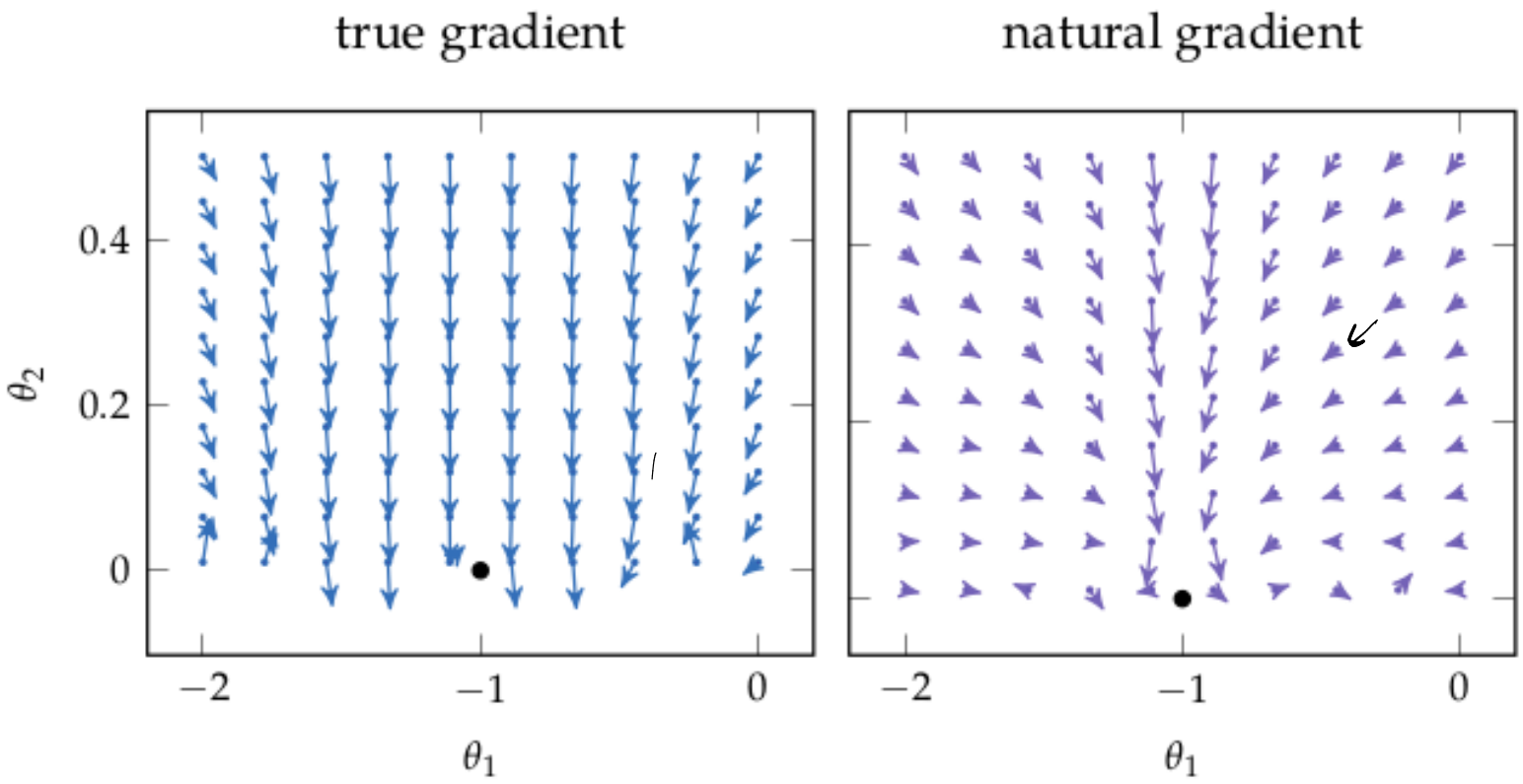


$$D_{KL}(p\|q) = \int p(x)\log\frac{p(x)}{q(x)}dx$$
$$= -\int p(x)\log\frac{q(x)}{p(x)}dx$$

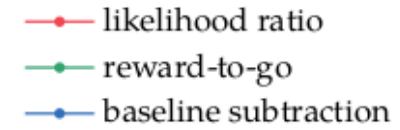
$$F_\theta = \int p(\tau|\theta)\nabla\log p(\tau|\theta)\nabla\log p(\tau|\theta)^T d\tau$$
$$= \mathop{E}_{\tau}\left[\nabla\log p(\tau|\theta)\,\nabla\log p(\tau|\theta)^T\right]$$

Natural Gradient

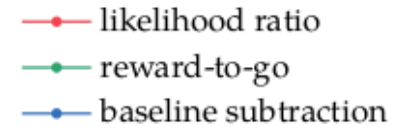
Natural Gradient



TRPO and PPO



TRPO and PPO



TRPO = Trust Region Policy Optimization

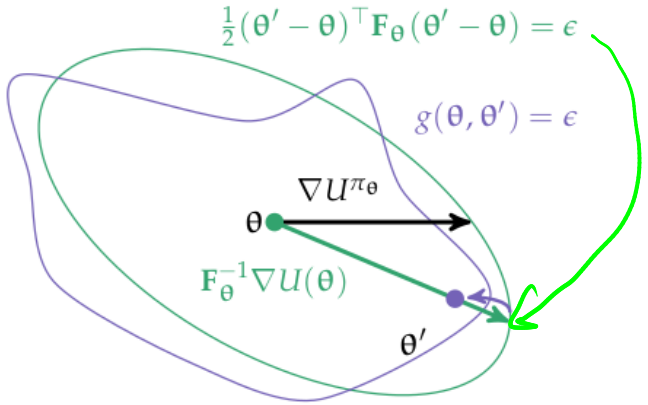
(Natural gradient + line search)

TRPO and PPO

likelihood ratio
reward-to-go
baseline subtraction

TRPO = Trust Region Policy Optimization

(Natural gradient + line search)



TRPO and PPO

TRPO = Trust Region Policy Optimization

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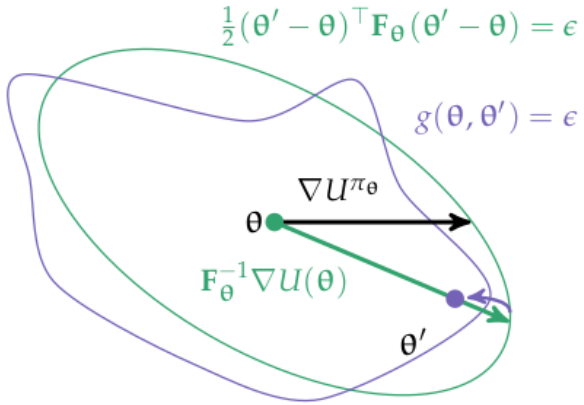
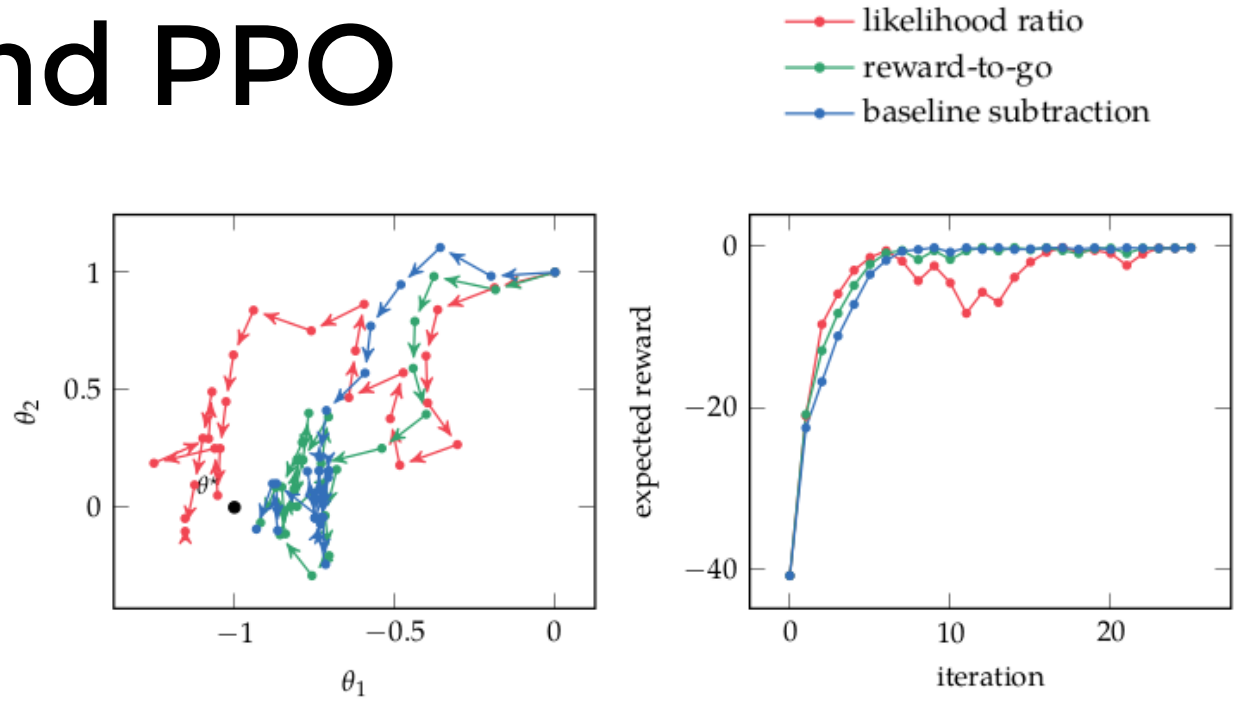
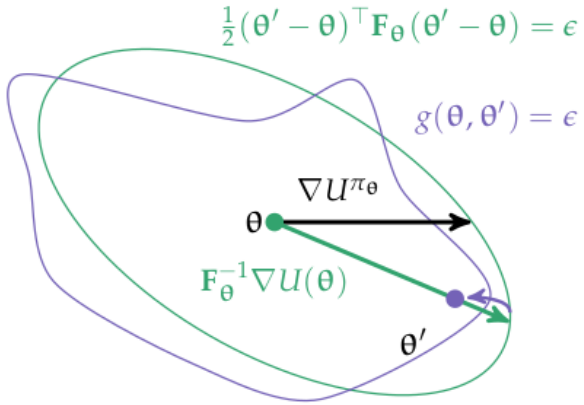


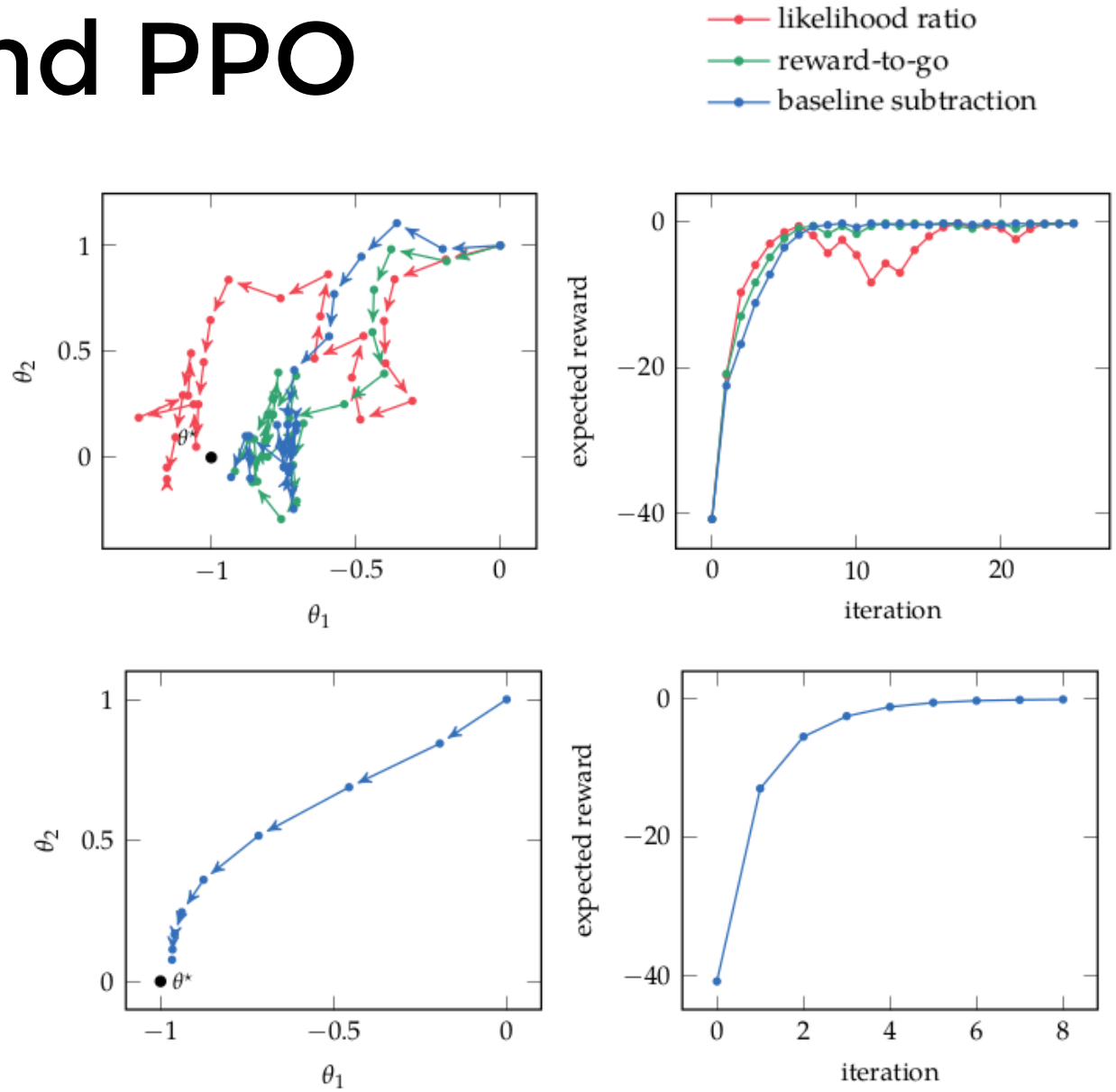


TRPO and PPO

TRPO = Trust Region Policy Optimization

(Natural gradient + line search)

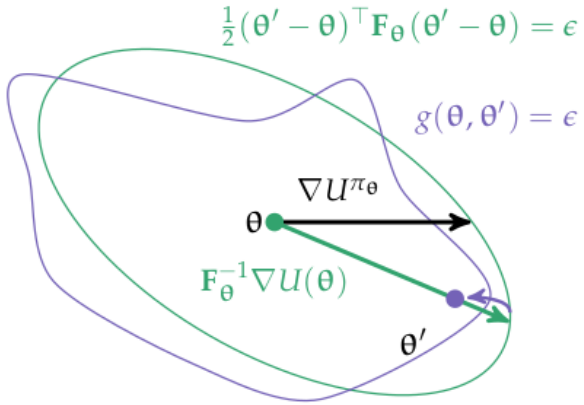


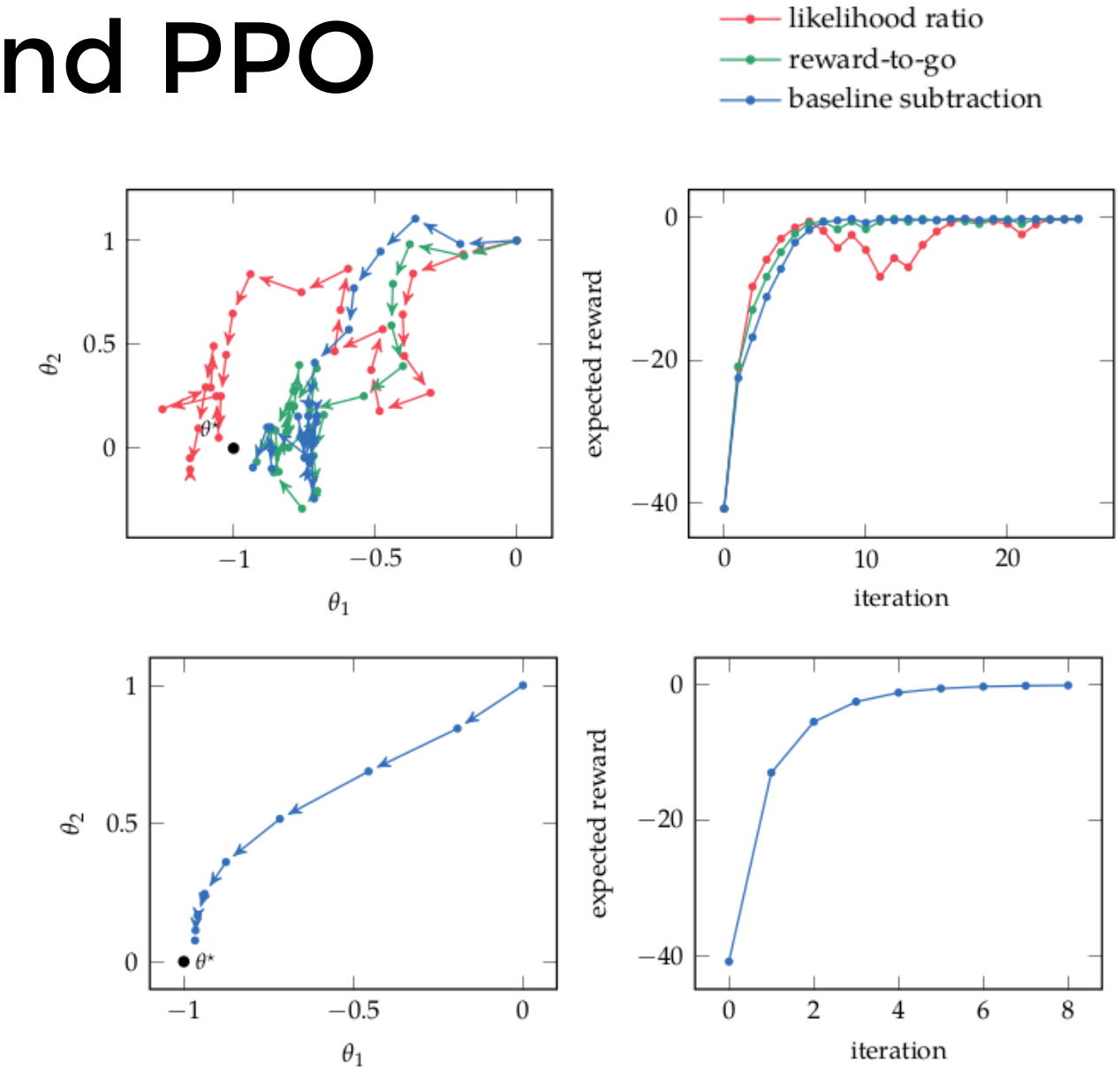


TRPO and PPO

TRPO = Trust Region Policy Optimization

(Natural gradient + line search)





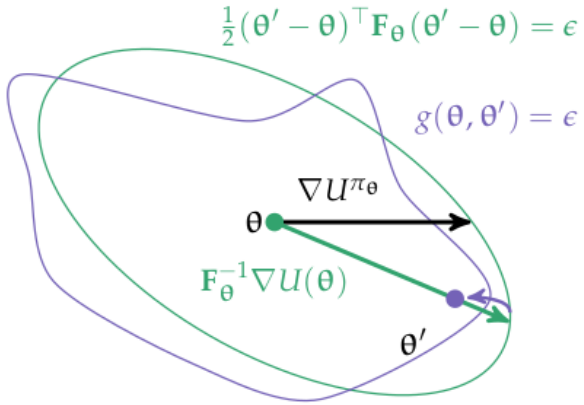
→ PPO = Proximal Policy Optimization

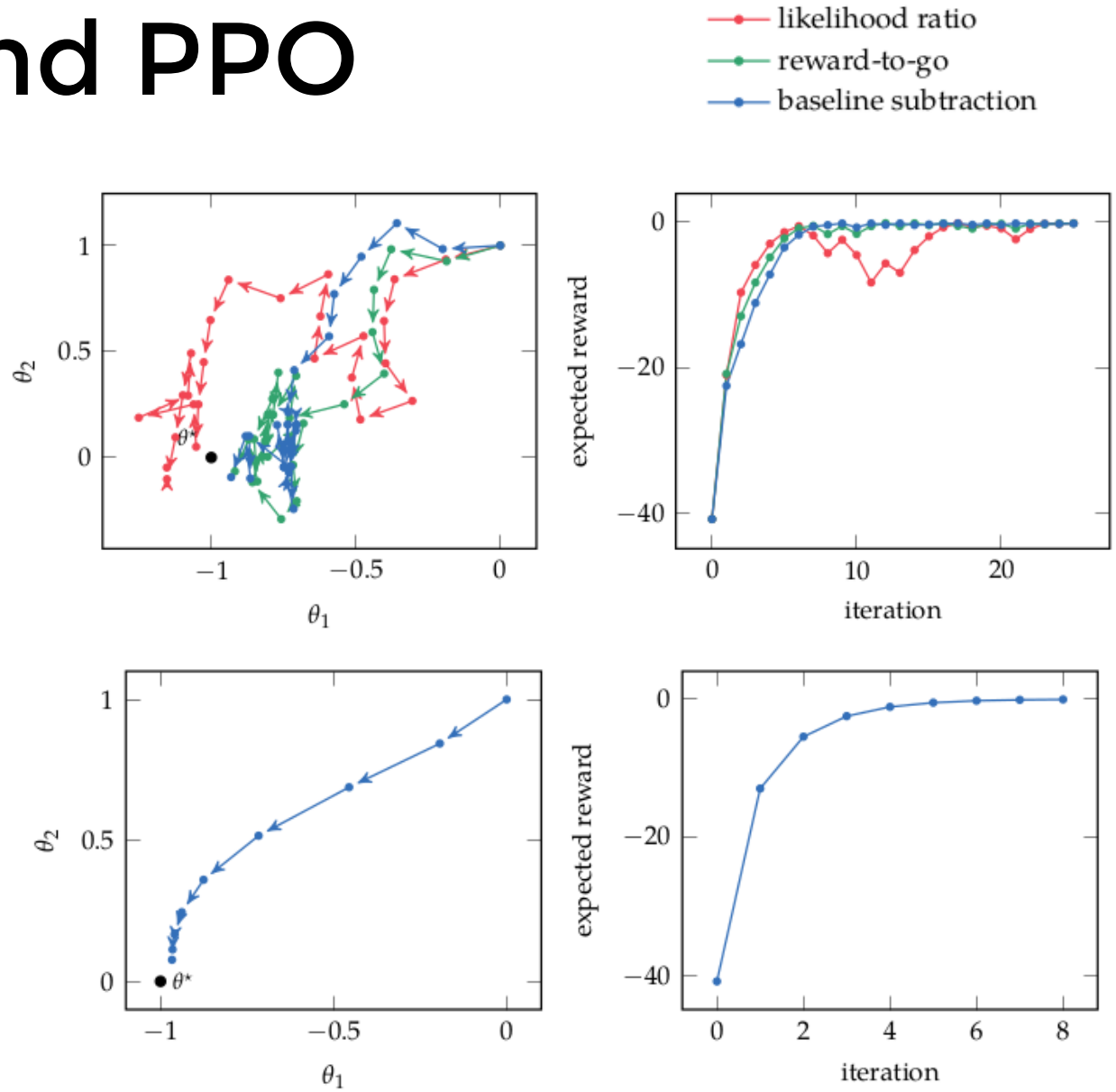
(Use clamped surrogate objective to remove the need for line search)

TRPO and PPO

TRPO = Trust Region Policy Optimization

(Natural gradient + line search)





PPO = Proximal Policy Optimization

(Use clamped surrogate objective to remove the need for line search)

https://openai.com/blog/openai-baselines-ppo/

Part III
Actor-Critic

Actor-Critic

$$\nabla U(\theta) = E_{\tau}\left[\sum_{k=0}^{d} \nabla_{\theta} \log \pi_{\theta}(a_k|s_k)\, \gamma^k \left(r_{to\text{-}go,k} - v_{base}^{(s_k)}\right)\right]$$

Actor

$Q_\phi(s,a) - V_\phi(s)$

$A_\phi(s,a)$

Critic

Actor-Critic

Which should we learn? A, Q, or V?

Actor-Critic

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$$\nabla U(\theta) = E_\tau \left[\sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) \gamma^k (r_k + \gamma V_\phi(s_{k+1}) - V_\phi(s_k)) \right]$$

Actor-Critic

Which should we learn? A, Q, or V?

$$\nabla U(\theta) = E_\tau \left[\sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) \gamma^k (r_k + \gamma V_\phi(s_{k+1}) - V_\phi(s_k)) \right]$$

$\sim Q(s,a)$

temporal difference residual

Actor-Critic

Which should we learn? A, Q, or V?

$$\nabla U(\theta) = E_\tau \left[\sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) \gamma^k (r_k + \gamma V_\phi(s_{k+1}) - V_\phi(s_k)) \right]$$

temporal difference residual

$$l(\phi) = E\left[\left(V_\phi(s) - V^{\pi_\theta}(s) \right)^2 \right]$$

Actor-Critic

Which should we learn? A, Q, or V?

$$\nabla U(\theta) = E_\tau \left[\sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) \gamma^k (r_k + \gamma V_\phi(s_{k+1}) - V_\phi(s_k)) \right]$$

temporal difference residual

$$l(\phi) = E\left[\left(V_\phi(s) - V^{\pi_\theta}(s) \right)^2 \right]$$

↑ estimate with reward to go from sims

Generalized Advantage Estimation

$$A(s_k, a_k) \approx r_k + \gamma V_\phi(s_{k+1}) - V_\phi(s_k)$$ ← High Bias

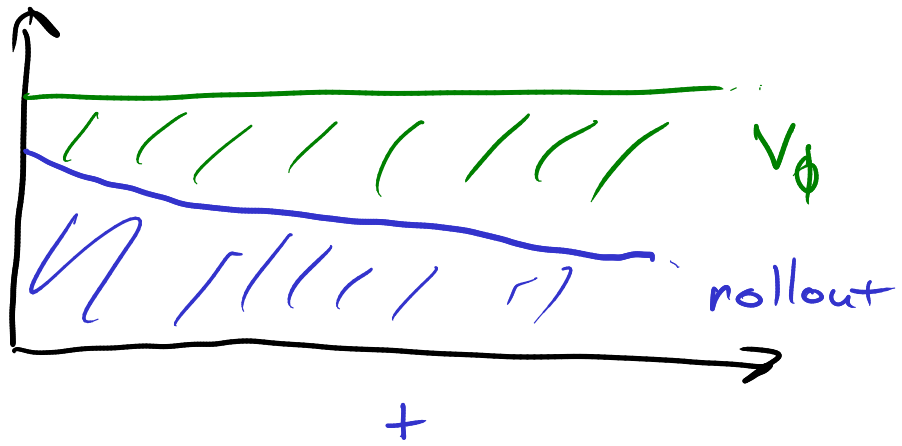
$$A(s_k, a_k) \approx \sum_{t=k}^{\infty} \gamma^{t-k} r_t - V_\phi(s_k)$$ ← High Variance

$$\approx \underbrace{r_k + \gamma r_{k+1} \cdots \gamma^d r_{k+d} + \gamma^{d+1} V_\phi(s_{k+d+1})} - V_\phi(s_k)$$

↑ How do we choose when to stop

let $\delta_t = r_t + \gamma V_\phi(s_{t+1}) - V_\phi(s_t)$

$$A^{GAE}(s_k, a_k) = \sum_{t=k}^{\infty} (\gamma\lambda)^{t-k} \delta_t$$



Recap

DQN

Policy Gradient → PPO

Actor - Critic

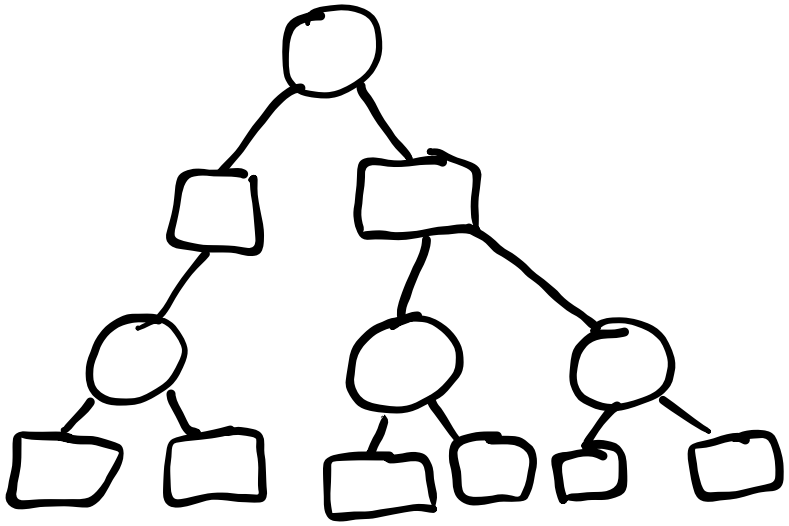
Alpha Zero: Actor Critic with MCTS

Alpha Zero: Actor Critic with MCTS

1. Use π_θ and U_ϕ in MCTS

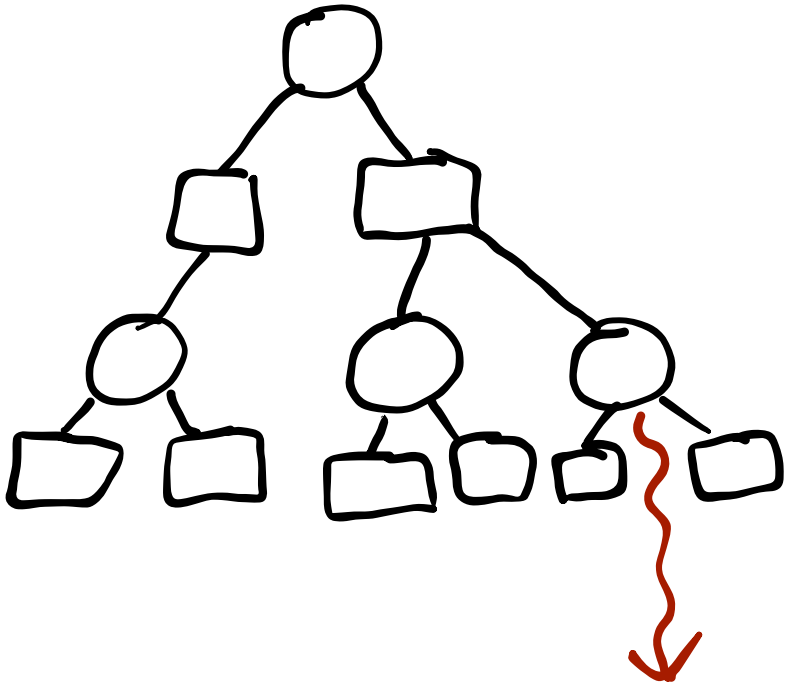
Alpha Zero: Actor Critic with MCTS

1. Use π_θ and U_ϕ in MCTS



Alpha Zero: Actor Critic with MCTS

1. Use π_θ and U_ϕ in MCTS



Alpha Zero: Actor Critic with MCTS

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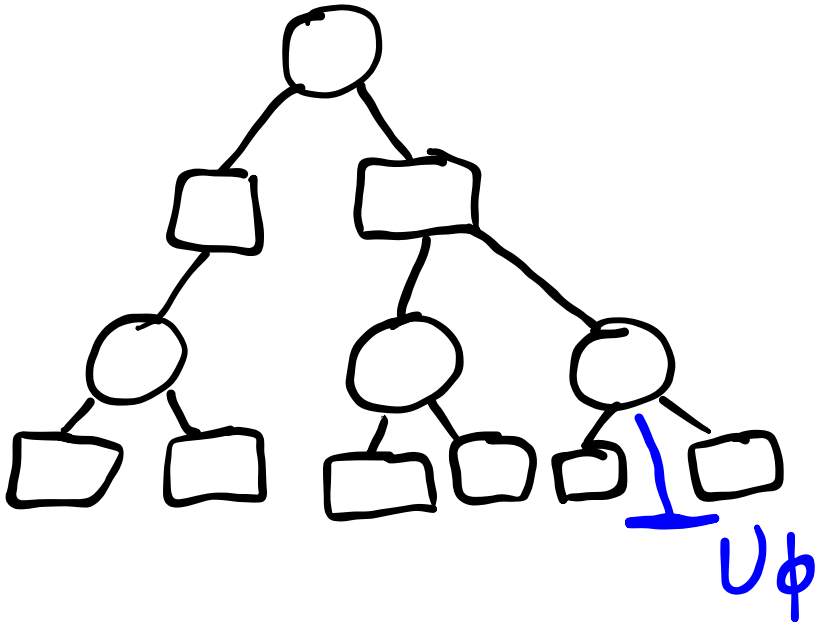

Alpha Zero: Actor Critic with MCTS

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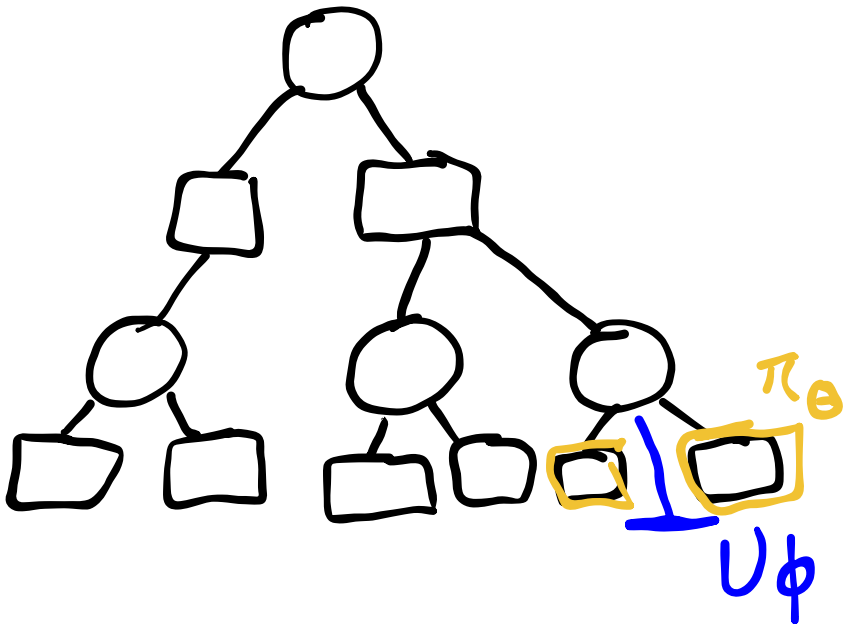
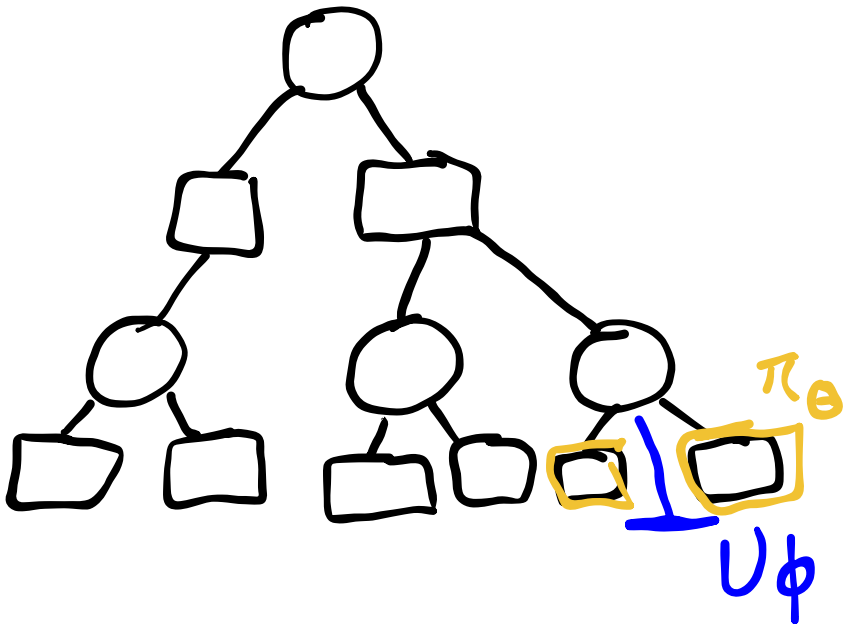


$$a = \arg\max_a Q(s,a) + c\pi_\theta(a \mid s)\frac{\sqrt{N(s)}}{1+N(s,a)}$$

Alpha Zero: Actor Critic with MCTS

1. Use π_θ and U_ϕ in MCTS
2. Learn π_θ and U_ϕ from tree



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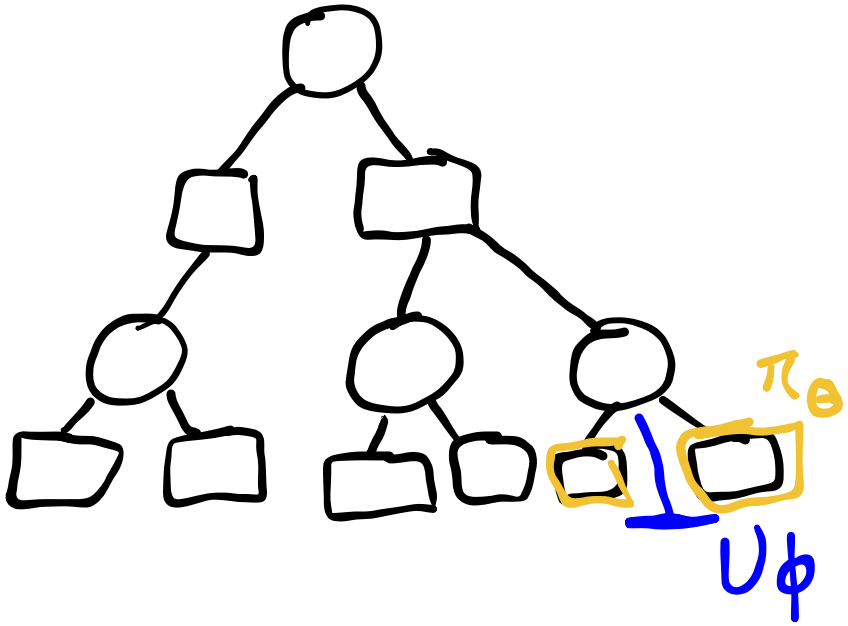
Alpha Zero: Actor Critic with MCTS

1. Use π_θ and U_ϕ in MCTS
2. Learn π_θ and U_ϕ from tree

$$\ell(\theta) = -\mathbb{E}_s \left[\sum_a \pi_{\text{MCTS}}(a \mid s) \log \pi_\theta(a \mid s) \right]$$

$$\pi_{\text{MCTS}}(a \mid s) \propto N(s,a)^\eta$$



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Alpha Zero: Actor Critic with MCTS

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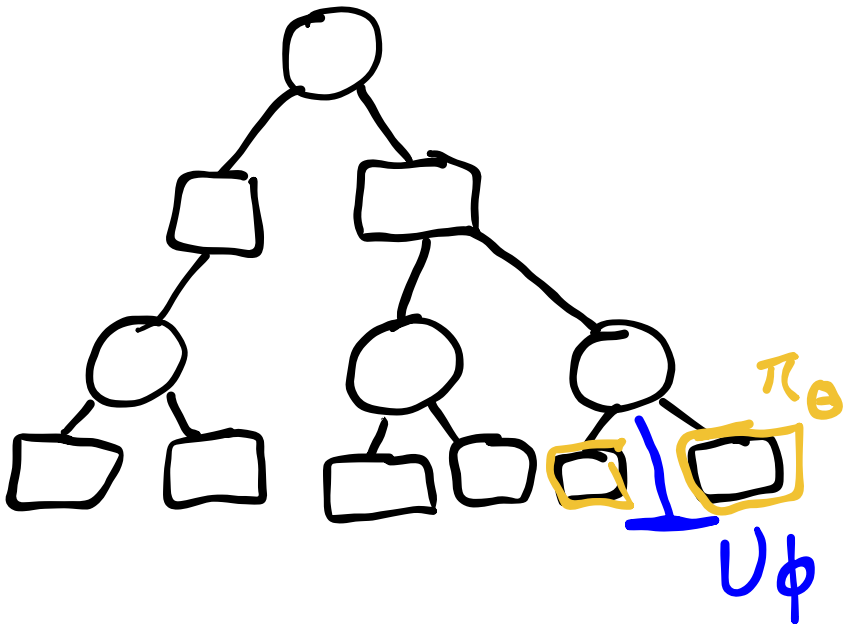

$$\ell(\phi) = \frac{1}{2}\mathbb{E}_s\left[\left(U_\phi(s) - U_{\text{MCTS}}(s)\right)^2\right]$$

$$U_{\text{MCTS}}(s) = \max_a Q(s,a)$$

$$a = \arg\max_a Q(s,a) + c\pi_\theta(a \mid s)\frac{\sqrt{N(s)}}{1+N(s,a)}$$