Question 1

a)

	a	b
a	5,4	2,3
b	4,1	3,2

As shown using the best-response arrows,

(a,a) and (b,b) are the pure Nash equilibria.

b) This game does not have a dominant strategy equilibrium because at least one player has no strategy that is a best response to any opponent strategy.

In particular if Player 1 plays a, Player 2's best response is a, but if Player 1 plays b, then Player 2's best response is b.

c)

	a	b
a	2,2	3,1
b	3,1	2,2

If $x=2$ and $y=2$, then, according to the best response arrows drawn at left, there is no pure Nash equilibrium.

d) Since all finite simple games have a Nash equilibrium, the game above has a Nash equilibrium.

Since there are no pure Nash equilibria, this game must have a fixed Nash equilibrium.

Question 2

a) $B \perp F \mid D$?

Enumerate all paths.

$B \to A \to F$ ← not d-separated by D

$B \to A \leftarrow D \to C \to F$ (no need to analyze this path)

Since a path is not d-separated, we $\boxed{\text{cannot}}$ conclude from the structure that $B \perp F \mid D$.

b) $B \perp F \mid A$?

Enumerate all paths

$B \to A \to F$ ← d-separated

$\underbrace{B \to A \leftarrow}_{\text{v-structure contains } A} D \to C \to F$ ← not d-separated

Since a path is not d-separated, we $\boxed{\text{cannot}}$ conclude from the structure that $B \perp F \mid A$

c) $B \perp E \mid A$

Enumerate all paths

$B \to A \to E$ ← d-separated

Since all paths are d-separated, we can conclude that $B \perp E \mid A$.

Question 3

a) $T(s'=2 \mid s=1, a^1 \sim \pi^1, a^2=1)$

$$= \sum_{a \in A^1} T(s'=2 \mid s=1, a^1=a, a^2=1)\, \pi^1(a)$$

$$= 0.8\, \overbrace{T(s'=2 \mid s=1, a^1=0, a^2=1)}^{1} + 0.2\, \cancelto{0}{T(s'=2 \mid s=1, a^1=1, a^2=1)}$$

$$\boxed{= 0.8}$$

b) 1) First we form an MDP, M', where the actions are the actions that player 2 takes and π^1 is integrated into the transition and reward functions.

2) Solving M' will yield a best response for player 2.

c) Transition matrices for M':

$$T'^{0} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.2 & 0.8 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad T'^{1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Perform Bellman Value Backup

S	V(S)	A	Q(S,a)
3	0	(terminal)	
2	1	0	$R^2(2,0) + \gamma(1 \cdot V(3)) = 1$
		1	$R^2(2,1) + \gamma(1 \cdot V(3)) = 1$
1		0	$R^2(1,0) + \gamma(0.2\, V(1) + 0.8\, V(2))$
		1	$R^2(1,1) + \gamma(0.2\, V(2) + 0.8\, V(3)) = 0.2$

$\rightarrow Q(1,0) = 0.2\, Q(1,0) + 0.8\, V(2)$

$0.8\, Q(1,0) = 0.8\, V(2)$

$\underline{Q(1,0) = 1} \longrightarrow$ The best response for player 1 at state 1 is $a^1 = 0$ (deterministic)

Question 4

$\boxed{\text{No}}$

If (π^{1*}, π^2) is a Nash equilibrium, then π^{1*} and π^2 must be best responses to each other. We know that π^{1*} is a best response to π^2, but π^2 may not be a best response to π^{1*}.

Question 5

Since player 2 can no longer distinguish whether they were dealt a King or Ace, $I_{2,1}$ and $I_{2,2}$ would be combined into a single information set.