ASEN 5264 Decision Making under Uncertainty
Homework 3: Online MDP Methods

February 6, 2026

A submission should consist of two or three files:
- A single PDF file containing answers to questions (typed, handwritten, or exported notebook).
- JSON file output from `HW3.evaluate`.
- A code listing *if the code is not included in the PDF.*

1 Conceptual Questions

Question 1. (10 pts)

a) Suppose that the optimal value function U^* for an MDP with discrete state and action spaces is known. Write an equation in terms of (S, A, T, R, γ) for extracting the optimal policy π^* from U^*.

b) Suppose you have an MDP defined by

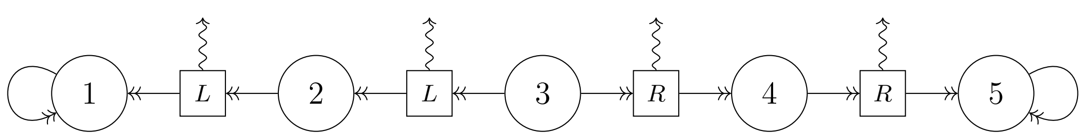
$$S = \mathbb{R}, \quad A = \{0, 0.1, 1\}, \quad \gamma = 1.0$$
$$T(s'|s,a) = \mathcal{U}([s+a, s+a+1])$$
$$R(s,a) = -\mathbb{1}(s \leq 1) - a$$

and a policy $\tilde{\pi}$ with $U^{\tilde{\pi}}(s) = -2 \cdot \mathbb{1}(s \leq 1)$. If the current state is 0.8, which action maximizes $Q^{\tilde{\pi}}(s,a)$? Justify your answer.

Question 2. (10 pts) Do similar Q values imply similar rewards? Consider the following claim:

> If a policy π satisfies $|Q^*(s,\pi^*(s)) - Q^*(s,\pi(s))| \leq \beta$ for all $s \in S$ for some $\beta > 0$, then it immediately follows that $|R(s,\pi^*(s)) - R(s,\pi(s))| \leq \beta$ for any $s \in S$.

It turns out that this claim is incorrect.[1] In this exercise, you will formulate a counterexample demonstrating that it is false. Consider the MDP below:



The state space is $S = \{1,\ldots,5\}$ and the action space is $A = \{L, R\}$ (but not all actions are available from each state). Transitions are deterministic as shown. The discount factor is $\gamma = 0.9$.

Choose a reward function, R, (i.e. values for the squiggly arrows), a policy, π, and a value β that constitute a counterexample to the claim above.[2] Justify your answer.

[1] Even seasoned researchers can be tripped up by this - this claim was erroneously made in the proof for Lemma 5 of the Sparse Sampling paper by Kearns, Mansour, and Ng `https://www.cis.upenn.edu/~mkearns/papers/sparsesampling-journal.pdf`.

[2] To demonstrate that you have found a counterexample, use the following steps: (1) Choose R, π and β (note that to choose π, you only have to choose $\pi(3)$ because all other actions are pre-determined. (2) Verify that $Q^*(s,\pi^*(s))$ and $Q^*(s,\pi(s))$ are closer than β for all states. (3) Find one state where the difference between $R(s,\pi^*(s))$ and $R(s,\pi(s))$ is greater then β. (4) If it is not possible, then revise R, π, and β and try again.

2 Exercises

`HW3.DenseGridWorld()` generates a 60x60 grid world MDP. There is a reward of +100 every 20 cells, i.e. at [20,20], [20,40], [40,20], etc. After the agent reaches one of these reward cells, the problem terminates. All cells also have a cost. Only a generative transition model is available. You will use the following functions from POMDPs.jl to interact with this problem (or larger versions) in the rest of this assignment:

- `actions(m)`
- `@gen(:sp, :r)(m, s, a)`
- `isterminal(m, s)`
- `discount(m)`
- `statetype(m)`
- `actiontype(m)`

Question 3. (15 pts) Monte Carlo Policy Evaluation

a) Write a rollout simulation function for an MDP starting with the following code:

```
r_total = 0.0
t = 0
while !isterminal(mdp, s) && t < max_steps
    a = :down # replace this with a policy
    s, r = @gen(:sp,:r)(mdp, s, a)
    r_total += discount(m)^t*r
    t += 1
end
```

Use this function to perform a Monte Carlo evaluation of a uniform random policy on an MDP created with `HW3.DenseGridWorld(seed=3)`. Report the mean discounted reward estimate and standard error of the mean (SEM). Run enough simulations so that the SEM is less than 5.

b) Create a heuristic policy that improves upon the random policy by at least 50 reward units. Report the mean and standard error from a Monte Carlo evaluation.

Question 4. (20 pts) Monte Carlo Tree Search

Write code that performs 7 iterations of Monte Carlo Tree Search on an MDP created with `HW3.DenseGridWorld(seed=4)`, starting at state $(19, 19)$. You will need to produce three dictionaries:

- `Q` maps (s, a) tuples to Q value estimates.
- `N` maps (s, a) tuples to N, the number of times the node has been tried.
- `t` maps (s, a, s') tuples to the number of times that transition was generated during construction of the tree.

Then visualize the resulting tree with `HW3.visualize_tree(Q, N, t, SA[19, 19])`[3]. **Submit an image of the tree, the code used to generate it, and a few sentences describing the tree after 7 iterations** (e.g. which actions have the highest Q values? Does this make sense?).

Question 5. (15 pts) Planning with MCTS

Use your Monte Carlo tree search from Question 4 to plan online in the simulation loop. Use 1000 iterations of MCTS to choose each action. Evaluate the MCTS planner with 100 100-step Monte Carlo simulations. Report the mean accumulated reward and standard error of the mean.

[3] `SA` is from the `StaticArrays.jl` package.

3 Challenge Problem

Question 6. (10 pts code and description, 20 pts score) Fast Online Planning

Create a function `select_action(m,s)` that takes in a 100×100 `DenseGridWorld`, `m`, and a state `s`, and returns a near-optimal action. You may wish to base this code on the MCTS code that you wrote for Question 4. Evaluate this function with `HW3.evaluate` and **submit the resulting `json` file along with the code and a one paragraph to one page description of your approach**, including tuning parameters that worked well, the rollout policy, etc. A score of 50 will receive full credit. In order to achieve a score above 50, you will be limited to 50ms of planning time per step. There are no restrictions on this problem - you may wish to use a different algorithm, multithreading, etc. Starter code on github will give suggestions for timing and other details.