Notation

Discrete Sets in DMU

- A *set* is a collection of *unique* elements. The elements can be numbers, tuples, vectors, or any other objects. Examples:
$$\{1, 2, 4\}, \{A, B, C\}$$
- Two sets are **equal** if they contain the *exact* same elements:
$$\{1, 2, 4\} = \{1, 4, 2\}, \text{ but } \{1, 2, 4\} \neq \{1, 2, 3\}$$
- The symbol $\in$ is read "in", and indicates that an object is in a set, i.e.
$$1 \in \{1, 2\}, \text{ but } 3 \notin \{1, 2\}$$
- The **union operator**, $\cup$, is used to combine two sets so that the new one contains all of the elements of both sets:
$$\{1, 2, 3\} \cup \{1, 2, 4\} = \{1, 2, 3, 4\}$$
- The **Cartesian product**, $\times$, creates a new set of tuples of all possible combinations of the elements of the argument sets:
$$\{1, 2, 3\} \times \{A, B\} = \{(1, A), (2, A), (3, A), (1, B), (2, B), (3, B)\}$$
- An exponent is used to denote multiple applications of the Cartesian product operator:
$$A^3 = A \times A \times A$$
- Example: The state space for the grid world used on the homework is
$$S_{\text{grid world}} = \{1, ..., 10\}^2 \cup (-1, -1)$$
(since $(-1, -1)$ is used as a terminal state).

Continuous Sets in DMU

- Some sets contain an uncountably infinite number of elements. These are often constructed with the set of all real numbers, $\mathbb{R}$, or intervals of real numbers. In DMU, we refer to these sets as "continuous". Examples:
  - $[0, 1]$ is the set of all real numbers between 0 and 1 including 0 and 1.
  - $(-5, -4)$ is the set of all real numbers between -5 and -4, *not* including -5 and -4. (Confusingly, this clashes with the notation for tuples. Use context clues!)
- Cartesian products of $\mathbb{R}$ or intervals can be used to construct vector spaces:
  - $\mathbb{R}^3$ is the space of all 3-dimensional real-valued vectors.
  - $[0, 1] \times [2, 3]$ is a "box" with the bottom left corner at $(0, 2)$ and top right corner at $(1, 3)$.
- We can also combine continuous sets with discrete ones:
  - $(0, 1) \cup \{0, 1\} = [0, 1]$
  - $[1, 2] \times \{3, 4.5\}$ is the set of all 2-dimensional vectors where the first entry is between 1 and 2 (inclusive) and the second entry is 3 or 4.5. The vectors $(1.42, 3)$ and $(2, 4.5)$ are in this set, but $(0.56, 3)$ and $(1.42, 4)$ are not.

At times, a vector might be expressed as $[1, 2]$