Bayesian Network Recap

Basic Distributions

Binary R.V. $\Rightarrow$ support is $\{0, 1\}$

Bernoulli(θ) Coin Flip

If $X \sim$ Bernoulli(θ) $\quad P(X=1) = \theta$, $P(X=0) = 1-\theta$

X	$P(X)$
0	$1-\theta$
1	θ

Categorical(ϕ) $\quad$ Cat(ϕ)

(ϕ: vector)

If $X \sim \text{Cat}(\phi)$ then $\text{support}(X) = \{1, 2, \ldots, \dim(\phi)\}$

$P(X=1) = \phi_1$

$P(X=2) = \phi_2$

$\vdots$

$P(X=n) = \phi_n$

X	$P(X)$
1	ϕ_1
2	ϕ_2
$\vdots$	$\vdots$
n	ϕ_n

Causal Bayesian Networks

Each Node: R.V.

Each Edge: Causal relationship

Informally, B is a function of A with some additional uncertainty

Example:

$$B \sim \begin{cases} \text{Bernoulli}(0.2) & \text{if } A=0 \\ \text{Bernoulli}(0.7) & \text{if } A=1 \end{cases}$$

$P(B=1 \mid A=1) = 0.7$

$P(B=1 \mid A=0) = 0.2$

If we have a BN with nodes $X_1, \ldots X_n$

$$P(X_1, X_2, \ldots X_n) = P(X_1 \mid pa(X_1))\, P(X_2 \mid pa(X_2)) \ldots P(X_n \mid pa(X_n))$$

$$= \prod_{i \in 1..n} P(X_i \mid pa(X_i))$$

X_1	X_2	X_3	$P(X_1, X_2, X_3)$
0	0	0	
0	0	1	

A	B	C	P(A,B,C)
0	0	0	θ_1
0	0	1	θ_2
⋮			

2^3

$2^3 - 1$ indep parameters

7 parameters

$P(A|pa(A)) = P(A)$ Bernoulli( ) — 1 param

$P(B|pa(B)) = P(B|A)$ Bernoulli(·) for each possible value of A — 2 param

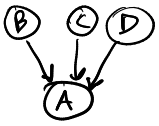
$P(C|pa(C)) = P(C|B)$ — 2 param

Since $P(A,B,C)$ is defined by $*$

Just need $P(X|pa(X))$

5 total parameters

A, B, C, D binary RV



$P(A|pa(A)) = P(A|B,C,D)$

$2^3 = 8$ possibilities

8 parameters

Independence $A \perp B$

$P(A|B) = P(A)$

$\frac{P(A,B)}{P(B)} = P(A)$

$P(A,B) = P(A)P(B)$

$P(A,B) = P(A)P(B)$

Conditional Independence

$A \perp B \mid C$

$P(A|B,C) = P(A|C)$

$\frac{P(A,B|C)}{P(B|C)} = P(A|C)$

$P(A,B|C) = P(A|C)P(B|C)$

$P(A,B|C) = P(A|C)P(B|C)$

$A \perp C \mid B$?

Yes

need to prove that

$P(A,C|B) = P(A|B)P(C|B)$

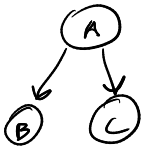
$P(A,B,C) = P(A|pa(A))P(B|pa(B))P(C|pa(C))$

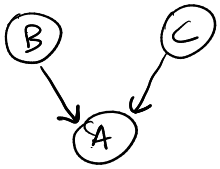
$= P(A)P(B|A)P(C|B)$

$\frac{P(A,B,C)}{P(B)} = \frac{P(A)P(B|A)P(C|B)}{P(B)}$

$P(A,C|B) = P(A|B)P(C|B)$

Similarly can prove $B \perp C \mid A$





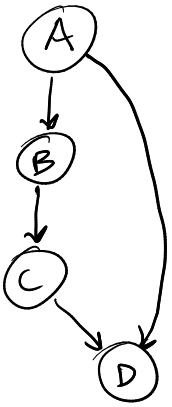
$B \perp C \mid A$? cannot prove from structure

but you could pick particular parameters where $B \perp C \mid A$

example: $A \sim \text{Bernoulli}(0.5)$ for any values of B, C

$B \sim$ " "

$C \sim$ " "



$A \perp C \mid B$? True

C

paths

$A \to B \to C$ rule 1: d-sep

$A \to D \leftarrow C$ rule 3: d-sep

all paths are d-sep

$A \perp C \mid B, D$? Inconclusive

paths

$A \to B \to C$ rule 1: d-sep

$A \to D \leftarrow C$ rule 3: not d-sep

not d-sep