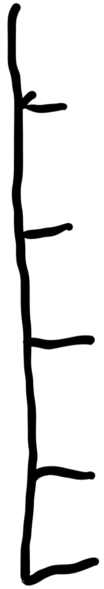
Recap

Recap

DMU



Recap

DMU

- Probabilistic Models
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Recap

DMU
- Probabilistic Models
- MDPs
- Reinforcement Learning
-
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Recap

DMU
- Probabilistic Models
- MDPs
- Reinforcement Learning
- POMDPs
- Games

Probabilistic Models

Probabilistic Models

3 Rules

$P(A)$

$P(A,B)$

$P(A|B)$

Probabilistic Models

$P(A)$

$P(A, B)$

$P(A|B)$

3 Rules

1. $0 \leq P(X \mid Y) \leq 1$

   $\sum_{x \in X} P(x \mid Y) = 1$

Probabilistic Models

$P(A)$
$P(A, B)$
$P(A \mid B)$

3 Rules

1. $0 \leq P(X \mid Y) \leq 1$
   $\sum_{x \in X} P(x \mid Y) = 1$

2. $P(X) = \sum_{y \in Y} P(X, y)$

Probabilistic Models

$P(A)$
$P(A, B)$
$P(A | B)$

3 Rules

1. $0 \leq P(X \mid Y) \leq 1$
   $\sum_{x \in X} P(x \mid Y) = 1$

2. $P(X) = \sum_{y \in Y} P(X, y)$

3. $P(X \mid Y) = \frac{P(X,Y)}{P(Y)}$

Probabilistic Models

$P(A)$
$P(A,B)$
$P(A|B)$

3 Rules

1. $0 \leq P(X \mid Y) \leq 1$
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3. $P(X \mid Y) \ = \frac{P(X,Y)}{P(Y)}$

Bayes Rule

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Probabilistic Models

$P(A)$
$P(A, B)$
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3 Rules

1. $0 \leq P(X \mid Y) \leq 1$
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Bayes Rule

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

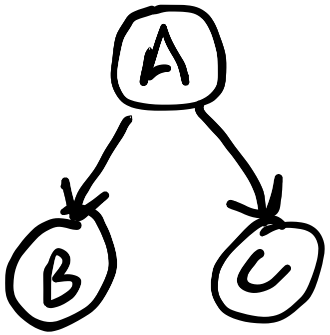
Independence

$$A \perp B \iff P(A, B) = P(A)P(B)$$

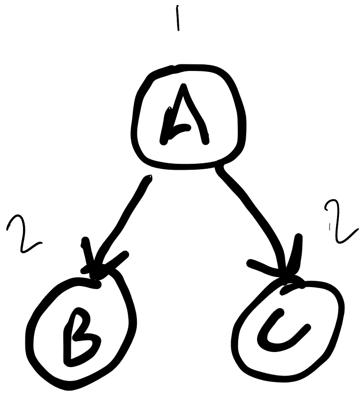
$$A \perp B \mid C \iff P(A, B \mid C) = P(A \mid C)P(B \mid C)$$

Bayesian Networks

Bayesian Networks



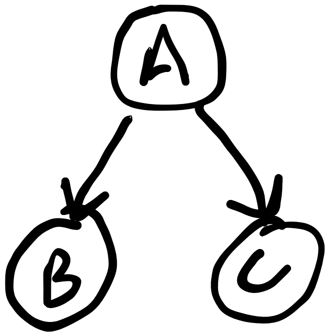
Bayesian Networks



Chain Rule

$$P(X_{1:n}) = \prod_i P(X_i \mid Pa(X_i))$$

Bayesian Networks



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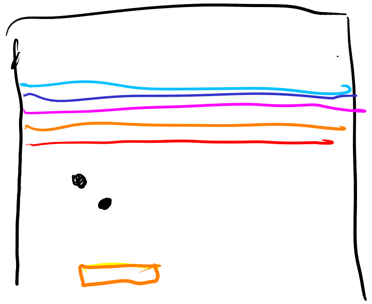
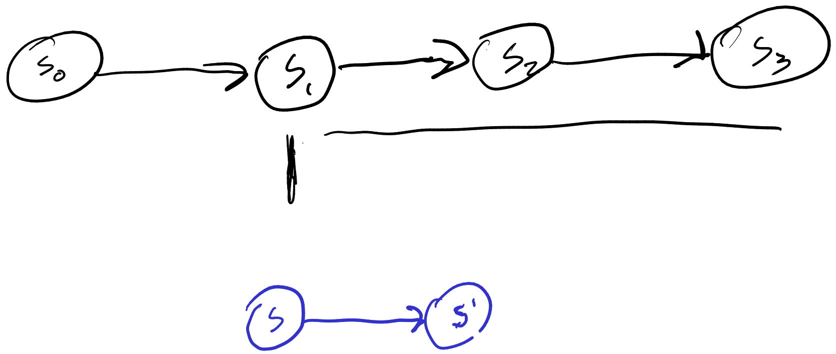
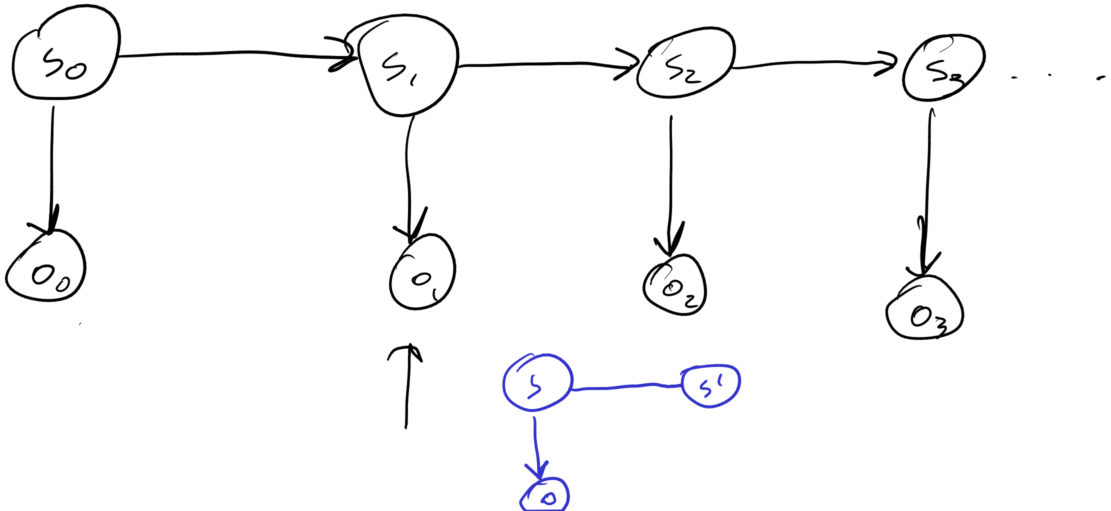


Conditional Independence

$X \perp Y \mid \mathcal{C}$ if all paths between X and Y are d-separated by $\mathcal{C}$



$o_2 \not\perp o_0 \mid o_1$



Markov Decision Processes

Markov Decision Processes

$$(S, A, R, T, \gamma)$$

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Examples: $S = \{1, 2, 3\}$ or $S = \mathbb{R}^2$

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Markov Decision Processes

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Examples: $S = \{1, 2, 3\}$ or $S = \mathbb{R}^2$

$$s = (x, \dot{x}) \in S = \mathbb{R}^2$$

$$\underset{\pi}{\text{maximize}} \; E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t)\right]$$

Markov Decision Processes

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Examples: $S = \{1, 2, 3\}$ or $S = \mathbb{R}^2$

$$s = (x, \dot{x}) \in S = \mathbb{R}^2$$

$$\underset{\pi}{\text{maximize}} \; E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t)\right]$$

$$Q^\pi(s,a) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \,\middle|\, s_0 = s, a_0 = a, a_t = \pi(s_t)\right]$$

Markov Decision Processes

$$(S, A, R, T, \gamma)$$

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$$V^\pi(s) = Q^\pi(s, \pi(s))$$

Markov Decision Processes

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$$V^\pi(s) = Q^\pi(s, \pi(s))$$

$$V^\pi(s) = R(s, \overset{\pi(s)}{\cancel{a}}) + \gamma E[V^\pi(s')]$$ ← policy evaluation

$$V^*(s) = \max_a \{R(s, a) + \gamma E[V^*(s')]\}$$ ← certificate of optimality "Bellman's Eq."

$$B[V](s) = \max_a \{R(s, a) + \gamma E[V(s')]\}$$ ← Bellman's operator

Markov Decision Processes

$$(S, A, R, T, \gamma)$$

Examples: $S = \{1, 2, 3\}$ or $S = \mathbb{R}^2$

$$s = (x, \dot{x}) \in S = \mathbb{R}^2$$

$$\text{maximize}_{\pi} \; E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t)\right]$$

$$Q^\pi(s,a) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \,\middle|\, s_0 = s, a_0 = a, a_t = \pi(s_t)\right]$$

$$V^\pi(s) = Q^\pi(s, \pi(s))$$

$$V^\pi(s) = R(s,a) + \gamma E[V^\pi(s')]$$ Policy Evaluation

$$V^*(s) = \max_a \{R(s,a) + \gamma E[V^*(s')]\}$$ Bellman's Equation: Certificate of Optimality

$$B[V](s) = \max_a \{R(s,a) + \gamma E[V(s')]\}$$ Bellman's Operator

Offline MDP Algorithms

Offline MDP Algorithms

Policy Iteration

loop

    Evaluate Policy

    Improve Policy

Offline MDP Algorithms

Policy Iteration

loop

  Evaluate Policy

  Improve Policy

Value Iteration

loop

  $V \leftarrow B[V]$

Offline MDP Algorithms

Policy Iteration

loop

    Evaluate Policy

    Improve Policy

Converges because policy always improves and there are a finite number of policies

Value Iteration

loop

    $V \leftarrow B[V]$

Offline MDP Algorithms

Policy Iteration

loop

Evaluate Policy

Improve Policy

Converges because policy always improves and there are a finite number of policies

Value Iteration

loop

$V \leftarrow B[V]$

Converges because B is a contraction mapping

Curse of Dimensionality $\rightarrow$ Large $|S|$

Online MDP Planning

Online MDP Planning

Monte Carlo Tree Search

Online MDP Planning

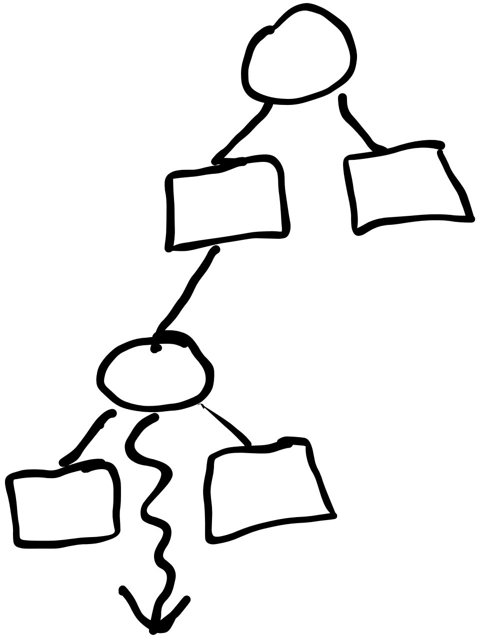
Monte Carlo Tree Search

Search

Expand

Rollout

Backup



Online MDP Planning

Monte Carlo Tree Search

Search

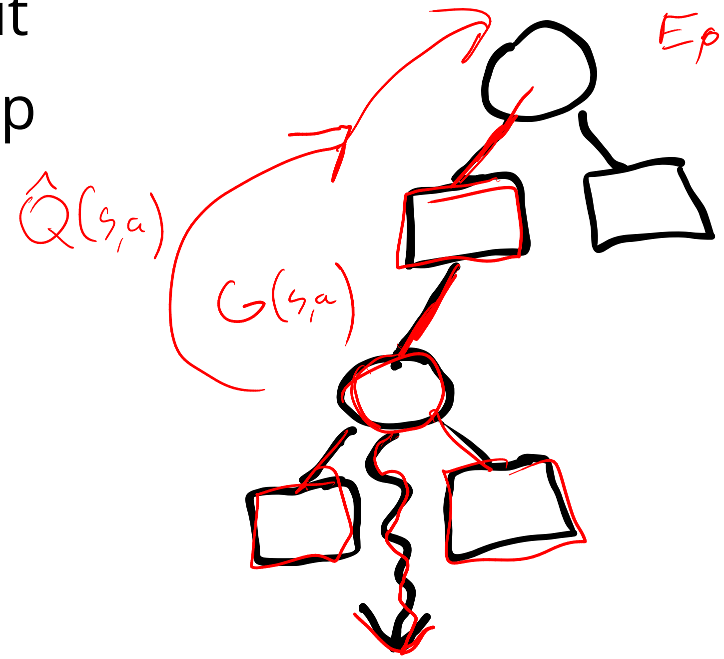
Expand

Rollout

Backup

Valuable

$$Q(s,a) + \underbrace{c\sqrt{\frac{\log N(s)}{N(s,a)}}}_{\text{Exploration Bonus}}$$



maximize $\hat{Q}(s,a)$ at root

or

maximize $N(s,a)$

Online MDP Planning

Monte Carlo Tree Search

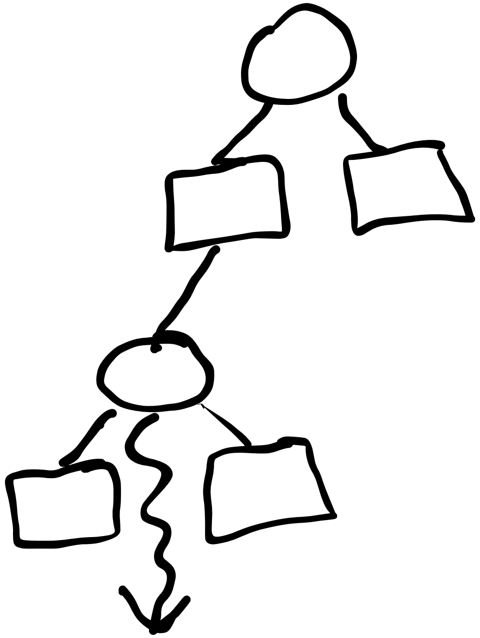
Search

Expand

Rollout

Backup

$$Q(s,a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$$



Sparse Sampling

Online MDP Planning

Monte Carlo Tree Search

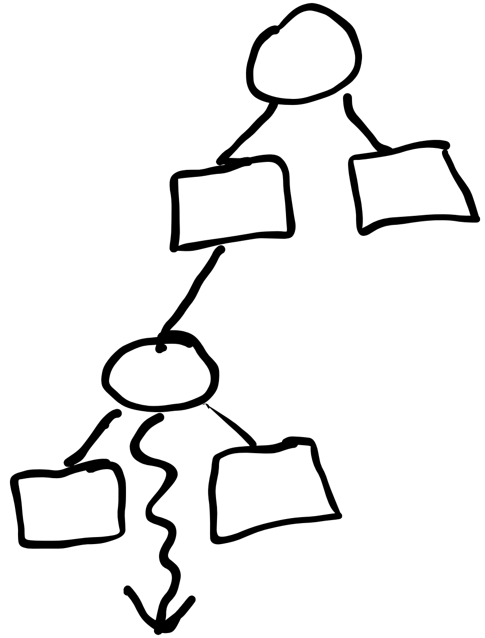
Search

Expand

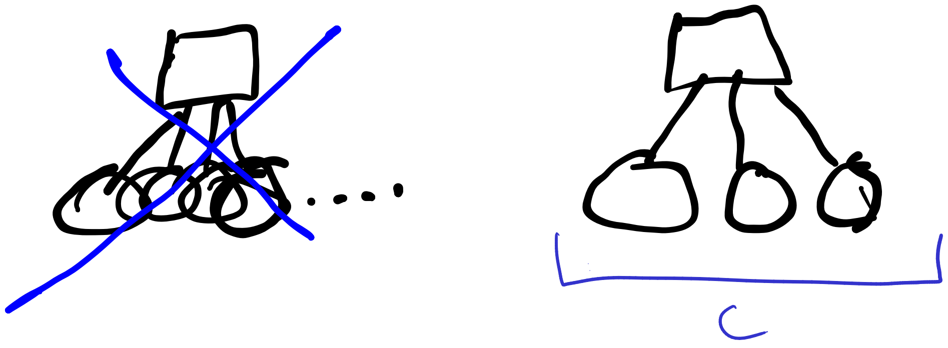
Rollout

Backup

$$Q(s,a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$$



Sparse Sampling



Online MDP Planning

Monte Carlo Tree Search

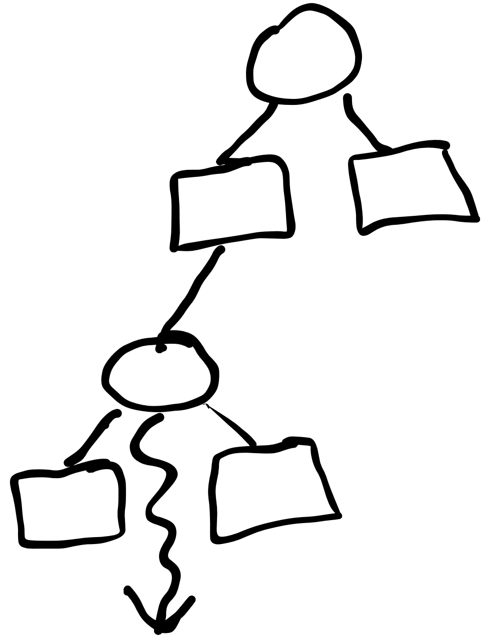
Search

Expand

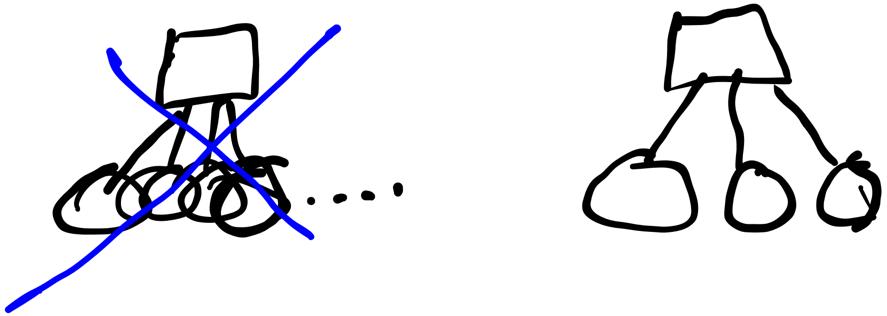
Rollout

Backup

$$Q(s,a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$$



Sparse Sampling

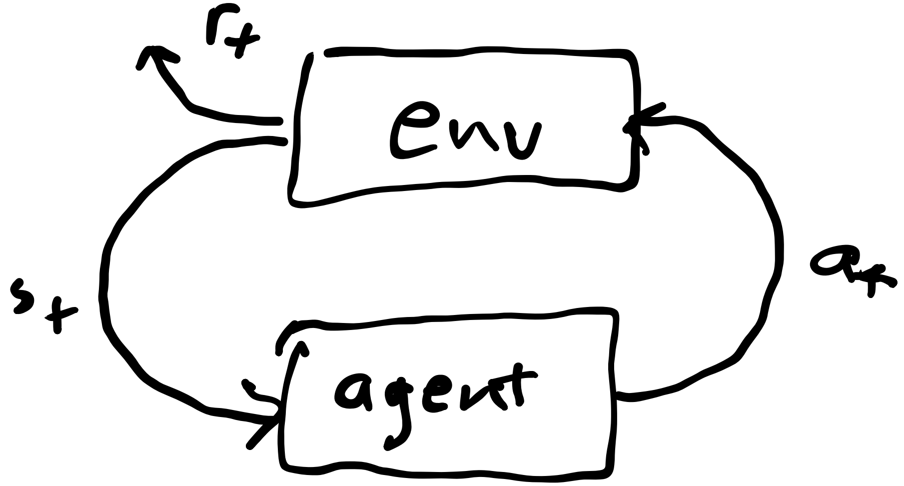


Guarantees *independent* of $|S|$!!

Reinforcement Learning

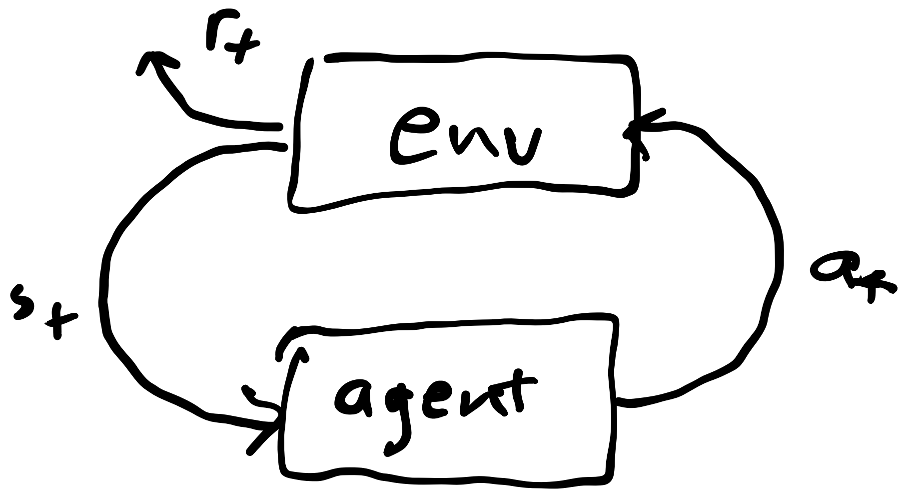
Challenges:

Reinforcement Learning



Challenges:

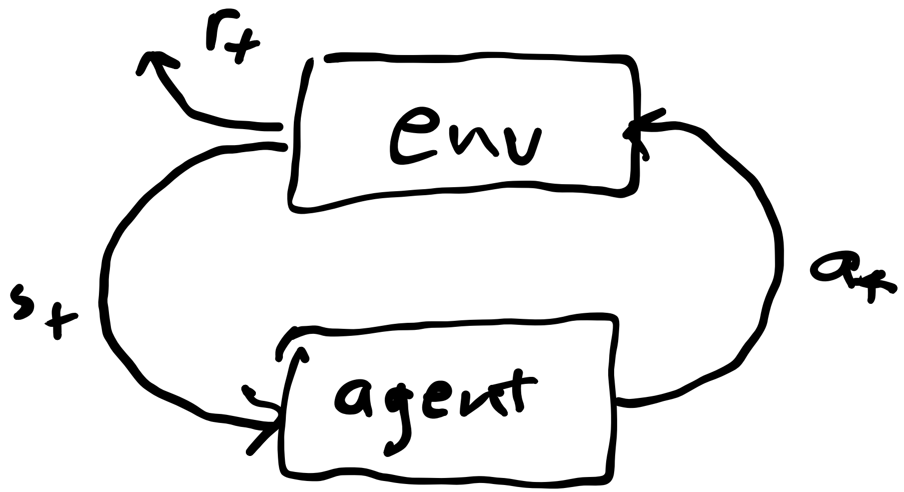
Reinforcement Learning



Challenges:

1. Exploration and Exploitation

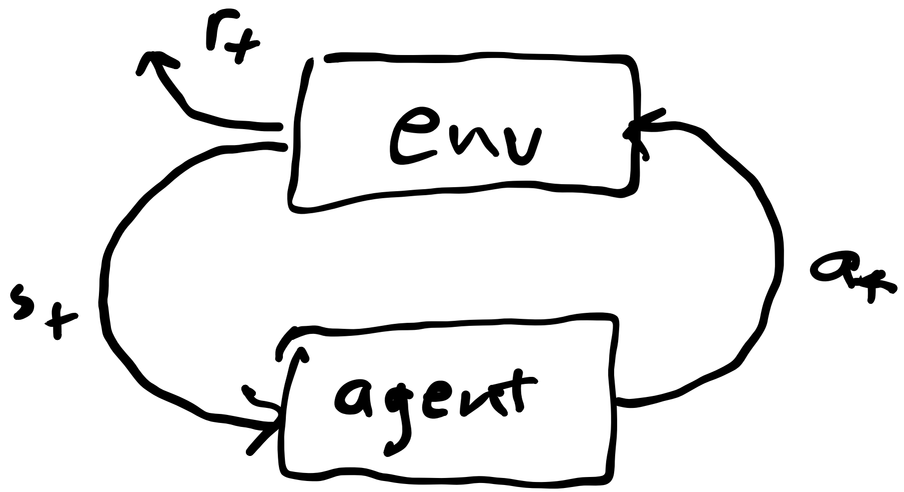
Reinforcement Learning



Challenges:

1. Exploration and Exploitation
2. Credit Assignment

Reinforcement Learning



Challenges:

1. Exploration and Exploitation
2. Credit Assignment
3. Generalization

Exploration

Exploration

Bandits

easiest

- ϵ-greedy
- softmax
- UCB $\hat{Q}(a) + c\sqrt{\frac{\log N}{N(a)}}$
- Thompson Sampling
- Optimal DP Solution (solving a POMDP!)

logarithmic regret

Exploration

Bandits

- ϵ-greedy
- softmax
- UCB
- Thompson Sampling
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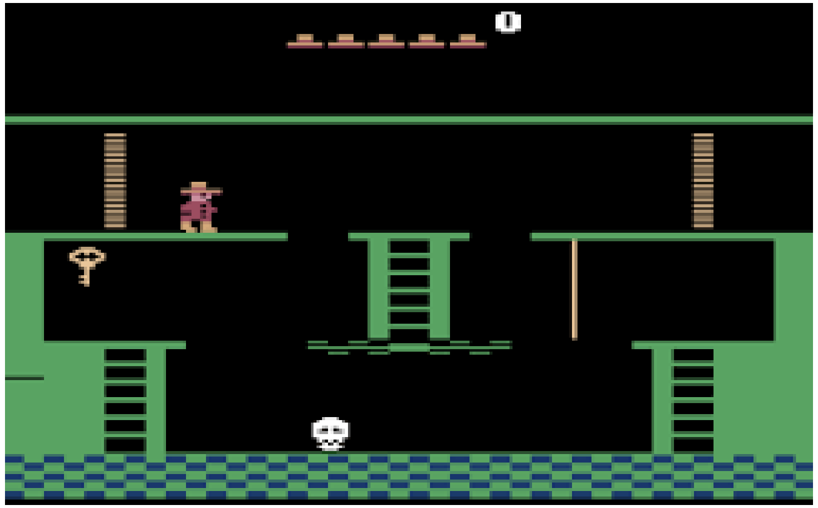
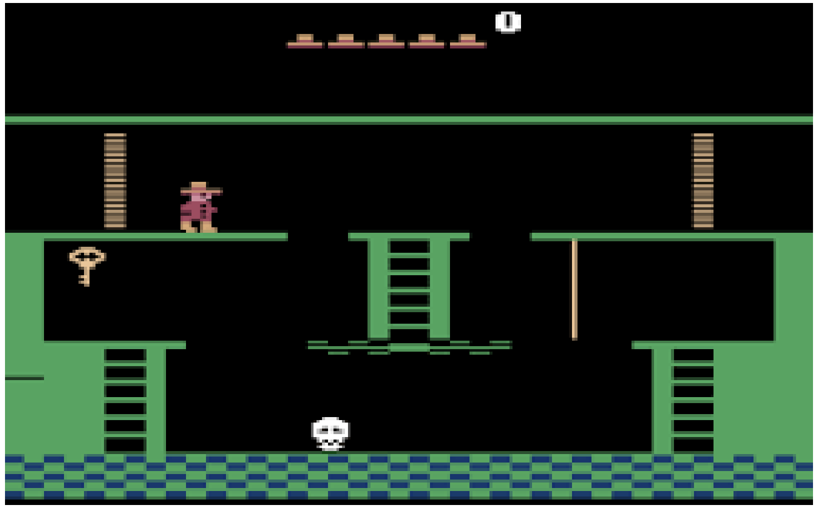


Montezuma's Revenge!

Exploration

Bandits

- ϵ-greedy
- softmax
- UCB
- Thompson Sampling
- Optimal DP Solution (solving a POMDP!)



Montezuma's Revenge!

- Pseudocounts

Exploration

Bandits

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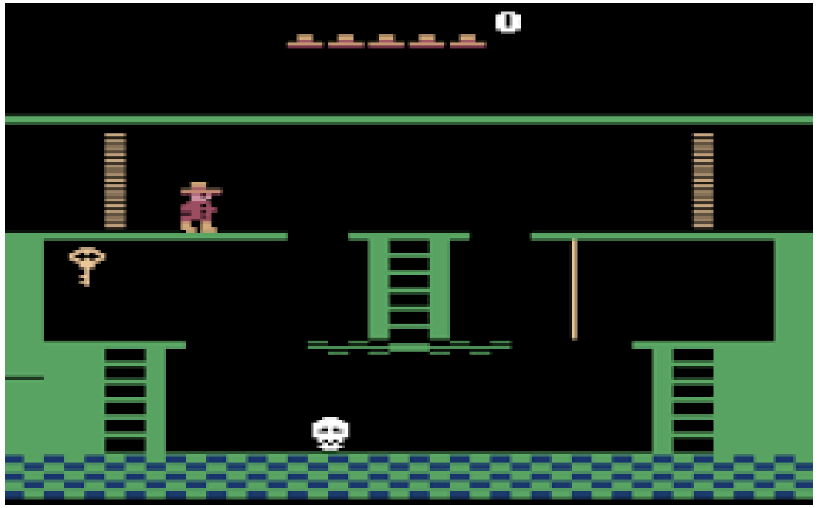


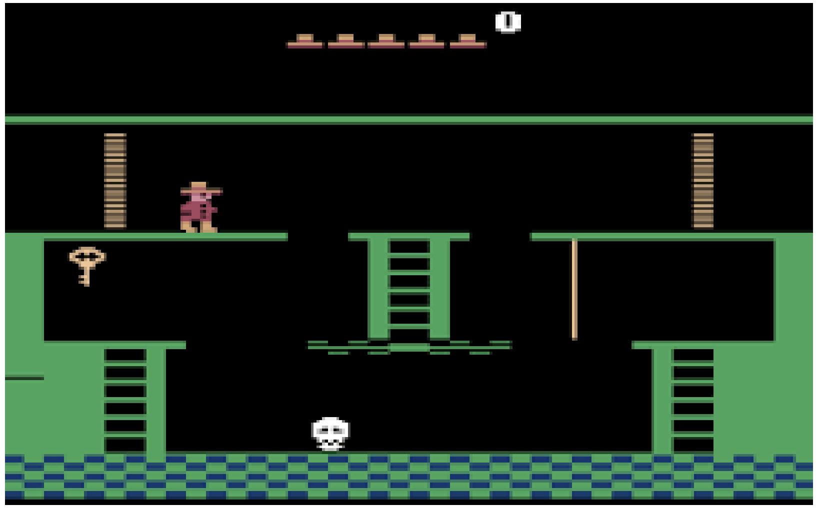
Montezuma's Revenge!

- Pseudocounts
- Curiosity: extra reward for bad prediction

Exploration

Bandits

- ϵ-greedy
- softmax
- UCB $Q(s,a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$
- Thompson Sampling
- Optimal DP Solution (solving a POMDP!)



Montezuma's Revenge!

- Pseudocounts
- Curiosity: extra reward for bad prediction
- Random network distillation

$s' = f_\theta(s,a)$

$f_\phi(s,a) = f_\theta(s,a)$

RL Algorithms

RL Algorithms

Model Based

Model Free

RL Algorithms

Model Based

Model Free

On Policy

Off Policy

RL Algorithms

Model Based

Model Free

On Policy

Off Policy

MLMBTRL
(learn T,R)

RL Algorithms

Model Based

Model Free

learn Q

learn π

On Policy

Off Policy

MLMBTRL
(learn T,R)

RL Algorithms

Model Based

Model Free

learn Q

learn π

Policy Gradient

On Policy

Off Policy

MLMBTRL
(learn T,R)

RL Algorithms

Model Based

Model Free

learn Q

SARSA

learn π

Policy Gradient

On Policy

Off Policy

MLMBTRL
(learn T, R)

RL Algorithms

Model Based ⟷ Model Free

learn Q	learn π
SARSA	Policy Gradient

On Policy ↕ Off Policy

MLMBTRL (learn T, R)

Q-learning

RL Algorithms

Model Based

Model Free

learn Q

SARSA

Q-learning

learn π

Policy Gradient

On Policy

Off Policy

MLMBTRL
(learn T, R)

BAMDP
(POMDP $\tilde{s} = (s, T, R)$)

Policy Gradient

Policy Gradient

- Likelihood ratio trick

$\nabla_\theta p_\theta(\tau) = p_\theta(\tau) \nabla_\theta \log p_\theta(\tau)$

Policy Gradient

- Likelihood ratio trick
- Causality

$\nabla_\theta p_\theta(\tau) = p_\theta(\tau) \nabla_\theta \log p_\theta(\tau)$

Policy Gradient

- Likelihood ratio trick
- Causality
- Baseline Subtraction

$$\nabla_\theta p_\theta(\tau) = p_\theta(\tau) \nabla_\theta \log p_\theta(\tau)$$

Policy Gradient

- Likelihood ratio trick
- Causality
- Baseline Subtraction

$\nabla_\theta p_\theta(\tau) = p_\theta(\tau) \nabla_\theta \log p_\theta(\tau)$

$$\nabla U(\theta) = \mathbb{E}_\tau \left[\sum_{k=1}^{d} \nabla_\theta \log \pi_\theta(a^{(k)} \mid s^{(k)}) \gamma^{k-1} \left(r_{\text{to-go}}^{(k)} - r_{\text{base}}(s^{(k)}) \right) \right]$$

Policy Gradient

- Likelihood ratio trick
- Causality
- Baseline Subtraction

$\nabla_\theta p_\theta(\tau) = p_\theta(\tau) \nabla_\theta \log p_\theta(\tau)$

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- Natural Gradient

Policy Gradient

- Likelihood ratio trick
- Causality
- Baseline Subtraction

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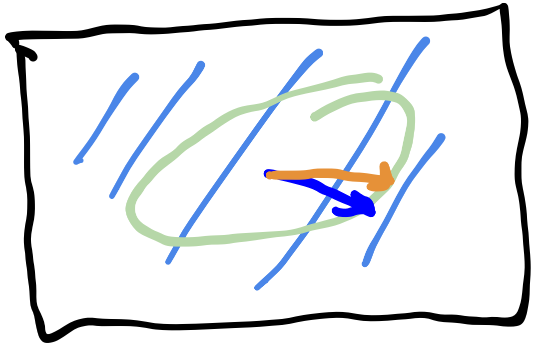
$$\nabla U(\theta) = \mathbb{E}_\tau\left[\sum_{k=1}^{d} \nabla_\theta \log \pi_\theta(a^{(k)} \mid s^{(k)}) \gamma^{k-1} \left(r_{\text{to-go}}^{(k)} - r_{\text{base}}(s^{(k)})\right)\right]$$

- Natural Gradient



KL div.
Bound

Q-Learning

Q-Learning

SARSA

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_t + \gamma Q(s',a') - Q(s,a))$$

Q-Learning

SARSA

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_t + \gamma Q(s',a') - Q(s,a))$$

Eligibility Traces

Q-Learning

SARSA

chosen by current policy

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_t + \gamma \underline{Q(s',\underline{a'})} - Q(s,a))$$

Eligibility Traces

Q-learning

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_t + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

Q-Learning

SARSA

$$Q(s,a) \leftarrow Q(s,a) + \alpha(r_t + \gamma Q(s',a') - Q(s,a))$$

Eligibility Traces

Q-learning

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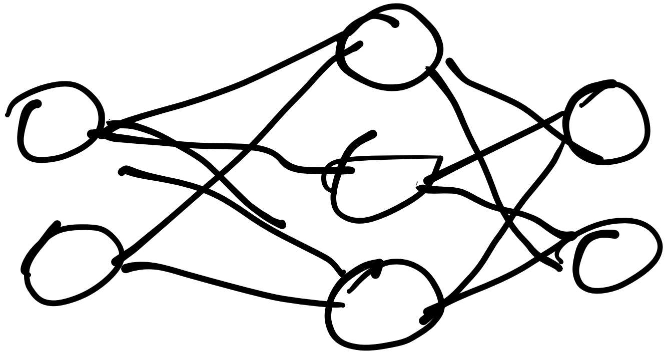
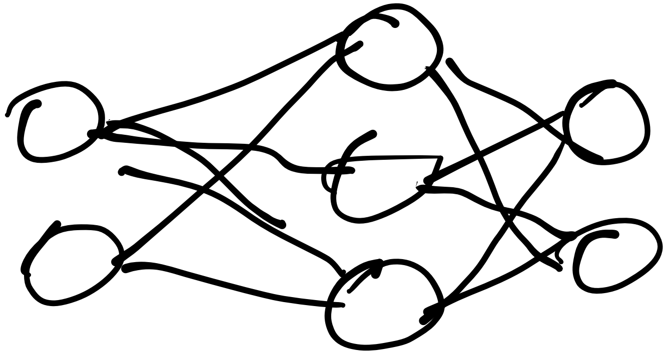
Double Q Learning

Neural Networks and DQN

Neural Networks and DQN

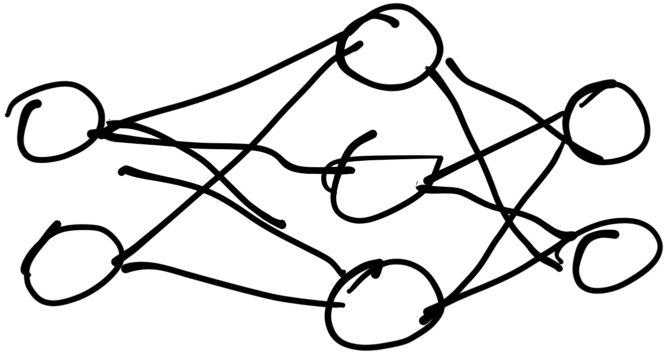


Neural Networks and DQN



$$f_\theta(x) = \sigma(W_2\sigma(W_1 x + b_1) + b_2)$$

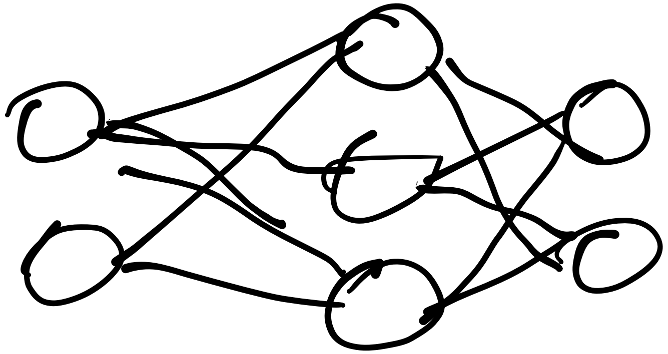
Neural Networks and DQN



$$f_\theta(x) = \sigma(W_2\sigma(W_1 x + b_1) + b_2)$$

Backprop

Neural Networks and DQN



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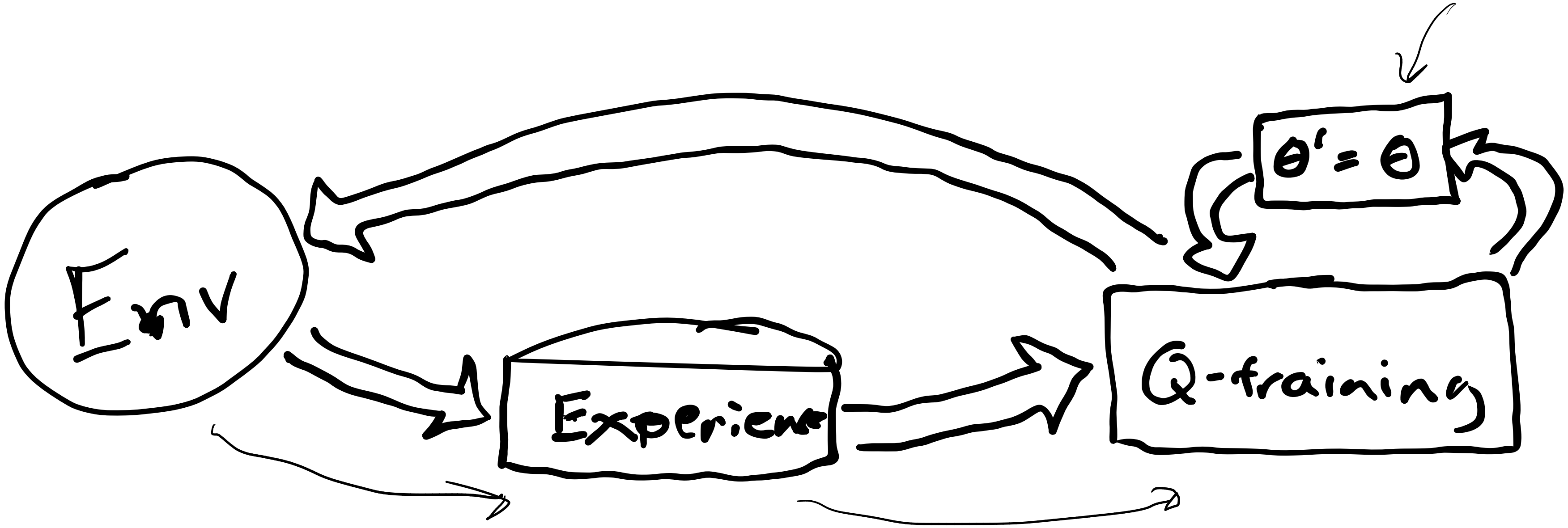
Backprop



Actor-Critic

Actor-Critic

- Actor: π_θ

Actor-Critic

- Actor: π_θ
- Critic: Q_ϕ

Actor-Critic

- Actor: π_θ
- Critic: Q_ϕ

Soft Actor Critic

Actor-Critic

- Actor: π_θ
- Critic: Q_ϕ

Soft Actor Critic

$$J(\pi) = E\left[\sum_{t=0}^{\infty} \gamma^t \left(r_t + \alpha \underline{\mathcal{H}(\pi(\cdot \mid s_t))}\right)\right]$$

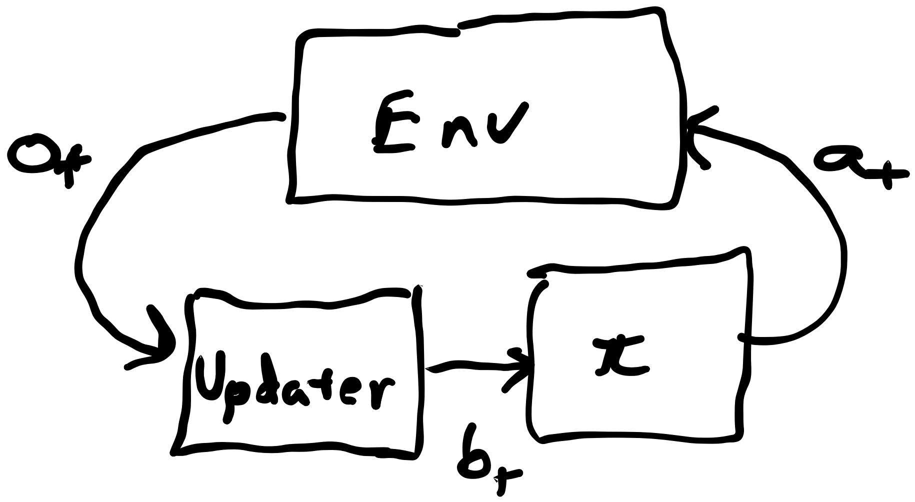
POMDPs

POMDPs

$$(S, A, T, R, O, Z, \gamma)$$

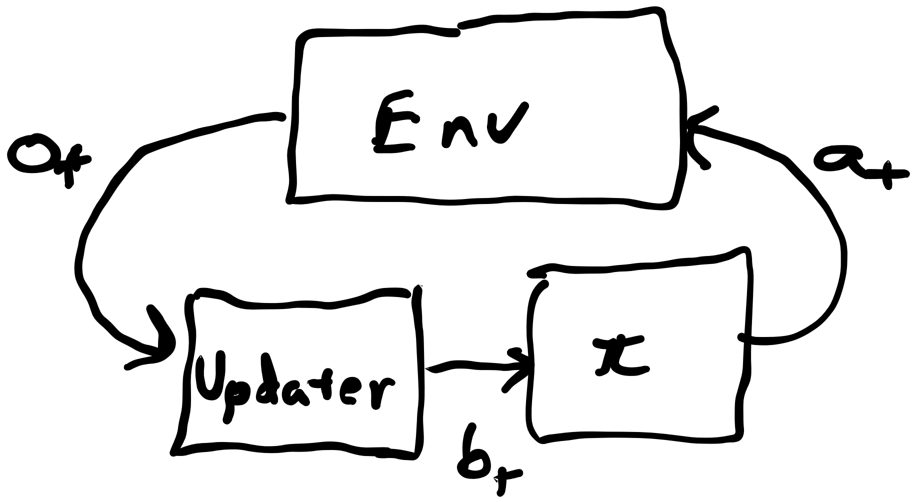
POMDPs

$$(S, A, T, R, O, Z, \gamma)$$



POMDPs

$$(S, A, T, R, O, Z, \gamma)$$

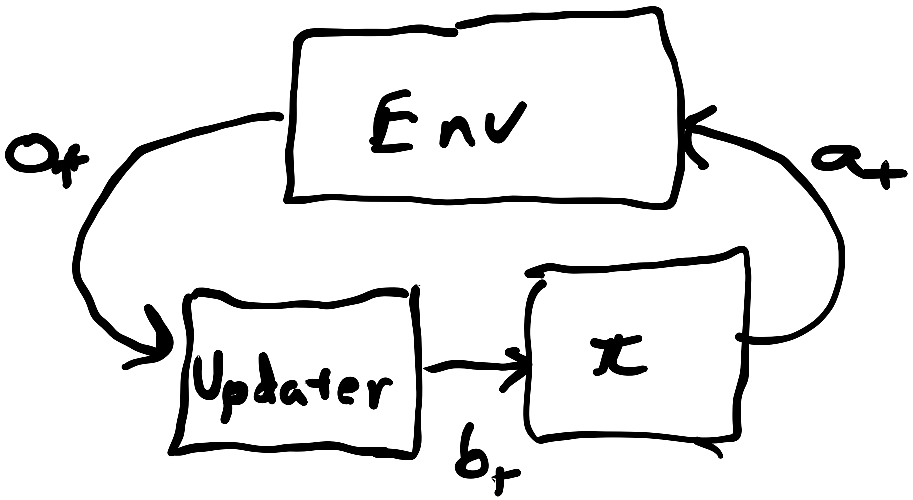


Belief Updates

- Discrete Bayesian Filter
- Particle Filter

POMDPs

$$(S, A, T, R, O, Z, \gamma)$$



Belief Updates

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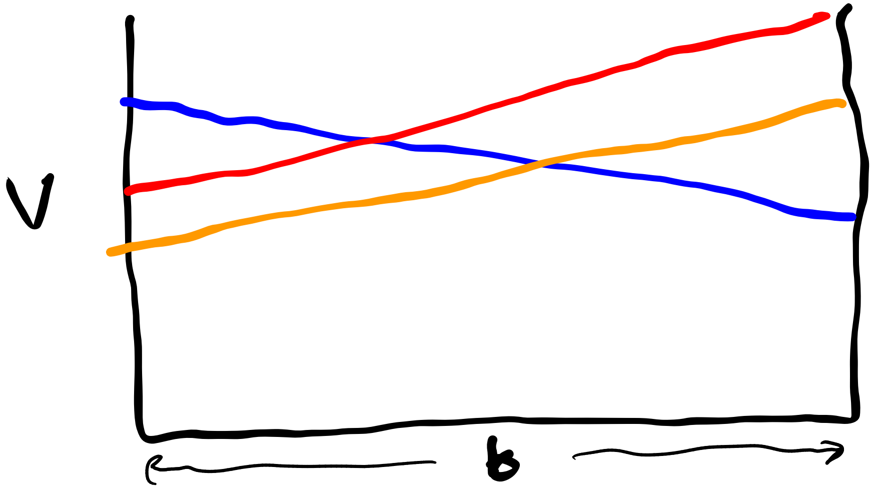
Alpha Vectors



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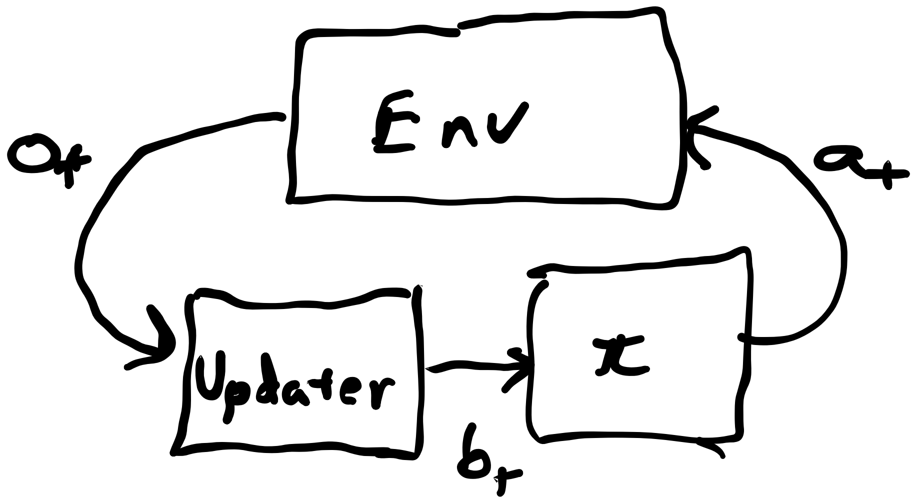

- Each alpha vector corresponds to a conditional plan

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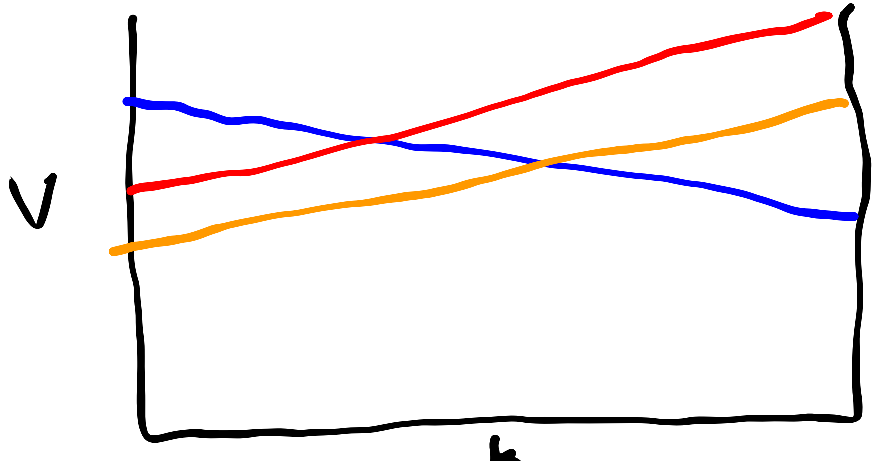

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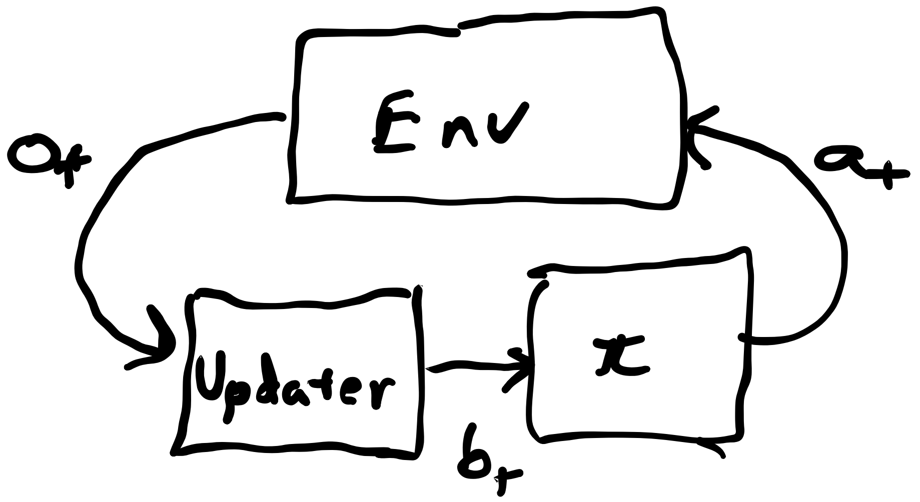

- Each alpha vector corresponds to a conditional plan
- You can prune alpha vectors by solving an LP

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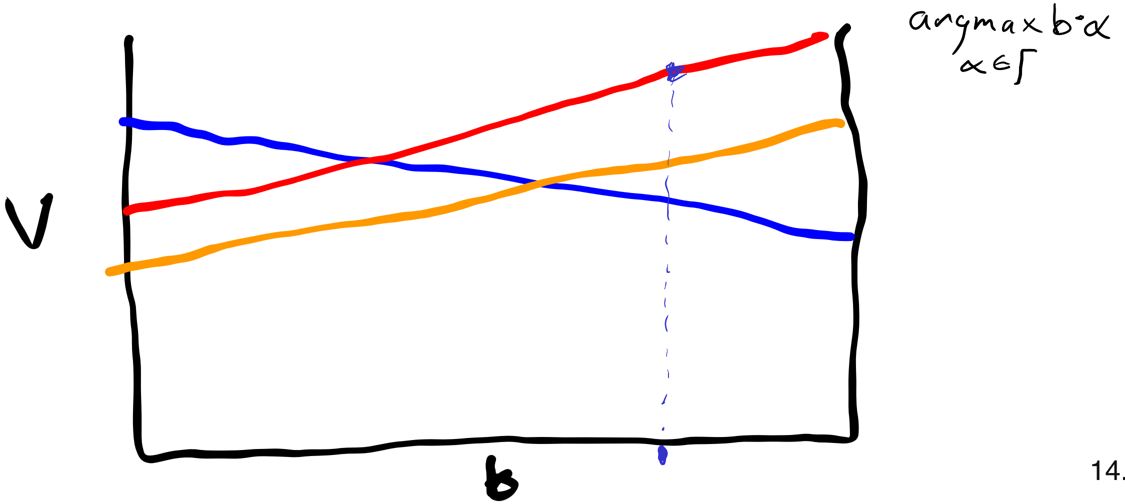

POMDP Approximations

POMDP Approximations

Formulation

- Certainty Equivalence
- QMDP

$\pi_{MDP}\left(\underset{s\sim b}{E}[s]\right)$

$\operatorname{argmax}_s b(s)$

optimal for LQG

$\pi_{QMDP}(b) = \underset{a}{\operatorname{argmax}} \underset{s\sim b}{E}[Q_{MDP}(s,a)]$

POMDP Approximations

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Numerical

POMDP Approximations

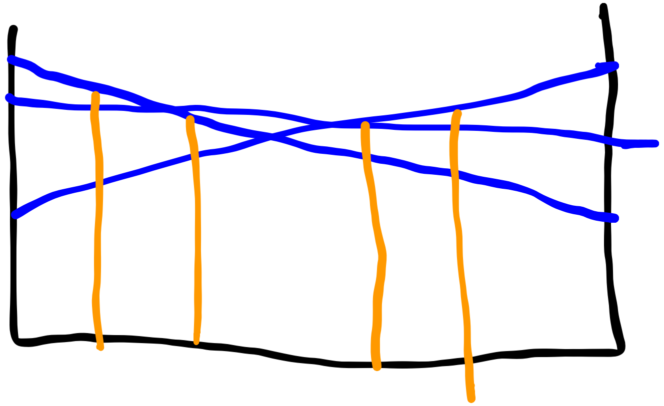
Formulation

- Certainty Equivalence
- QMDP

Numerical

Offline

- Point-Based Value Iteration
- SARSOP



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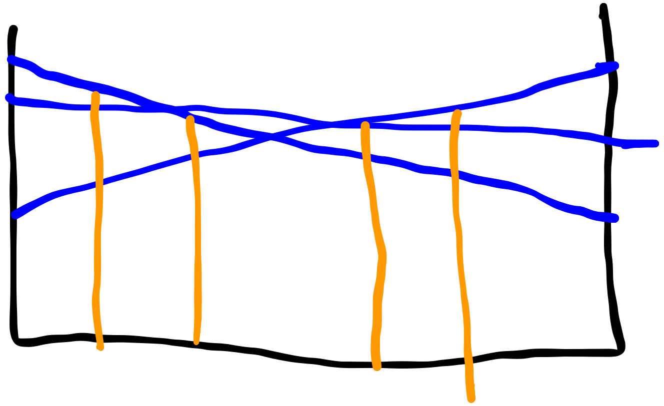

Online

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- DESPOT

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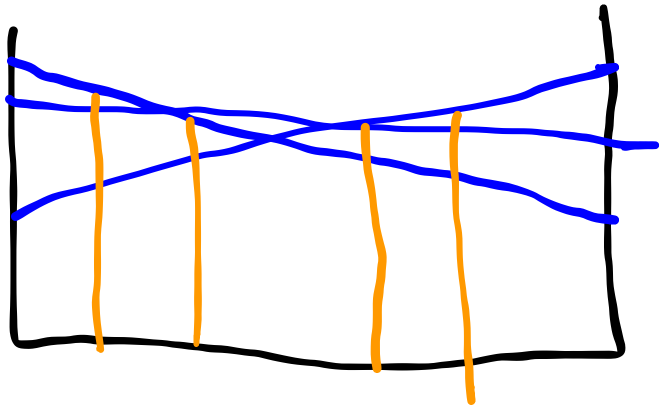

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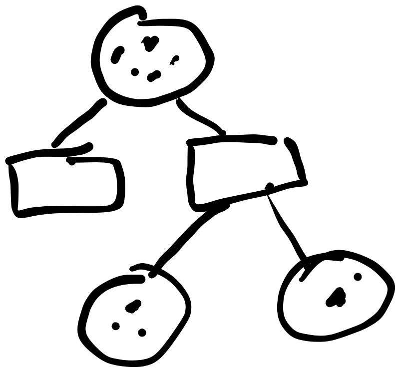

Simple Games

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NE: every player plays a best response

	L	R
up	-1,-1	-3,0
down	0,-3	$\boxed{-2,-2}$

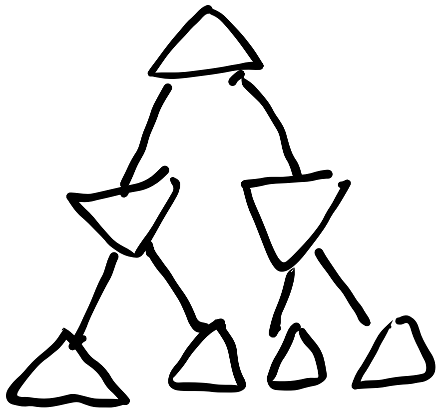
- Every finite game has at least 1 Nash Equilibrium
- Might be pure or mixed
- Algorithms like fictitious play converge in special cases

Dominant Strategy Equilibrium

Correlated Equilibrium → NE is a correlated Eq. where all policies are independent

Turn Taking Games

Turn Taking Games



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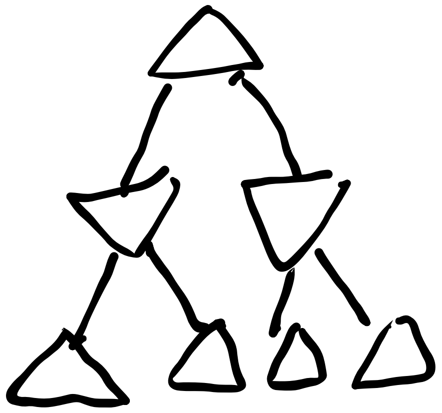

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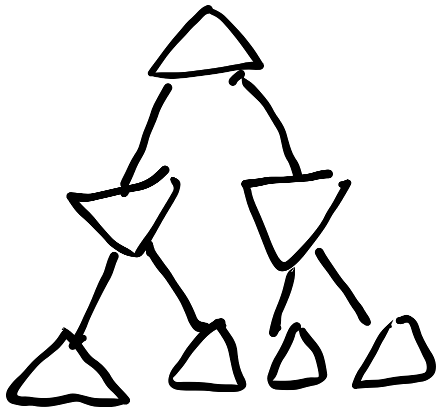
Turn Taking Games



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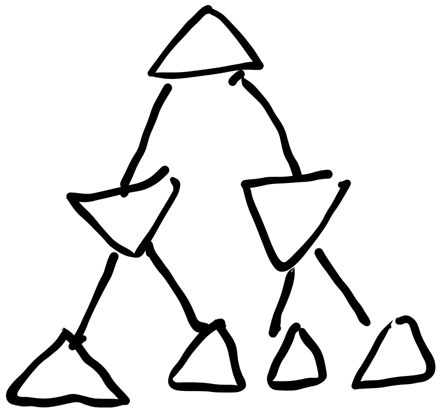

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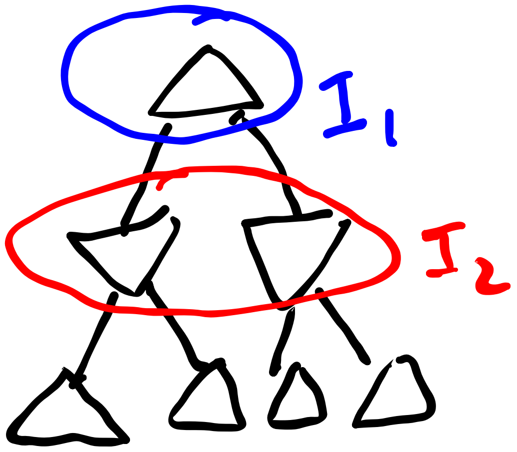
Turn Taking Games



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- All players play simultaneously
- Transitions are stochastic
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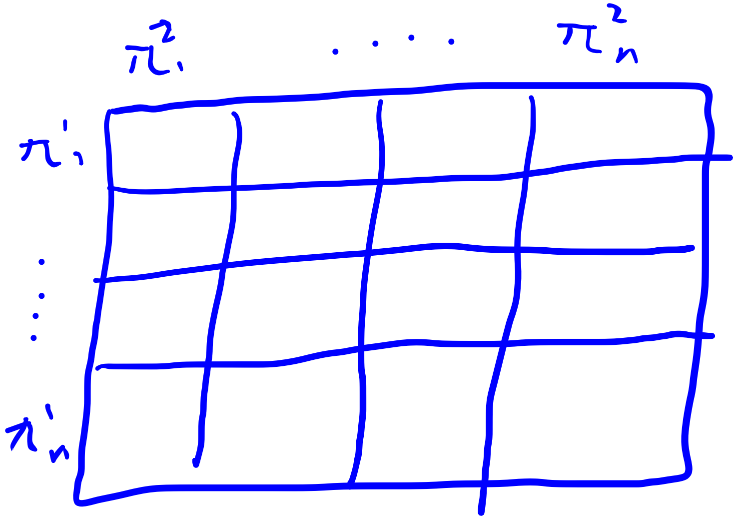

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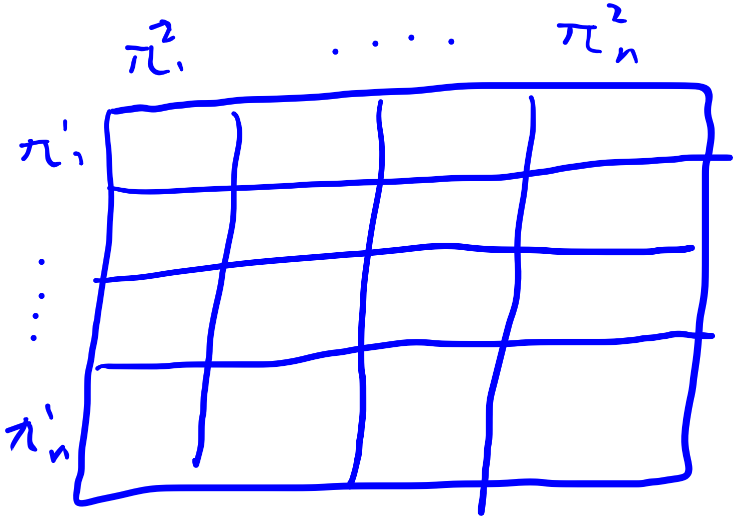

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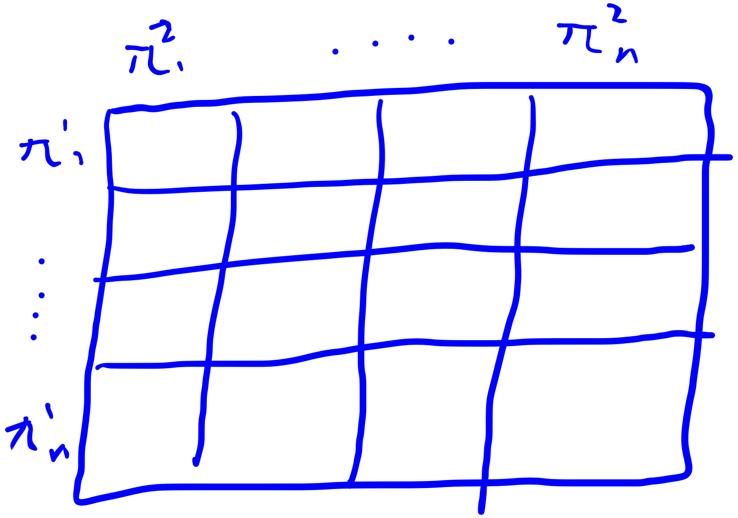

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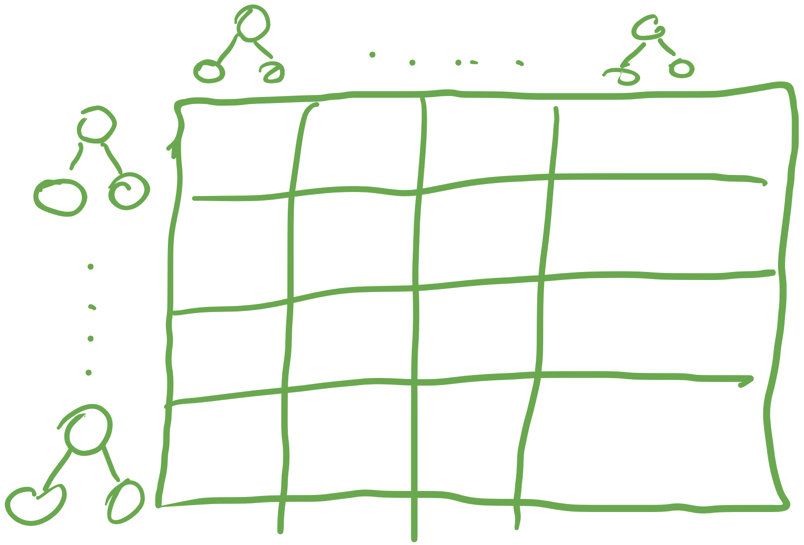

Fictitious Play in Markov Games

initialize $N(j, a^j, s)$ to 0
initialize π^{BR}

Loop:

1. Simulate MG with π^{BR}
2. Update $N(j, a^j, s)$ ← N should reflect all past simulations
3. $\pi^j(a^j | s) \propto N(j, a^j, s) \quad \forall j$
4. π^{BR} ← best response to π ← converges to NE for some classes of games

exploitability

solve an MDP with other player's strategies integrated into it

will π^{BR} converge? No

Recap

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After DMU you have basic tools to deal with 4 Big Problems:

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After DMU you have basic tools to deal with 4 Big Problems:

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4. Other Agents