Causal Bayesian Networks

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Today:

- Causal Bayesian Networks
- How do we reason about independence in Bayesian Networks?

Review: Distributions of Discrete R.V.s

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Joint	$P(X=x, Y=y)$ Shorthand: $P(x, y)$	Single number	"Probability that $X = x$ *and* $Y = y$"
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Conditional	$P(X = x \mid Y = y)$ Shorthand: $P(x \mid y)$	Single number	"Probability that $X = x$ *if* $Y = y$"
	$P(X \mid Y)$	A collection of tables for each y	"Conditional distribution of X given Y"

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Marginal	$P(X = x)$ Shorthand: $P(x)$	Single number	"Probability that $X = x$"
	$P(X)$	A table	"Marginal distribution of X"

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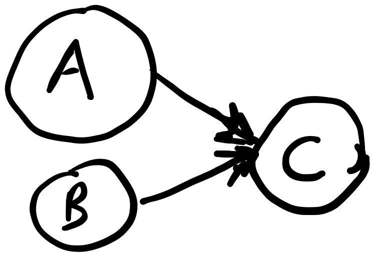

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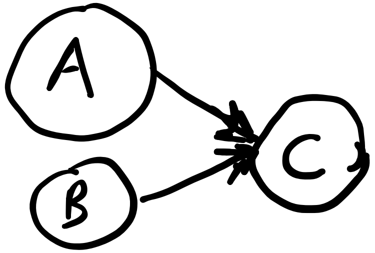

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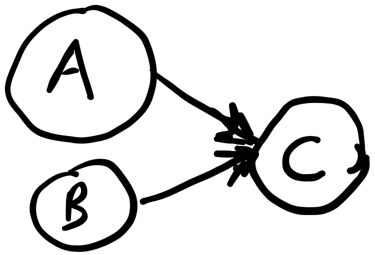

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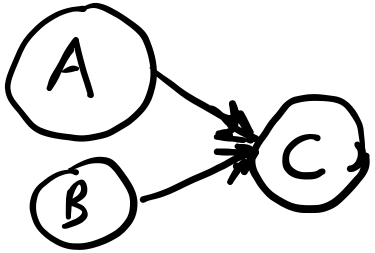

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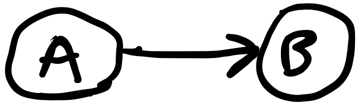

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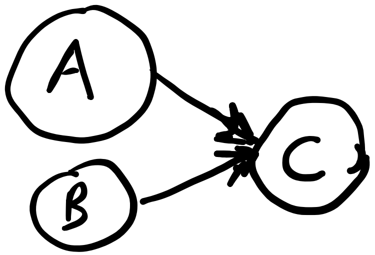

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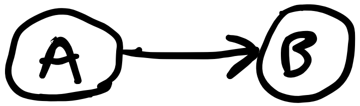

B is a result of A (and some aleatory uncertainty)

Chain rule for Bayesian Networks

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$$P(X_{1:n}) = \prod_{i=1}^{n} P(X_i \mid \mathrm{pa}(X_i))$$

Simple Causal Bayes Net Example

Naive Inference on Bayes Nets

(Book introduces unnormalized "factors", but process is the same.)

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Bayes Net with 3 Random Variables: $A \to C \to B$

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$$P(A = a, B = b) = \sum_{c} P(A = a, B = b, C = c)$$

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3. $P(A = a \mid B = b) = \frac{P(A=a, B=b)}{P(B=b)}$

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Conditional Independence in Bayes Nets

Conditional Independence: Fork

https://kunalmenda.com/2019/02/21/causation-and-correlation/

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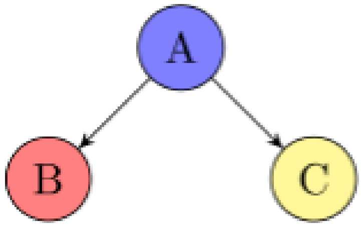

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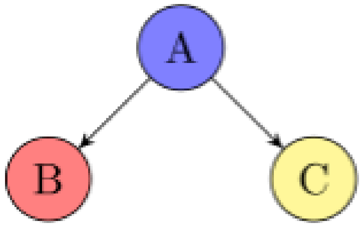

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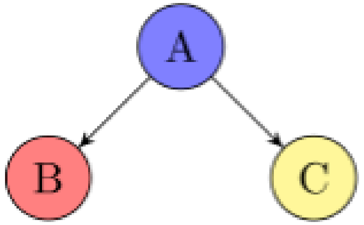

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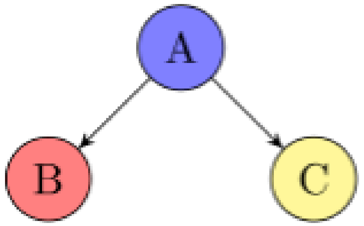

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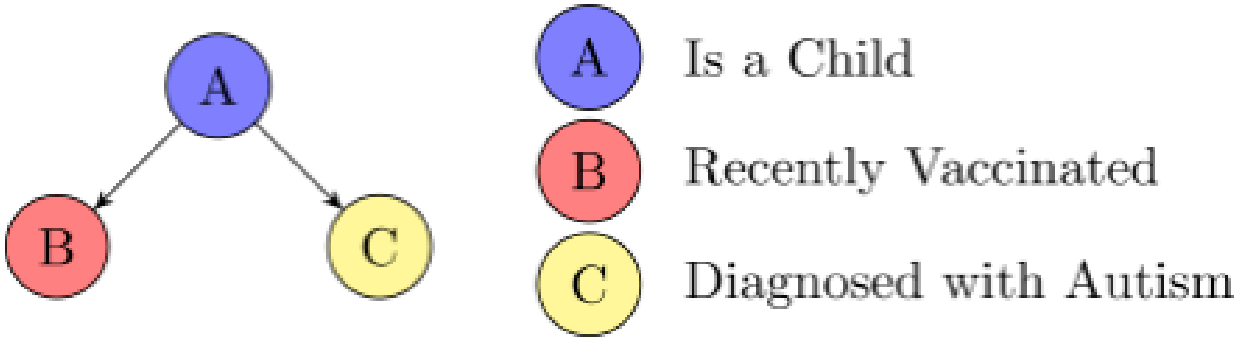

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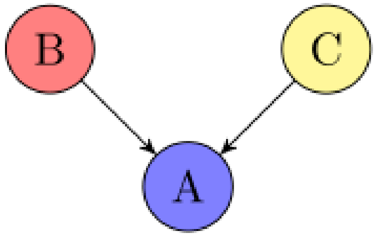

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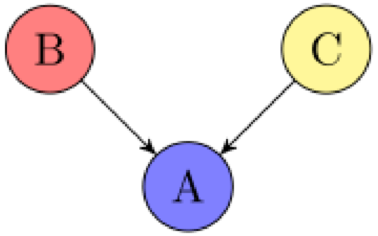

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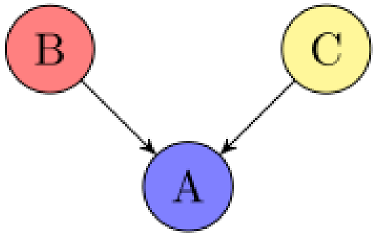

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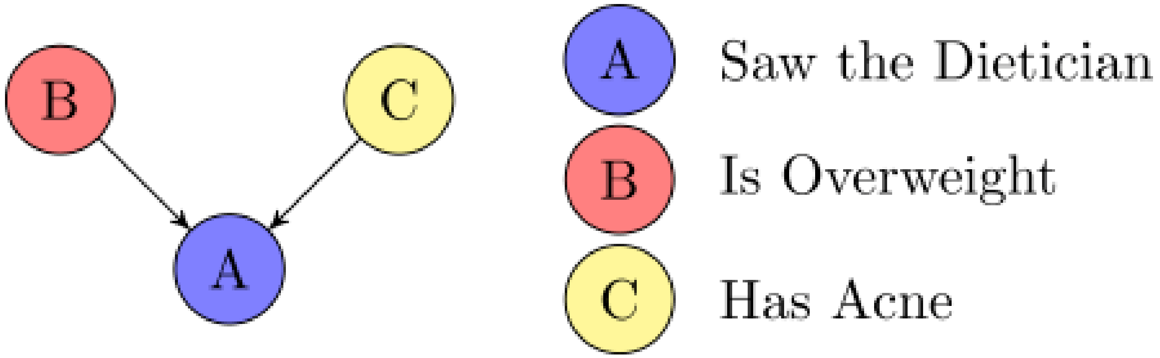

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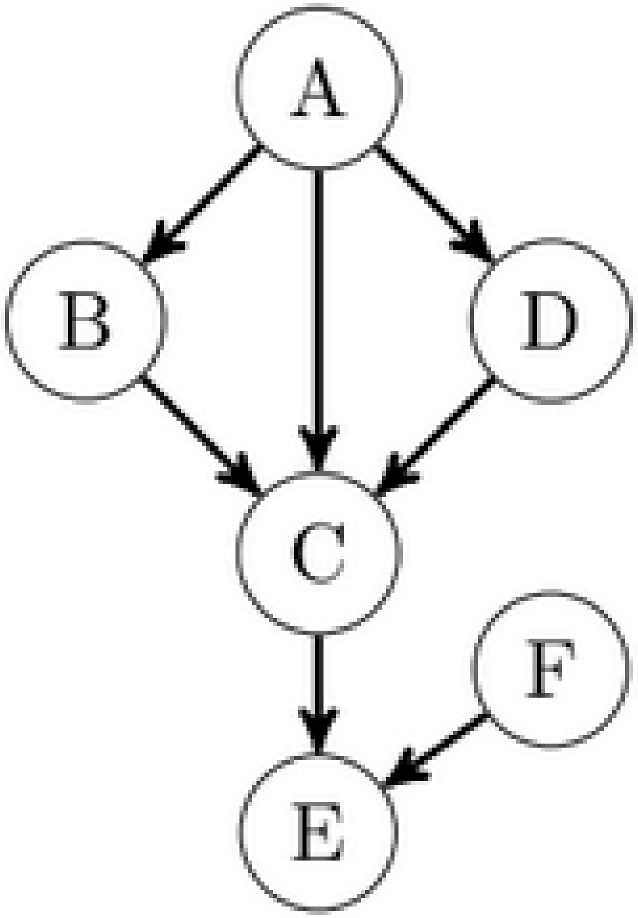

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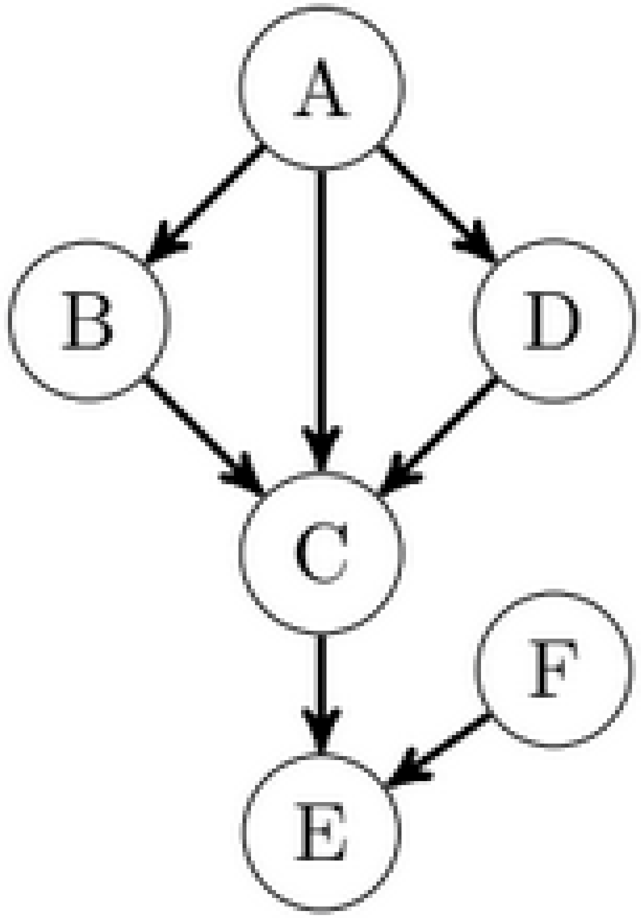

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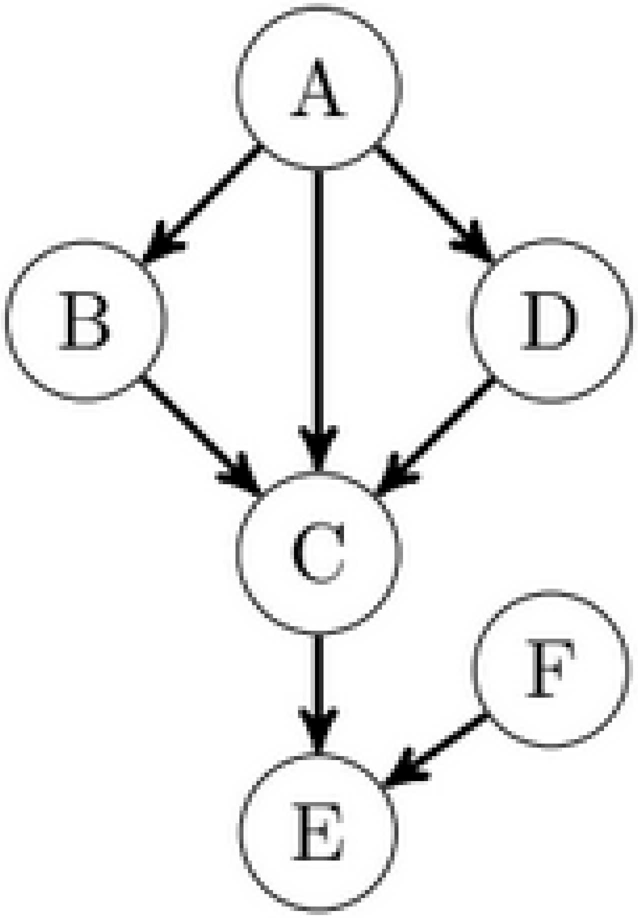

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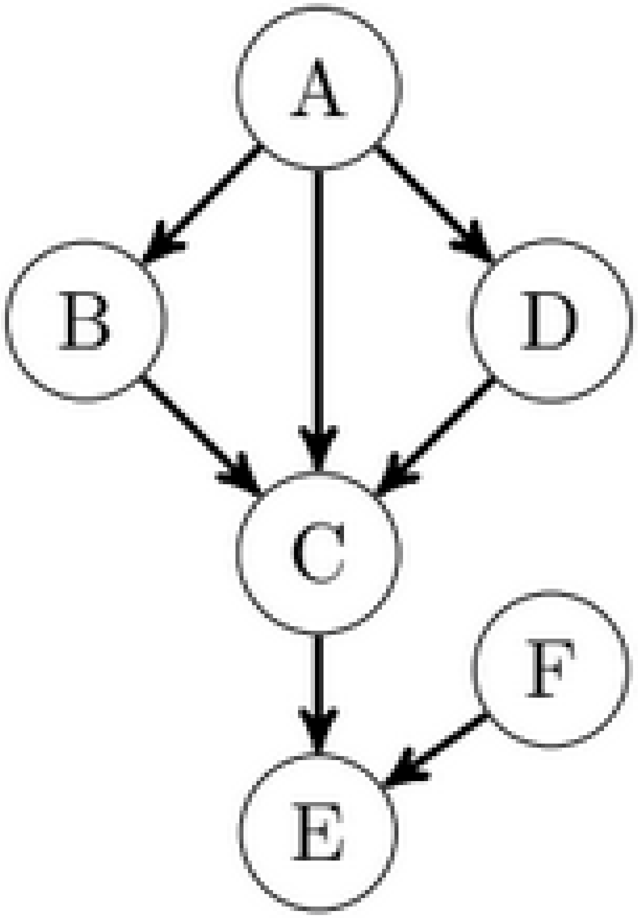

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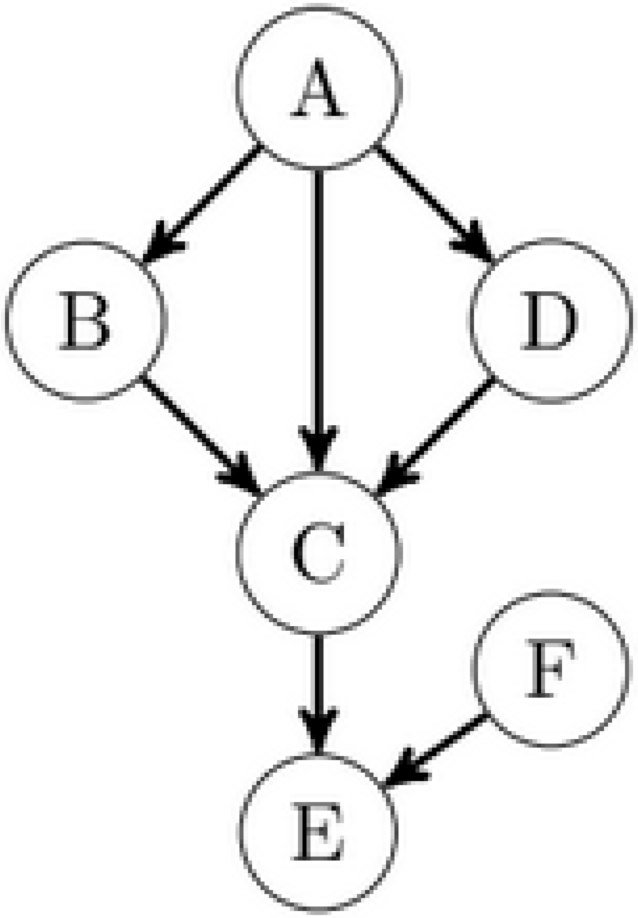

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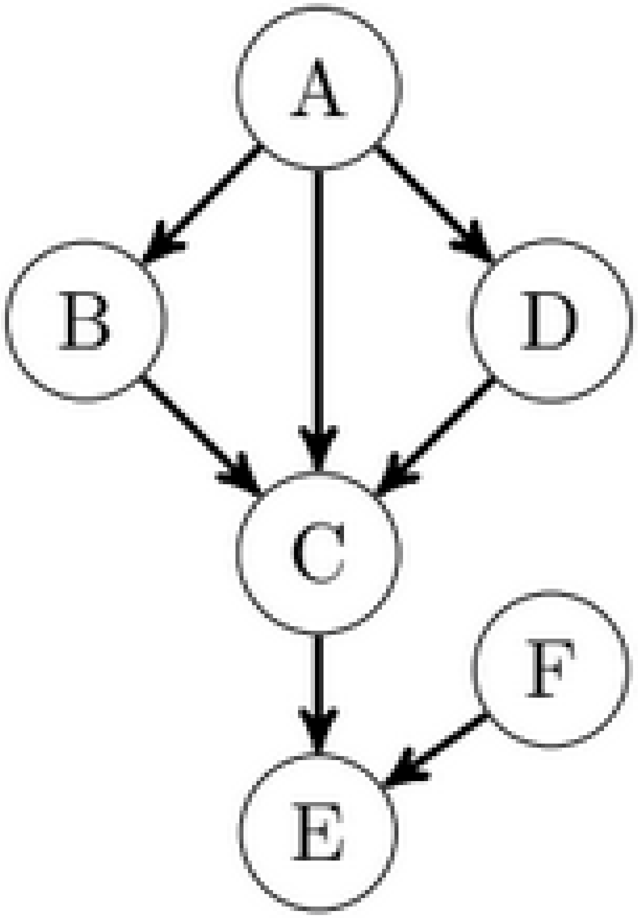

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Why is this relevant to decision making?

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Are these paths d-separated by $\mathcal{C} = \{C\}$?

d-Separation for Bayes Nets

*short for "directionally separated"

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If A and B are d-separated by $\mathcal{C}$ then $A \perp B \mid \mathcal{C}$

In other words, if there is any active path w.r.t. $\mathcal{C}$ between A and B, we *cannot* conclude that $A \perp B \mid \mathcal{C}$ based on the structure alone.

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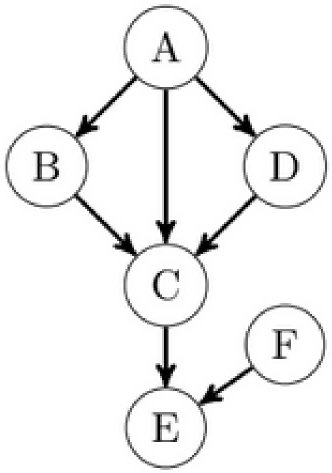

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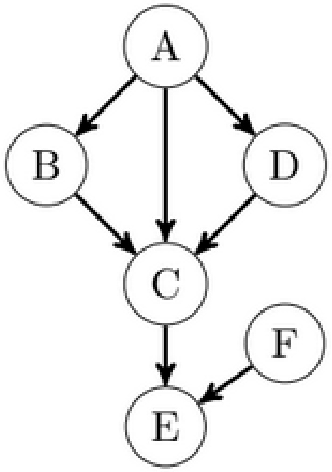

Example: $(B \perp D \mid C, E)$?

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If $\mathcal{B}$ is the Markov blanket of $\mathcal{X}$, you can treat analyze $\mathcal{B} \cup \mathcal{X}$ alone, and ignore any other nodes.

Exercise

1. The path contains a *chain* $X \rightarrow Y \rightarrow Z$ such that $Y \in \mathcal{C}$
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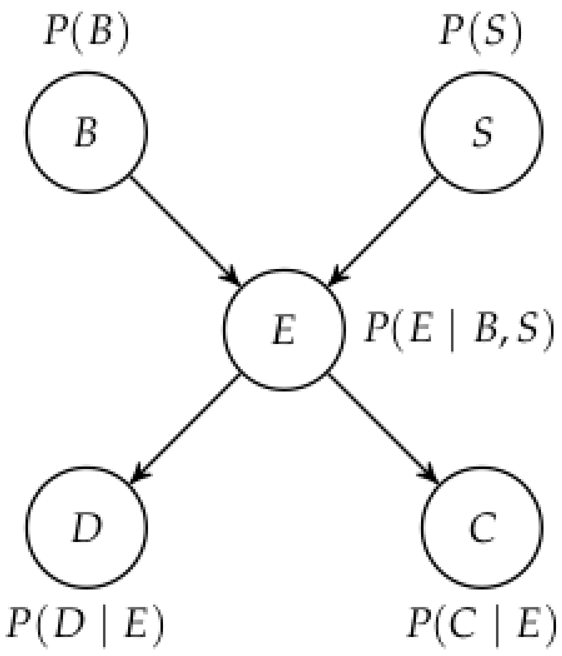

B battery failure
S solar panel failure
E electrical system failure
D trajectory deviation
C communication loss

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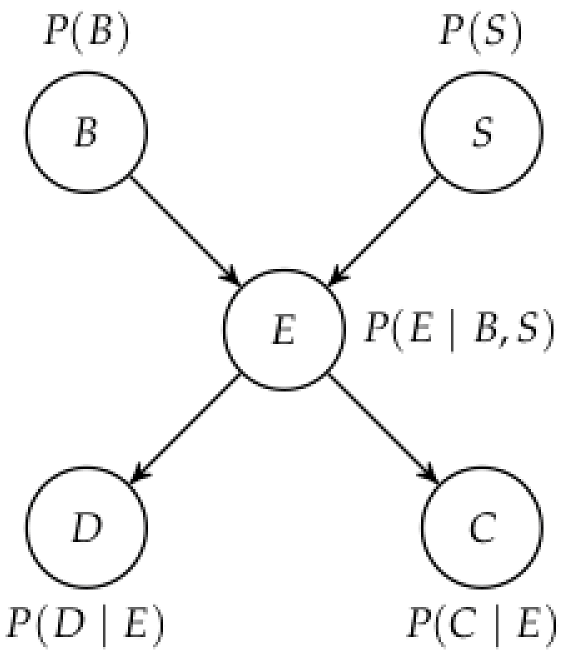
3. The path contains an *inverted fork* (v-structure) $X \to Y \leftarrow Z$ such that $Y \notin \mathcal{C}$ and no descendant of Y is in $\mathcal{C}$.

Exercise

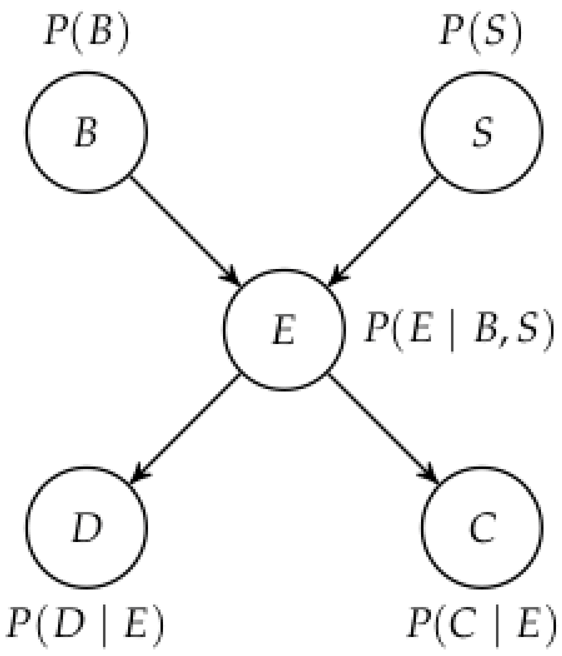


B battery failure
S solar panel failure
E electrical system failure
D trajectory deviation
C communication loss

$$D \perp C \mid B\,?$$

1. The path contains a *chain* $X \to Y \to Z$ such that $Y \in \mathcal{C}$
2. The path contains a *fork* $X \leftarrow Y \to Z$ such that $Y \in \mathcal{C}$
3. The path contains an *inverted fork* (v-structure) $X \to Y \leftarrow Z$ such that $Y \notin \mathcal{C}$ and no descendant of Y is in $\mathcal{C}$.

Exercise



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C communication loss

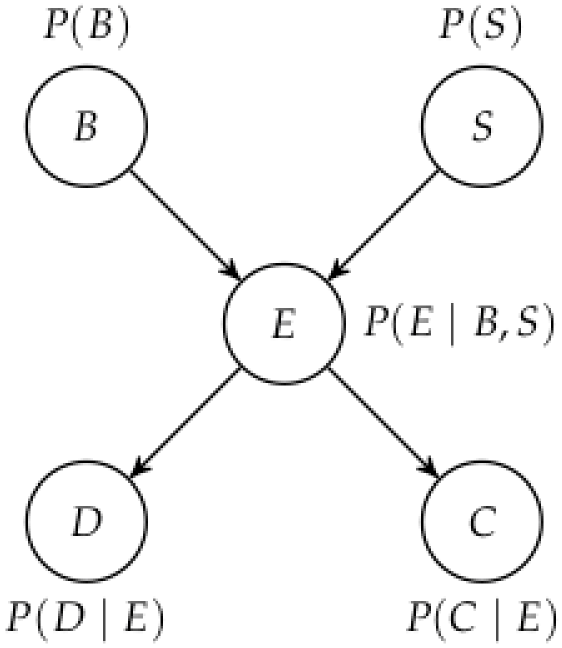
$D \perp C \mid B$?

$D \perp C \mid E$?

1. The path contains a *chain* $X \to Y \to Z$ such that $Y \in \mathcal{C}$
2. The path contains a *fork* $X \leftarrow Y \to Z$ such that $Y \in \mathcal{C}$
3. The path contains an *inverted fork* (v-structure) $X \to Y \leftarrow Z$ such that $Y \notin \mathcal{C}$ and no descendant of Y is in $\mathcal{C}$.

Approximate Inference

Approximate Inference: Direct Sampling



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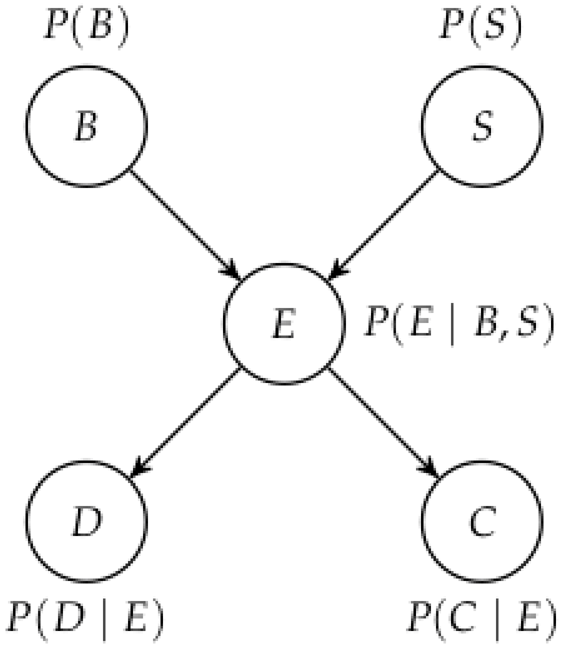
Approximate Inference: Direct Sampling



B battery failure
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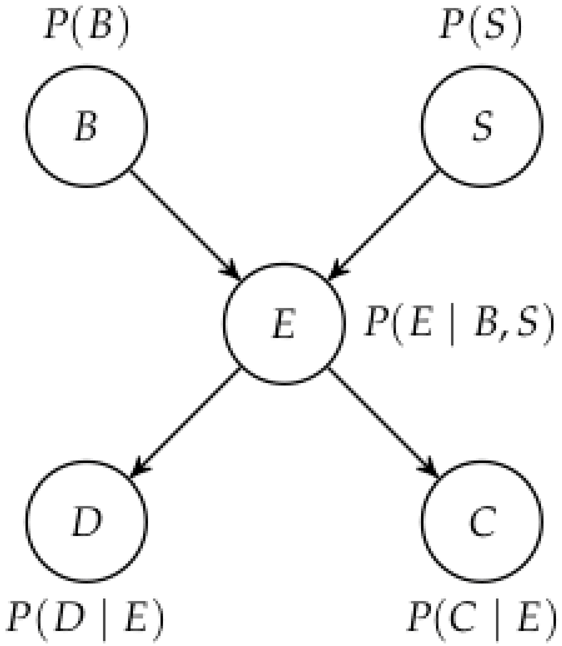
D trajectory deviation
C communication loss

$$P(b^1 \mid d^1, c^1) \approx \frac{\sum_i (b^{(i)} = 1 \land d^{(i)} = 1 \land c^{(i)} = 1)}{\sum_i (d^{(i)} = 1 \land c^{(i)} = 1)}$$

Approximate Inference: Direct Sampling



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E electrical system failure
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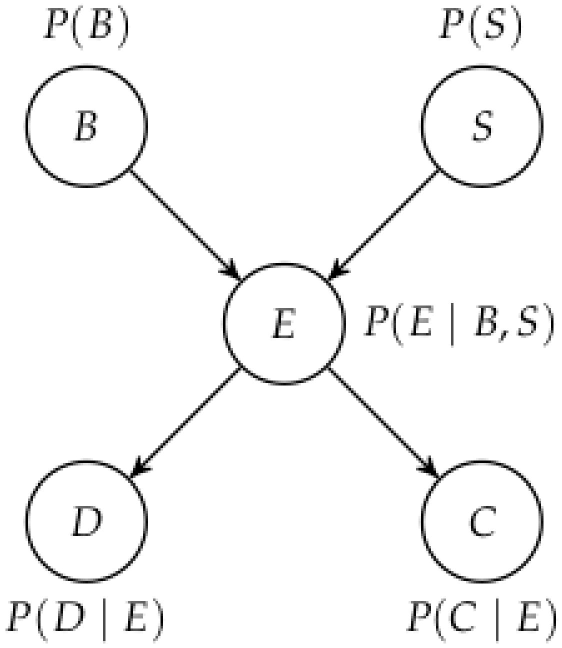
$$P(b^1 \mid d^1, c^1) \approx \frac{\sum_i (b^{(i)} = 1 \wedge d^{(i)} = 1 \wedge c^{(i)} = 1)}{\sum_i (d^{(i)} = 1 \wedge c^{(i)} = 1)}$$

B	S	E	D	C
0	0	1	1	0
0	0	0	0	0
1	0	1	0	0
1	0	1	1	1
0	0	0	0	0
0	0	0	1	0
0	0	0	0	1
0	1	1	1	1
0	0	0	0	0
0	0	0	1	0

Approximate Inference: Direct Sampling



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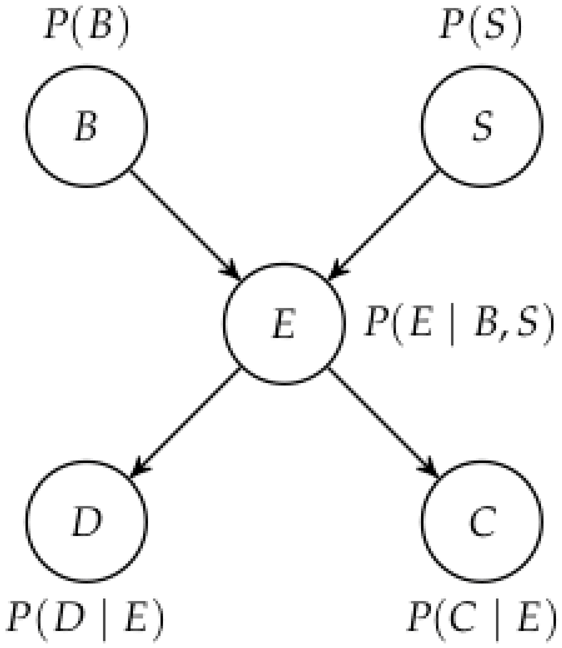
$$P(b^1 \mid d^1, c^1) \approx \frac{\sum_i (b^{(i)} = 1 \land d^{(i)} = 1 \land c^{(i)} = 1)}{\sum_i (d^{(i)} = 1 \land c^{(i)} = 1)}$$

B	S	E	D	C
0	0	1	1	0
0	0	0	0	0
1	0	1	0	0
1	0	1	1	1
0	0	0	0	0
0	0	0	1	0
0	0	0	0	1
0	1	1	1	1
0	0	0	0	0
0	0	0	1	0

Analogous to

Approximate Inference: Direct Sampling



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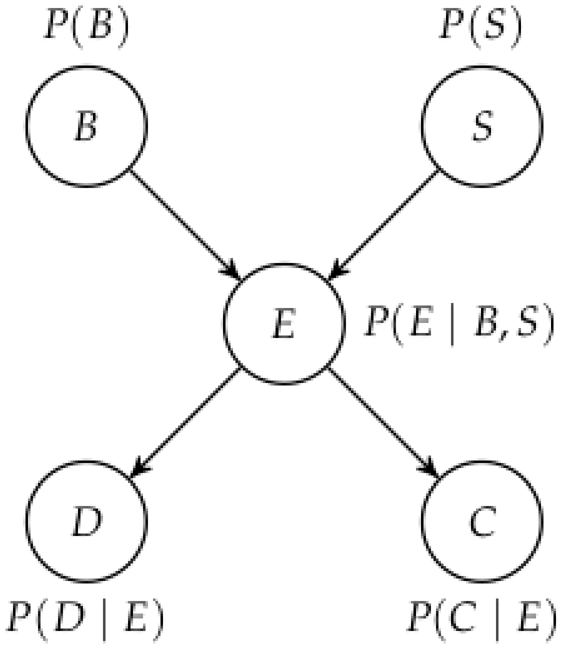
$$P(b^1 \mid d^1, c^1) \approx \frac{\sum_i (b^{(i)} = 1 \land d^{(i)} = 1 \land c^{(i)} = 1)}{\sum_i (d^{(i)} = 1 \land c^{(i)} = 1)}$$

B	S	E	D	C
0	0	1	1	0
0	0	0	0	0
1	0	1	0	0
1	0	1	1	1
0	0	0	0	0
0	0	0	1	0
0	0	0	0	1
0	1	1	1	1
0	0	0	0	0
0	0	0	1	0

Analogous to **unweighted particle filtering**

Approximate Inference: Weighted Sampling



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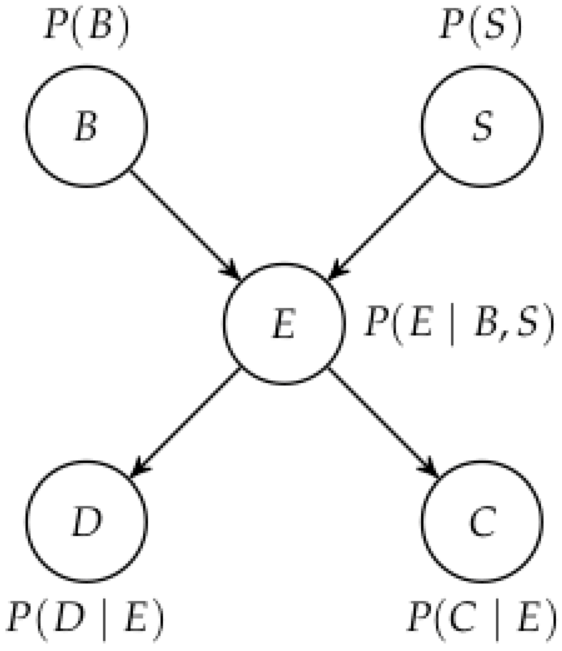
C communication loss

Approximate Inference: Weighted Sampling



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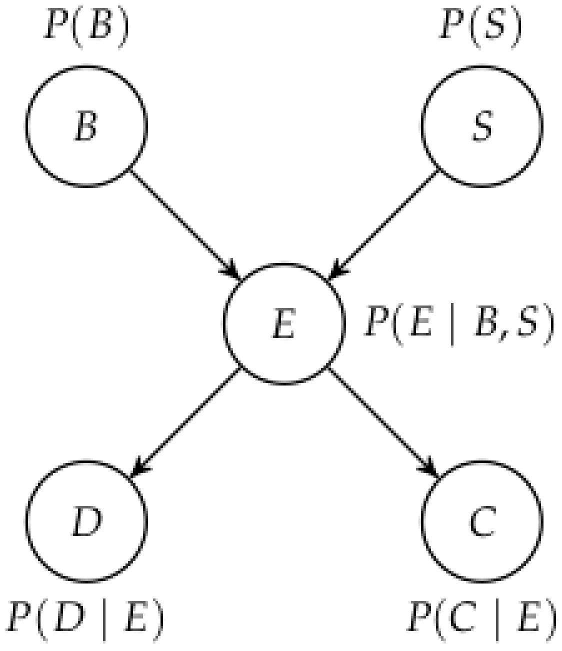
E electrical system failure
D trajectory deviation
C communication loss

$$P(b^1 \mid d^1, c^1) \approx \frac{\sum_i w_i(b^{(i)} = 1 \wedge d^{(i)} = 1 \wedge c^{(i)} = 1)}{\sum_i w_i(d^{(i)} = 1 \wedge c^{(i)} = 1)}$$
$$= \frac{\sum_i w_i(b^{(i)} = 1)}{\sum_i w_i}$$

Approximate Inference: Weighted Sampling



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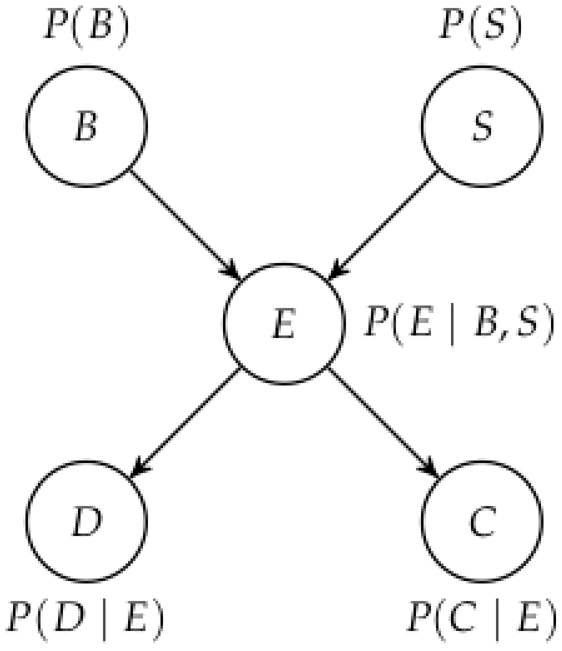
D trajectory deviation
C communication loss

$$P(b^1 \mid d^1, c^1) \approx \frac{\sum_i w_i(b^{(i)} = 1 \land d^{(i)} = 1 \land c^{(i)} = 1)}{\sum_i w_i(d^{(i)} = 1 \land c^{(i)} = 1)} = \frac{\sum_i w_i(b^{(i)} = 1)}{\sum_i w_i}$$

B	S	E	D	C	Weight
1	0	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$
0	1	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$
0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
0	0	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$

Approximate Inference: Weighted Sampling



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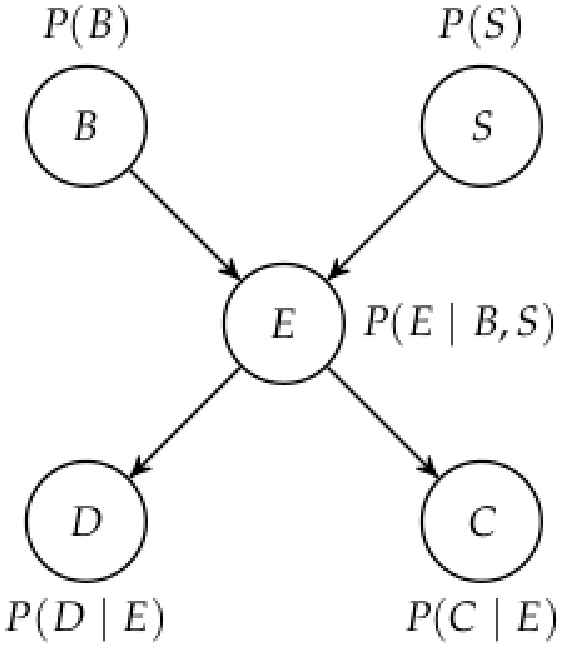
$$= \frac{\sum_i w_i(b^{(i)} = 1)}{\sum_i w_i}$$

B	S	E	D	C	Weight
1	0	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$
0	1	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$
0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
0	0	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$

Analogous to

Approximate Inference: Weighted Sampling



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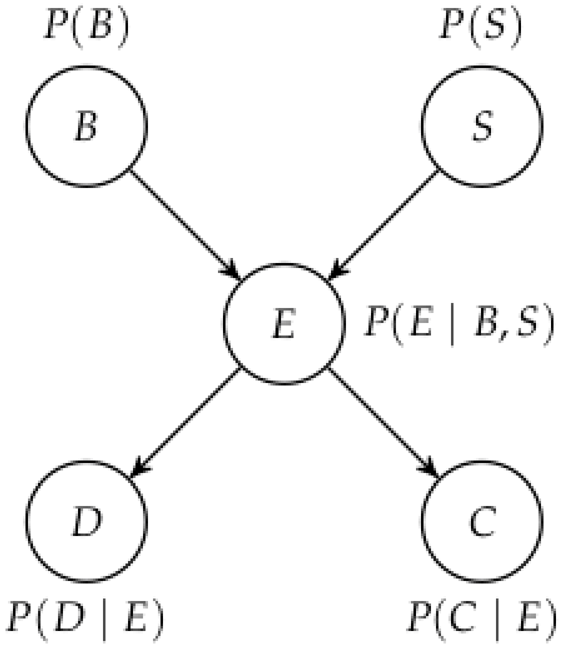
B	S	E	D	C	Weight
1	0	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$
0	1	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$
0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
0	0	0	1	1	$P(d^1 \mid e^0)P(c^1 \mid e^0)$
0	0	1	1	1	$P(d^1 \mid e^1)P(c^1 \mid e^1)$

Analogous to **weighted particle filtering**

Approximate Inference: Gibbs Sampling



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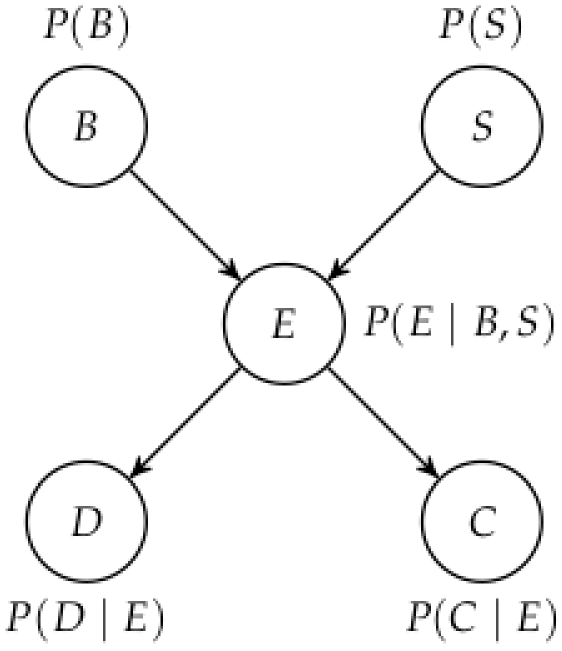
Approximate Inference: Gibbs Sampling



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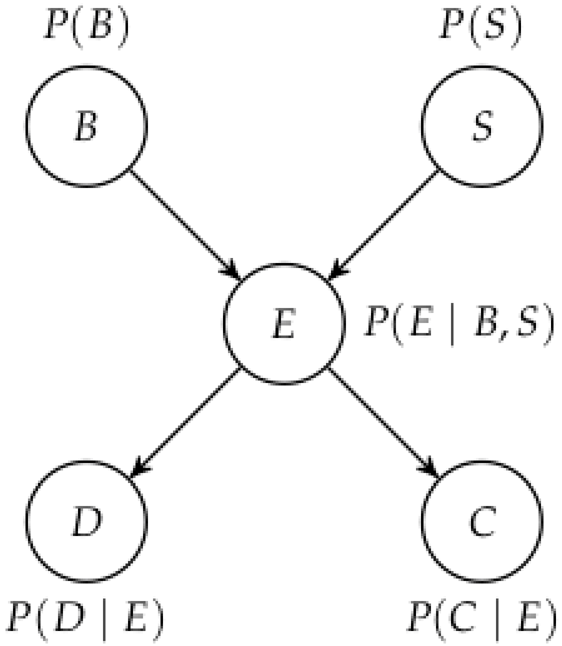
Markov Chain Monte Carlo (MCMC)

Approximate Inference: Gibbs Sampling



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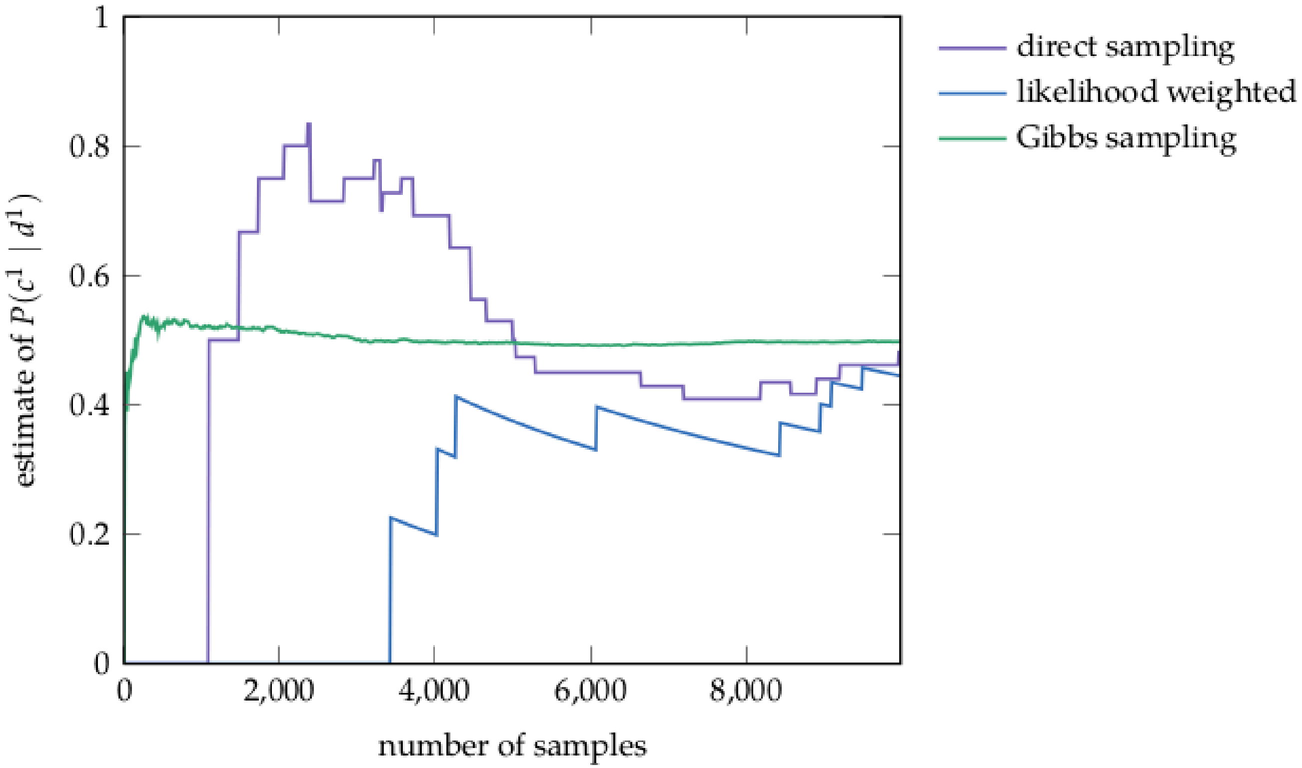
Markov Chain Monte Carlo (MCMC)



Recap