Policy and Value Iteration

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(MDP notebook)

Guiding Questions

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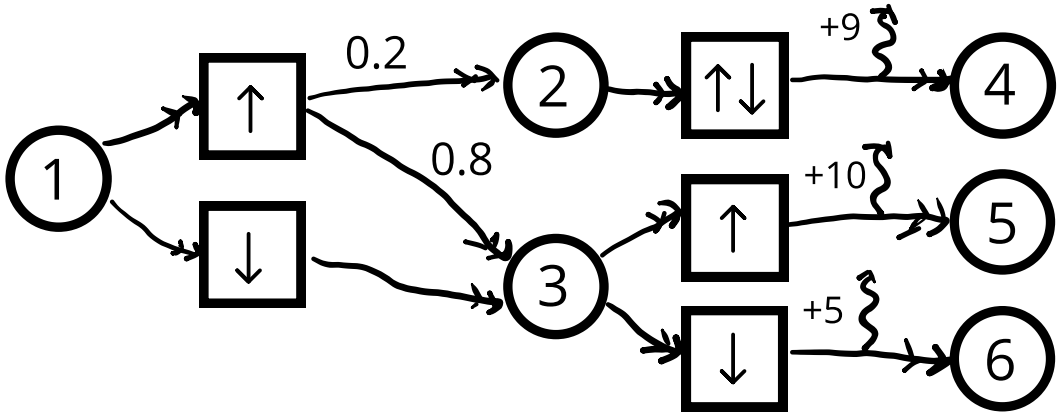
- How do we reason about the **future consequences** of actions in an MDP?
- What are the basic **algorithms for solving MDPs**?

Value-Based Policy Evaluation

MDP Example: Up-Down Problem

From *Dynamic Programming and the Calculus of Variations*, 1965



Algorithm: Bellman Backup

Given: MDP $(S, A, R, T, S_T, \gamma)$

1. $U^*(s) \leftarrow 0 \quad \forall s \in S_T$
2. Repeat until $U^*(s)$ known for all states:
    1. Choose s where U^* is known for all children
    2. Calculate $U^*(s)$
3. Extract $\pi^*(s) = \operatorname{argmax} Q^*(s, a)$

Break: DIA Run

Policy Iteration

Algorithm: Policy Iteration

Given: MDP (S, A, R, T, γ, b)

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2. while $\pi \neq \pi'$
3. $\quad \pi \leftarrow \pi'$
4. $\quad U^\pi \leftarrow (I - \gamma T^\pi)^{-1} R^\pi$
5. $\quad \pi'(s) \leftarrow \underset{a \in A}{\operatorname{argmax}} \left(R(s,a) + \gamma \sum_{s' \in S} T(s'|s,a) U^\pi(s') \right) \quad \forall s \in S$

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(Policy iteration notebook)

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- Returned U' will be close to U^*!
- π^* is easy to extract: $\pi^*(s) = \arg\max(R(s,a) + \gamma E[U^*(s)])$

Bellman's Equations

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"In any small change he will have to consider only these quantitative indices (or "values") in which all the relevant information is concentrated; and by adjusting the quantities one by one, he can appropriately rearrange his dispositions without having to solve the whole puzzle ab initio, or without needing at any stage to survey it at once in all its ramifications."

-- F. A. Hayek, "The use of knowledge in society", 1945