Online Methods

Last Time

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- Does value iteration always converge?
- Is the optimal value function unique?

Guiding Questions

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- What are the differences between *online* and *offline* solutions?
- Are there solution techniques that require computation time *independent* of the state space size?

Why Do We Need Something Else?

- Problems Policy and Value Iteration may struggle with?

- Why are these problems hard?

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  - More realistic car dynamics (continuous states)
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Why Do We Need Something Else?

- Problems Policy and Value Iteration may struggle with?
  - Path planning across the country, or interplanetary
  - More realistic car dynamics (continuous states)
- Why are these problems hard?
  - State Space is massive (or infinite)

Curse of Dimensionality

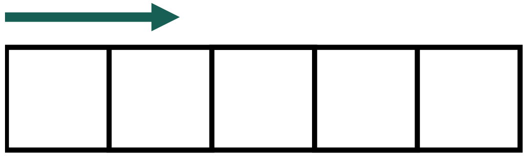
Curse of Dimensionality

1 dimension

e.g. $s = x \in S = \{1, 2, 3, 4, 5\}$

$|S| = 5$

(Discretize each dimension into 5 segments)



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2 dimensions

e.g. $s = (x, y) \in S = \{1, 2, 3, 4, 5\}^2$

$|S| = 25$

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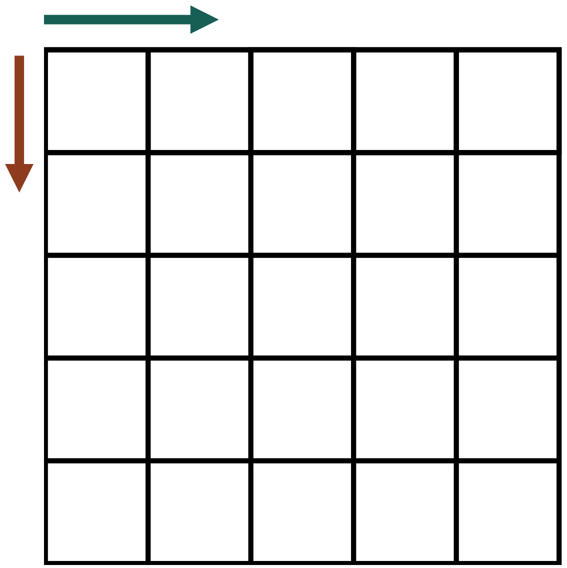

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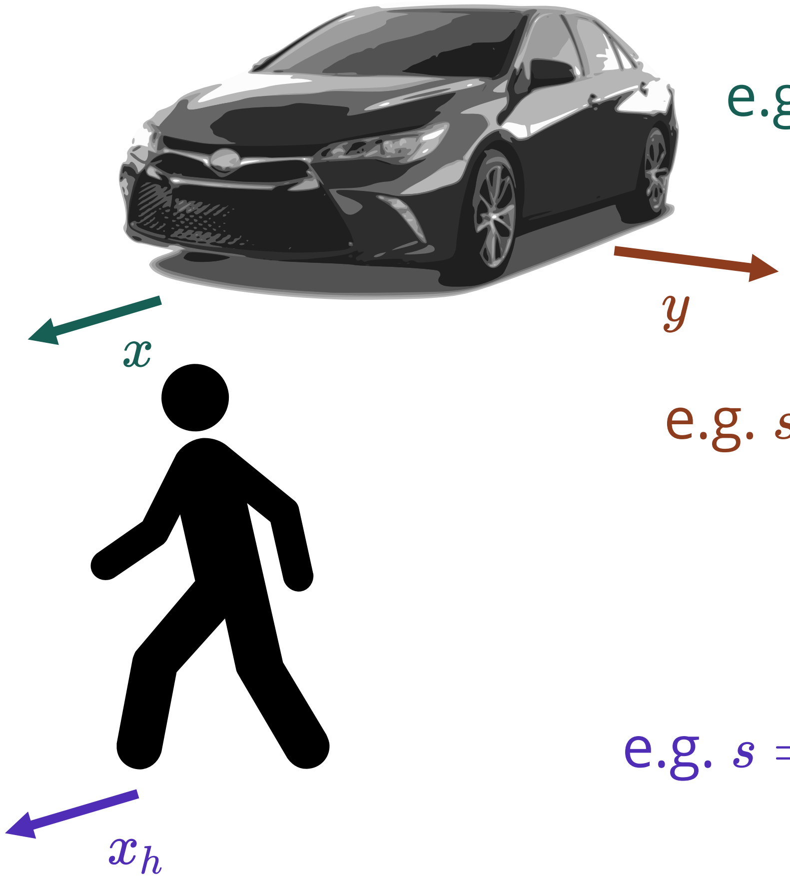

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3 dimensions

e.g. $s = (x, y, x_h) \in S = \{1, 2, 3, 4, 5\}^3$

$|S| = 125$

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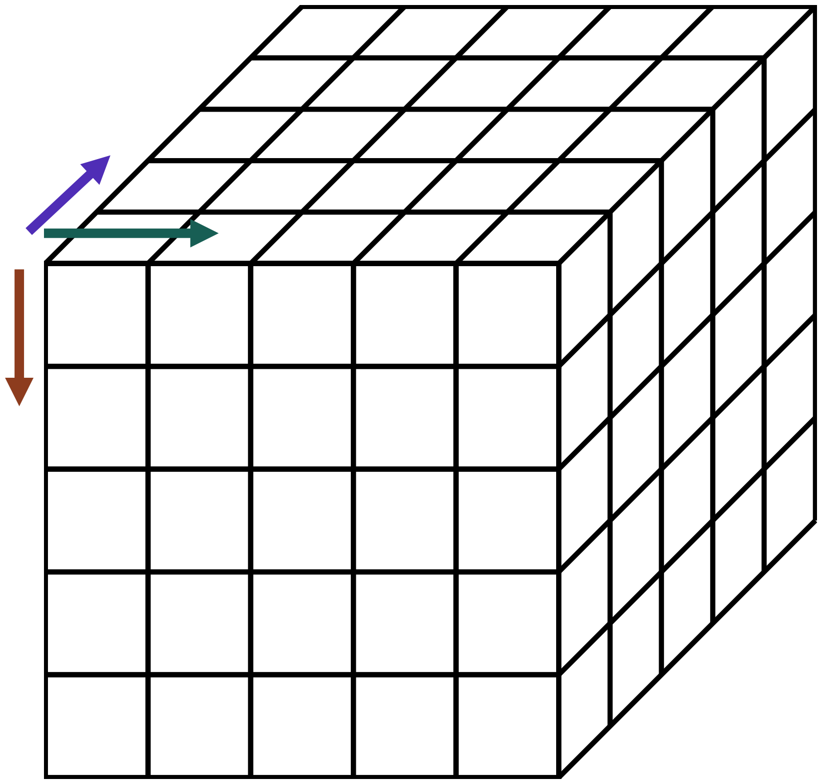

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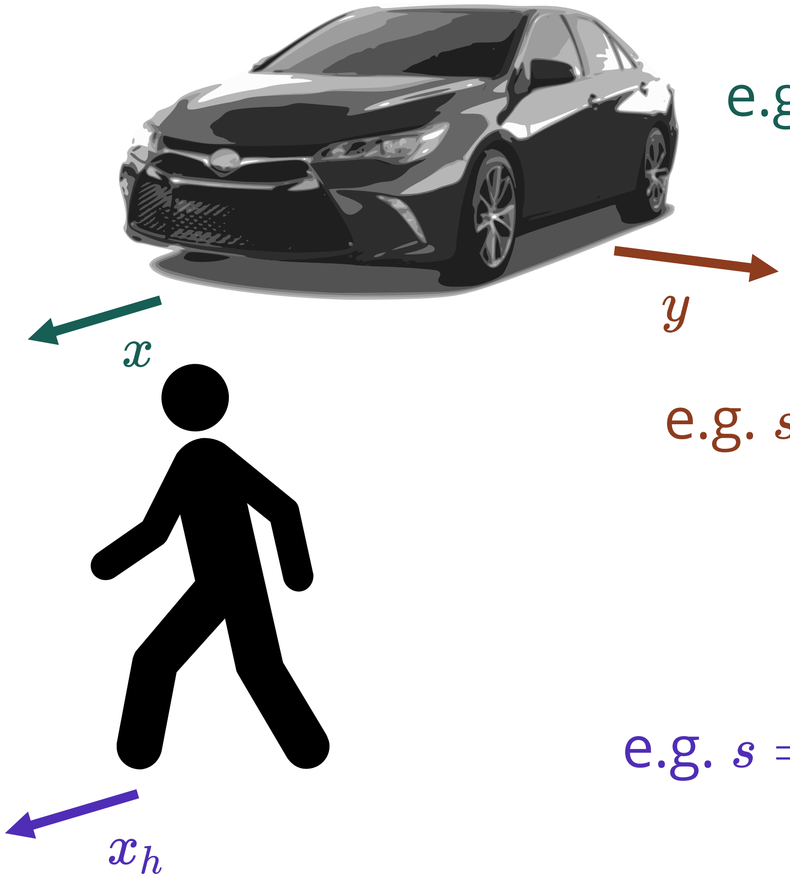

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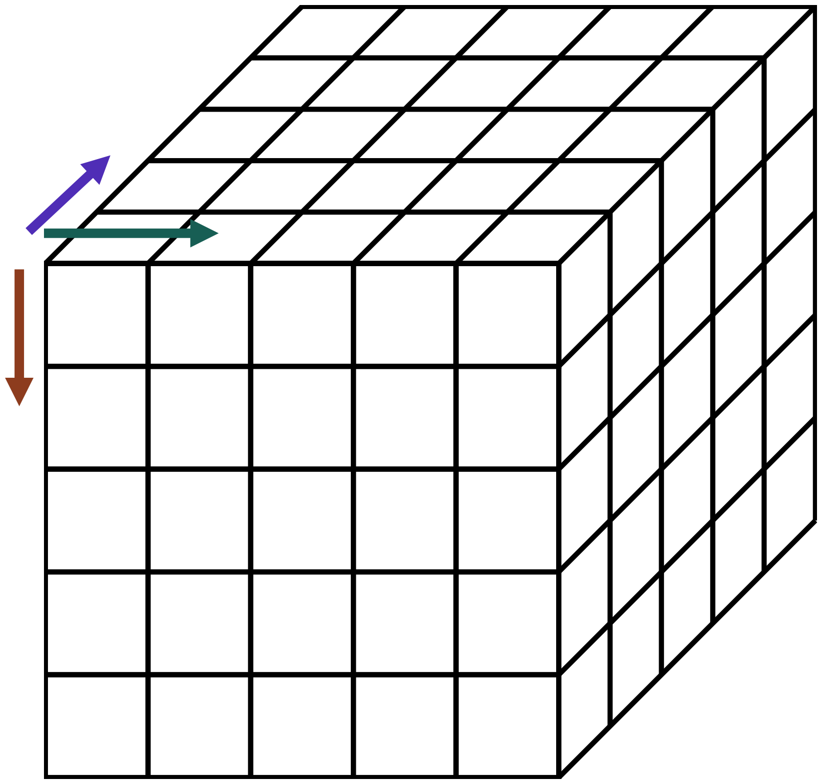

d dimensions, k segments $\rightarrow |S| = k^d$

Offline vs Online Solutions

Offline

Online

Offline vs Online Solutions

<u>Offline</u>

- Before Execution: find V^*/Q^*

<u>Online</u>

Offline vs Online Solutions

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- Before Execution: find V^*/Q^*
- During Execution: $\pi^*(s) = \operatorname{argmax} Q^*(s, a)$

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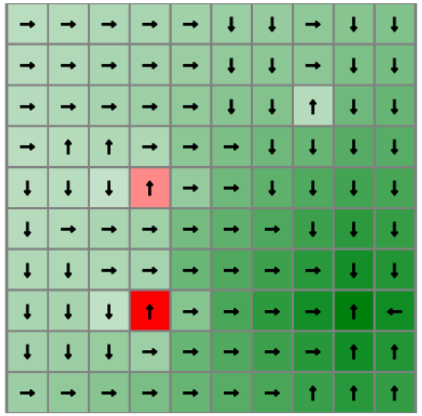
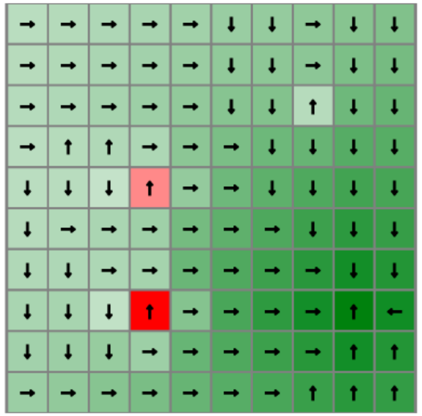


<u>Online</u>

Offline vs Online Solutions



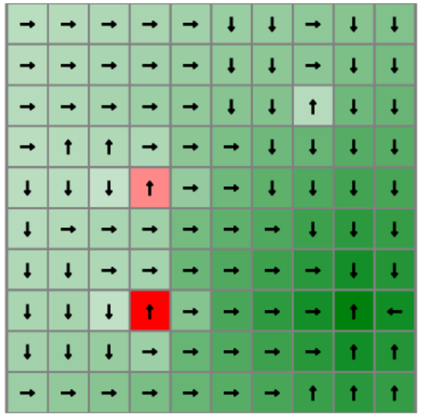
Offline

- Before Execution: find V^*/Q^*
- During Execution: $\pi^*(s) = \operatorname{argmax} Q^*(s,a)$

Online

- Before Execution: <nothing>

Offline vs Online Solutions



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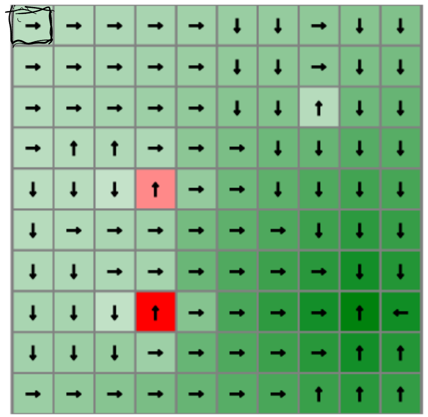

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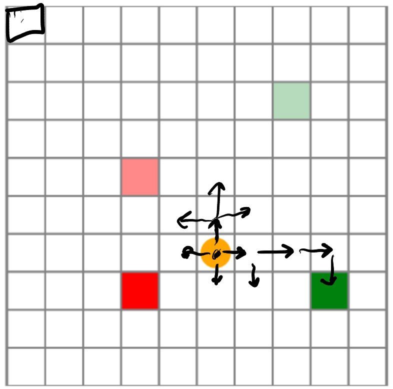
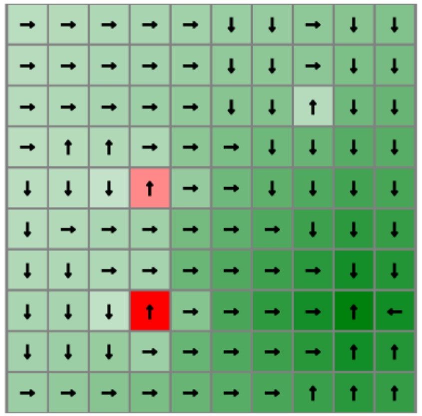


Offline vs Online Solutions



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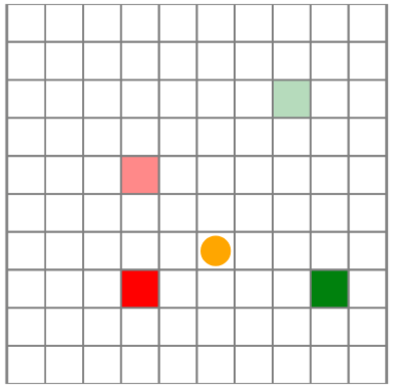

Online

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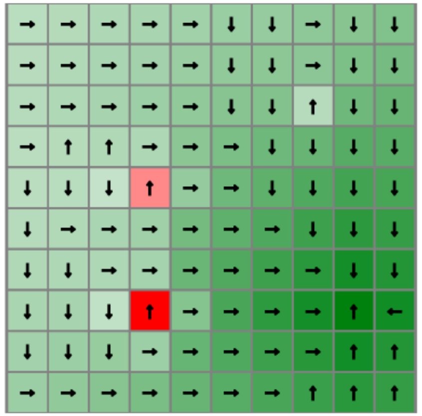
- During Execution: Consider actions and their consequences (everything)

- Why?

Offline vs Online Solutions



Offline

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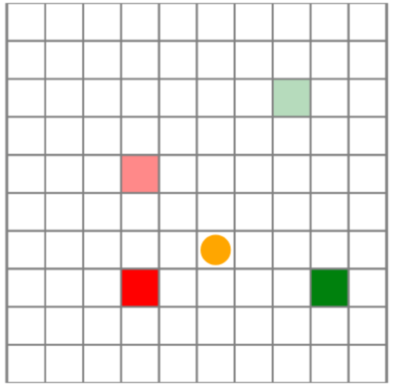


Online

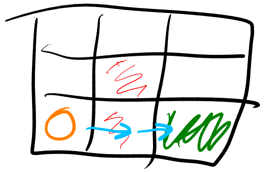
- Before Execution: <nothing>
- During Execution: Consider actions and their consequences (everything)

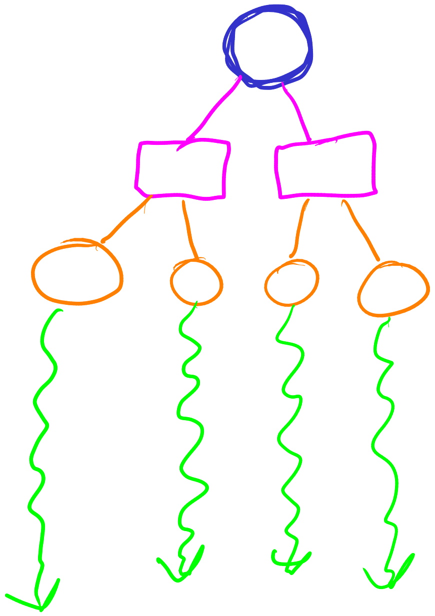
- Why?
- Online methods are insensitive to the size of S !

```julia
struct MDP
    γ  # discount factor
    𝒮  # state space
    𝒜  # action space
    T  # transition function
    R  # reward function
    TR # sample transition and reward
end
```

s', r ← G(s,a)

Rollout Lookahead



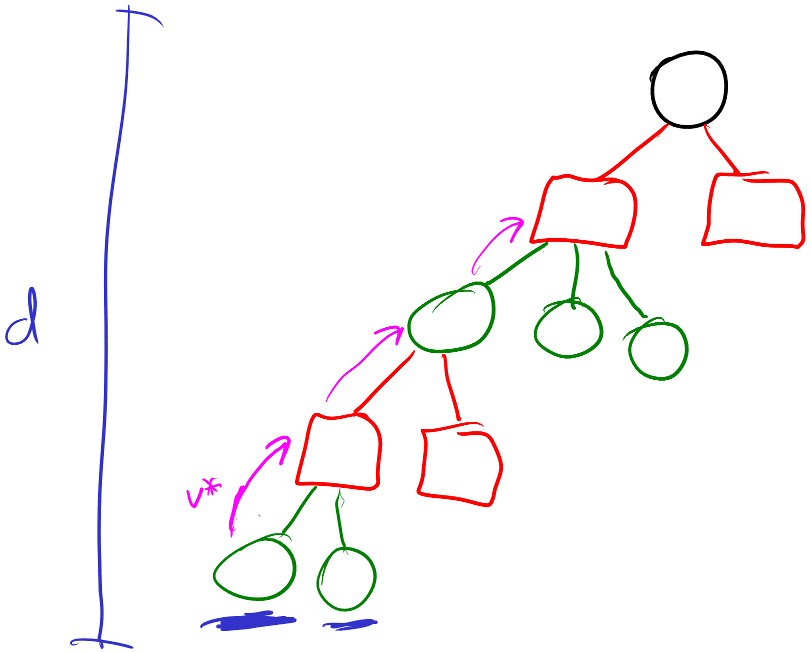


```julia
struct RolloutLookahead
    𝒫 # problem
    π # rollout policy
    d # depth
end

randstep(𝒫::MDP, s, a) = 𝒫.TR(s, a)

function rollout(𝒫, s, π, d)
    ret = 0.0
    for t in 1:d
        a = π(s)
        s, r = randstep(𝒫, s, a)
        ret += 𝒫.γ^(t-1) * r
    end
    return ret
end

function (π::RolloutLookahead)(s)
    U(s) = rollout(π.𝒫, s, π.π, π.d)
    return greedy(π.𝒫, U, s).a
end
```

```julia
function greedy(𝒫::MDP, U, s)
    u, a = findmax(a->lookahead(𝒫, U, s, a), 𝒫.𝒜)
    return (a=a, u=u)
end
```

```julia
function lookahead(𝒫::MDP, U, s, a)
    𝒮, T, R, γ = 𝒫.𝒮, 𝒫.T, 𝒫.R, 𝒫.γ
    return R(s,a) + γ*sum(T(s,a,s′)*U(s′) for s′ in 𝒮)
end
```

Forward Search



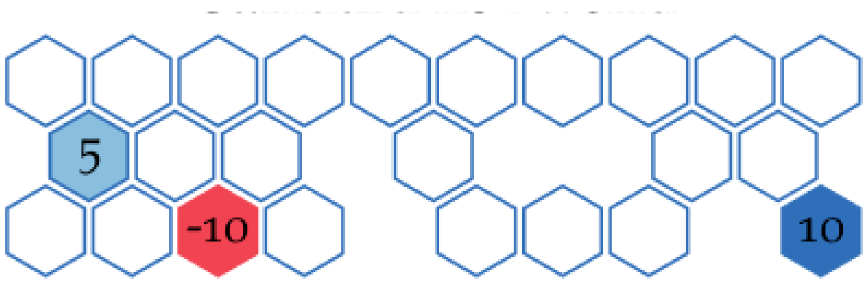
Algorithm 4.6 Forward search

function SELECTACTION(s, d)
  if $d = 0$
    return (NIL, 0)
  $(a^*, v^*) \leftarrow (\text{NIL}, -\infty)$
  for $a \in A(s)$
    $v \leftarrow R(s, a)$
    for $s' \in S(s, a)$
      $(a', v') \leftarrow$ SELECTACTION$(s', d-1)$
      $v \leftarrow v + \gamma T(s' \mid s, a) v'$
    if $v > v^*$
      $(a^*, v^*) \leftarrow (a, v)$
  return (a^*, v^*)

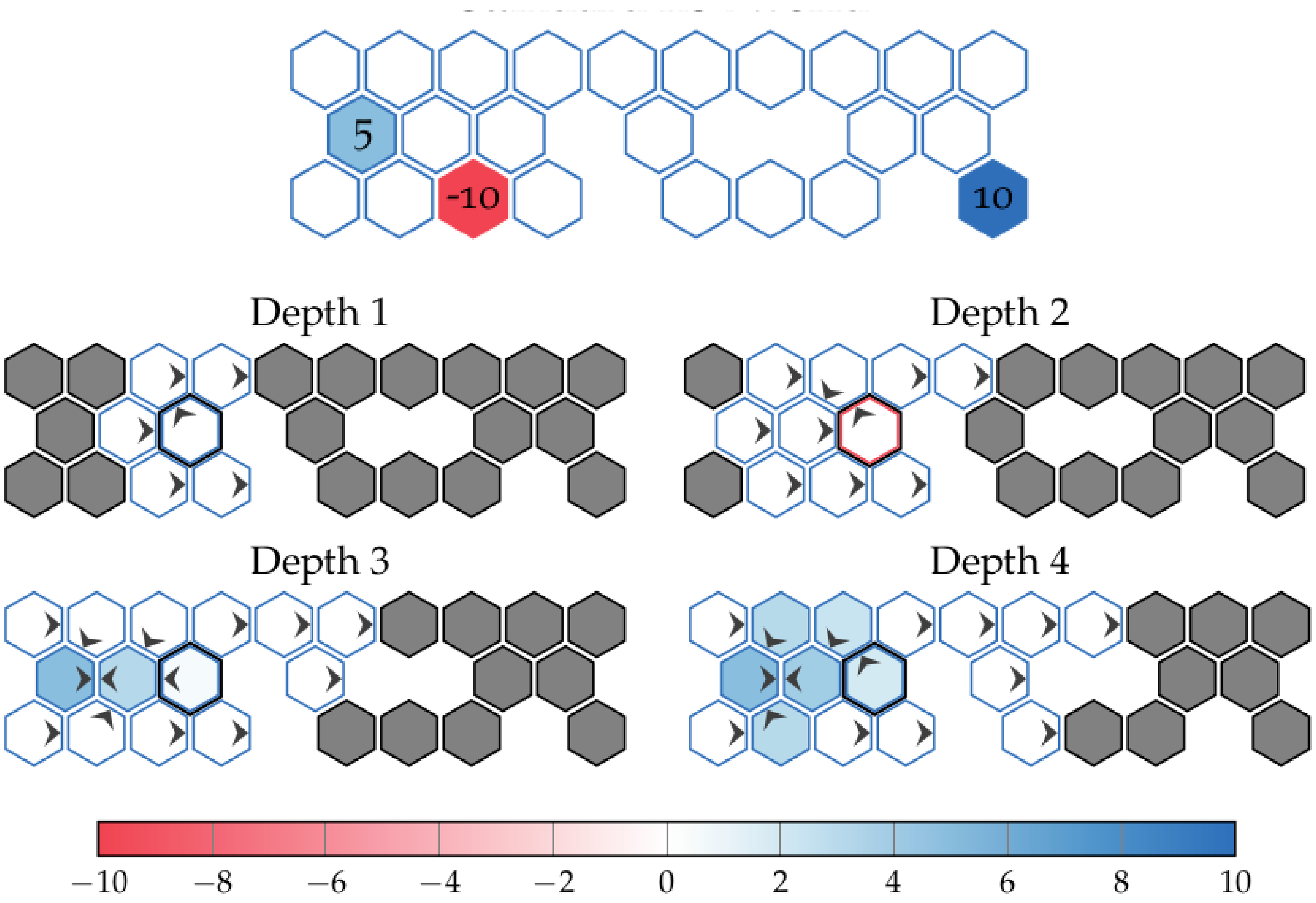
$$(|S| \times |A|)^d$$

Forward Search depth

Forward Search depth

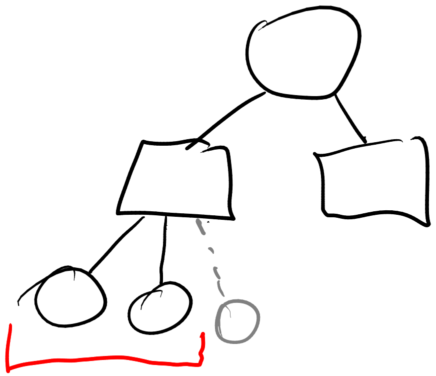


Forward Search depth



Sparse Sampling

$n = 2$



Algorithm 4.8 Sparse sampling

```
function SELECTACTION(s, d)
    if d = 0
        return (NIL, 0)
    (a*, v*) ← (NIL, −∞)
    for a ∈ A(s)
        v ← 0
        for i ← 1 to n
            (s′, r) ~ G(s, a)
            (a′, v′) ← SELECTACTION(s′, d − 1)
            v ← v + (r + γv′)/n
        if v > v*
            (a*, v*) ← (a, v)
    return (a*, v*)
```

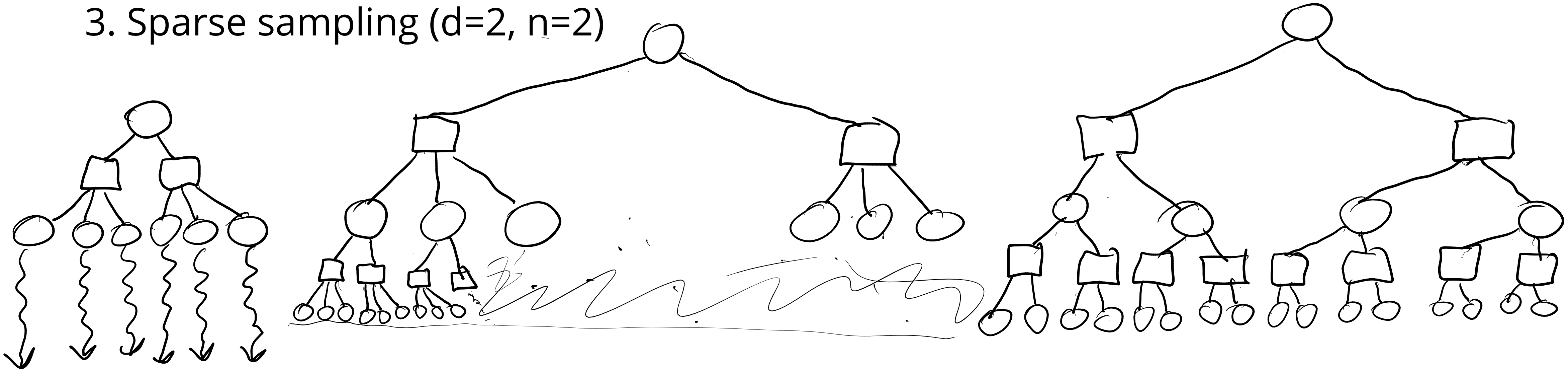
$\underbrace{(n|A|)^d}_{\text{size of tree}}$ $\quad$ $\underbrace{|V^{\text{SS}}(s) - V^*(s)|}$ $\leq \epsilon$ $\quad$ n, ϵ, and d related, but independent of $|S|$

https://www.cis.upenn.edu/~mkearns/papers/sparsesampling-journal.pdf

Break

Draw the trees produced by the following algorithms for a problem with 2 actions and 3 states:

1. One-step lookahead with rollout
2. Forward search (d=2)
3. Sparse sampling (d=2, n=2)



Monte Carlo Tree Search (MCTS/UCT)

Keep track of:

$Q(s, a)$: Value estimate of that state and action combo

$N(s, a)$: Number of times we visit a state and action combo

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$$\underline{Q(s,a)} + \underbrace{c\sqrt{\frac{\log N(s)}{N(s,a)}}}_{\text{Exploration}}$$

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$Q(s,a)$: Value estimate of that state and action combo

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$$Q(s,a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$$

low $N(s,a)/N(s)$ = high bonus

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start with $c = 2(\bar{V} - \underline{V})$, $\beta = 1/4$

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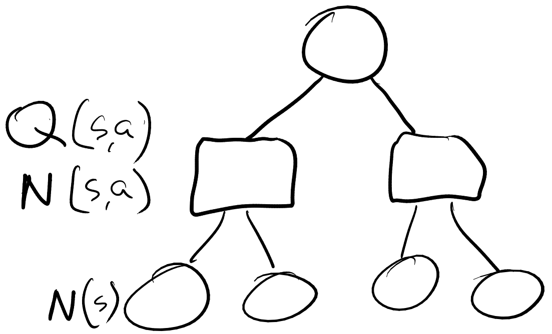
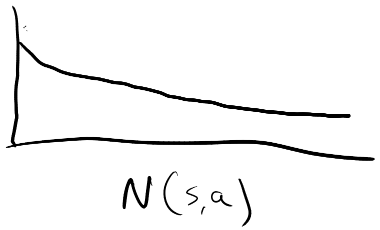


Original

$$Q(s, a) + c\sqrt{\frac{\log N(s)}{N(s, a)}}$$

New PolyUCT

$$Q(s, a) + c\frac{N(s)^{\beta}}{\sqrt{N(s, a)}}$$



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Full story can be found in https://arxiv.org/pdf/1902.05213.pdf

Monte Carlo Tree Search (MCTS/UCT)

```julia
function (π::MonteCarloTreeSearch)(s)
    for k in 1:π.m
        simulate!(π, s)
    end
    return argmax(a->π.Q[(s,a)], π.𝒫.𝒜)
end
```

```julia
function simulate!(π::MonteCarloTreeSearch, s, d=π.d)
    if d ≤ 0
        return π.U(s)
    end
    𝒫, N, Q, c = π.𝒫, π.N, π.Q, π.c
    𝒜, TR, γ = 𝒫.𝒜, 𝒫.TR, 𝒫.γ
    if !haskey(N, (s, first(𝒜)))
        for a in 𝒜
            N[(s,a)] = 0
            Q[(s,a)] = 0.0
        end
        return π.U(s)
    end
    a = explore(π, s)
    s′, r = TR(s,a)
    q = r + γ*simulate!(π, s′, d-1)
    N[(s,a)] += 1
    Q[(s,a)] += (q-Q[(s,a)])/N[(s,a)]
    return q
end
```

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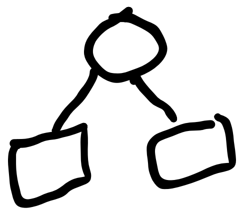

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m iterations

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        return π.U(s)
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    q = r + γ*simulate!(π, s′, d-1)
    N[(s,a)] += 1
    Q[(s,a)] += (q-Q[(s,a)])/N[(s,a)]
    return q
end
```

2. expansion

maximizes $Q(s,a) + c\sqrt{\frac{\log N(s)}{N(s,a)}}$

3. Rollout/Value Estimate

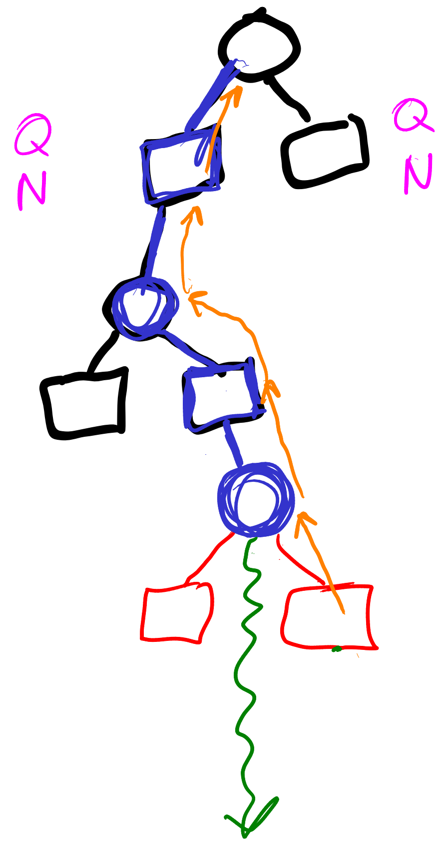
$G(s,a)$

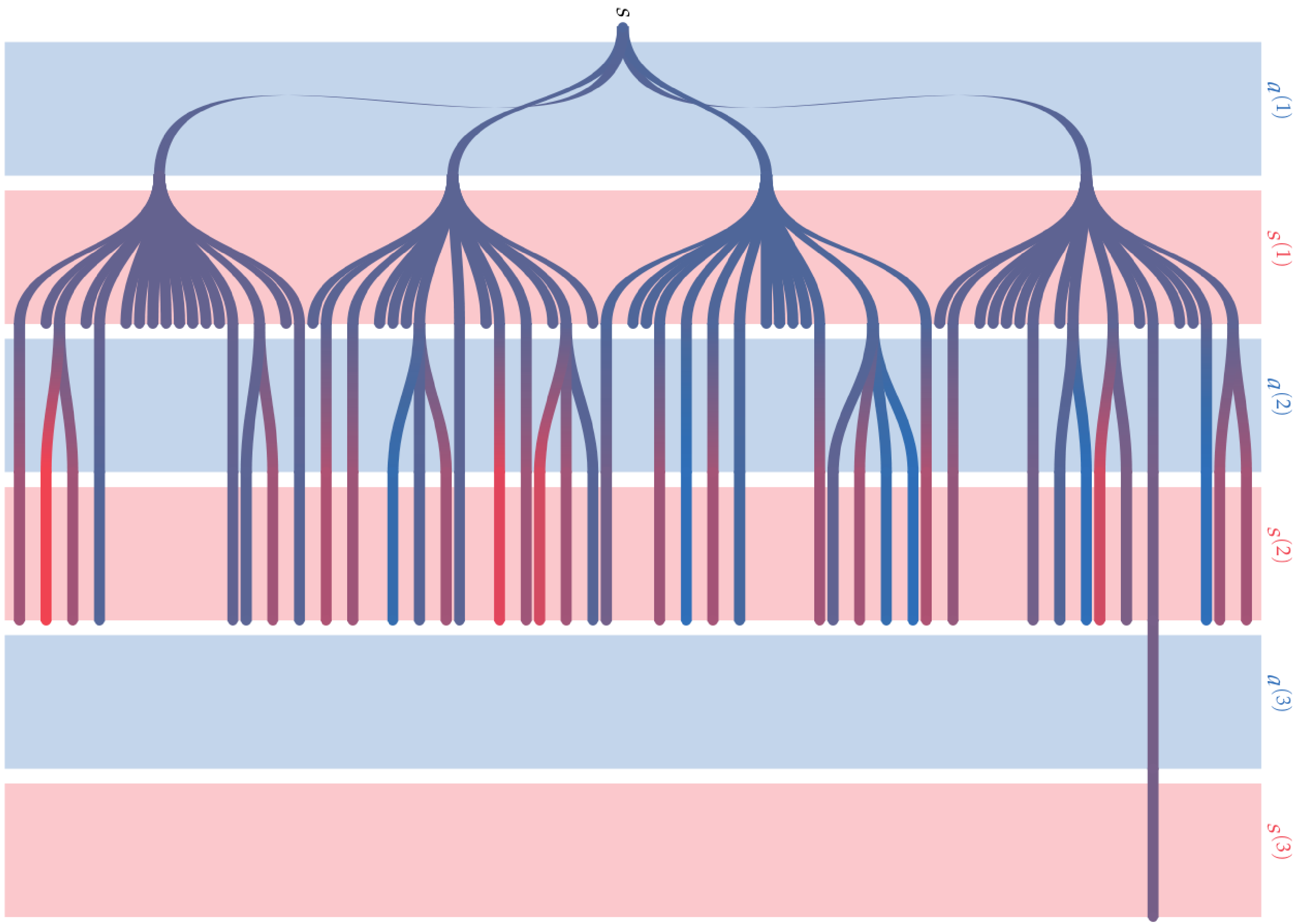
1. Search

4. Backup

want $U(s)$

rollout gives a sample estimate of U




s
$a^{(1)}$
$s^{(1)}$
$a^{(2)}$
$s^{(2)}$
$a^{(3)}$
$s^{(3)}$

Using Online Methods in a Simulation

Using Online Methods in a Simulation

<u>Algorithm: Rollout Simulation</u>

Given: MDP (S, A, R, T, γ, b)

$s \leftarrow \text{sample}(b)$

$\hat{u} \leftarrow 0$

for t in $0 \ldots T-1$

$\quad a \leftarrow \pi(s)$

Create tree (run more imaginary rollouts)
Choose best action based on tree

$\quad s', r \leftarrow G(s, a)$

take step in real environment

$\quad \hat{u} \leftarrow \hat{u} + \gamma^t r$

$\quad s \leftarrow s'$

return $\hat{u}$

Guiding Questions

Guiding Questions

- What are the differences between online and offline solutions?
- Are there solution techniques that are *independent* of the state space size?

Forward Search Sparse Sampling

(FSSS)

Paper: https://cdn.aaai.org/ojs/7689/7689-13-11219-1-2-20201228.pdf

- Sparse Sampling, but only look at potentially valuable states

Forward Search Sparse Sampling

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- Sparse Sampling, but only look at potentially valuable states

Things it keeps track of:

$Q(s, a)$: Estimate of the value for the state action pair

$U(s)$: Upper bound for value of state s

$L(s)$: Lower bound for value of state s

$U(s, a)$: Upper bound for value of state-action

$L(s, a)$: Lower bound for value of state-action

Forward Search Sparse Sampling

Algorithm 3 FSSS(s, d)

if $d = 1$ (leaf) **then**
  $L^d(s,a) = U^d(s,a) = R(s,a), \forall a$
  $L^d(s) = U^d(s) = \max_a R(s,a)$
else if $n_{sd} = 0$ **then**
  for each $a \in A$ **do**
    $L^d(s,a) = V_{\min}$
    $U^d(s,a) = V_{\max}$
    for C times **do**
      $s' \sim T(s,a,\cdot)$
      $L^{d-1}(s') = V_{\min}$
      $U^{d-1}(s') = V_{\max}$
      $K^d(s,a) = K^d(s,a) \cup \{s'\}$
$a^* = \operatorname{argmax}_a U^d(s,a)$
$s^* = \max_{s' \in K^d(s,a^*)} (U^{d-1}(s') - L^{d-1}(s'))$
FSSS(s^*, $d-1$)
$n_{sd} = n_{sd} + 1$
$L^d(s,a^*) = R(s,a^*) + \gamma \sum_{s' \in K^d(s,a^*)} L^{d-1}(s')/C$
$U^d(s,a^*) = R(s,a^*) + \gamma \sum_{s' \in K^d(s,a^*)} U^{d-1}(s')/C$
$L^d(s) = \max_a L^d(s,a)$
$U^d(s) = \max_a U^d(s,a)$

Forward Search Sparse Sampling

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    for C times **do**
      $s' \sim T(s,a,\cdot)$
      $L^{d-1}(s') = V_{\min}$
      $U^{d-1}(s') = V_{\max}$
      $K^d(s,a) = K^d(s,a) \cup \{s'\}$
$a^* = \operatorname{argmax}_a U^d(s,a)$
$s^* = \max_{s' \in K^d(s,a^*)} (U^{d-1}(s') - L^{d-1}(s'))$
FSSS(s^*, $d-1$)
$n_{sd} = n_{sd} + 1$
$L^d(s,a^*) = R(s,a^*) + \gamma \sum_{s' \in K^d(s,a^*)} L^{d-1}(s')/C$
$U^d(s,a^*) = R(s,a^*) + \gamma \sum_{s' \in K^d(s,a^*)} U^{d-1}(s')/C$
$L^d(s) = \max_a L^d(s,a)$
$U^d(s) = \max_a U^d(s,a)$

If $L(s,a*) \geq \max_{a \neq a^*} U(s,a)$ for best action ($a^* = \arg\max_a U(s,a)$):
then, the node is closed because the best action is found.