Math2411 - Calculus II
Guided Lecture Notes
Integration – Basic Approaches

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Integration – Basic Approaches:

We are looking at some basic approaches to integration used when a function does not exactly match one of our known formulas. Often the strategy is to use identities and/or manipulate the integrand algebraically so that it fits into a know form. We will work through a number of examples below.

Integration Examples:

Let's work an example together.

Example 1. Evaluate the integral $\int \cos^2(x)\, dx$

Workspace:

Solution:

We integrate as follows.

$$\begin{aligned}
\int \cos^2(x)\,dx &= \frac{1}{2}\int 1+\cos(2x)\,dx \\
&= \frac{1}{2}\left(\, x+\frac{1}{2}\sin(2x) \,\right)+C \\
&= \frac{1}{2}x+\frac{1}{4}\sin(2x)+C
\end{aligned}$$

Example 2. Evaluate the integral $\int \tan(x)\,dx$

Workspace:

Solution:

We integrate as follows.

$$\begin{aligned} \int \tan(x)\,dx &= \int \frac{\sin(x)}{\cos(x)}\,dx \qquad \text{Let } u = \cos(x) \text{ and } du = -\sin(x)\,dx. \\ &= -\ln\big|\cos(x)\big| + C \\ &= \ln\big|\sec(x)\big| + C \end{aligned}$$

Example 3. Evaluate the integral $\int \sec(x)\,dx$

Workspace:

Solution:

We integrate as follows.

$$\begin{aligned}
\int \sec(x)\,dx &= \int \sec(x)\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\,dx \\
&= \int \frac{\sec^2(x)+\sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx \qquad \text{Let } u=\sec(x)+\tan(x),\ du=\sec^2(x)+\sec(x)\tan(x)dx \\
&= \ln\big|\sec(x)+\tan(x)\big|+C
\end{aligned}$$

Example 4. Evaluate the integral $\displaystyle\int \frac{1}{e^x+e^{-x}}\,dx$

Workspace:

Solution:

We integrate as follows.

$$\begin{aligned}
\int \frac{1}{e^x + e^{-x}}\, dx &= \int \frac{e^x}{e^x} \cdot \frac{1}{e^x + e^{-x}}\, dx \\
&= \int \frac{e^x}{e^{2x} + 1}\, dx \qquad \text{Let } u = e^x \text{ and } du = e^x\, dx \\
&= \frac{1}{u^2 + 1}\, du \\
&= \tan^{-1}(u) + C \\
&= \tan^{-1}\left(e^x\right) + C
\end{aligned}$$

Example 5. Evaluate the integral $\displaystyle\int \frac{1}{x^2 + 4x + 5}\, dx$

Workspace:

Solution:

We integrate as follows.

$$\begin{aligned}
\int \frac{1}{x^2+4x+5}\,dx &= \int \frac{1}{x^2+4x+4+1}\,dx \\
&= \int \frac{1}{(x+2)^2+1}\,dx \qquad \text{Let } u = x+2 \text{ and } du = dx \\
&= \int \frac{1}{u^2+1}\,du \\
&= \tan^{-1}(u) + C \\
&= \tan^{-1}(x+2) + C
\end{aligned}$$

Please let me know if you have any questions, comments, or corrections!