Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #8 Solutions

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Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. **Transformations of Random Variables**

   (a) Let $X \sim N(10, 4)$. Define $Y = 3X - 5$. Compute the mean and variance of Y.

   (b) Let $X \sim N(5, 9)$. Define $Y = -2X + 7$. Compute the mean and variance of Y.

   (c) Standardize $X \sim N(\mu, \sigma^2)$ to $Z = (X - \mu)/\sigma$. Compute $P(X < 15)$ for $X \sim N(12, 16)$ using standardization.

2. **Simulation of Linear Transformations**

   (a) Simulate 50, 100, 1000, and 50000 draws from $X \sim N(5, 2^2)$ and $Y = -3X + 2$. Plot histograms overlaying X and Y.

   (b) Plot the empirical CDFs of X and Y and compare with theoretical CDFs.

Solutions

1.
```
# Example 1
X1 <- 10
VarX1 <- 4
Y1_mean <- 3*X1 - 5
Y1_var <- 3^2 * VarX1

# Example 2
X2 <- 5
VarX2 <- 9
Y2_mean <- -2*X2 + 7
Y2_var <- (-2)^2 * VarX2

# Example 3
mu <- 12
sigma <- 4
```

```r
   Z <- (15 - mu)/sigma
   pnorm(Z)

2. set.seed(123)
   mu <- 5; sigma <- 2; a <- -3; b <- 2
   for(n in c(50,100,1000,50000)){
     X <- rnorm(n, mu, sigma)
     Y <- a*X + b
     hist(X, breaks=50, prob=TRUE, col=rgb(0,0,1,0.5), main=paste("n=",n))
     hist(Y, breaks=50, prob=TRUE, col=rgb(1,0,0,0.5), add=TRUE)
     # Empirical CDF
     ecdf_X <- ecdf(X)
     ecdf_Y <- ecdf(Y)
     plot(ecdf_X, verticals=TRUE, col="blue", main=paste("CDF, n=",n))
     lines(ecdf_Y, col="red", verticals=TRUE)
   }
```

Please let me know if you have any questions, comments, or corrections!