Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #2 Solutions

University of Colorado Denver / College of Liberal Arts and Sciences

Department of Mathematics - Dr. Robert Rostermundt

Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. **Vector and Sequence Operations**
   (a) Create a numeric vector containing the numbers 1 through 10.
   (b) Compute the sum and mean of the vector.
   (c) Create a sequence from 0 to 5 in increments of 0.5.
   (d) Compute the sum of the squares of this sequence.

2. **Simulating Two Dice Rolls**
   (a) Simulate a single roll of two fair six-sided dice.
   (b) Simulate 1000 rolls of two dice.
   (c) Compute the proportion of times the sum is 7.
   (d) Compare your result to the theoretical probability.

3. **Empirical Distribution of a Die**
   (a) Simulate 600 rolls of a fair six-sided die.
   (b) Compute the empirical probability of each face.
   (c) Create a barplot of the frequencies.
   (d) Compare the empirical probabilities to the theoretical probability of 1/6 for each face.

4. **Simulating a Biased Coin**
   (a) Simulate 100 coin tosses where $P(\text{Heads}) = 0.7$.
   (b) Compute the proportion of heads and tails.
   (c) Repeat this simulation three times and observe the variation.
   (d) Discuss why the results differ slightly across trials.

5. **Conceptual Question** In your own words, explain:
   - The difference between theoretical and empirical probabilities,
   - Why simulations can be used to estimate probabilities,
   - How increasing the number of trials affects the estimates.

Solutions

1. ```
   v <- 1:10
   sum(v)
   mean(v)

   seq1 <- seq(0, 5, by = 0.5)
   sum(seq1^2)
   ```

2. ```
   roll <- sample(1:6, 2, replace = TRUE)

   rolls <- replicate(1000, sum(sample(1:6, 2, replace = TRUE)))
   mean(rolls == 7)
   ```

   Theoretical probability: $6/36 = 1/6 \approx 0.1667$.

3. ```
   rolls <- sample(1:6, 600, replace = TRUE)
   table(rolls) / 600
   barplot(table(rolls), col = "skyblue", main = "Empirical␣Frequencies␣of␣Die␣Faces
   ```

   Theoretical probability for each face: $1/6 \approx 0.1667$.

4. ```
   coin <- sample(c("H","T"), 100, replace = TRUE, prob = c(0.7, 0.3))
   mean(coin == "H")
   mean(coin == "T")

   for(i in 1:3){
     coin <- sample(c("H","T"), 100, replace = TRUE, prob = c(0.7,0.3))
     print(mean(coin == "H"))
   }
   ```

   Results differ slightly due to random variation.

5. Simulations approximate theoretical probabilities. Empirical probabilities vary because each trial is random. Increasing the number of trials reduces variation, stabilizing estimates near theoretical values.

Please let me know if you have any questions, comments, or corrections!