Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #7 Solutions

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Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. **Joint Distribution Simulation**
   (a) Simulate 5000 pairs (X, Y) where $X \sim N(0, 1)$, $Y \sim N(0, 1)$ independent.
   (b) Plot the scatterplot of (X, Y) and comment on independence.
   (c) Compute sample covariance and correlation.
2. **Dependent Variables**
   (a) Create $Z = X + Y$. Plot histogram of Z.
   (b) Compute sample mean and variance.
   (c) Compare with theoretical mean and variance.
3. **Bivariate Normal**
   (a) Simulate 5000 pairs from bivariate normal with $\rho = 0.7$.
   (b) Plot scatterplot and overlay marginal densities.
   (c) Compute sample correlation.
4. **Conditional Probability**
   (a) Estimate $P(Y > 1 \mid X > 0)$ from simulations.
   (b) Compare with theoretical conditional probability for independent X, Y.
5. **Discussion**
   - Explain how correlation affects scatterplots.
   - Discuss the effect on variance of the sum $Z = X + Y$.

Solutions

1. ```
   set.seed(123)
   X <- rnorm(5000)
   Y <- rnorm(5000)
   plot(X, Y)
   cov(X,Y)
   cor(X,Y)
   ```

2. ```
   Z <- X + Y
   hist(Z, prob=TRUE)
   mean(Z); var(Z)
   ```

3. ```
   library(MASS)
   Sigma <- matrix(c(1,0.7,0.7,1),2,2)
   biv <- mvrnorm(5000, mu=c(0,0), Sigma=Sigma)
   plot(biv[,1], biv[,2])
   cor(biv[,1], biv[,2])
   ```

4. ```
   mean(Y[X>0] > 1)
   ```

5. Correlation affects direction and shape of scatterplot; positive correlation tilts upward. Variance of sum increases with positive correlation.

Please let me know if you have any questions, comments, or corrections!