Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #5

University of Colorado Denver / College of Liberal Arts and Sciences

Department of Mathematics - Dr. Robert Rostermundt

Name: **Student Number:**

Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. **Joint Distributions**
   (a) Simulate 500 pairs (X, Y) from a bivariate normal distribution with $\mu_X = 0, \mu_Y = 0$, $\sigma_X^2 = 1, \sigma_Y^2 = 1$, and $\rho = 0.5$.
   (b) Compute the sample covariance and correlation.
   (c) Plot the scatterplot of Y versus X.
2. **Marginal Distributions**
   (a) Compute the empirical marginal distributions of X and Y.
   (b) Compare histograms with theoretical marginal densities.
3. **Conditional Distribution**
   (a) Compute $Y|X > 0$ and $Y|X < 0$.
   (b) Plot histograms of conditional distributions.
   (c) Comment on how the mean and variance differ conditionally.
4. **Covariance Transformation**
   (a) Define $U = 2X - Y$ and $V = X + 3Y$.
   (b) Compute the covariance matrix of (U, V).
   (c) Compare with theoretical result using linear transformation formula.
5. **Discussion**
   - Explain the effect of correlation on joint scatterplots.
   - Describe the effect of linear transformations on covariance.