Math3810 - Probability
Section 001 - Fall 2025
Practice Problems : Conditional Probability

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1. Yang Xuelian has lost her dog in either forest A (with *a priori* probability 0.4) or in forest B (with *a priori* probability 0.6). On any given day, if the dog is in A and Yang Xuelian spends a day searching for it in A, the conditional probability that she will find the dog that day is 0.25. Similarly, if the dog is in B and Yang Xuelian spends a day looking for it there, the conditional probability that she will find the dog that day is 0.15. The dog cannot go from one forest to the other. Moreover, Yang Xuelian can only search in one forest during a given day.

   (a) Draw a tree-diagram describing all possible outcomes and conditional probabilities in the sample space.

   (b) In which forest should Yang Xuelian look to maximize the probability she finds her dog on the first day of the search?

   (c) Given that Yang Xuelian looked in forest A on the first day but didn't find her dog, what is the probability that the dog is in forest A?

   (d) If Yang Xuelian flips a fair coin to determine where to look on the first day and finds the dog on the first day, what is the probability that she looked in forest A?

   (e) If the dog is alive and not found by the Nth day of the search, it will die that evening with probability $N/N + 2$. Yang Xuelian has decided to look in forest A for the first two days. What is the probability that she will find a live dog for the first time on the second day?

2. Let A be the event that it rains today and B be the event that we carry an umbrella today. It is clear that $\mathbb{P}\big(B\big|A\big) \neq \mathbb{P}(B)$ and so the events are dependent. Then we must have $\mathbb{P}\big(A\big|B\big) \neq \mathbb{P}(A)$. Does this imply that our choice to carry an umbrella effects whether it rains or doesn't rain? Explain your reasoning.

3. Consider the following random experiment. From a jar containing five indistinguishable coins we randomly select a coin. Then flip the coin, record the result. Assume that for each coin we have the following probability of flipping heads:

$$p_1 = 0.1 \qquad p_2 = 0.65 \qquad p_3 = 0.9 \qquad p_4 = 0.4 \qquad p_5 = 0$$

   (a) Determine the probability of flipping heads.

   (b) Now suppose that we flip the coin a second time, and then record the result. If we know that the coin showed heads on the first toss, what is the probability that obtain heads on this second toss.

   ***Note*:** Remember that the coins are indistinguishable.

4. Consider the following experiment. A container has $r > 1$ red balls and $b > 1$ black balls. A ball is drawn and the color is noted. Again, a ball is drawn and the color is noted. This process is repeated until we have chosen $k \geq 2$ times.

   Let $k = 2$ so that two balls are chosen. What is the probability that the sampling was done with replacement given that one red ball was chosen and one black ball was chosen? Show that this probability depends on $n = r + b$, but not on r or b separately.

5. We consider now a problem called the ***Monty Hall problem***. This problem but was made popular by a letter from Craig Whitaker to Marilyn vos Savant for consideration in her column in Parade Magazine. In his letter to the column, Craig wrote:

   "Suppose you're on Monty Hall's *L*et's Make a Deal! You are given the choice of three doors, behind one door is a car, the others, goats. You pick a door, say 1, Monty opens another door, say 3, which has a goat. Monty says to you 'Do you want to pick door 2?' Is it to your advantage to switch your choice of doors?"

   The solution given by Marilyn vos Savant implicitly assumes the following: the car was put behind a door by rolling a three-sided die which made all three choices equally likely; Monty knows where the car is and always opens a door with a goat behind it; finally, we assume that if Monty has a choice of doors (i.e., the contestant has picked the door with the car behind it), he chooses each door with probability 1/2. Surprisingly, under these assumptions, given that we switch to door 2, the probability of winning the car equals 2/3, while if we stay with our original choice of door 1 the probability of winning the car equals 1/3. So it is to our advantage to switch to door 2.

   Consider the following events:

$$\begin{aligned} D_i &= \{\omega \in \Omega : \text{ the car is behind door } i\} \\ C_i &= \{\omega \in \Omega : \text{ the contestant chooses door } i\} \\ M_i &= \{\omega \in \Omega : \text{ Monte Hall chooses door } i\} \end{aligned}$$

   (a) [5 points] Under the assumptions given above, draw a probability tree diagram that includes all possible single outcome events $\{\omega\} \in \mathcal{F}$ with non-zero probability. Then make a separate list of all single outcome events $\{\omega\} \in \mathcal{F}$ with zero probability.

   (b) [10 points] Next suppose instead that when Monte Hall has a choice, he chooses the door having the larger number with probability equal to 3/4. Draw a probability tree diagram that includes all possible single outcome events $\{\omega\} \in \mathcal{F}$ with non-zero probability. Then make a separate list of all single outcome events $\{\omega\} \in \mathcal{F}$ with zero probability and draw a Venn diagram representing the event $E = C_1 \bigcap M_3$ and showing all single outcome events in E. Finally, determine the following: the path-probability of the contestant winning when switching to door 2; the path-probability of the contestant winning if he does not switch doors; the probability of the event $E = C_1 \bigcap M_3$; the conditional probabilities $\mathbb{P}\big(D_i \,\big|\, C_1 \bigcap M_3\big)$ for $i = 1, 2, 3$. What does this say about the contestants chances of winning if he switches to door 2 under these new assumptions?

6. A man is known to speak the truth 3 out of 4 times. He throws a 6-sided die and reports that he rolled a five. Find the probability that it he actually rolled a five.

7. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space where $B \in \mathcal{F}$ and $\mathbb{P}(B) \neq 0$. Then define a function $Q : B \to \mathbb{R}$ where $Q(A) = \mathbb{P}\big(A \,\big|\, B\big)$ for every event $A \subseteq B$. Show that the function Q satisfies the three probability axioms and is therefore a probability function. Then decide which of the following properties hold for Q.

   (a) $Q(\emptyset) = 0$.

   (b) $Q\big(A^c\big) = 1 - Q\big(A\big)$ for all $A \in \mathcal{F} \cap B$.

   (c) If $A, C \in \mathcal{F} \cap B$ and $A \subset C$, then $Q\big(A\big) \leq Q\big(C\big)$.

   (d) $Q\big(A \cup C\big) = Q\big(A\big) + Q\big(C\big) = Q\big(A \cap C\big)$ for all $A, C \in \mathcal{F} \cap B$.

   (e) $Q\big(A\big) = Q\big(A \cap C\big) + Q\big(A \cap C^c\big)$ for all $A, C \in \mathcal{F} \cap B$.

   (f) For any increasing sequence of events $\{A_k : A_k \in \mathcal{F} \cap B\}$ where $A = \bigcup_k A_k$, then we have $Q\Big(\bigcup_k\Big) = \lim_{n\to\infty} Q(A_n)$.

8. A child mixes twelve good and five dead batteries. To find the dead batteries, his father tests them one-by-one and without replacement. If the first four batteries tested are all good, what is the probability that the fifth one is dead?

9. Your neighbor has 2 children. You learn that he has a son. What is the probability that the other child is a boy?

10. Your neighbor has 2 children. He picks one of them at random and comes by your house; he brings a boy named Joe (his son). What is the probability that Joe's sibling is a brother?

11. (Ross, Section 3.4, Example 4h) Consider independent trials consisting of rolling a pair of fair dice, over and over. What is the probability that a sum of 5 appears before a sum of 7?

12. Let three fair coins be tossed. Let A be the event all heads or all tails, Bbe the event at least two heads, and C be the event at most two tails. Of the pairs of events $\{A, B\}$, $\{A, C\}$, and $\{B, C\}$, which are independent and which are dependent? (Justify your answers.)

13. Consider the game of Let's Make a Deal in which there are five doors (numbered 1, 2, 3, 4, and 5), one of which has a car behind it and four of which are empty. You initially select Door 1, then, before it is opened, Monty Hall opens two of the other doors that are empty (selecting the two at random if there are three empty doors among 2,3,4,5). (We are assuming that Monty Hall knows where the car is and that he selects doors to open only from among those that are empty.) You are then given the option to switch your selection from Door 1 to one of the two remaining closed doors. Given that Monty opens Door 2 and Door 4, what is the probability that you will win the car if you switch your door selection to Door 3? Also, compute the probability that you will win a car if you do not switch. (What would you do?)

14. Consider a game where a bag contains three \$1 bills, two \$5 bills, and one \$10 bill. Bills are selected randomly until the \$10 bill is selected and then the game stops. What is the probability that \$16 are selected in the game.

15. In a card game, suppose a player needs to draw two cards of the same suit in order to win. Of the 52 cards, there are 13 cards in each suit. Suppose first the player draws a heart. What is the probability the player draws a second heart?

16. Consider the college applicant who has determined that he has 0.80 probability of acceptance and that only 60% of the accepted students will receive dormitory housing. Of the accepted students who receive dormitory housing, 80% will have at least one roommate. What is the probability of being accepted and receiving dormitory housing and having no roommates?

17. Suppose a voter poll is taken in three states. In state A, 50% of voters support the liberal candidate, in state B, 60% of the voters support the liberal candidate, and in state C, 35% of the voters support the liberal candidate. Of the total population of the three states, 40% live in state A, 25% live in state B, and 35% live in state C. Given that a voter supports the liberal candidate, what is the probability that she lives in state B?