Math3810 - Probability
Section 001 - Fall 2025
Notes: Joint Distribution Practice

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A. Discrete Joint Distributions

Problem 1. Two fair dice are rolled. Let X be the minimum and Y the maximum.

(a) Find the joint pmf $p_{X,Y}(x,y)$.

(b) Compute the marginal pmfs $p_X(x)$ and $p_Y(y)$.

(c) Are X and Y independent?

Problem 2. Let N be Bernoulli($1/2$). Conditional on $N = n$:

- if $n = 0$: X and Y are independent Bernoulli($1/4$),

- if $n = 1$: X and Y are independent Bernoulli($3/4$).

(a) Find the joint pmf $p_{X,Y}(x,y)$.

(b) Find the marginals.

(c) Are X and Y independent?

Problem 3. Suppose (X,Y) has pmf $p_{X,Y}(x,y) = c(x+y)$ over $x \in \{1,2\}, \quad y \in \{1,2,3\}$.

(a) Find c.

(b) Find the marginal pmfs.

(c) Compute $P(X < Y)$.

Problem 4. A biased coin has probability p of heads. Let X be the number of heads in the first two flips and Y the number in the first three.

(a) Find the joint pmf $p_{X,Y}(x, y)$.

(b) Find the marginals.

(c) Find $p_{Y|X}(y \mid x)$.

Problem 5. Let (X, Y) have joint pmf $1/8$ on the eight points

$$(0, 0), (0, 1), (1, 0), (1, 1), (0, 2), (2, 0), (2, 1), (1, 2).$$

(a) Create the joint distribution table.

(b) Compute marginals.

(c) Are X and Y independent?

B. Continuous Joint Distributions

Problem 6. Suppose $f(x, y) = k(x + y)$, $\quad 0 < x < 1, \ 0 < y < 1$.

(a) Find k.

(b) Find the marginals.

(c) Compute $P(X < Y)$.

Problem 7. Let (X, Y) uniform on $0 < y < x < 2$.

(a) Find $f_{X,Y}(x, y)$.

(b) Find the marginals.

(c) Compute $P(Y < 1)$.

(d) Find $f_{Y|X}(y \mid x)$.

Problem 8. Let $X \sim \text{Exp}(1)$, $Y \sim \text{Exp}(2)$ be independent random variables.

(a) Find the joint pdf.

(b) Compute $P(X < Y)$.

(c) Find the pdf of $Z = X + Y$.

(d) Find $f_{X,Z}(x, z)$.

Problem 9. Suppose $f_{X,Y}(x,y) = 6(1-y), \quad 0 < x < y < 1$.

(a) Verify the pdf is valuid.

(b) Find both marginals.

(a) Compute $P(Y - X > 1/4)$.

Problem 10. (X,Y) jointly normal with

$$E[X] = 0, \ E[Y] = 2, \ \text{Var}(X) = 1, \ \text{Var}(Y) = 4, \ \text{Cov}(X,Y) = 1.$$

(a) Write the joint pdf.

(b) Find the marginals.

(c) $X \mid Y = y$.

(d) Compute $P(X > 0, Y > 2)$.

C. Mixed Discrete–Continuous

Problem 11. $N \sim \text{Poisson}(\lambda)$ and $X|N = n \sim \text{Gamma}(n+1, 1)$.

(a) Find the joint distribution.

(b) Find the marginal of X.

(c) Compute $P(N = n \mid X = x)$.

Problem 12. With prob. 1/2: $(X,Y) = (0,U)$, $U \sim \text{Unif}(0,2)$. With prob. 1/2: $(X,Y) = (V,0)$, $V \sim \text{Unif}(0,2)$.

(a) Find the joint distribution.

(b) Find both marginals.

(c) Compute $P(X = 0 \text{ or } Y = 0)$.

D. Additional Problems (Double Integrals / Non-Rectangular Regions)

Problem 13. Let $f_{X,Y}(x,y) = k(x^2 + y^2)$ on the quarter disk

$$x \geq 0, \ y \geq 0, \ x^2 + y^2 \leq 4.$$

(a) Find k.

(b) Find $f_X(x)$.

(c) Compute $P(X + Y < 1)$.

da) Find $f_{Y|X}(y \mid x)$.

Problem 14. Let $f(x, y) = c(2x + 3y)$ on the triangle

$$0 < x < 2, \quad 0 < y < 1 - x/2.$$

(a) Find c.

(b) Find $f_Y(y)$.

(c) Compute $P(Y < X/2)$.

(d) Find $f_{X|Y}(x \mid y)$.

Problem 15. Let $p(x, y) = c(x + y)$ for integer lattice points

$$x \geq 1, \ y \geq 1, \ x + y \leq 6.$$

(a) Find c.

(b) Find $p_Y(y)$.

(c) Compute $P(X > Y)$.

(d) Are X, Y independent?

Problem 16. Suppose $f_{X,Y}(x, y) = \lambda^2 e^{-\lambda(x+2y)}$ on the wedge

$$0 < x, \ 0 < y < x.$$

(a) Normalize the pdf.

(b) Compute $f_X(x)$.

(c) Compute $P(Y < X/3)$.

(d) Compute $E[Y]$.