Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #3 Solutions

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Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. **Binomial Simulation**
   (a) Simulate 100 Bernoulli trials with $p = 0.3$.
   (b) Compute the empirical mean and variance.
   (c) Compare to theoretical values.
2. **Multiple Experiments**
   (a) Repeat 3 times with 100 trials each.
   (b) Record empirical proportions each time.
   (c) Comment on variability.
3. **Poisson Approximation**
   (a) Simulate 1000 Poisson($\lambda = 2$) observations.
   (b) Plot histogram and overlay theoretical PMF.
   (c) Compute sample mean and variance.
4. **Law of Large Numbers**
   (a) Simulate sums of n i.i.d. Bernoulli($p = 0.5$) for $n = 10, 100, 1000, 10000$.
   (b) Plot the running proportion of successes.
   (c) Comment on convergence to true probability.
5. **Discussion**
   - Explain why empirical frequencies converge as n increases.
   - How does randomness affect small samples?

Solutions

1.
```
x <- rbinom(100, size=1, prob=0.3)
mean(x)
var(x)
```

2.
```
for(i in 1:3){
  x <- rbinom(100,1,0.3)
  print(mean(x))
}
```

3.
```
y <- rpois(1000, lambda=2)
hist(y, prob=TRUE)
points(0:10, dpois(0:10, 2), col="red", pch=19)
mean(y)
var(y)
```

4.
```
nvec <- c(10,100,1000,10000)
for(n in nvec){
  x <- rbinom(n,1,0.5)
  plot(cumsum(x)/seq(1,n), type="l", main=paste("n␣=", n))
}
```

5. Empirical frequencies converge due to the Law of Large Numbers. Small samples show more variability because randomness has larger influence.

Please let me know if you have any questions, comments, or corrections!