Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #1 Solutions

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Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. **Basic R Warm-Up**

   (a) Use R to compute:
   $$2^5, \qquad \frac{17}{3}, \qquad \sqrt{2}.$$

   (b) Store the value of $\sqrt{2}$ in a variable `x` and compute $x^2 - 2$.

2. **Simulating a Coin Toss**

   (a) Use `sample()` to simulate one fair coin toss with outcomes H and T.

   (b) Simulate 10 independent coin tosses and store the outcomes.

   (c) Count the number of heads.

3. **Empirical Probability**

   (a) Simulate 100 independent coin tosses.

   (b) Compute the empirical proportion of heads.

   (c) Repeat the experiment three times.

   (d) Comment on whether the empirical proportions are identical or different.

4. **Dice Experiment**

   (a) Simulate 60 rolls of a fair six-sided die.

   (b) Estimate the probability that the outcome is greater than 4.

   (c) Compare your estimate to the theoretical probability.

5. **Conceptual Question**

   In your own words, explain:

- What an empirical probability is,
- Why empirical probabilities vary from run to run,
- How this connects to the idea of long-run frequency.

Solutions

1.
```r
2^5
17 / 3
sqrt(2)

x <- sqrt(2)
x^2 - 2
```

2.
```r
sample(c("H","T"), 1)

tosses <- sample(c("H","T"), 10, replace = TRUE)
sum(tosses == "H")
```

3.
```r
t1 <- sample(c("H","T"), 100, replace = TRUE)
mean(t1 == "H")

t2 <- sample(c("H","T"), 100, replace = TRUE)
mean(t2 == "H")

t3 <- sample(c("H","T"), 100, replace = TRUE)
mean(t3 == "H")
```

   The proportions differ slightly due to randomness but cluster near 0.5.

4.
```r
rolls <- sample(1:6, 60, replace = TRUE)
mean(rolls > 4)
```

   Theoretical probability: $P(X > 4) = 2/6 = 1/3$.

5. Empirical probabilities are computed from observed data rather than exact formulas. They vary due to random sampling, but as the number of trials increases, they stabilize near the true probability, illustrating the Law of Large Numbers.

Please let me know if you have any questions, comments, or corrections!