# Math3810 - Probability
# Section 001 - Fall 2025
# Introductory Homework #4 Solutions

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## Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

## Problems

1. **Continuous Random Variable Simulation**
    (a) Simulate 500 samples from $X \sim N(5, 4)$.
    (b) Compute sample mean and variance.
    (c) Plot histogram and overlay the theoretical density curve.
2. **Linear Transformation of RV**
    (a) Define $Y = 3X - 2$.
    (b) Compute mean and variance of $Y$ empirically.
    (c) Compare to theoretical $E[Y]$ and $Var(Y)$.
3. **CDF Comparison**
    (a) Compute empirical CDF of $X$ and $Y$.
    (b) Plot both on the same graph with theoretical CDF curves.
4. **Standardization**
    (a) Standardize $X$ to $Z = (X - \bar{X})/s_X$.
    (b) Verify mean and variance of $Z$.
    (c) Plot histogram of $Z$ and overlay standard normal density.
5. **Discussion**
    - Explain the effect of linear transformations on mean and variance.
    - Explain why standardization produces mean 0 and variance 1.

## Solutions

1. ```
   X <- rnorm(500, mean=5, sd=2)
   mean(X)
   var(X)
   hist(X, prob=TRUE)
   curve(dnorm(x,5,2), add=TRUE, col="red")
   ```

2. ```
   Y <- 3*X - 2
   mean(Y)
   var(Y)
   3*mean(X) - 2    # Theoretical
   3^2 * var(X)     # Theoretical
   ```

3. ```
   ecdfX <- ecdf(X)
   ecdfY <- ecdf(Y)
   plot(ecdfX, col="blue")
   lines(ecdfY, col="red")
   curve(pnorm(x,5,2), add=TRUE, lty=2)
   curve(pnorm(x,3*5-2,3*2), add=TRUE, lty=2)
   ```

4. ```
   Z <- (X - mean(X)) / sd(X)
   mean(Z)
   var(Z)
   hist(Z, prob=TRUE)
   curve(dnorm(x,0,1), add=TRUE, col="green")
   ```

5. Linear transformations scale and shift mean and variance. Standardization shifts mean to 0 and scales variance to 1.

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**Please let me know if you have any questions, comments, or corrections!**