Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #9 Solutions

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Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. **Poisson Random Variable**
   (a) Simulate 100, 500, 1000, 50000 Poisson($\lambda = 4$) random variables.
   (b) Plot histograms and compare with theoretical PMF.
   (c) Compute empirical mean and variance; compare to λ.
2. **Exponential Random Variable**
   (a) Simulate Exponential($\lambda = 0.5$) random variables.
   (b) Plot histogram with theoretical density overlay.
   (c) Compute sample mean and variance; compare to theory.
3. **Transformation**
   (a) If $Y = 3X + 2$ where $X \sim$ Poisson(4), simulate and compare mean/variance.
   (b) Plot histogram of Y and overlay X histogram scaled.
4. **Empirical CDF**
   (a) Compute empirical CDFs of X and Y from above simulations.
   (b) Compare to theoretical CDF.
5. **Discussion**
   - Explain how sample size affects the empirical distribution.
   - Discuss the difference between discrete and continuous distributions.

Solutions

1.
```
lambda <- 4
for(n in c(100,500,1000,50000)){
  X <- rpois(n, lambda)
  hist(X, breaks=0:(max(X)+1)-0.5, prob=TRUE, main=paste("Poisson, n=",n))
  mean(X); var(X)
}
```

2.
```
lambda <- 0.5
X <- rexp(5000, rate=lambda)
hist(X, prob=TRUE)
curve(dexp(x, rate=lambda), add=TRUE, col="red", lwd=2)
mean(X); var(X)
```

3.
```
Y <- 3*X + 2
hist(Y, prob=TRUE, col=rgb(1,0,0,0.5))
hist(X, prob=TRUE, col=rgb(0,0,1,0.5), add=TRUE)
mean(Y); var(Y)
```

4.
```
ecdf_X <- ecdf(X)
ecdf_Y <- ecdf(Y)
plot(ecdf_X)
lines(ecdf_Y, col="red")
```

5. Larger sample sizes make empirical distribution closer to theoretical. Discrete histograms have gaps; continuous histograms are smooth.

Please let me know if you have any questions, comments, or corrections!