

Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #6 Solutions

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Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. Simulating Exponential Random Variables

- (a) Simulate 1000 samples from an exponential distribution with rate $\lambda = 0.5$.
- (b) Plot the histogram and overlay the theoretical density.
- (c) Compute the sample mean and compare with theoretical mean.

2. Transformations

- (a) Let $Y = 2X + 3$. Simulate Y from your X samples.
- (b) Compute the sample mean and variance of Y and compare with theoretical values.
- (c) Plot the histogram of Y .

3. Probability Computation

- (a) Estimate $P(X > 3)$ using your simulated samples.
- (b) Compare with the exact probability $P(X > 3) = \exp(-0.5 * 3)$.

4. CDF Comparison

- (a) Plot the empirical CDF of X .
- (b) Overlay the theoretical CDF.
- (c) Comment on the convergence.

5. Discussion

- Explain how linear transformations affect the exponential distribution.
- Discuss the difference between empirical and theoretical probabilities.

Solutions

1.

```
set.seed(123)
X <- rexp(1000, rate=0.5)
hist(X, prob=TRUE)
curve(dexp(x, rate=0.5), add=TRUE, col="red")
mean(X)
```
2.

```
Y <- 2*X + 3
mean(Y); var(Y)
hist(Y, prob=TRUE)
```
3.

```
mean(X > 3)
1 - pexp(3, rate=0.5)
```
4.

```
ecdf_X <- ecdf(X)
plot(ecdf_X)
curve(pexp(x, rate=0.5), add=TRUE, col="blue")
```
5. Linear transformation scales the mean and variance: $E[Y] = 2E[X] + 3$, $Var(Y) = 4Var(X)$. Empirical probabilities converge to theoretical as n increases.

Please let me know if you have any questions, comments, or corrections!