Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #5 Solutions

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Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. **Joint Distributions**
   (a) Simulate 500 pairs (X, Y) from a bivariate normal distribution with $\mu_X = 0, \mu_Y = 0$, $\sigma_X^2 = 1, \sigma_Y^2 = 1$, and $\rho = 0.5$.
   (b) Compute the sample covariance and correlation.
   (c) Plot the scatterplot of Y versus X.

2. **Marginal Distributions**
   (a) Compute the empirical marginal distributions of X and Y.
   (b) Compare histograms with theoretical marginal densities.

3. **Conditional Distribution**
   (a) Compute $Y|X > 0$ and $Y|X < 0$.
   (b) Plot histograms of conditional distributions.
   (c) Comment on how the mean and variance differ conditionally.

4. **Covariance Transformation**
   (a) Define $U = 2X - Y$ and $V = X + 3Y$.
   (b) Compute the covariance matrix of (U, V).
   (c) Compare with theoretical result using linear transformation formula.

5. **Discussion**
   - Explain the effect of correlation on joint scatterplots.
   - Describe the effect of linear transformations on covariance.

Solutions

1. ```
   library(MASS)
   set.seed(123)
   Sigma <- matrix(c(1,0.5,0.5,1),2,2)
   samples <- mvrnorm(500, mu=c(0,0), Sigma)
   X <- samples[,1]
   Y <- samples[,2]
   cov(X,Y)
   cor(X,Y)
   plot(X,Y)
   ```

2. ```
   hist(X, prob=TRUE)
   curve(dnorm(x,0,1), add=TRUE, col="blue")
   hist(Y, prob=TRUE)
   curve(dnorm(x,0,1), add=TRUE, col="red")
   ```

3. ```
   Y_pos <- Y[X>0]
   Y_neg <- Y[X<0]
   hist(Y_pos, prob=TRUE, main="Y | X>0")
   hist(Y_neg, prob=TRUE, main="Y | X<0")
   mean(Y_pos); var(Y_pos)
   mean(Y_neg); var(Y_neg)
   ```

4. ```
   U <- 2*X - Y
   V <- X + 3*Y
   cov(cbind(U,V))
   # Theoretical: use linear transform formula
   ```

5. Correlation stretches or rotates scatterplots. Linear transformations change the covariance according to $Cov(A * X) = A * Cov(X) * A^T$.

Please let me know if you have any questions, comments, or corrections!