Math3810 - Probability
Section 001 - Fall 2025
Introductory Homework #10 Solutions

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Instructions

Show all reasoning clearly. All simulation results should be reproducible and clearly labeled. You may use R for all computations.

Problems

1. **Covariance and Correlation**
   (a) Simulate 5000 pairs (X, Y) with $X \sim N(0,1)$, $Y \sim N(0,1)$ independent.
   (b) Compute sample covariance and correlation.
   (c) Create $Z = X + Y$ and compute mean, variance, and correlation with X.
2. **Dependent Variables**
   (a) Simulate $Y = 0.5X + \epsilon$ with $\epsilon \sim N(0,1)$.
   (b) Compute sample covariance and correlation between X and Y.
   (c) Plot scatterplot.
3. **Linear Combinations**
   (a) Compute $W = 2X - 3Y$ and its sample mean and variance.
   (b) Compare with theoretical values using variance formulas.
4. **Discussion**
   - Explain how covariance and correlation describe linear dependence.
   - Discuss how correlation affects variance of sums and differences.

Solutions

```
set.seed(123)
X <- rnorm(5000)
Y <- rnorm(5000)
cov(X,Y); cor(X,Y)
Z <- X+Y
mean(Z); var(Z); cor(Z,X)
```

```
epsilon <- rnorm(5000)
Y2 <- 0.5*X + epsilon
cov(X,Y2);  cor(X,Y2)
plot(X,Y2)

W <- 2*X - 3*Y2
mean(W);  var(W)
```

Correlation measures linear dependence. Covariance sign indicates direction. Variance of sums increases with positive correlation; variance of differences decreases.

Please let me know if you have any questions, comments, or corrections!