

The True Role of Accelerometers in Quadrotor's Inertial Navigation System

Vahid Khorani

Mechatronics Research Laboratory (MRL)
Qazvin Islamic Azad University (QIAU)
Qazvin, Iran
vahid.khorani@gmail.com

Alireza Mohammad Shahri

Electrical Engineering Department
Iran University of Science and Technology (IUST)
Tehran, Iran
shahri@iust.ac.ir

Abstract—Quadrotors are Vertical Take-Off and Landing (VTOL) vehicles which use Inertial Measurement Units (IMU) in the heart of their navigation and instrumentation systems. Mostly, IMUs containing three accelerometers are used to measure the tilt angles and robot's position (attitude and location) when they are integrated in an Inertial Navigation System (INS). The role of the accelerometers in this INS is measuring both the vehicle's linear accelerations and the gravity. As an exception for VTOL platforms, in this paper it is shown that the values measured by the accelerometers of a quadrotor are affected by neither the vehicle's linear accelerations nor the gravity. Therefore, different elements affecting on the quadrotor's accelerometers are studied and it is shown that the most significant element is the quadrotor's linear velocity expressed in the earth frame. So, it is shown that using the accelerometers to measure the vehicle's linear velocity is possible in quadrotors, whereas their measurements are not suitable to be used to estimate the tilt angles and position. The simulation results are presented in a previous version of this paper [1] and, following that, the experimental results are shown here to validate the proposed idea. This research is developed using a Genetic Algorithm (GA) based method.

Index Terms—Quadrotor, INS, GA, MEMS Accelerometers.

I. INTRODUCTION

Quadrotors are an emerging rotorcraft concept for unmanned aerial vehicle (UAV) platforms. The vehicle consists of four rotors in total, with two pairs of counter-rotating, fixed-pitch blades located at four corners of the aircraft [2]. Having four rotors makes the quadrotor able to be controlled without using swash plates. This simplifies both the design and maintenance of the quadrotor.

A quadrotors is controlled using an Inertial Navigation System (INS) in which an Inertial Measurement Unit (IMU) is the key sensory unit. IMU may be used alone (as far as horizontal stabilization is concerned) or supplemented by other sensors which provide position-related information [3-5]. In the case of inertial navigation, three gyroscopes and three accelerometers are used to calculate the odometric position (attitude and location) of the robot [6]. In order to overcome odometric errors, a set of absolute sensors such as tilt sensor [7,

8], compass [9], sonar range finder and GPS receiver are applied to correct INS's measurements using proper sensor fusion algorithms such as Kalman filter [10]. It worth notifying that the accelerometers are also used in the tilt sensors to measure the gravity.

Some researchers have used accelerometers for both inertial navigation and tilt estimation [2, 11-17] and none of them used these sensors to measure the vehicle's linear velocity. Based on Martin and Salaun's research in [3], acceleration measured by an accelerometer installed on a quadrotor is not caused by the vehicle's linear acceleration or the gravity. Indeed, researchers of [3] show that the drag force is the main element measured by the accelerometers. Moreover, in a recent paper of the author [1], it is shown that the acceleration measured by the quadrotor's accelerometer will be zero while the drag force is neglected. It was shown that the quadrotor's linear accelerations are against the gravity and the overall acceleration affecting on its center of mass is zero. Therefore, it seems that the values measured by the accelerometers are not suitable for inertial navigation and tilt estimation and they are independent to the vehicle's linear acceleration and the gravity.

In this research a rigorous amount of study is carried out to discover the true role of the accelerometers on a quadrotor. As a result of this study, it is shown that the most significant element affecting on the vehicle's accelerometers is its linear velocity relative to the earth frame. Indeed, this element is indirectly produced by the drag force and can be used to estimate the robot's linear velocity.

The main concentration of this research will be on accelerometers which are installed on a quadrotor horizontally. We name these sensors the Horizontal accelerometers. After introducing the frames and sensors, quadrotor's dynamic model, and the inertial navigation equations in sections II, III and IV, in section V it is shown that the overall linear acceleration on the vehicle's mass center will be zero when the drag force is neglected. Then, a GA-based method is proposed in section VI to show what elements make a real accelerometer on a quadrotor measuring non-zero values. In section VII an experimental test validates the idea proposed in previous

sections and finally an accelerometer is used to measure the quadrotor's linear velocity in section VIII.

II. FRAMES AND SENSORS

In this paper two frames are used to show dynamic and inertial equations of the quadrotor. As shown in Fig. 1, three axes x_b , y_b and z_b form the body-frame which is rigidly connected to the mass center of the robot. Three axes X , Y and Z form the earth-frame which is connected to the earth and it is assumed to be fixed.

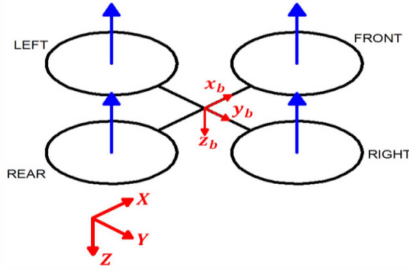


Fig. 1. Quadrotor's frames

Three accelerometers are rigidly connected to the center of the quadrotor where their directions are the same as body-frame axes as seen in Fig. 2. Two accelerometers installed on x_b and y_b directions are named Horizontal accelerometers and the other one is name Vertical accelerometer.

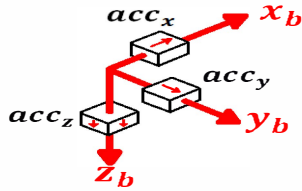


Fig. 2. Three accelerometers installed on a quadrotor

III. QUADROTOR'S DYNAMIC MODEL

Neglecting the drag force, the quadrotor's dynamic behavior can be modeled as a 6-DOF rigid body that is presented as follows [11]:

$$\begin{cases} \ddot{X} = (-\sin\varphi \sin\psi - \cos\varphi \sin\theta \cos\psi)U_1 \\ \ddot{Y} = (\sin\varphi \cos\psi - \cos\varphi \sin\theta \sin\psi)U_1 \\ \ddot{Z} = g - (\cos\varphi \cos\theta)U_1 \\ \dot{p} = I_x q r - \frac{J_{TP}}{I_{XX}} q \Omega + U_2 \\ \dot{q} = I_y p r - \frac{J_{TP}}{I_{YY}} p \Omega + U_3 \\ \dot{r} = I_z p q + U_4 \end{cases} \quad (1)$$

where \ddot{X} , \ddot{Y} and \ddot{Z} are linear accelerations of the quadrotor presented in the earth-frame, p , q and r are angular velocities presented in the body-frame, φ , θ and ψ are ZYX-Euler angles between body-frame and earth-frame, g is the acceleration due to the gravity, I_x , I_y , I_z , I_{xx} , I_{yy} and I_{zz} are inertia matrix components, and U_1 , U_2 , U_3 and U_4 are the movement vector components that are generated due to four rotary propellers and are presented in equation set below:

$$\begin{cases} U_1 = \frac{b}{m}(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = \frac{lb}{I_{XX}}(-\Omega_2^2 + \Omega_4^2) \\ U_3 = \frac{lb}{I_{YY}}(\Omega_1^2 - \Omega_3^2) \\ U_4 = \frac{d}{I_{ZZ}}(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{cases} \quad (2)$$

where Ω_1 to Ω_4 are the turn rate of the front, right, rear and left propellers respectively, b and d are propeller's trust and drag factors respectively, l is the distance between the center of the quadrotor's structure and the center of a propeller, and m is the mass of the quadrotor.

Ω in equation (1) is the propellers' turn rate vector that is presented in equation below:

$$\Omega = (-\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4) \quad (3)$$

IV. INERTIAL NAVIGATION EQUATIONS

Inertial navigation equations can be shown as follows [6]:

$$\begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix} = C_b^e \begin{bmatrix} f_{x_b} \\ f_{y_b} \\ f_{z_b} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad (4)$$

where $\begin{bmatrix} f_{x_b} & f_{y_b} & f_{z_b} \end{bmatrix}^T$ is the vector of the accelerations measured by the accelerometers and C_b^e is the rotation matrix from body-frame to earth-frame which is shown in equation below:

$$C_b^e = \begin{bmatrix} c_\theta c_\psi & -c_\theta s_\psi + s_\varphi s_\theta c_\psi & s_\varphi s_\psi + c_\varphi s_\theta c_\psi \\ c_\theta s_\psi & c_\theta c_\psi + s_\varphi s_\theta s_\psi & -s_\varphi c_\psi + c_\varphi s_\theta s_\psi \\ -s_\theta & s_\varphi c_\theta & c_\varphi c_\theta \end{bmatrix} \quad (5)$$

V. ACCELERATIONS MEASURED BY A DRAG-FREE QUADROTOR'S ACCELEROMETERS

Using linear part of equation set (1) and equation (4) it is possible to write:

$$\begin{bmatrix} (-s_\varphi s_\psi - c_\varphi s_\theta c_\psi)U_1 \\ (s_\varphi c_\psi - c_\varphi s_\theta s_\psi)U_1 \\ g - (c_\varphi c_\theta)U_1 \end{bmatrix} = C_b^e \begin{bmatrix} f_{x_b} \\ f_{y_b} \\ f_{z_b} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad (6)$$

Solving this equation results:

$$\begin{bmatrix} f_{x_b} \\ f_{y_b} \\ f_{z_b} \end{bmatrix} = C_b^{e-1} \begin{bmatrix} (-s_\varphi s_\psi - c_\varphi s_\theta c_\psi) \\ (s_\varphi c_\psi - c_\varphi s_\theta s_\psi) \\ -(c_\varphi c_\theta) \end{bmatrix} U_1 = \begin{bmatrix} 0 \\ 0 \\ -U_1 \end{bmatrix} \quad (7)$$

Equation (7) shows, neglecting the drag force, the values of the accelerations measured by the horizontal accelerometers will be zero. Considering this theoretical result in one hand and the fact that the real accelerometers show some non-zero values in the other hand, we are going to study the constitutive elements of the real values measured by a practical accelerometer.

VI. GA-BASED METHOD PROPOSED TO STUDY THE ACCELERATION ELEMENTS

In previous section it was shown that, neglecting the drag force, the horizontal accelerometers will measure zero values. However, practical sensors on a quadrotor measure non-zero values. Therefore, in this section all elements affecting on a horizontal accelerometer installed on a real quadrotor are studied and a GA-based formula is proposed to evaluate the proposed idea.

Based on our study, the measured acceleration consists of five major elements:

- 1) The gravity
- 2) The quadrotor's linear accelerations expressed in the body-frame
- 3) Effects of the quadrotor's angular accelerations when the accelerometer is not installed exactly at the robot's mass center
- 4) Effects of centrifugal forces when the accelerometer is not installed exactly at the robot's mass center
- 5) Effects of the drag forces and air resistance which are proportional to robot's linear velocity expressed in the earth-frame

From previous section it is clear that the first and second elements are against each other and will be eliminated. So, the main elements which are measured by a real accelerometer are the third, fourth, and fifth elements. Since these elements are not useful for inertial navigation and tilt estimation, let name theme Undesired Accelerations. The third and fourth elements

are produced when the sensor is installed at a place far from the vehicle's mass center. Therefore, in this study it is assumed that the IMU is installed at a place far from robot's mass center. This assumption makes it easier to test and validate the proposed idea.

It is possible to calculate the undesired accelerations using measurements of a gyroscope. This possibility is clear from forthcoming formulas. Therefore, we propose a formula to calculate the undesired accelerations and use the GA to fit the result of this formula to the acceleration measured by a real accelerometer. Whenever we could show this fitness, our idea is correct and the elements considered in our formula are correctly chosen.

Fig. 3 shows an IMU installed on a quadrotor in a place far from its mass center in an imprecise manner. The center of the IMU has a distance r from the center of the body frame and it is installed in angle of β with respect to the x_b axis. The IMU also has an angle of α with respect to the x_b axis. In a precise installation, these physical parameters (r, β, α) should be $(0, \pi/2, 0)$. θ in this figure shows the pitch angle around y_b axis, ω_y shows the angular rate measured by y_b axis gyroscope and \bar{f}_{x_b} shows our calculation and estimation from the undesired accelerations measured by the accelerometer in x_b direction. The undesired accelerations affecting on x_b accelerometer are determined by green color in this figure. U_1 is the acceleration produced by the propellers, $r\dot{\omega}_y$ is the angular acceleration due to the quadrotor's rotation around its y_b axis and $r\omega_y^2$ is centrifugal force applied on the IMU because of quadrotor's rotations around its y_b axis.

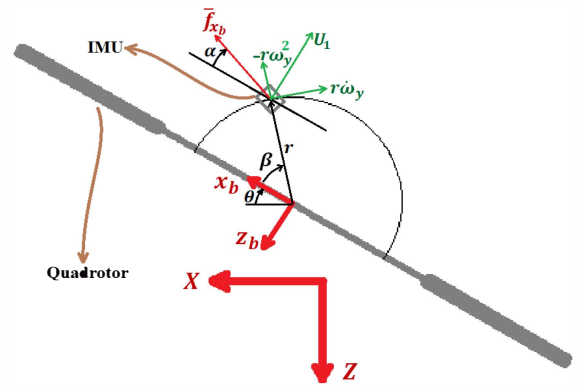


Fig. 3. Model of an IMU installed far from quadrotor's mass center

\bar{f}_{x_b} is calculated using equation below:

$$\bar{f}_{x_b} = U_1 \sin \alpha - (r \omega_y^2 \cos \beta + r \dot{\omega}_y \sin \beta) \cos \alpha \quad (8)$$

Assuming that the height of the quadrotor is fixed during the test, U_1 may be approximated by g . So the equation (8) may be rewritten as follows:

$$\bar{f}_{x_b} = g \sin \alpha - r (\omega_y^2 \cos \beta + \dot{\omega}_y \sin \beta) \cos \alpha \quad (9)$$

The other source which causes acceleration on the horizontal accelerometers is the air resistance against the quadrotor's horizontal motions or the drag force. This effect can be estimated by second power of the quadrotor's horizontal velocity [18]. The quadrotor's horizontal velocity can be calculated by integrating the first equation of the equation set (1). Since the x_b axis acceleration is studied in this test, the first equation from equation set (1) can be written as equation (10) while assuming the roll and yaw angles are zero:

$$\ddot{X} = \sin \theta U_1 \quad (10)$$

in which, U_1 may be approximated by g when the height of the robot is assumed to be stationary and θ can be calculated using y_b axis gyroscope as shown in equation below:

$$\theta = \int_0^t \omega_y dt + \theta_0 \quad (11)$$

Integrating \ddot{X} from equation (10) results:

$$v_X = g \int \sin \left(\int_0^t \omega_y dt + \theta_0 \right) dt + v_{X0} \quad (12)$$

where v_X is the robot's horizontal linear velocity in X axis direction and initial values θ_0 and v_{X0} can be considered to be zero.

As mentioned above, air resistance against the quadrotor's horizontal motion is proportional to the second power of its horizontal velocity and can be calculated using equation below:

$$R_{air} = \mu \text{sign}(v_X) v_X^2 \quad (13)$$

where μ is a coefficient that shows the impressibility of the quadrotor's structure against the air resistance.

Finally, undesired accelerations applied on the x_b axis accelerometer can be calculated using equation blow which only uses one gyroscope as angular velocity measuring sensor and has four parameters which should be determined using experimental data gathered by the gyroscope:

$$\bar{f}_{x_b} = g \sin \alpha - r (\omega_y^2 \cos \beta + \dot{\omega}_y \sin \beta) \cos \alpha + R_{air} \quad (14)$$

where r , β , α and μ are parameters that should be identified. In this research an evolutionary optimization algorithm, GA, is applied to carry out the task of parameters identification. This algorithm needs an objective function to determine the correct values of the parameters (r, β, α, μ) during its optimization. The objective function is introduced based on root mean square error (RMSE) method and is written as follows:

$$OF = \sqrt{\sum_{k=1}^n (f_{x_b(k)} - \bar{f}_{x_b(k)})^2} \quad (15)$$

where f_{x_b} is the acceleration measured by the x_b axis accelerometer and $\bar{f}_{x_b(k)}$ is the acceleration calculated using equation (14) and the data measured by the gyroscope.

During the optimization, the GA tries to fit \bar{f}_{x_b} to the f_{x_b} while determining correct values for parameters (r, β, α, μ) . In case of successful termination of the optimization algorithm, it is possible to conclude that parameters are correctly identified and f_{x_b} which is measured by horizontal accelerometer is equal to the undesired accelerations \bar{f}_{x_b} . The idea proposed in this section will be validated if we can identify parameters which fit the \bar{f}_{x_b} to the f_{x_b} . Indeed, a successful fitting shows that the accelerations measured by the horizontal accelerometer exactly consists of the undesired accelerations.

VII. EXPERIMENTAL VALIDATION

In this research a home-made quadrotor (shown in Fig. 4) is used to experimentally test the proposed ideas. The mechanical structure of this quadrotor is made of carbon fiber tubes and polyamide joints in order to have a light (320 grams) and safe quadrotor experimental setup. Because of the special design of the light and quite sturdy guard of the quadrotor, there is a minimum of chance to hurt or damage the people or objects around in case of crash of the quadrotor. This design also has minimum effect against of the blowing of wind in the environment.

For horizontal stabilization, the roll and pitch angles of the robot are controlled using MPC method. Yaw angle of the robot is also controlled using PID method and its height is controlled using state feedback method. The main board of the robot as shown in Fig. 5 contains two ATMEGA128 microcontrollers. One of them is used just for processing the



Fig. 4. The quadrotor implemented in MRL.

control algorithms and the other is used for sensory unit and instrumentation. Sensory unit contains a set of sonar range finders (MaxSonar-EZ0) which are used for collision avoidance purposes and one extra sonar range finder is also used to measure height of the robot. An onboard XBee-PRO module is used for telecommunication of the robot and an RF remote controller is used for teleoperation.

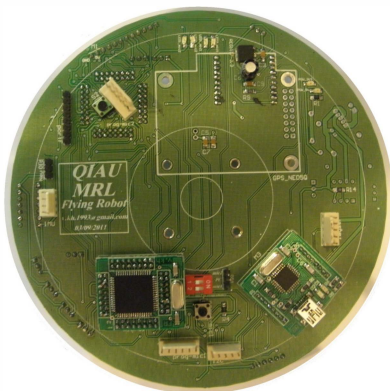


Fig. 5. Robot main board.

The navigation system is based on an IMU which is designed in MRL. As it is shown in Fig. 6, the designed IMU consists of a three-axis accelerometer ADXL327, a three-axis gyroscope ITG3200 and compass sensor HMC5843 with a 32 bit Cortex ARM based microcontroller (STM32F107). The data gathered using this module is transferred to the main board via serial communication.

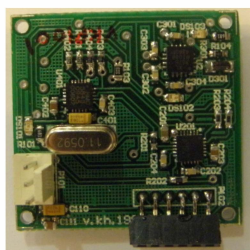


Fig. 6. The IMU designed in MRL.

In this test, the IMU is installed in a place far from the robot's mass center as it is clear from Fig. 7 and data are collected while the robot is flying in the laboratory environment with several agile maneuvering with high accelerations.

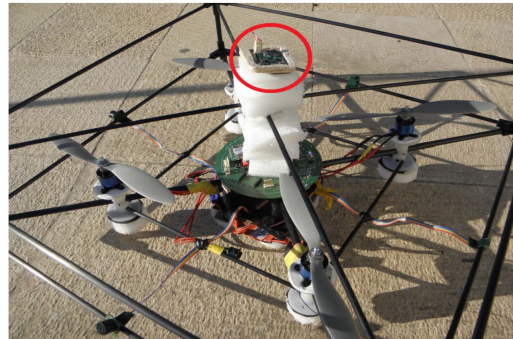


Fig. 7. Installing the IMU in a place far from the mass center of the quadrotor.

Acceleration measured by the x_b axis accelerometer and its filtered value are shown in Fig. 8. Angular velocity measured by y_b axis gyroscope (ω_y) is shown in Fig. 9. Angular acceleration ($\dot{\omega}_y$) calculated by differentiation of the ω_y is shown in Fig. 10. Linear velocity of the quadrotor which is calculated using equation (12) from angular velocity measured by gyroscope is shown in Fig. 11.

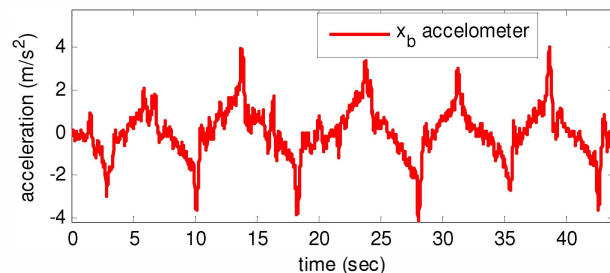


Fig. 8. Acceleration measured by accelerometer when the IMU is installed far from the robot's mass center

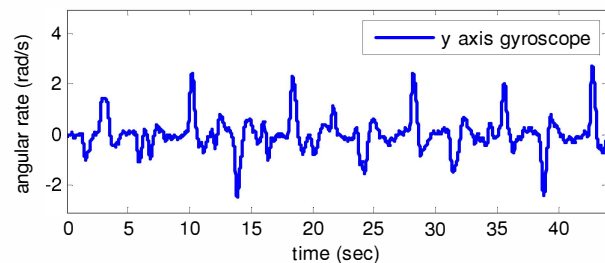


Fig. 9. Angular rate measured by gyroscope when the IMU is installed far from the robot mass center

Using data collected (as it is seen from Fig. 8 to Fig. 11), the GA algorithm tries to optimize the objective function which is shown in Fig. 12. In this optimization process, GA uses the data collected for the first 33 seconds of the test flight (left part of the green vertical line in Fig. 13) as parameter identification data. As shown in the right part of the green vertical line in Fig. 13 parameters resulted from this

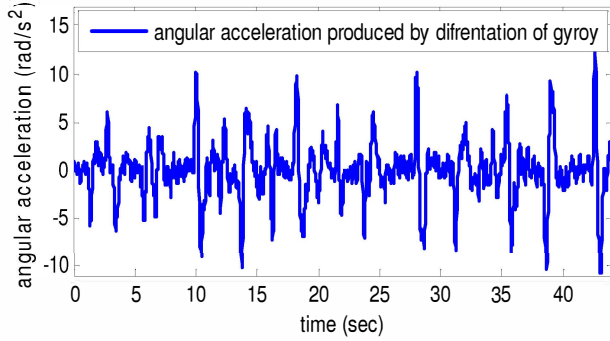


Fig. 10. Angular acceleration calculated by differentiation of the ω_y

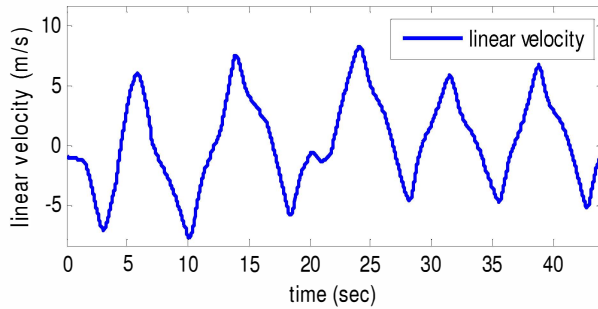


Fig. 11. Linear velocity of the quadrotor calculated based on gyroscope angular velocity

optimization process are valid for the other part of the data and it proves that parameters are identified correctly. During this optimization, values of parameters (r, β, α, μ) are identified as $(0.179, 85.12, -0.3, 0.0386)$ while the optimized OF value is 0.578. Now, using the identified parameter values, \bar{f}_{x_b} may be calculated and compared to the acceleration measured by the x_b axis accelerometer (f_{x_b}) which is shown in Fig. 13. As it is clear from this figure, the acceleration calculated using the proposed GA-based formula is the same as the acceleration measured by the x_b axis accelerometer and this proves that the acceleration measured by this accelerometer is due to the undesirable accelerations.

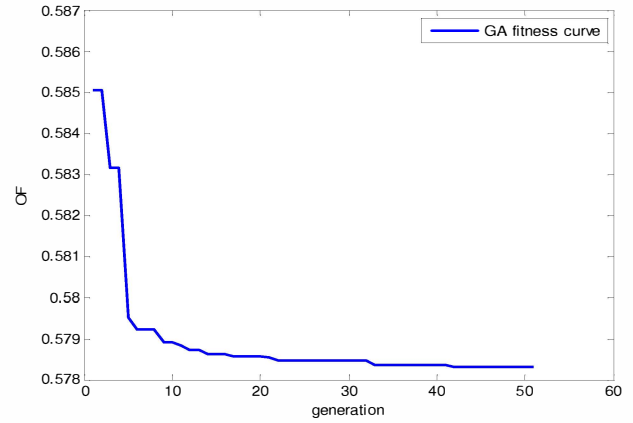


Fig. 12. Convergence curve for the objective function using GA

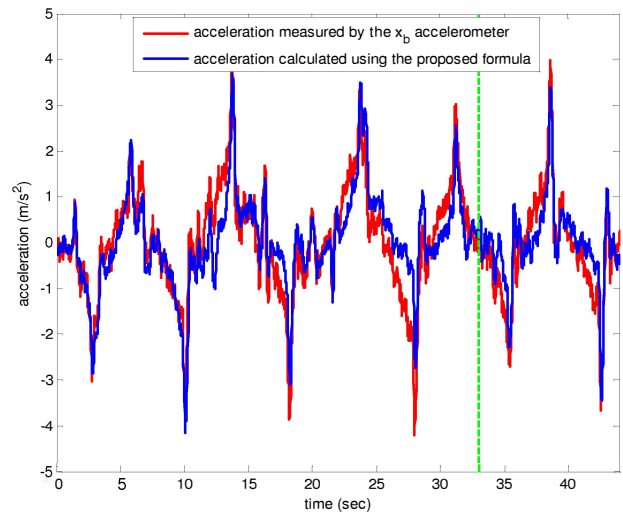


Fig. 13. Comparing the acceleration measured by accelerometer and the acceleration calculated by the proposed GA based method

VIII. MEASURING LINEAR VELOCITY USING A HORIZONTAL ACCELEROMETER

In previous section it was shown and proven that the values measured by a real accelerometer are equal to the undesired accelerations described in section VI as the IMU is installed in a place far from the vehicle's mass center. In this section we replaced the IMU to a place close to the quadrotor's mass center. Replacing the IMU makes it possible to weaken the third and fourth elements and as a result the acceleration measured by this IMU can be used to estimate the vehicle's linear velocity. Fig. 14 shows the acceleration measured by the IMU installed close to mass center. These values are R_{air}

from equation (13) and we use them to estimate the v_x while the μ is identified by the GA.

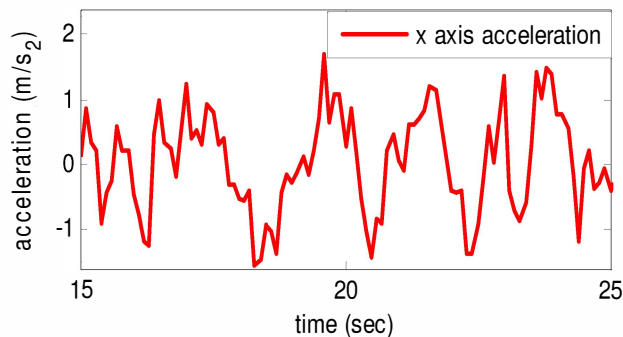


Fig. 14. Acceleration measured by a horizontal accelerometer while the IMU is installed at the mass center

IX. CONCLUSION

In this research it was shown that the values measured by horizontal accelerometers installed on a quadrotor can be used to estimate the vehicle's linear velocity expressed in the earth frame. Besides, the different elements measured by the accelerometers were studied and it was proven that it is not a suitable choice to use the acceleration measured by a sensor installed on a quadrotor for calculating the robot's position and tilt angles. A GA-based method was applied on practical data to validate the proposed idea.

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