ECEN 5458 Project Summary 2

Kyle Harlow, Akshay Iyer

May 18, 2018

1 Introduction

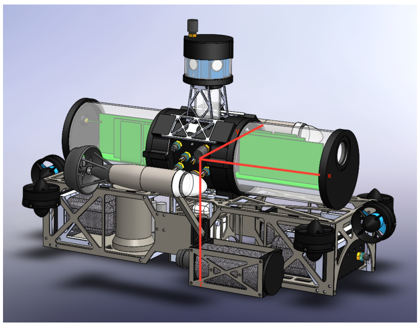

Figure 1: CAD Rendering of the AUV with x, y, and z axes labeled

1.1 Definitions

The standard 6DOF model of an AUV uses a left hand model. In the left hand model x, y, and z, represent thrust, strafe, and depth, and the rotations about each axis p, q, r represent roll, pitch, and yaw respectively. Each of these coordinates are part of the body frame of the vehicle, but we can also describe the pose in terms of the world frame. The world frame will be represented using North, East, and Down coordinate N,E, and D, with ϕ, θ, and ψ being the rotations around each of these axes. This allows us to define the following vectors (all vectors are assumed to be column vectors):

$$\eta = [N, E, D, \phi, \theta, \psi]^T \tag{1}$$

$$v = [\dot{x}, \dot{y}, \dot{z}, \dot{p}, \dot{q}, \dot{r}]^T \tag{2}$$

These two equations can be related by the equation

$$\dot{\eta} = J(\eta) * v \tag{3}$$

where $J(\eta)$ is the standard Euler Jacobian Matrix. Note that the Euler Jacobian Matrix is singular at any rotation $= \pm\pi/2$.

2 Dynamics Model

2.1 6DOF Model

Given these vectors we can begin to define the model of our vehicle in more detail, and then apply some assumptions about the vehicle in order to simplify and linearize the model.

From our last project summary we listed the initial equation

$$\boldsymbol{M\dot{v}} + \boldsymbol{C(v)v} + \boldsymbol{D(v)v} + \boldsymbol{g(\eta)} = \boldsymbol{\tau} \tag{4}$$

This equation describes the 6DOF model and is given by the book "Handbook of Marine Craft Hydrodynamics and Motion Control" by Thor I. Fossen, referenced from this point forward as [1].

M, C, and D describe the rigid body and added mass, Coriolis effect, and drag matrices of the AUV all of which are $\in \mathbb{R}^{6x6}$ and $g \in \mathbb{R}^6$ describes the effects of gravity and buoyancy on the vehicle.

2.2 Assumptions

For any vehicle model there are a number of assumptions that can be made in order to reduce the complexity of the system. We will list and justify the assumptions made here.

1. Only Small Roll and Pitch Angles are possible
   - Due to the vehicles arrangement, with the center of bouyancy, approximated by the center of the main tube, being above the center of gravity, approximated by the bottom of the center of the frame, the vehicle naturally rests with 0 pitch or roll, and will attempt naturally attempt to maintain small values during operation.
2. The vehicle is low speed
   - The vehicle moves at a maximum speed of 1m/s based on previous tests with full motor power in the x direction
3. The vehicle is symmetrical across the xz-plane, the yz-plane.
   - Throughout the mechanical design of the vehicle, effort was taken to balance the enclosures held in the frame such that they are approximately symmetrical on both the right and left as well as the front and back. Any significant outliers are offset using either buoyant foam or with weights in order to maintain this symmetry.
4. The vehicle is neutrally buoyant
   - The vehicle at the moment is slightly positively buoyant floating upwards at 0.1m/s based on in water observations. Due to the slow buoyancy relative to the speed of the system overall we can approximate this as neutrally buoyant.

2.3 Reduced Model

Under Assumption 1) we can make the following estimations:

The Jacobian can be simplified if $\theta, \phi \approx 0$

$$\dot{\eta} = J(\eta) * v \approx P(\psi) * v \tag{5}$$

Where

$$P(\psi) = \begin{bmatrix} R(\psi) & 0_{3x3} \\ 0_{3x3} & I_{3x3} \end{bmatrix} \tag{6}$$

and R is the 3x3 rotation matrix [1, p. 173].

Note that P is also a projection matrix, and $P^T = P^{-1}$. We can define a new frame $\eta_p = P^T(\psi) * \eta$. This frame under our low speed assumption 2) as well as assumption 1) mean that $\dot{\eta_p} \approx v$ In addition, based on our low speed assumption and low we can ignore the forces of lift on the vehicle, as well as rotational changes to buoyancy and gravity meaning we can approximate $g(\eta) \approx G * \eta_p$ where due to the systems neutral buoyancy

$$G = diag\{0, 0, 0, (z_g - z_b) * W, (z_g - z_b) * W, 0\} \tag{7}$$

where z_g and z_b are the forces of gravity and buoyancy on the z axis and W is the weight of the vehicle [1, p. 174].

Next due to our slow speed we can approximate the Coriolis affect on the vehicle as negligible variables and call them zero for the purpose of linearizing the model.

And we can also linearize drag using the approximation of $D * v^2 \approx D * v$ for $v < 1m/s$.

Finally based on the vehicles symmetry we can define M, which is the combination of the rigid body moments M_{RB} and added mass from submersion in a fluid M_A. Typically the matrix is a full 6x6 matrix but under xz, and yz symmetry we can simplify the matrix [1, p. 172].

$$M = \begin{bmatrix} m_{11} & 0 & 0 & 0 & m_{15} & 0 \\ 0 & m_{22} & 0 & m_{24} & 0 & 0 \\ 0 & 0 & m_{33} & 0 & 0 & 0 \\ 0 & m_{42} & 0 & m_{44} & 0 & 0 \\ m_{51} & 0 & 0 & 0 & m_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{66} \end{bmatrix} \tag{8}$$

Combining all of these simplifications we get the following simplified model

$$M\dot{v} + Dv + G\eta_p = \tau \tag{9}$$

$$\dot{\eta_p} = v \tag{10}$$

Finally, we can turn this into a continuous time, LTI state space model by letting

$$x = [\eta_p^T, v^T]^T \tag{11}$$

$$u = L\tau \tag{12}$$

$$A = \begin{bmatrix} \mathbf{0} & \boldsymbol{I} \\ -M^{-1} * G & -M^{-1} * D \end{bmatrix} \tag{13}$$

$$B = \begin{bmatrix} \mathbf{0} \\ M^{-1} \end{bmatrix} \tag{14}$$

$$\dot{x} = Ax + Bu \tag{15}$$

The final matrix that we need to define in order to use this state space model is L, where L maps the output of the 8 motors to our 6 degrees of freedom. This will be based on the length from the center of each motor to the center of the vehicle, and the primary direction of thrust provided by each motor.

In order to simulate this model we need to determine constants for M, D, and G, as well as the physical characteristics of L. Our first priority is to determine these parameters based on the SolidWorks model of the full assembly.

3 Hardware and State Estimation Model

To correct some false assumptions from the last project summary:

1. Sensors
   - The IMU and the DVL are not quantized, and are currently outputting 64 Bit floating values
2. State Estimation
   - The state estimator is outputting the current state at 40Hz. The states aline with x defined in equation 11 such that we will receive the vehicles full body and global positions and velocities.
   - The state estimator is most accurate at 8Hz due to the DVL being the sensor with the least drift or noise. We may choose to design our system to operate at 8Hz to take advantage of this sensor.

4 Timeline and Goals Update

Most of the project goals remain the same, however, our timeline has been pushed back by about half a week as we determine the parameters for M, D, and G, as well as measure the vehicle in order to determine L, for the simulink model of the vehicle. This means our complete plant should still be done by the first week of March, and implemented in Simulink. The digital controller design will begin then, completed by the second week of March, with the design and testing in the simulink model completed by the first week of April project summary.Finally we will test our hardware from mid April until the presentation.

References

[1] T. I. Fossen, *Handbook of Marine Craft Vehicle Hydrodynamics and Motion Control.* The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom: John Wiley and Sons Ltd., 2011.