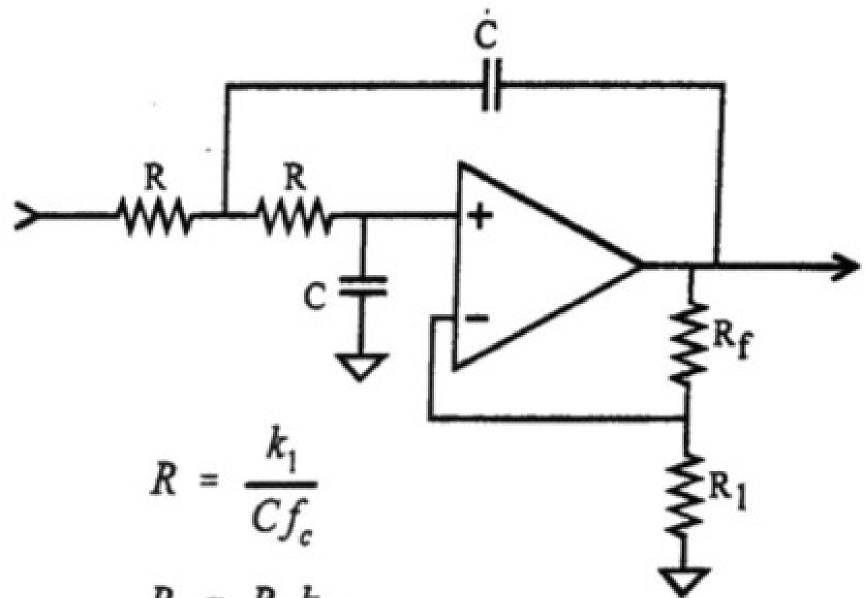
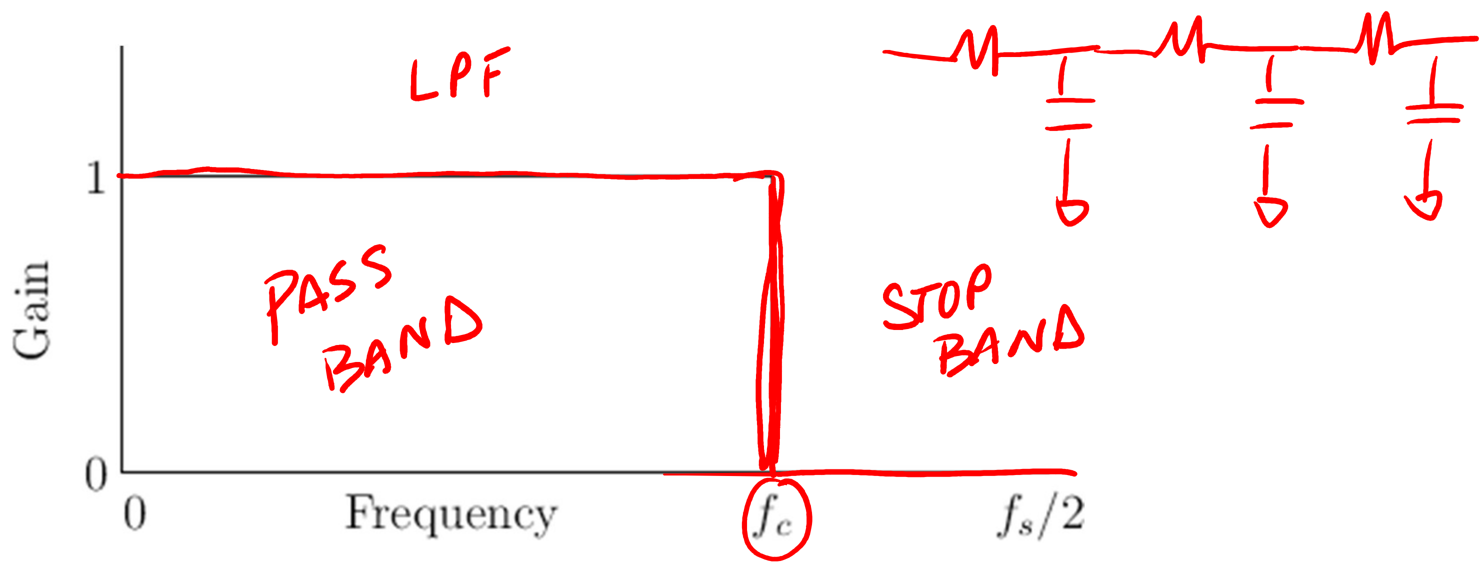
Active Filters



$$R = \frac{k_1}{Cf_c}$$

$$R_f = R_1 k_2$$

Ideal Low Pass Filter



What is it that we want from a filter:

(1) Flat gain in the pass band

(2) Low distortion in the pass-band (low or even phase shift)

(3) Sharp corner and steep fall-off above the cut-off frequency

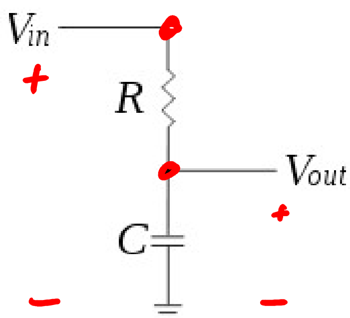
Low Pass Filter Revisted

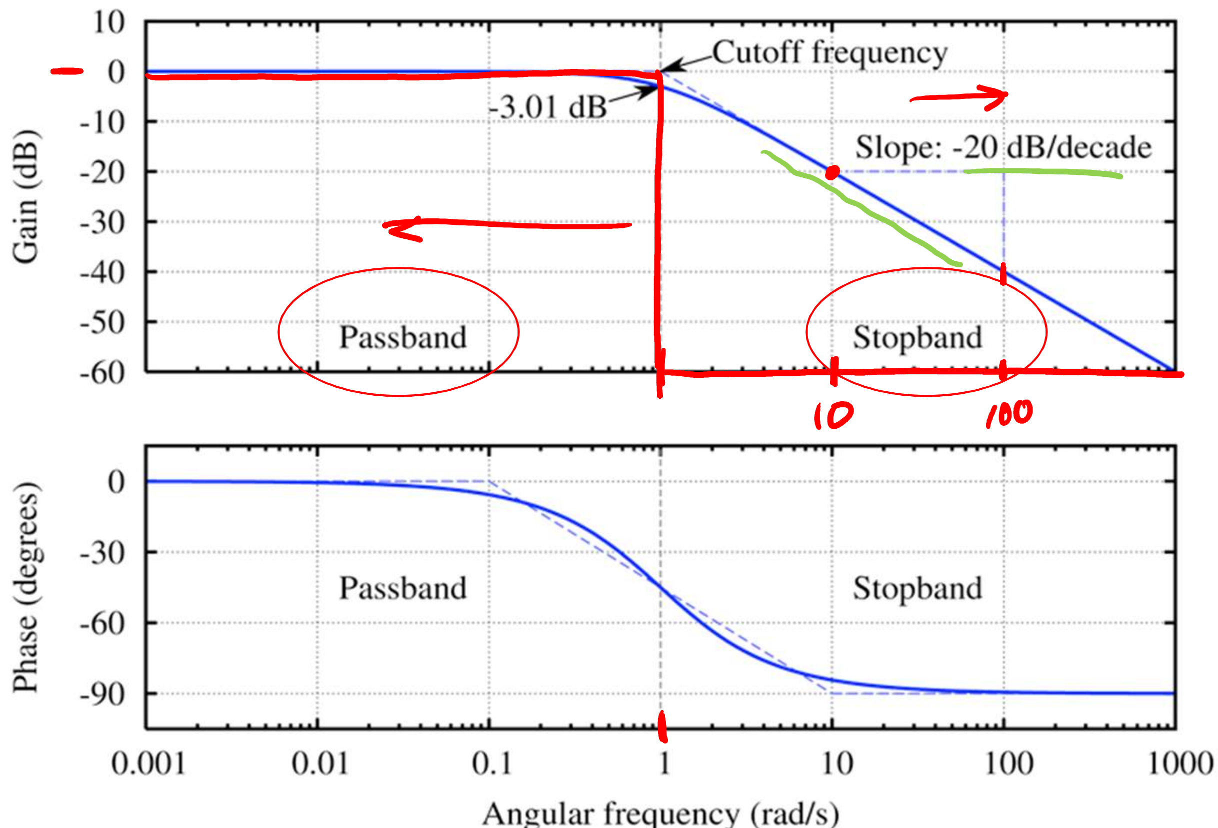
$H(j\omega)$

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}$$

1st Order low pass filter

Bode diagram





$\omega = 0 \quad H(0) = 1$

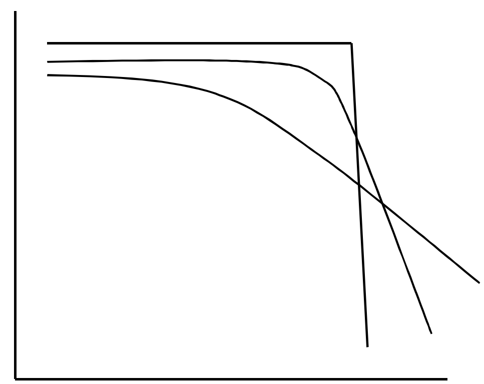
$\omega \to \infty \quad H(\infty) \to 0$

$\omega_0 = 1$ rad/se

Higher Order Filters

2nd Order Active Filter

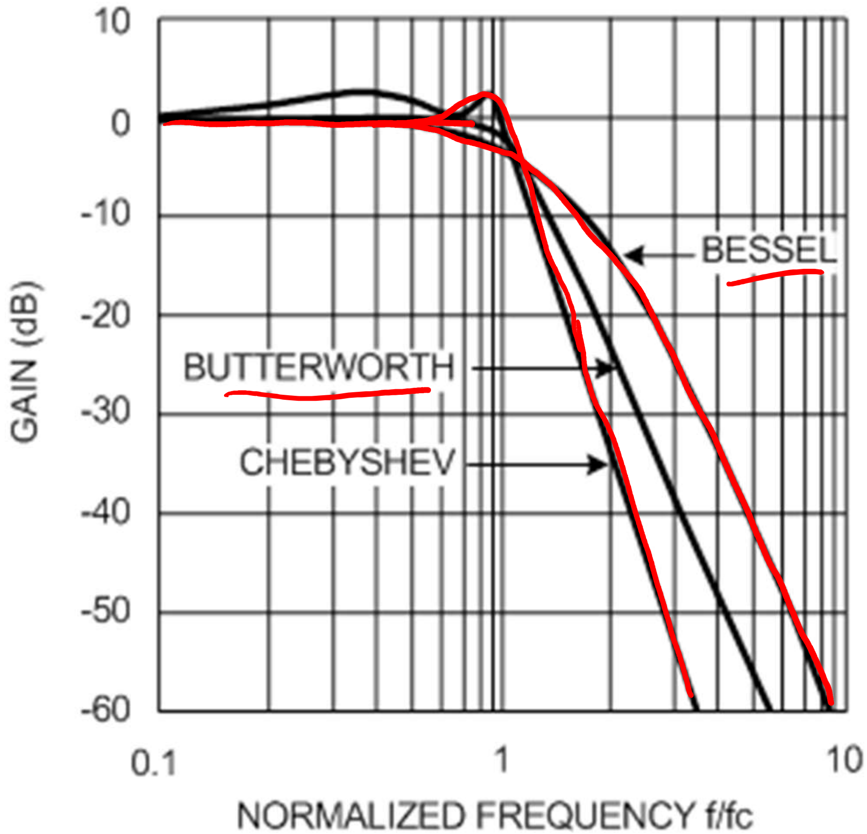
- You can easily design a filter with -40 dB/decade
  - 1 Op-Amp, 2 *Resistors* and 2 *Capacitors*
  - Compare to -20 dB/decade 1 Resistor and 1 Capacitor
- Filter designs are available in the text book
  - Rarely necessary to design from scratch
- There are many filter designs
  * Butterworth, Bessel, Chebyshev, etc.
  Some designs are flatter near the cutoff.



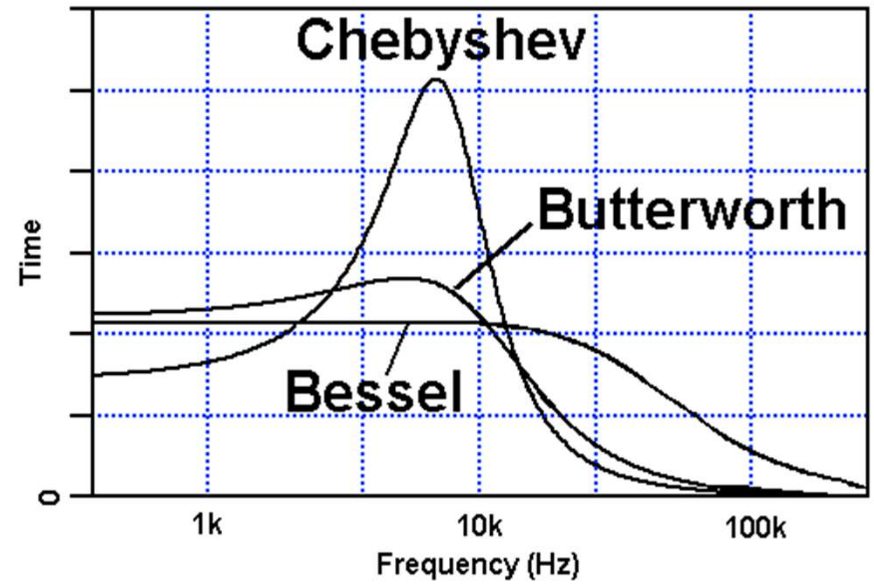
- We will examine more closely the Sallen-Key filter

Butterworth, Chebyshev, Bessel

$c_1 = c_2 \Rightarrow$ Bessel
$c_1 \neq c_2 \Rightarrow$ chebyshev

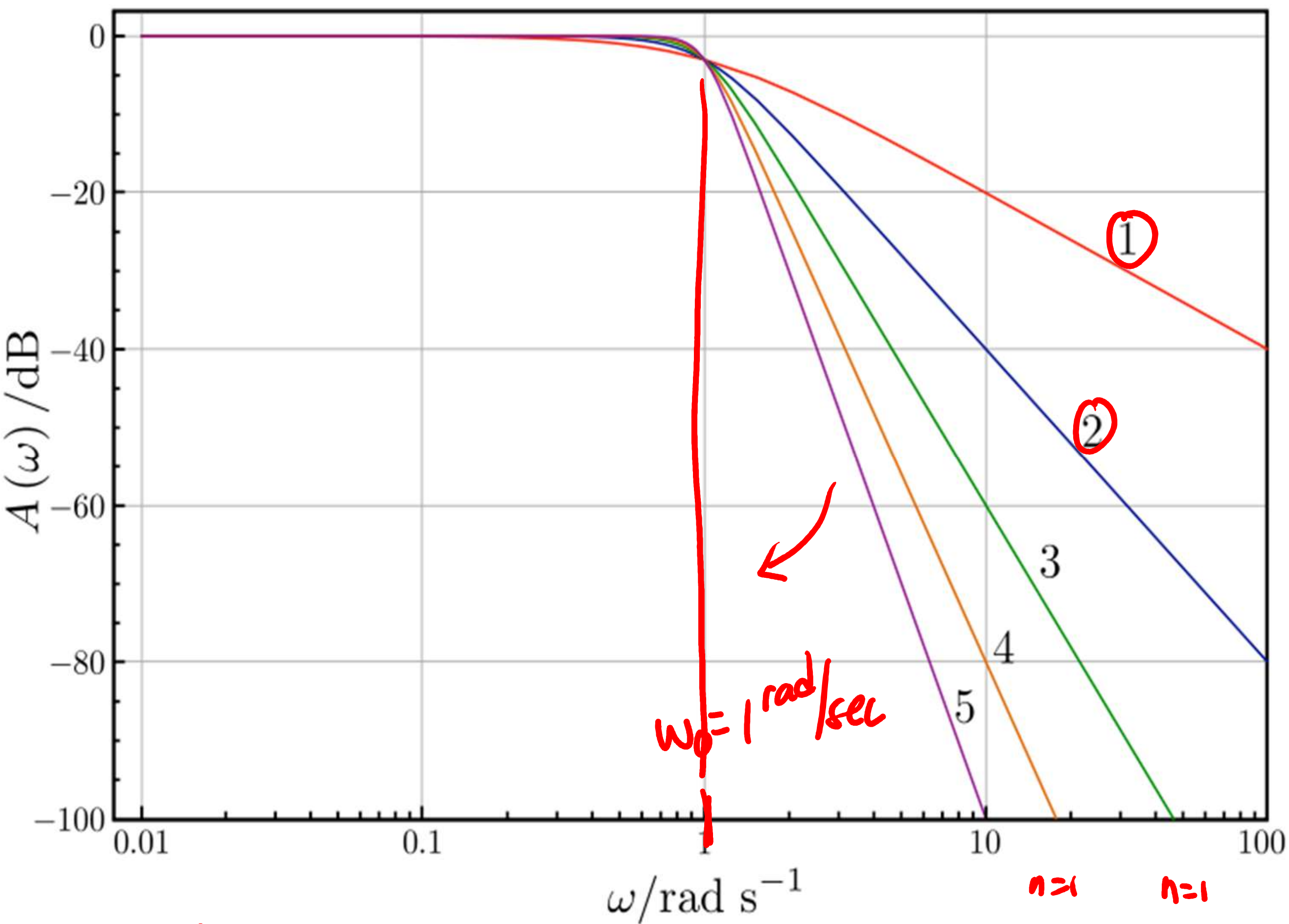


Comparison of Gain Responses of Fourth Order Low Pass Filters



Group Delay

Order of filters (Butterworth)



$|H(\omega)| = 1 / (1+\omega/\omega_0)^n$

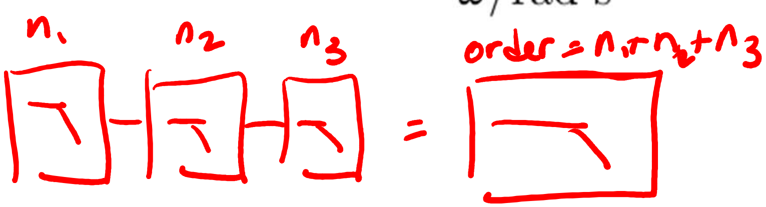
N = order of filter

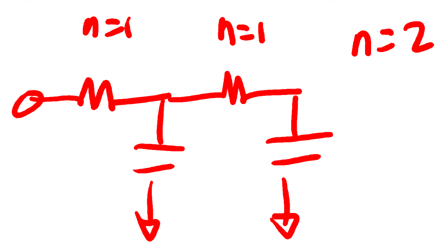
N = 1 (-20dB/decade)

N = 2 (-40 dB/decade)

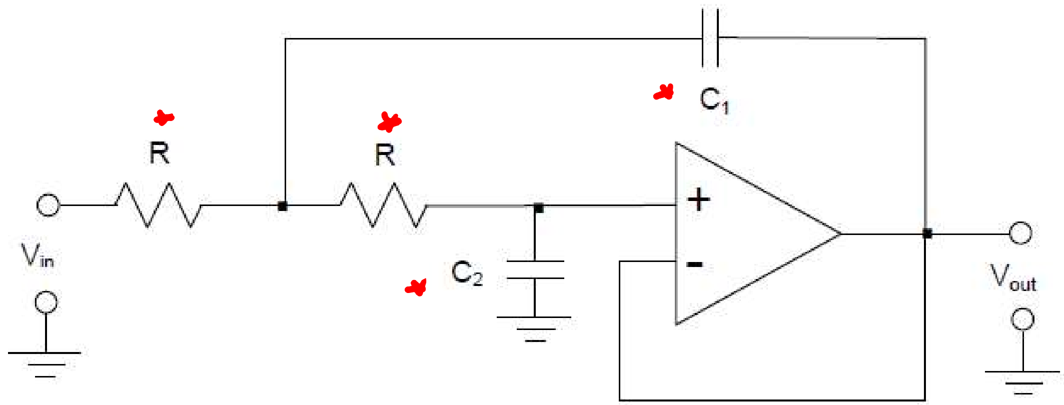
N = 5 (-100 dB/decade)

RC Filter n=1





Sallen-Key 2nd Order Filter



$C_1 = 10C$

$C_2 = \frac{1}{10}C$

$$\frac{Vout}{Vin} = \frac{\omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$$

$$\omega_o = \frac{1}{R\sqrt{C_1 C_2}}; \quad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

$C_1 = C_2$

$w_0 = \frac{1}{RC}$

$C_1 = 10 \cdot C_2$

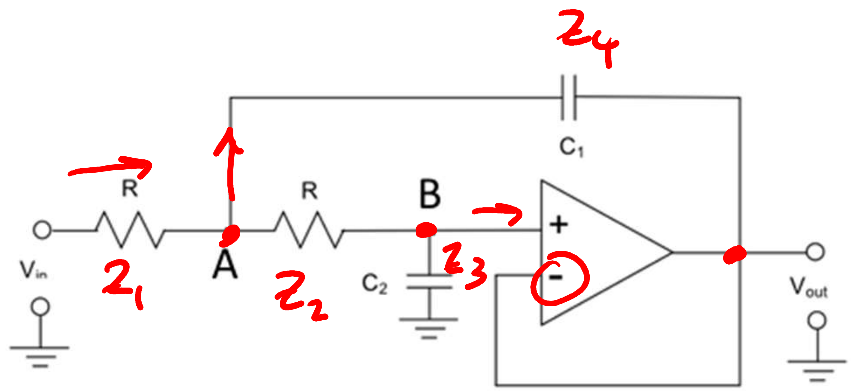
$C_1 = 100 C_2$

$Q = \frac{1}{2}$

$Q = \frac{\sqrt{10}}{2} = \frac{3}{2} \approx 1.5$

$Q = \frac{10}{2} = 5$

Sallen-Key Filter Derivation



$Z_1 = Z_2 = R$, $Z_3 = 1/j\omega C_1$, and $Z_4 = 1/j\omega C_2$.

$$\frac{V_{in} - V_A}{Z_1} = \frac{V_A - V_{out}}{Z_3} + \frac{V_A - V_B}{Z_2}$$

$$V_B = V_{out}$$

$$\frac{V_A - V_{out}}{Z_2} = \frac{V_{out}}{Z_4}$$

- Revisiting op-amps
- Exploiting basic rules:
  - KCL
  - Op-Amp rule #1 ($V^+ = V^-$)
  - Op-Amp rule #2 ($i_{in} = 0$)

Sallen-Key Filter Transfer Function

$$\frac{V_{out}}{V_{in}} = \frac{1}{1-(\omega^2/\omega_0^2)+j(\omega/Q\,\omega_0)} = \frac{\omega_0^2}{\omega_0^2-\omega^2+j\frac{\omega_0}{Q}\omega}$$

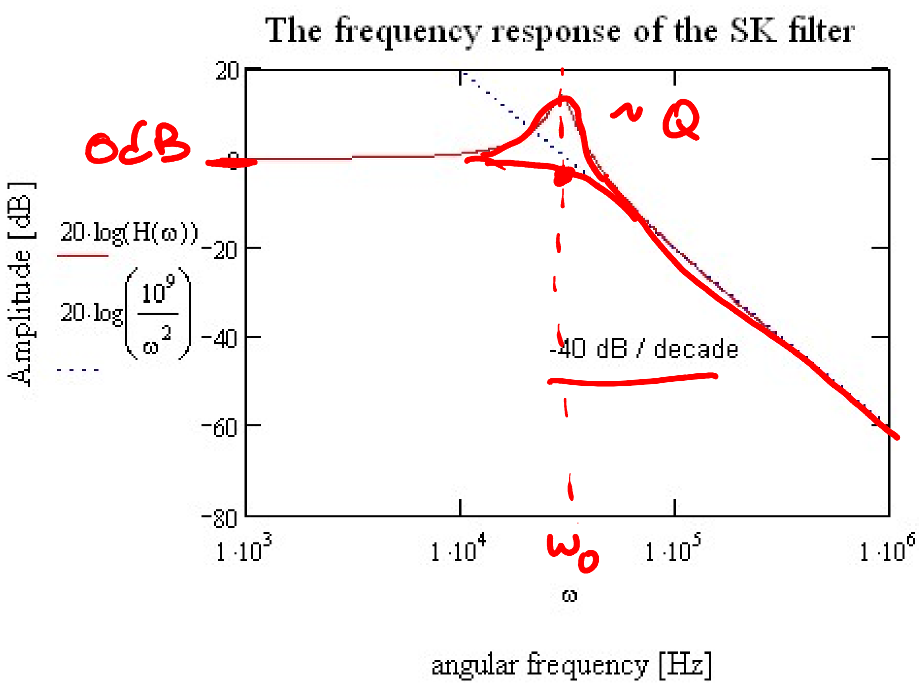
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2+\frac{\omega_0^2\omega^2}{Q^2}}}$$

$$\omega_0^2 = \left(1/R^2C_1C_2\right)$$

$$Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

$C_1 = C_2$

$Q = \frac{1}{2}$



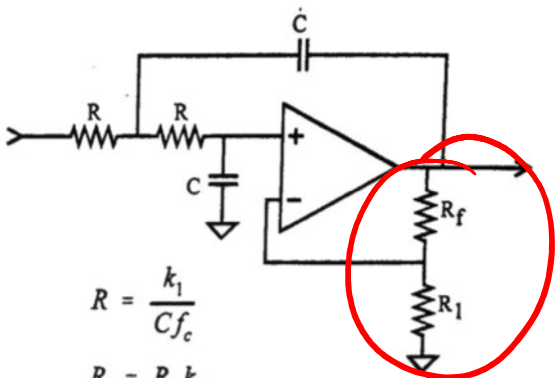
Observations

- Gain = 1 at DC & low freq.
- Gain = Q at $\omega = \omega_0$
- -40dB/decade stop band (2nd order filter)

- The SK filter is one of the options in the final practical

Cascaded Filters

6th order filter



$$R = \frac{k_1}{Cf_c}$$

$$R_f = R_1 k_2$$

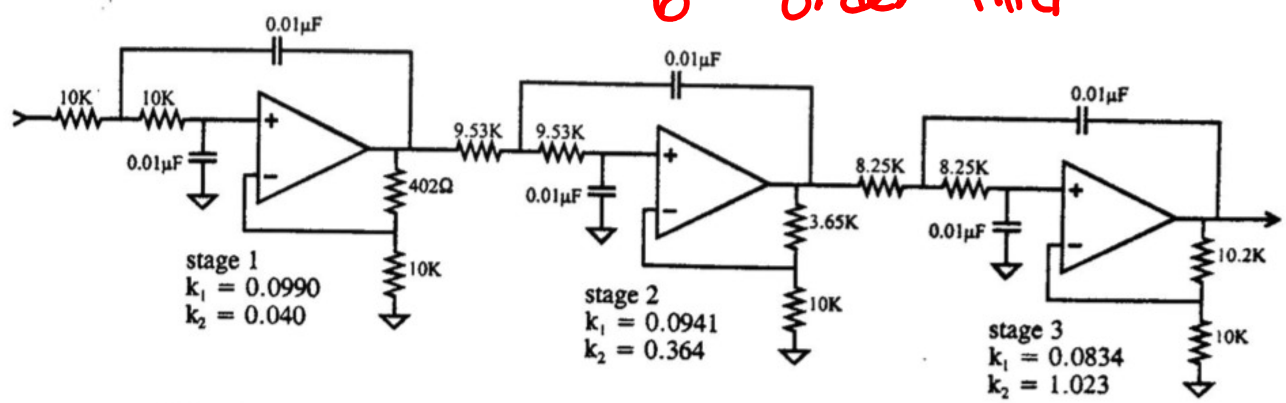


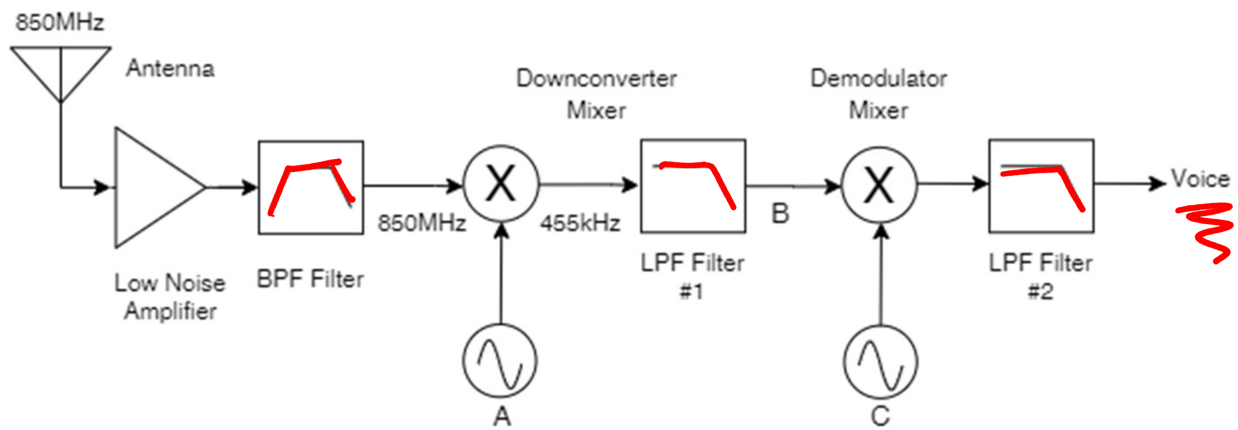
FIGURE 3-9
A six-pole Bessel filter formed by cascading three Sallen-Key circuits. This is a low-pass filter with a cutoff frequency of 1 kHz.

TABLE 3-1
Parameters for designing Bessel, Butterworth, and Chebyshev (6% ripple) filters.

# poles		Bessel k_1	Bessel k_2	Butterworth k_1	Butterworth k_2	Chebyshev k_1	Chebyshev k_2
2	stage 1	0.1251	0.268	0.1592	0.586	0.1293	0.842
4	stage 1	0.1111	0.084	0.1592	0.152	0.2666	0.582
	stage 2	0.0991	0.759	0.1592	1.235	0.1544	1.660
6	stage 1	0.0990	0.040	0.1592	0.068	0.4019	0.537
	stage 2	0.0941	0.364	0.1592	0.586	0.2072	1.448
	stage 3	0.0834	1.023	0.1592	1.483	0.1574	1.846
8	stage 1	0.0894	0.024	0.1592	0.038	0.5359	0.522
	stage 2	0.0867	0.213	0.1592	0.337	0.2657	1.379
	stage 3	0.0814	0.593	0.1592	0.889	0.1848	1.711
	stage 4	0.0726	1.184	0.1592	1.610	0.1582	1.913

- Bessel, Butterworth, Chebyshev filters can be constructed from Sallen-Key
- Multiple stages → higher order
- Generic filter design – Do not start from scratch!

Importance of filters in communication



- Get rid of unwanted frequencies (e.g. harmonics generated in the mixing process)
- Limit the bandwidth (frequency range)
  - Bandwidth is precious
  - Stations/communication channels are closely packed
  - Need to avoid interference (overlap of stations)

Frequency Channelization

850 kHz

97.3 MHz

850 KOA

97.3 KBCO