Global analysis of exclusive kaon and pion electroproduction

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The p$(e, e'\pi^+)$n and p$(e, e', K^+)\Lambda$ (Σ^0) reactions are important tools in the study of hadron structure. In particular, the flavor degree of freedom introduced with the addition of the strange quark helps us understand the reaction mechanism underlying strangeness production, and the transition from hadronic to partonic degrees of freedom in exclusive processes. In this study, we examine the world's data on exclusive p$(e, e'\pi^+)$n and p$(e, e'K^+)\Lambda$ cross sections. The data were combined into a superset with one global uncertainty, and examined for $-t$ dependence of the longitudinal and transverse components of the cross section as function of Q^2 and the longitudinal momentum fraction, x_B. The data suggest that the importance of t-channel meson exchange decreases at higher values of x_B. The Q^2-dependence of the longitudinal to transverse cross section ratio was compared with the Q^2-scaling expectation for hard exclusive processes.

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Depending on the virtuality of the photon, Q^2, exclusive processes give access to different physics regimes. At low Q^2, the process is described by hadronic fluctuations [1] and the spatial structure of the meson enters through the meson form factor. At asymptotic values of Q^2, the process becomes effectively pointlike and can be understood in terms of a virtual photon scattering from the quark-gluon degrees of freedom in Quantum Chromo Dynamics (QCD). At sufficiently large values of Q^2, at fixed longitudinal momentum fraction x_B carried by the struck quark, and fixed momentum transfer $-t$ to the nucleon, a QCD factorization theorem allows one to calculate the electroproduction amplitude in terms of Generalized Parton Distributions (GPDs), which encode the long-distance (soft) physics of hadron structure. GPDs have been shown to factorize from perturbative (hard) QCD processes for longitudinally polarized virtual photons [2]. Though it is expected that the factorization theorem is valid for large Q^2, for instance, $Q^2 > 10$ GeV^2, to date it is unclear whether it may already be approximately valid at moderate values of Q^2 under certain conditions [3]. Detailed descriptions of the GPD formalism can be found in Refs. [4–6].

The goal of the present analysis is to evaluate the world's data on exclusive p$(e, e'\pi^+)$n and p$(e, e'K^+)\Lambda$ cross sections for phenomena signaling the transition from the non-perturbative to the hard regime. Examining these data requires forming a superset with a single global uncertainty, taking into account the individual uncertainties and the differences in kinematic coverage of the individual experiments. This would allow for examining the global uncertainty in the Q^2- and t-dependences of the world cross section.

Longitudinal/transverse (L/T) separated cross sections are generally accepted as the best way to test the predictions of the factorization theorem for hard exclusive processes. To leading order, the meson electroproduction cross section for longitudinally polarized virtual photons, σ_L, scales like $\sim Q^{-6}$ at fixed $-t$ and x_B [7]. The contribution of transversely polarized photons is suppressed by at least an additional power of $1/Q$ in the amplitude [2].

A study of the non-perturbative (soft) region and its transition to the hard regime hold important information for our understanding of the transverse spatial structure of hadrons. In particular, data for systems with different quark flavors can provide important information about the exclusive production mechanism and the onset of factorization, which thus far is not well understood.

The t-dependence of the cross section is of similar interest as the Q^2-dependence. A feature of π^+ production, and to some extent K^+ production, is the existence of a "pole term", which gives the cross section a characteristic exponential rise as $-t$ decreases. The applicability of the t-pole dominance over the full dynamic range can be tested through the t-dependence of the cross sections for different values of x_B. Furthermore, by comparing the t-dependences of π^+ and K^+ production, one can also get an indication of the transition to the region where the pole term is believed not to be dominant. At sufficiently large values of Q^2, measurements of the t-dependence of the cross section may also give access to the hadronic transverse spatial structure.

At present, precise L/T separated data at sufficiently large Q^2 and center-of-mass energy, W, are lacking. Furthermore, the sparse existing L/T separated data are often at disparate values of Q^2, W, and t. To assess what can be learned from the current world data, and to guide future measurements at the Thomas Jefferson National Accelerator Facility (JLab) and elsewhere, an analysis of the global features and uncertainty is thus required.

To address the physics questions above using all suitable π^+ and K^+ electroproduction data requires an evaluation of the global uncertainty of trends in the cross section. To date, most analyses were carried out using data sets focusing on particular aspects relevant to an individual experiment, but as standalone experiments could not address more global physics features.

To study the global uncertainty one needs to create a superset of data combining individual measurements, also taking into account the differences in kinematic coverage. In our analysis, we use the previously published unseparated and L/T separated $^1\mathrm{H}(e,e'\pi^+)n$ and $^1\mathrm{H}(e,e'K^+)\Lambda$ cross sections [8–19]. However, the existing data are at disparate values of Q^2, W, and t. We will thus first examine the Q^2, W, and t dependence of unseparated data. Then we will examine the Q^2 dependence at fixed x_B and t of separated pion and kaon data, and evaluate whether it is possible to draw conclusions regarding phenomena expected in the transition to the hard regime.

We first generate supersets of unseparated pion and kaon data. The electroproduction cross sections depend on W, Q^2, x_B, and t (three of which are independent). To generate a cross section superset and evaluate the global uncertainty of the t-dependence one thus has to scale the experimental cross section to common values of W, Q^2, and x_B. This procedure, where we followed the factorized cross section model as outlined in Refs. [18, 20], is described below.

To combine and compare the t-dependence of the results from recent JLab measurements with those of earlier experiments, we scaled the experimental cross sections according to the empirical form $(W^2-M_p^2)^{-n}$, where M_p is the proton mass [14]. This W-dependence provides a good parameterization of earlier photo- and electroproduction data [14, 20]. The data were scaled through an iterative procedure, using the fit forms described below, to a central Q^2=1.6 GeV^2 for the pion and Q^2=1.9 GeV^2 for the kaon data, and binned in three x_B bins centered at x_B=0.15, 0.25, and 0.4. A fit using the empirical form above for W with n=2.41±0.01 (n=2.25±0.11 for the kaon data) describes the pion data to about 10% for the bin with x_B=0.2-0.3 and a central value of $-t$=0.15 GeV^2. The deviations from the empirical form in W are similar for the x_B=0.3-0.5 bin, but become slightly larger as W decreases. For the x_B=0.1-0.2 bin, the deviations from the fit form at small W can be up to 30%. Compared to the pion data, the kaon data for the x_B=0.2-0.3 bin have a larger central value of $-t$ of 0.35 GeV^2, and thus lie at slightly lower values of W. Most kaon data fall into the x_B=0.3-0.5 bin with a central value of $-t$=0.55 GeV^2. The empirical form in W describes the data in this bin to about 10%, and the data in the other two x_B bins to about 30%. In fact, the fit above describes all these data to within about 30%. This may provide a parameterization for guiding ongoing and future experiment considerations.

The smooth Q^2-dependence of the data allows to also scale to a common value of Q^2. No simple prediction exists for the Q^2 dependence of the unseparated cross section where the contributions from longitudinal and transverse cross sections are unknown. While σ_L is dominated by the pion-pole contribution there is no equivalent prediction for σ_T. The transverse cross section was, however, observed to follow the form $C(1+DQ^2)^{-1}$ [20]. The unseparated cross sections in the present analysis lie on a universal curve when scaled with this empirical form of the Q^2-dependence, $C(1+DQ^2)^{-1}$. A fit with C=6400±300 and D=0.17±0.02 (C=1200±20 and D=0.53±0.13 for the kaon data) gives a very acceptable description. For the bin with x_B=0.2-0.3, and central values of t = 0.15 GeV^2 and W=2.2 GeV, the pion data are described to about 10%, while other bins in the pion data are described to about 30%. The kaon data using the described empirical fit form are described to 30% as well. The range in Q^2 of the kaon data is, however, more limited than that of the pion data.

The smooth W- and Q^2-dependencies also allow looking at the features of the t-dependence in pion and kaon data. Figure 1 shows the t-dependence of the pion and kaon cross sections after scaling to W=2.2 GeV, using

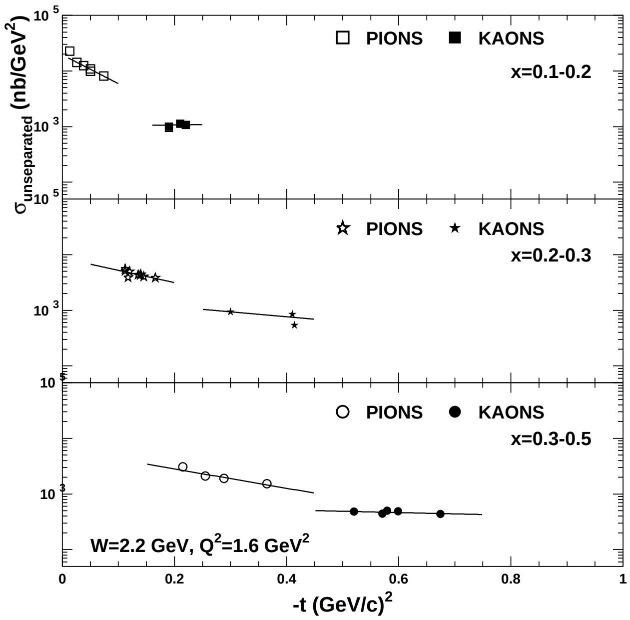

FIG. 1: *(Color online) The t-dependence of the unseparated pion and kaon production cross sections for three bins in x_B at fixed Q^2 and W. The error bars include the statistical and systematic uncertainty combined in quadrature.*

x_B	$-t$ (GeV/c)^2	B	b
Pions			
0.1-0.2	0.01-0.1	19±3x10^3	-11.9±2.9
0.2-0.3	0.1-0.2	9±3x10^3	-5.1±2.6
0.3-0.5	0.2-0.4	6±3x10^3	-4.0±1.7
Kaons			
0.1-0.2	0.1-0.2	10±2x10^2	0.3±0.7
0.2-0.3	0.3-0.4	17±3x10^2	-2.0±0.4
0.3-0.5	0.5-0.6	6±4x10^2	-0.5±0.9

TABLE I: *The t-slopes for π^+ and K^+ production. Here, B and b are the fit parameters of the form Be^{-bt}.*

the functional form $(W^2 - M_p^2)^{-n}$, and to Q^2=1.6 GeV^2 according to $\frac{C}{1+DQ^2}$. The data are binned in x_B with bin centers x_B=0.15, 0.25, and 0.4. Fitting the data with an exponential of the form $Be^{-b|t|}$ results in the slope parameters and normalization factors listed in Table I. Such a form describes all cross section data at approximately the 10% level, with one or two outliers at low $-t$, and to 40% at the largest values of $-t$. This could thus provide a guiding parameterization of unseparated charged pion data for ongoing and future experiments.

We now generate pion and kaon data supersets using only L/T separated data to evaluate the global uncertainty of the Q^2-dependence of the cross section. Given that the specific W, Q^2, and t dependences may differ in L/T separated data as compared to the total cross section we follow the procedure outlined in Ref [17]. We then again generate a superset scaling the cross section to common values of the kinematic variables x_B and t.

The data were scaled to constant values of W=2.2 GeV using the functional form $(W^2 - M_p^2)^{-2}$, and to Q^2=1.6 GeV^2 according to the form $\frac{C}{1+DQ^2}$. The data were bin-centered in $-t$ and x_B using a Regge-based model that describes the longitudinal component, and the shape of the transverse component, of the cross section reasonably well [21, 22]. The additional uncertainty due to this kinematic scaling was determined from comparisons of the resulting cross section with ones obtained using a GPD model prediction (for σ_L only) [23], and a parameterization based on pion electroproduction data [20]. The kinematic scaling uncertainty in σ_L ranges between 7% and 13% for both x_B kinematics, while the σ_T uncertainty is larger by a factor of three. The Q^2-dependencies of the longitudinal to transverse cross section ratio σ_L/σ_T, where we have used only L/T separated data, are shown in Figure 2.

The t-dependence as shown in Figure 1 exhibits the characteristic exponential fall-off, which, for σ_L, would be associated with t-pole dominance. At values of $x_B \sim 0.15$, the t-slopes differ by about an order of magnitude albeit with large uncertainties. Nonetheless, the pion and kaon t-slopes are different at x_B=0.15 with 3 σ confidence. At values of $x_B > 0.2$, the pion and kaon t-slopes become more similar, although uncertainties are large in the bin centered at $x_B = 0.4$. However, the cross sections shown here are unseparated, making it more difficult to identify the pole term, which is known to be best isolated through the separated longitudinal cross section.

Even if the t-dependence should be studied with separated cross sections, at present such data are too sparse to do so. To assess what can be learned from the current world data and to provide some guidance for ongoing and future experiment considerations, it may be instructive, however, to examine the global uncertainty of the t dependence with unseparated cross sections. It is remarkable that these data can be described relatively well, $\sim$ 10%, by functional forms based on relatively few parameters. In addition, the t dependence of pion and kaon unseparated cross sections seems to be drastically different, which may hint at significant differences in the pole terms.

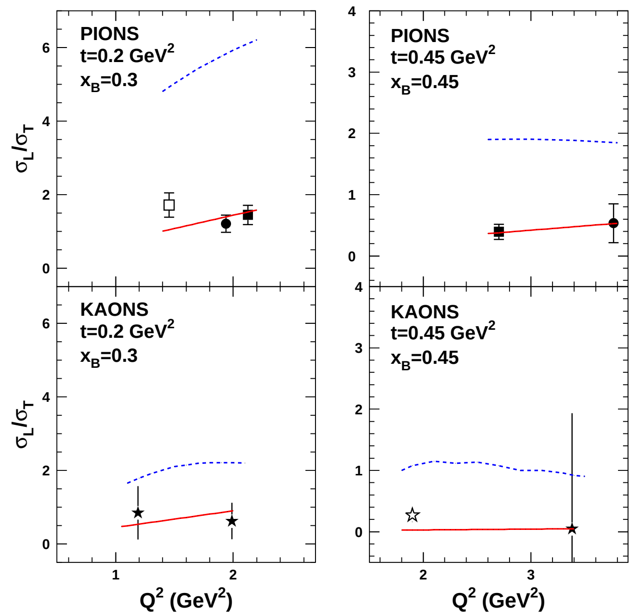

FIG. 2: *(Color online) The Q^2-dependence of the separated cross section ratio at fixed values of $-t$ and x_B. The error bars denote the statistical and systematic uncertainty combined in quadrature. The solid red curve shows a fit of the form $(\sigma_L/\sigma_T \sim Q^2$. The dashed blue line in the upper two panels is a VGL/Regge calculation using Λ_π^2 from a global fit to F_π. For the kaon data in the lower two panels we used the value Λ_K^2 as given in [22].*

The t-dependence in hard exclusive processes may in principle be interpreted as a measure of the average transverse size of the nucleon [24]. Indeed, there is a pole term in the GPD in which the QCD operator measuring the quark density is connected to the nucleon by the t-channel meson exchange [25, 26]. A priori it is not clear if strange-quark systems will show similar behavior as the lighter ones. However, to access the physics contained in the GPDs one first needs to show that hard-soft factorization applies.

A stringent and model-independent test of such hard-soft QCD factorization is the leading order Q^2 power-law scaling of the separated electroproduction cross sections. Recent fully separated π^+ data indicate a Q^{-6}-scaling of σ_L that is consistent with the hard scattering mechanism already at values of Q^2 >1 GeV^2, but σ_T does not show a corresponding Q^{-8} behavior [17]. The ratio of cross sections may thus provide additional essential information on the Q^2-dependence, and it is thought that higher-twist effects may largely cancel in the super-ratio of pions to kaons.

The extracted ratios are compared to a Regge-based

x_B	$-t$	$\sigma_{\rm L}/\sigma_{\rm T}\sim Q^{-n}$ n	$\sigma_{\rm L}/\sigma_{\rm T}\sim Q^{2}$ χ^2/ν (P)	$\sigma_{\rm L}/\sigma_{\rm T}\sim Q^{-2}$ χ^2/ν (P)
Pions				
0.3	0.2	1.32 ± 1.07	2.40 (0.10)	0.55 (0.58)
0.45	0.45	-1.86 ± 4.40	0.002 (0.95)	0.59 (0.44)
Kaons				
0.3	0.2	1.48 ± 4.59	0.42 (0.52)	0.01 (0.91)
0.45	0.45	2.87 ± 15.2	0.04 (0.84)	0.002 (0.48)

TABLE II: *The central fit values for the Q^2-scaling R=($\sigma_{\rm L}/\sigma_{\rm T}$) $\sim Q^{-n}$ for both pion and kaon production for both x_B settings. Also shown are the χ^2 per degree of freedom, ν, values for fitting the data ratio with the hard scattering prediction $\sigma_{\rm L}\sim Q^{-6}$ and the hard scattering expectation $\sigma_{\rm T}\sim Q^{-8}$. In the last column, $\sigma_{\rm T}$is assumed to scale as in DIS. Together with the hard scattering prediction for $\sigma_{\rm L}$, the ratio is expected to scale as Q^{-2}. The corresponding probability for both cases, P, for finding this value of χ^2 or larger in the sampling distribution assuming Poisson statistics is shown in parentheses.*

calculation by Vanderhaeghen, Guidal, and Laget [21, 22]. The VGL Regge model underestimates the magnitude of the transverse cross section data, which explains the large overshoot in the calculated ratios.

The hard scattering prediction for the σ_L/σ_T ratio is fitted to, and indicated by, the solid red lines in Figure 2. To investigate the statistical impact of the existing data, we have fitted the ratio σ_L/σ_T at each value of x_B to the form Q^n, where n is a free parameter. The experimental fit values are listed in Table II. The best fit values from a two-parameter fit of σ_L/σ_T seem to favor a negative Q^2 exponent. This may point to that soft reaction mechanism contributions continue to play a substantial role in the cross sections at these values of Q^2. However, the quality of these fits is limited by the very small samples of available data. Indeed, within these large uncertainties, the best fit values for σ_L/σ_T would also largely be consistent with the hard scattering prediction, and agree with the results from the fully separated pion cross sections [17]. The χ^2-results of Q^2-exponents fixed to the hard scattering expectation (n=2) and to n=-2 are shown in columns three and four. The latter dependence may be expected if σ_L scaled according to the hard scattering prediction and σ_T as in Deep Inelastic Scattering (DIS). This would be consistent with recent theoretical work, which suggested that σ_T could be described as the semi-inclusive production limit through the fragmentation mechanism [27]. For both pions and kaons, at small values of $-t$ and x_B this fit form seems to agree better with the data, while bins with larger $-t$ or x_B seem to favor the hard scattering fit form.

In summary, we have analyzed the global uncertainties of the kinematic dependencies in exclusive p$(e,e'\pi^+)$n and the p$(e,e',K^+)\Lambda$ (Σ^0) data. Since the existing data are at disparate values of Q^2, W, and t we created two separate supersets to examine the Q^2, W, and t dependence of unseparated and the Q^2 dependence at fixed x_B and t of separated pion and kaon data. An empirical fit form of the W, Q^2, and t-dependence describes the unseparated cross sections to about 30% and for most bins to better than 10%. The relatively simple parameterization could be useful for ongoing and future experiment considerations. The t-dependence of the unseparated data shows a characteristic exponential fall-off, which, for σ_L, would be associated with t-pole dominance. This trend is most pronounced in the pion data and is less noticable in the kaon data. At values of $x_B > 0.2$ both the pion and kaon t-slopes seem to flatten out. The Q^2-dependence of the current separated pion and kaon longitudinal to transverse cross section ratios was found to be in agreement with the Q^2-scaling prediction within the experimental uncertainty. This would suggest that it may be possible to access GPDs at relatively low values of Q^2. However, higher order contributions may still be significant in this kinematic regime and mimic the expected Q^2-scaling behavior. It is interesting to note that the data at larger t and x_B seem to favor a fit form that assumes that σ_L scales according to the hard scattering prediction and σ_T as in DIS.

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