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1 using LinearAlgebra
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1 using Random
```

```
1 using Noise
```

```
inputs = []
```

```
1 # Define XOR points...
2 inputs = Any[]
```

```
outputs = []
```

```
1 # ... and corresponding labels
2 outputs = Any[]
```

```
n = 500000
```

```
1 # Define number of data points
2 n = 500000
```

```
1 for _ = 1 : n
2
3     # define XOR input point
4     xor_point = bitrand(2)
5
6     # define the two cases in which XOR outputs 0, otherwise 1
7     if (xor_point[1] == 1 && xor_point[2] == 1) || (xor_point[1] == 0 &&
8         xor_point[2] == 0)
9         label = 0
10    else
11
12        label = 1
13    end
14
15    # add small gaussian noise
16    dp = add_gauss(Float64.(xor_point), 0.05)
17
18    # add to dataset
19    push!(inputs, [dp; 1.0]) # absorb bias into datapoint
20    push!(outputs, label)
21
22 end
```

```
sigmoid (generic function with 1 method)
```

```
1 # Activation function sigmoid
2 function sigmoid(z)
3     return 1 / (1 + exp(-z))
4 end
```

relu (generic function with 1 method)

```

1 # Activation function ReLU
2 function relu(z)
3     if z > 0
4         return z
5     else
6         return 0
7     end
8 end

```

MSEloss (generic function with 1 method)

```

1 # Mean-Squared Error loss function
2 function MSEloss(y_tilde, y)
3     return (y_tilde - y)^2
4 end

```

feedforward (generic function with 1 method)

```

1 # Takes one data point, produces prediction based on current state of weights
2 function feedforward(x, W1, W2)
3
4     # Repackage in desired way (make x the operator on vector w)
5     X = [x' zeros(6)'; zeros(3)' x' zeros(3)'; zeros(6)' x']
6     w1 = [W1[1, 1:3] ; W1[2, 1:3]; W1[3, 1:3]]
7
8     # Feedforward starts here
9     z1 = X * w1
10
11     r = map(relu, z1)
12
13     x2 = [r; 1]
14
15     z2 = W2 * x2
16
17     y_tilde = map(sigmoid, z2)
18
19     return w1, X, z1, x2, z2, y_tilde
20
21 end

```

d_sigmoid (generic function with 1 method)

```

1 # Derivative of sigmoid activation function
2 function d_sigmoid(z)
3     return sigmoid(z) * (1 - sigmoid(z))
4 end

```

d_relu (generic function with 1 method)

```
1 # Derivative of ReLU activation function
2 function d_relu(z)
3     if z > 1
4         return 1
5     else
6         return 0
7     end
8 end
```

d_MSEloss (generic function with 1 method)

```
1 # Derivative of Mean-Squared Error loss function
2 function d_MSEloss(y, y_tilde)
3     return 2 * (y_tilde - y)
4 end
```

backprop (generic function with 1 method)

```
1 function backprop(w1, w2, X, z1, x2, z2, y_tilde, y)
2
3     dL_dz2 = d_MSEloss(y, y_tilde) * map(d_sigmoid, z2)
4
5     # Compute update of W2:
6     dL_dw2 = dL_dz2 * x2'
7
8     # Compute update of W1:
9     dL_dw1 = dL_dz2 * w2 * [I ; [0 0 0]] * diagm(map(d_relu, z1)) * X
10    dL_dw1 = [dL_dw1[1:3]' ; dL_dw1[4:6]'; dL_dw1[7:9]']
11
12    return dL_dw1, dL_dw2
13
14 end
```

```
1 let
2
3     # Number of epochs (times to iterate over dataset)
4     num_epochs = 10
5
6     # Learning rate
7      $\alpha$  = 0.05
8
9     # Initialize weights
10    W1 = add_gauss(zeros(3, 3), 0.5)
11    W2 = add_gauss(zeros(1, 4), 0.5)
12
13    # Record loss
14    training_loss = Any[]
15
16    function train()
17
18        for _ = 1 : num_epochs
19            for i = 1 : n
20
21                dp = inputs[i]
22                y = outputs[i]
23
24                w1, X, z1, x2, z2, y_tilde = feedforward(dp, W1, W2)
25
26                dL_dW1, dL_dW2 = backprop(w1, W2, X, z1, x2, z2, y_tilde[1], y)
27
28                W1 -=  $\alpha$  * dL_dW1
29                W2 -=  $\alpha$  * dL_dW2
30
31                loss = MSEloss(y, y_tilde[1])
32                push!(training_loss, loss)
33
34            end
35        end
36    end
37
38    function predict(x)
39        _, _, _, _, _, pred = feedforward([x ; 1], W1, W2)
40
41        if pred[1] >= 0.5
42            return pred, 1
43        else
44            return pred, 0
45        end
46    end
47
48    train()
49
50    # Predict actual XOR points
51    pred1, label1 = predict([0; 0])
52    pred2, label2 = predict([1; 0])
53    pred3, label3 = predict([0; 1])
54    pred4, label4 = predict([1; 1])
```

```
55  
56     println(pred1, label1)  
57     println(pred2, label2)  
58     println(pred3, label3)  
59     println(pred4, label4)  
60
```

```
[0.010927996838302154]0  
[0.9898135233251393]1  
[0.9998436152871761]1  
[0.0003025395204761653]0
```

