You cannot use any books or calculators. You have **80** minutes to complete the test.

1. (20 pts) Which of the following subsets of $\mathbb{R}^3$ are subspaces?
   (a) $\{(x, y, z) \mid x + y = 0 \text{ and } y + 2z = 0\}$
   (b) $\{Y \in \mathbb{R}^3 \mid AX = Y \text{ for some } X \in \mathbb{R}^3, A \text{ fixed } 3 \times 3 \text{ matrix}\}$
   (c) $\{X \in \mathbb{R}^3 \mid AX = (0, 0, 0)^t, A \text{ fixed } 3 \times 3 \text{ matrix}\}$
   (d) $\{(x, y, z) \mid x = 0 \text{ or } y = 0 \text{ or } z = 0)\}$

2. (20 pts) Find a basis of the solution space of

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = 0$$

3. (a) (10 pts) Show that the vectors $\alpha = (1, 1, 0)^t, \beta = (1, 2, 0)^t$ and $\gamma = (1, 1, 1)^t$ in $\mathbb{R}^3$ are linearly independent.
   (b) (10 pts) Assume for the linear map $T : \mathbb{R}^3 \to \mathbb{R}$ that $T(\alpha) = 1, T(\beta) = 2$ and $T(\gamma) = 3$. Let $\delta = (3, 4, 2)^t$. What is $T(\delta)$?

4. (10 pts) Let A be an $m \times n$ matrix. Prove that the solution set of $Ax = 0$ is a subspace of R^n

5. (10 pts) Let H be the subspace of P_3 made up of only those polynomials $p(x)$ such that $p(0) = 0$, $H = \{p(x) \in P_3 | p(0) = 0\}$. Find a basis for H.

6. (20 pts) Let M_{nxn} be the vector space of all $n \times n$ matrices, and let $C \in$M$_{nxn}$ be a fixed $n \times n$ matrix. Let H be the set of all $n \times n$ matrices that commute with A. Is H a subspace of M_{nxn}? Justify your answer.