

**CSCI 2820 - Linear Algebra with Computer Science Applications - Final Exam.**

You can use your textbook and notes but no internet/chatgpt You have **80** minutes.

1. (20 pts) Consider the matrix,  $M = \begin{bmatrix} -4 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ . Are its columns linearly dependent or linearly independent vectors? Justify your answer and use it to find all solutions to the linear system  $Mx = 0$ , where  $x$  is a  $3 \times 1$  real vector and  $0$  is a  $3 \times 1$  vector of zeros. Explain your answer fully!
2. (20 pts) Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a linear transformation such that it maps the vectors  $\mathbf{v}_1, \mathbf{v}_2$  as indicated in the figure below.

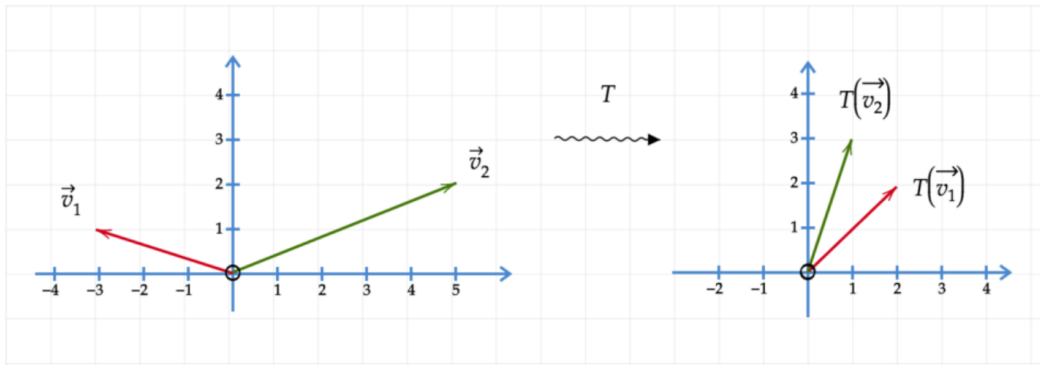


Figure 1: Linear Transformation

Find the matrix representation  $A$  of the linear transformation  $T$ .

3. (20 pts) Let  $A = \begin{bmatrix} -5 & 2 \\ -7 & 4 \end{bmatrix}$ . Find its eigenvalues and eigenvectors. **Show ALL work!**
4. (20 pts) . Let  $\gamma : \alpha \rightarrow \beta$  be a linear transformation from a vector space  $\alpha$  into a vector space  $\beta$ . Prove that the range of  $\gamma$  is a subspace of  $\beta$ .
5. (20 pts) Describe all least-squares solutions of the system:  
 $x + y = 2$   
 $x + y = 4$ .

**Hint:** You will need to get the matrix into reduced row echelon form to solve.