HW1 Manually Graded: Upload PDF here

Graded

Student

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Total Points

17 / 17 pts

Question 1

Question 9

3 / 3 pts

Question 9 (3 pts)

```
def summation(n):
    """Compute the summation i^3 + 3 * i^2 for 1 <= i <= n."""
    # BEGIN SOLUTION
    return sum((np.arange(1, n + 1) ** 3) + (3 * np.arange(1, n + 1) ** 2))
    # END SOLUTION

#Do not change the following cells:
print("summation(5) =  ", summation(5))
print("summation(200) = ", summation(200))

```

```
summation(5) =   390
summation(200) =  412070100
```

✔ **– 0 pts** Completely correct

Question 2

Question 10

3 / 3 pts

SOLUTION:

Option 1:

Option 1:

Find the definite integral (need to make sure you change your integration limits when using a u-sub)

$$\begin{aligned}
&\int_0^\infty \lambda e^{-\lambda x}\,dx \\
&= \lim_{b\to\infty}\int_0^b \lambda e^{-\lambda x}\,dx && \text{let u} = \lambda x \implies du = \lambda dx \\
&= \lim_{b\to\infty}\int_0^{\lambda b} e^{-u}\,du \\
&= \lim_{b\to\infty} -e^{-u}\Big|_{u=0}^{u=\lambda b} \\
&= \lim_{b\to\infty}(-e^{-\lambda b}-(-e^{-\lambda(0)})) \\
&= \lim_{b\to\infty}(-\frac{1}{e^{\lambda b}})+1 \\
&= 0+1 \\
&= \boxed{1}
\end{aligned}$$

Option 2:

Option 2:

Find the antiderivative first and then use the Fundamental Theorem of Calculus

$$\begin{aligned}
\text{Let } F(x) &= \int \lambda e^{-\lambda x}\,dx && \text{let u} = \lambda x \implies du = \lambda dx \\
&= \int e^{-u}\,du \\
&= -e^{-u}+C \\
&= -e^{-\lambda x}+C
\end{aligned}$$

$$\begin{aligned}
&\int_0^\infty \lambda e^{-\lambda x}\,dx \\
&= \lim_{b\to\infty}\int_0^b \lambda e^{-\lambda x}\,dx \\
&= \lim_{b\to\infty}(F(b)-F(0)) && \text{By the Fundamental Theorem of Calculus} \\
&= \lim_{b\to\infty}(-e^{-\lambda b}-(-e^{-\lambda(0)})) \\
&= \lim_{b\to\infty}(-\frac{1}{e^{\lambda b}})+1 \\
&= 0+1 \\
&= \boxed{1}
\end{aligned}$$

✔ **– 0 pts** Correct!

Question 3

Question 11

11 / 11 pts

✔ **– 0 pts** Question 11 correct

Question assigned to the following page: 1

Back to top

0.1 Question 9 (5 pts)

Write a function `summation(n)` that uses vectorization in Numpy to evaluate the following summation for $n \geq 1$:

$$\sum_{i=1}^{n} (i^3 + 3i^2)$$

Note: You should **NOT** use ANY `for` loops in your solution. You may find `np.arange` helpful for this question!

```
In [98]: def summation(n):
             return np.sum(np.arange(n+1)**3 + 3 * np.arange(n+1)**2)

         #Do not change the following cells:
         print("summation(5) =  ", summation(5))
         print("summation(200) = ", summation(200))
```

```
summation(5) =   390
summation(200) =  412070100
```

```
In [99]: grader.check("q9")
```

```
Out[99]: q9 results: All test cases passed!
```

Question assigned to the following page: 2

Question 10 Solution) **Type your answer to Question 10** in this cell (You can copy the LaTeX from the first 2 lines above as a start and then complete the rest of the problem). Show all of your steps and fully justify your answer. Do not add any additional cells to this part.

$$\begin{aligned}
&\int_0^\infty \lambda e^{-\lambda x}\,dx &(1)\\
&= \lim_{b\to\infty}\int_0^b \lambda e^{-\lambda x}\,dx &(2)\\
&= \lim_{b\to\infty} \lambda \frac{e^{-\lambda x}}{-\lambda}\Bigg|_0^b \text{ By using the antiderivative of } e^u\text{, where } u=-\lambda x &(3)\\
&= \lim_{b\to\infty} -e^{-\lambda x}\Bigg|_0^b &(4)\\
&= \lim_{b\to\infty} -e^{-\lambda b}+e^0 &(5)\\
&= \lim_{b\to\infty} -e^{-\lambda b}+1 &(6)\\
&= 1 &(7)
\end{aligned}$$

Question assigned to the following page: 3

Question 11abc Solutions) Use LaTeX (not code) in the cells below to show all of your steps for parts 11a, 11b and 11c and fully justify your answers. Do not add any additional cells to this part.

Enter your answer for part 11a) in this cell (double click on this cell and write all steps using Markdown and LaTex): $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Enter your answer for part 11b) in this cell (double click on this cell and write all steps using Markdown and LaTex):

$$\begin{align}
A &= \text{Both flips are heads} \tag{8}\\
B &= \text{At least one of the flips is a heads} \tag{9}\\
P(A \cap B) &= 1/4 \text{ because there are 4 possible permutations and only one has both 2 heads and at least 1 head} \tag{10}\\
P(B) &= 3/4 \text{ because there are 4 possible permutations and 3 have at least one head} \tag{11}\\
P(A|B) &= \frac{1/4}{3/4} \tag{12}\\
&= 1/3 \tag{13}
\end{align}$$

Enter your answer for part 11c) in this cell (double click on this cell and write all steps using Markdown and LaTex):
Part i:
A signifies that the 2nd marble is red and B signifies that the 1st marble is red. Therefore, $P(A \cap B)$ is the probability that both marbles are red. The total number of permutations is $P\binom{8}{2} = \frac{8!}{6!} = 56$. The probability that both marbles are red is $(P\binom{5}{2} = \frac{5!}{3!} = 20)/56$. Therefore, $P(A \cap B) = 20/56$. The total number of permutations for B is $P\binom{8}{1} = \frac{8!}{7!} = 8$. There are 5 red marbles, so $P(B) = 5/8$. Using the formula for conditional probability, $P(A|B) = \frac{20/56}{5/8} = 16/28 = 4/7$
Part ii:
The total number of permutations possible is, as established in part i, 56. The number of permutations for picking two blue marbles is $P\binom{3}{2} = \frac{3!}{1!} = 6$. Therefore, the probability is 6/56.