Modern Physics Letters B
2050394 (17 pages)

DOI: 10.1142/S0217984920503947

Key node identification of wireless sensor networks based on cascade failure

Xiao Wang* and Jian Du†

School of Mechanical Engineering,
Southwest Petroleum University, Chengdu 610500, China
**educationxiao@163.com*
†d19940522729@163.com

Rongcheng Zou

School of Aeronautics and Astronautics,
Northwestern Polytechnical University,
Xi'an 710000, Shaanxi, China
zourongcheng@nwpu.edu.cn

Zebo Zhou

School of Aeronautics and Astronautics,
University of Electronic Science and Technology of China,
Chengdu 610500, China
klinsmann.zhou@gmail.com

Received 15 April 2020
Revised 27 May 2020
Accepted 28 May 2020
Published 20 August 2020

Wireless sensor networks (WSNs) have become one of the core technologies of the internet of things (IoT) system. They are information generation and acquisition systems used by the IoT to sense and identify the surrounding environment. They are also sensor technology, embedding computing technology, communication technology and important product in the development of Internet technology, which have made the whole society more intelligent and humanized. WSNs are multi-hop self-organizing networks consisting of a large number of micro-sensor nodes deployed in the monitoring area. They can collaboratively sense, collect and process the monitored objects and transmit them to the observers. In this paper, we use the cascade failure method to find the key nodes in the WSNs. First, a complex network cascade failure model based on load redistribution is proposed. Differences from the existing model are as follows: (1) for each node, an overload function is defined; (2) the evolution of the network topology is replaced by node weight evolution. Based on the cascade failure model, a method for evaluating the importance of complex load network nodes is proposed and a new definition of node importance is given. This method helps to discover some potential "critical nodes" in the network. The final experimental analysis verifies the effectiveness and feasibility of the proposed method.

Keywords: Cascade failure; node importance; invulnerability; load; wireless sensor networks.

†Corresponding author.

1. Introduction

Wireless sensor networks (WSNs) consist of small and compact devices which have been distributed over a distant area to monitor various events such as border, temperature and humidity. In Refs. 1–5, these small nodes require resources support such as battery and computation. Reliability evaluation of WSN is critical to ensure the functioning of such networks.[6] Therefore, they should be preserved as long as possible to reach the monitoring goals in Refs. 7 and 8. It is also crucial to design the algorithms that consume less resources from sensor devices.

In Refs. 9 and 10, the scale-free topology in WSNs is widely used in social networks, transportation networks and communication networks. In Ref. 11, weighted scale-free networks refer to networks constructed by nodes and edges based on weights. Dynamics on complex networks are thoroughly investigated, such as spreading and evolutionary dynamics.[12,13] Nodes and edges in different networks carry different forms of load, and the load bearing capacity is limited. In WSNs, the load on the nodes and edges is evolving and has certain dynamic characteristics. When the load of a node or an edge is greater than its own capacity and causes a failure, the failed load is reassigned to the relevant node through the interconnection of the network or on the side, causing other nodes or edges to fail, resulting in a cascading effect, which may cause problems in the entire network in Refs. 14 and 15, such phenomenon caused by a small event is referred to as cascade failure. In the WSNs, the failure of nodes due to energy exhaustion is the main factor of cascade failure. Therefore, it is important to construct a reasonable and integrate WSNs cascade failure model and analyze the influence of parameters on cascade failure, which is essential to improve the robustness of the network in Refs. 16–18.

In recent years, the study of cascade failure is mainly based on complex network theory, in which the scale-free structure is used, from the initial load of the node, the relationship among the node capacity, the initial load, and the load redistribution rule after the node fails. The initial load of a node is usually defined as a power function with respect to degrees, medians, or neighbor degrees. The power exponent is a parameter used to control the initial load strength. The load redistribution mainly considers the capacity, load, degree, and distance from the failed node. The capacity of the node must be greater than the initial load of the node, generally defined as a proportional function with respect to the initial load. In Ref. 19, compared with the importance evaluation in the network, the important evaluation of nodes in WSNs has not received much attention, and the existing evaluation methods are limited. In Ref. 20, the degree of connectivity of a node is taken as a measure of the importance of a node, where the more edges that are connected to a node, the more important the node is. Reference 21 evaluates the importance of nodes based on the shortest path from the source point to the sink point. The most important node is defined as removing the node to maximize the distance from the source point to the shortest path of the sink point. In Ref. 22, a node deletion method based on the number of spanning trees is proposed. The most important node is defined as removing the node to minimize the number of spanning trees. In Ref. 23, the literature proposed a node contraction method, which defines the most important node as shrinking the node to maximize network cohesion. These node importance evaluation methods almost assume that node failures are independent of each other and are static, without considering the load on the network.[24]

In fact, most networks are loaded. These loads can be matter, information, or energy. The load on the network is dynamically changing, especially when the network structure changes, such as the joining and removal of nodes, the load on the network will be redistributed.[25] In general, the ability of a node in a wireless sensor network to withstand a load is limited, that is, the load capacity of the sensor is limited. The limited load capacity and load redistribution make the load network's invulnerability problem more complicated: the failure of one node leads to the redistribution of network load, and the redistribution of the load causes the load on some nodes to exceed its capacity or fail. As a result, the failure of these nodes may lead to cascading failure of other nodes. If we start to remove an important "key node", its removal may trigger a crash of the entire network, which is called cascading break-down.[26] Therefore, it is of great practical significance to evaluate node importance under the condition of "cascade failure". It will help us identify some potential "critical" nodes to better secure the network.

In this paper, a cascade failure model with overload function is proposed. Based on this, the importance evaluation method of complex load network nodes considering cascade failure is proposed and its algorithm steps are given. Finally, the effectiveness of our proposed method is verified by experimental analysis.

2. Wireless Sensor Miscellaneous Network Cascading Failure Model

The wireless sensor network can be represented by the graph $G(V, E)$ (see Fig. 1). Suppose G is an undirected connected graph with n nodes and m edges, where $V = \{v_1, v_2, \ldots, v_n\}$ represents the set of nodes and $E = \{e_1, e_2, \ldots, e_n\}$ represents the collection of edges.

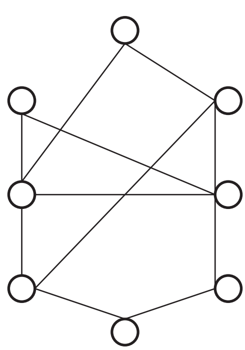

Fig. 1. Topology of WSNs with eight sensors and 11 edges.

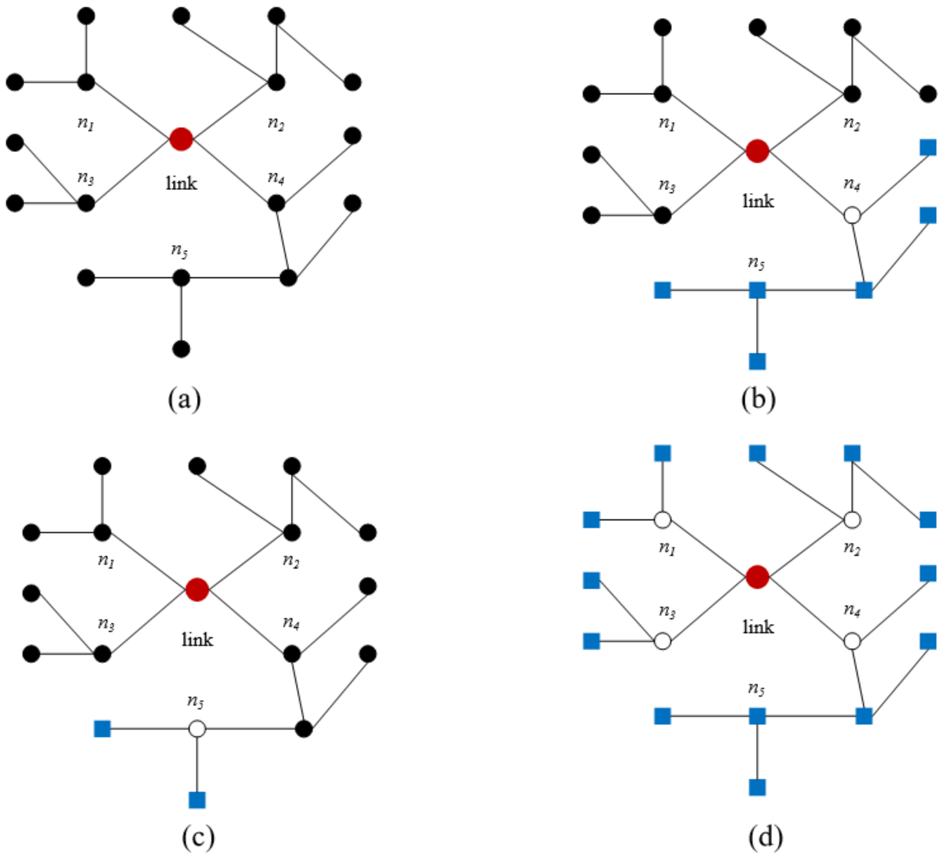

Fig. 2. (Color online) The influence of node degree and distance for network performance.

Figure 2(a) is the initial complete topology and the nodes are shown in black. In Fig. 2(b), when the n_4 node near the sink node fails due to energy exhaustion (indicated in white), node such as n_5 that needs to be transmitted to the sink node through the n_4 node cannot be transmitted (in blue), which will cause network segmentation. In Fig. 2(c), after n_5 node with a large degree of failure fails, the two nodes connected to it cannot be connected to the sink node. In Fig. 2(d), when the n_1–n_4 nodes around the sink node fail, the entire network cannot continue to communicate with the sink node.

2.1. *Distribution of load*

The study of cascade failure is based on the relationship between node load and its capacity.[27] Therefore, the definition of node load plays a crucial role in the change law among subsequent derivation parameters. In the network, the load of the node is affected by many aspects. On one hand, from the global structure analysis, it is affected by the transmission path, the number of the shortest path through the node has a direct relationship with the transmission load of the node; on the other hand, the node average degree and node weight are also inseparable from the transmission load of the node, and the node interface and node degree are inseparable from the degree distribution of the network. The load based on the median analysis ignores the result deviation caused by the difference in the actual transmission path.[28] The load based on the node degree analysis ignores the bridge node with a small degree. Therefore, how to combine the global and local factors reasonably and effectively, and build a comprehensive node initial load model is the key to this paper. In addition, the parameters and function expressions involved in each factor are very different. How to derive the variation law of each parameter and network degree distribution is more difficult for theoretical research.[29]

The load distribution on the network is determined by several factors, among which the network topology is one of the main factors. In order to distinguish from the actual "physical load", we refer to the load determined entirely by the topology as the "structural load".[30] In this paper, we define the structural load of a node as the between ness of the node. In case that it is difficult to determine the actual physical load on the network, it is reasonable and effective to use this dimensionless "structural load" to study the invulnerability and node importance evaluation of complex networks. The central measure of the median as a node was originally proposed in sociological research. This paper uses the simplest and most commonly used shortest path between, that is to say, the more the shortest paths pass through the node, the higher the load on the node:

$$L_k = C_B(k) = \frac{\sum_{w \neq w'} \frac{\sigma_{ww'}(k)}{\sigma_{ww'}}}{n \cdot (n-1)}\,, \tag{1}$$

where $\sigma_{ww'}$ is the number of all shortest-paths between w and w', $\sigma_{ww'}(k)$ represents the shortest path number between node w and w' through node k. In order to avoid the load on the node is zero, let w, w' be k in Eq. (1). Actually, we have

$$\begin{aligned} \sum_{w \neq w' \in V} \frac{\sigma_{ww'}(k)}{\sigma_{ww'}} &= \sum_{w = k \neq w'} \frac{\sigma_{ww'}(k)}{\sigma_{ww'}} + \sum_{w \neq k = w'} \frac{\sigma_{ww'}(k)}{\sigma_{ww'}} + \sum_{w \neq k \neq w'} \frac{\sigma_{ww'}(k)}{\sigma_{ww'}} \\ &= \sum_{w \neq k \neq w'} \frac{\sigma_{ww'}(k)}{\sigma_{ww'}} + 2(n-1)\,. \end{aligned} \tag{2}$$

So, $2/n \leq Lk \leq 1$. In a star network, the shortest path between all pairs of nodes must pass through the central node, so the load of the central node is at a maximum of 1.

Cascading failures occur in the real world. Most of them are caused by environmental impacts or human factors, and load shunting occurs. These distributed loads cause the assigned nodes to fail due to overloading, and the phenomenon of reloading again occurs. This phenomenon continues to cause network paralysis. The load redistribution diagram after cascading failure is shown in Fig. 3. When the node

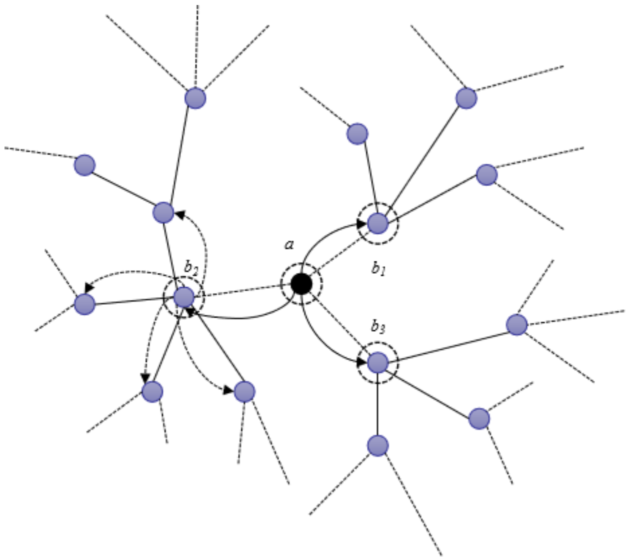

Fig. 3. Load redistribution after cascading failure.

fails, the load of a is redistributed and allocated to its neighbor nodes b_1, b_2 and b_3 in the corresponding allocation ratio. The b_2 node is re-allocated due to overload, and the loop continues until the network no longer appears. The phenomenon of node overload.

2.2. *Redistribution of load*

In previous cascading failure models, nodes usually had only two states, "normal" and "failed", and when the load on the node exceeded its capacity and was in the "failed" state, the node was immediately removed from the network. In fact, the nodes in the network are often in the "intermediate state" between "normal" and "failed". For example, in the traffic network, there are not many cases of real "blocking".[31] Only in a certain period of time, the traffic speed of these sections is slower, and when the load on the node is reduced, the node can return to the normal state. Therefore, it is not appropriate to remove it from the network.

In response to the above problems, we define an "overload function" F_k for each node in the network, which is equivalent to assigning each node a dynamic weight, which represents the "difficulty" of the load through the node, defining the edge. The weight is the sum of weights of each nodes which are connected to other nodes. We define F_k as the monotonic non-decreasing function of the load L_k on the node, thus the load L_k on the node and its overload function value F_k are mutually constrained in the load redistribution process: when the overload function value of the node changes, the load on the network will be reallocated, thus the load redistribution will change the node's overload function value.

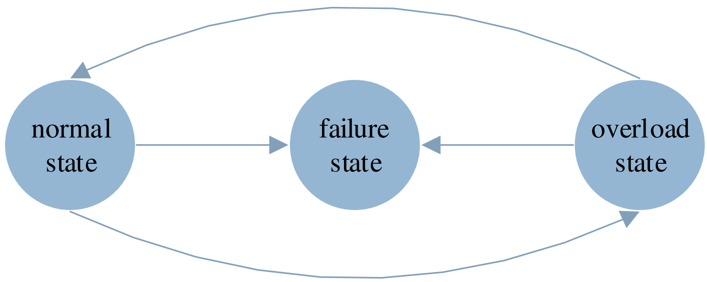

Fig. 4. Three states of sensor.

According to the actual situation, the "overload function" can take many forms. In the following, we present an iterative form of the "overload function" expression:

$$F_k = \begin{cases} 1 & L_k(t) \le L_k(0) \\ 1 + \dfrac{L_k(t-1) - L_k(0)}{C_k - L_k(0)} & L_k(0) < L_k(t) < C_k \\ n & L_k(t) \ge C_k \end{cases}, \tag{3}$$

where L_k is the initial load and C_k is the load capacity. The three cases on the right side of the medium number correspond to the three states of the node: normal state, overload state and failure state. We assume that when the node is in a failed state, its state no longer changes. The transition between the three states is shown in Fig. 4.

The load capacity of nodes in the network is cost-constrained. When determining the load capacity of these nodes, they are "on-demand capacity", so the load capacity of the node is generally considered to be proportional to its initial load $L_k(0)$.

$$C_k = L_k(0) \cdot (1 + a)\,, \tag{4}$$

where a is the "tolerance factor" and can be given according to the actual situation of the network.

Capacity and load in the actual network exhibit a non-linear behavior. Therefore, we present a new load capacity model that is close to the real system. The model uses two parameters for more flexibility, as follows:

$$C_k = L_k(0) + \beta L_k^{\alpha}(0)\,, \tag{5}$$

where $\alpha \ge 0$ and $\beta \ge 0$. When $\alpha = 1$, the model degenerates into an ML model.

2.3. *Cascade failure process*

In the previous cascading failure model, "initial damage" was processed to delete one or more nodes. In this paper, we only need to change the value of the "overload function" to be attached to n. This paper defines the end of the cascading failure process as follows: no new nodes will be converted to a failure state. Because the node cannot be converted to normal and overload state after the failure, the cascading failure process will be repeated at most one step. In this paper, we measure the consequences of cascade failure using the average weighted efficiency of the network after cascade failure, i.e.

$$E = \frac{1}{n(n-1)} \sum_{i \neq j \in V} \frac{1}{d_{ij}}\,, \tag{6}$$

where d_{ij} is the weighted shortest distance between node u_i and node u_j. This not only reflects the number of nodes that the "initial attack" caused the cascade to fail, but also indicates the location of the failed node in the network.

Crucitti[32] gave the conditions for cascading failures. They found that the failure of some nodes with high load in the network is enough to affect the efficiency of the network and cause the whole system to collapse. Therefore, we consider removing the node with the highest load in the initial network, which will cause the traffic in the network to redistribute, which in turn causes the load of each node to change. Since the capacity of a node is limited, it is possible to invalidate the load of some nodes which is beyond their capacity. To simplify the discussion, we remove those failed nodes from the network, triggering a new round of traffic distribution until the load on all nodes does not exceed their capacity.

In previous work, the degree of network damage caused by cascading failures was expressed by

$$g = \frac{N'}{N}\,, \tag{7}$$

where N and N' represent the size of the largest connected component in the network before and after the cascade failure, respectively. In this paper, g is used to describe the robustness of the network. When $g \approx 1$, the network is connected almost everywhere, while $g \approx 0$ means the network is completely collapsed.

We examine the relationship between the parameters α, β and g on a scale-free network with nodes $N = 4500$ and averaging degree $\langle k \rangle \approx 5$, as shown in Fig. 5. g is a function of α and β, taking $\alpha = 0.70$, 0.75, 0.80, 0.85, 0.90, 0.95, 1 (from right to left) to simulate the process of network cascade failure on the interval of $\beta \in [0, 20]$ to obtain the extent of network damage. It can be seen from the figure that when the network connectivity represented by α and g suddenly appears on a small β interval, that is, g is shifted from a smaller value to a larger value. For example, when $\alpha = 0.70$, there are $\beta = 12.5$, $g = 0.0188$; $\beta = 13.0$ and $g = 0.9672$. For example, when $\alpha = 0.95$, there are $\beta = 0$, $g = 0.0062$, $\beta = 0.5$ and $g = 0.8421$. This shows that we can effectively resist the cascading failure and obtain better robust performance by increasing the relatively small capacity. However, after this critical phenomenon occurs, even if the β value is greatly increased, g does not increase significantly but gradually approaches 1. We can see that $\beta = 3.5$, $g = 0.9446$ when

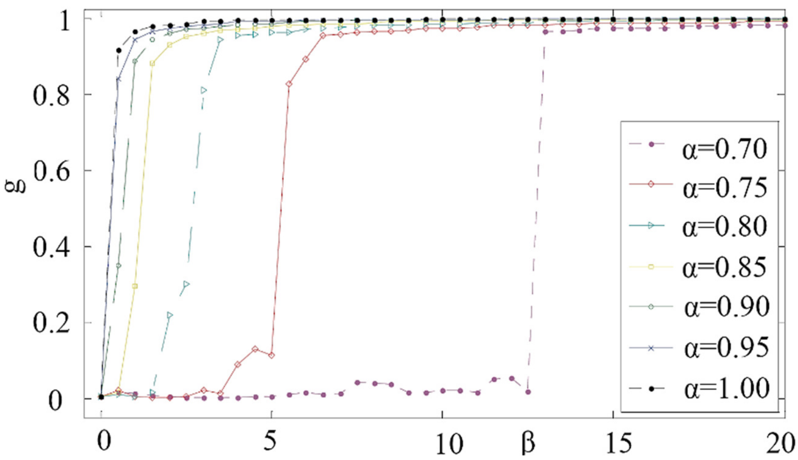

Fig. 5. (Color online) Cascading failure on B-A scale-free WSN. Network nodes $N = 4500$, average degree $\langle k \rangle \approx 5$.

$\alpha = 0.80$; and the value of g reaches 0.9768 when β is doubled. At this time, it indicates that the failure of a node with a large load will not cause a wide range of failures, and the network maintains good connectivity. Therefore, we can select specific model parameters according to the requirements for system robustness.

In this paper, even if the node fails and is not removed from the network, the network topology remains unchanged during the cascading failure process. Since the "cost" through the failed node is too large ($F_k = n$), the shortest path that passed through it will automatically avoid it (equivalent to the node deleted). If the load still passes through the node after it fails (the node is a must-have between other nodes), at this time, due to the high value of the node overload function, the network performance degradation caused by the node failure will influence the network average degree. The weighting efficiency is clearly reflected. This method of replacing the topology evolution with node weight evolution makes the cascading failure model operation simpler.

In Fig. 6, after the cascading failure occurs on the B-A WSN model, the relationship between α and β when the network robustness $g > 0.95$ with $N = 1000$, 3000, 5000, 7000 and average degree $\langle k \rangle \approx 4$ is considered. The differently identified points in the figure represent the results of actual network simulations in the case of different nodes, and the curves of different styles and colors are the fits to these points. It can be seen from the figure that the curve is basically consistent with the point, so under the given conditions, we can get the conclusion that α and β have a negative exponential relationship $\beta = c_1 e^{-c_2 \alpha}$, $\alpha \in [0, 1]$, c_1 and c_2 are the constants. Further work will focus on whether the theoretical relationship between α and β can be obtained theoretically.

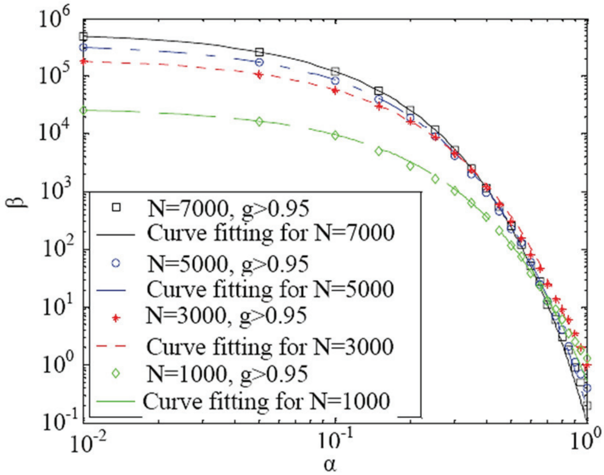

Fig. 6. (Color online) Relationship between the parameters α and β of the model in B-A scale-free WSN.

3. Wireless Sensor Load Network Node Importance Evaluation Method Considering Cascading Failure

In the scale-free structure in which the previous construction degree distribution satisfies the power rate characteristic, most of the node-based selection connections are used, but in many cases, the node degree cannot be used as the sole criterion for measuring energy consumption or load, and in the case that the degree of existence is small but the amount of transmitted data is large, some nodes in special positions, such as bridge nodes, often bear a large number of data transmission tasks. Once these nodes fail, it is easy to break down the network, it produces huge loss to the network transmission. Therefore, the bridge nodes play decisive roles in the network as shown in Fig. 7.

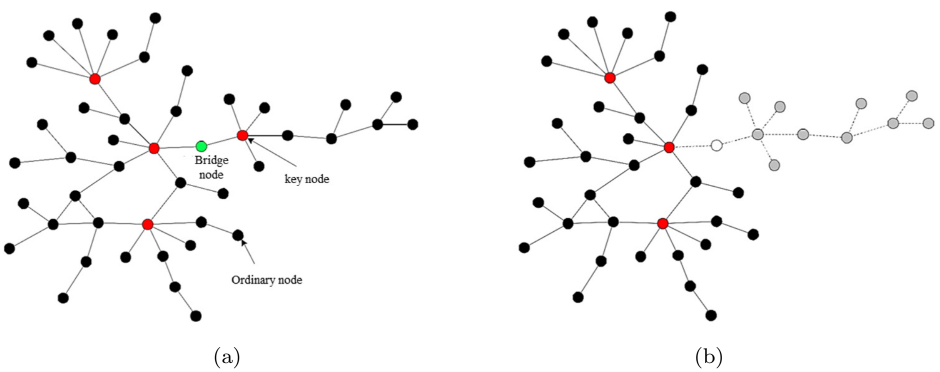

Fig. 7. (Color online) Failure of bridge node. (a) The initial topology and (b) After the green failure.

In Fig. 7, red nodes are important nodes in the network, with a large degree; the green node is a bridge node, which is a bridge for communication between different groups; black nodes are common nodes. Red node failure is bound to trigger the embarrassment of the entire network, so the role of the node with a large degree in the network is self-evident. However, the role of the bridge node cannot be ignored. In Fig. 7(b), when the green node fails, its neighbor nodes and the entire packet are segmented with the network, causing large-area failures, and the consequences are even greater than a certain degree. Therefore, the importance of node energy and bridge nodes should be considered in the process of building the network.

Considering the cascading failure, assuming a node fails, the node "overload function" value is n. If the node is an important "critical node", the failure of the node triggers the cascade failure of the network, thereby this leads to the decline in network performance, in other words, the average weighting efficiency becomes smaller. Therefore, it can be considered that the smaller the average weighting efficiency of the network after cascading is failure, the more important the node is.

Definition 1. In the network $G = (V, E)$, let u_k be invalid, and $F_k = n$, then $I_k = 1 - E_k/E_0$ is the importance of the node u_k, E_0 is the initial average weighted efficiency of the network, and E_k is the node u_k failure. The average weighted efficiency of the network after the triggered cascading failure process ends.

Obviously $< I_k$<1.

The algorithm steps for evaluating the importance of all nodes as shown in Algorithm 1.

Algorithm 1. Algorithm of the Importance of All Sensors.

Input: G, a, β
Output: I

1. Calculate all initial load L_k and initial load energy C_k;
2. Calculate the initial average weighted efficiency E in Eq. (5);
3. *for* $k = 1$ *to* n
   {*for* $i = 1$ *to* n *do* $F_i(0) = 1$;
      $F_k(1) = n$;
      $T = 1$;
      while
      $t = t + 1$
      for $i = 1$ *to* n
      {*if* $(F_i(t-1) = n)$ *then* $F_i(t) = n$;
      Else calculate $F_i(t)$ *in Eq.* (3)
      }
        Calculating the average weighting efficiency A *according to formula* (5);
        Calculate $Fi(t)$ *according to Definition* 1;
      }

Assume that there are N nodes in the wireless sensor network and perform time complexity analysis in the worst case. Firstly, the time complexity of the mediator is analyzed. Each node needs to find the shortest path between other nodes. The time complexity of Dijkstra algorithm is $O(N^2)$, and the time complexity of node degree and node weight is $O(1)$. The node load is composed of the median, node degree and node weight. The calculated time complexity is $O(N^2)$, the node capacity is proportional to the node load, and the time complexity is also $O(N^2)$. Therefore, the node performs load redistribution. The time complexity required is $O(N^2)$.

4. Experimental Analysis

As shown in Fig. 8, a load network contains 20 nodes and 38 edges. The size of the nodes in the graph is proportional to its initial load. Choose $a = 0.7$, $\beta = 13.0$ and evaluate the importance of each node according to the algorithm steps in Sec. 3. As a comparison, the importance of each node is evaluated and ordered using the node contraction method without considering the cascading failure. The evaluation results are shown in Table 1.

It can be seen from Table 1 that considering the cascade failure has a significant impact on the evaluation of the importance of complex network nodes. Nodes 13, 20, 2, and 11 are important "key nodes" without considering cascading failure conditions. The importance of node 12 is not prominent, but under the condition of cascading failure, due to nodes 12, 2, 19 and 3 failures trigger the cascading failure

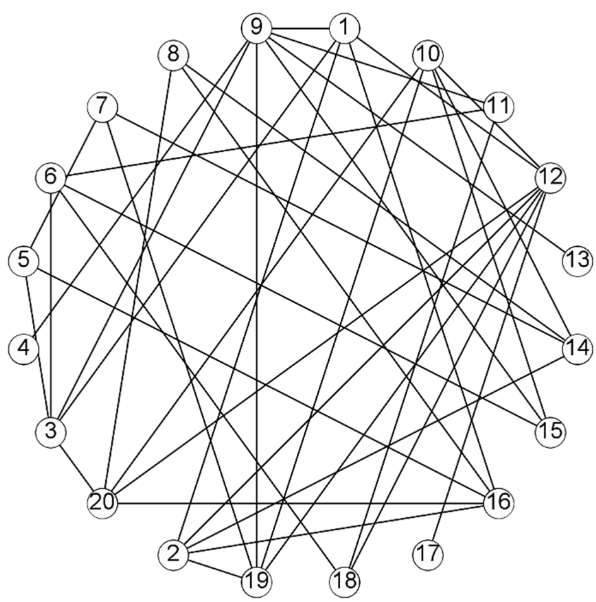

Fig. 8. Illustration of a network ($n = 20$, $m = 38$) edges.

Table 1. Results of evaluation of node importance

	Our way		Traditional division method	
Sensor	Importance	Ranking	Importance	Ranking
1	0.943	4	0.611905	6
2	0.9593	2	0.741911	3
3	0.9421	5	0.494904	7
4	0.4235	20	0.098892	20
5	0.8322	9	0.630295	5
6	0.7023	10	0.28932	16
7	0.7001	11	0.253246	18
8	0.6925	12	0.451484	8
9	0.9021	6	0.354732	13
10	0.8736	7	0.435355	9
11	0.5328	17	0.676174	4
12	0.9601	1	0.205916	19
13	0.5233	18	0.9624	1
14	0.6659	13	0.259193	17
15	0.6654	14	0.384195	11
16	0.6338	15	0.304021	15
17	0.9594	19	0.363046	12
18	0.5421	16	0.385873	10
19	0.9586	3	0.321078	14
20	0.8449	8	0.79888	2

of other nodes, the performance of the entire network is greatly reduced, so the importance of these nodes is greatly improved. In particular, node 12, we can see from Fig. 3 that the node connectivity and initial load are very low, but after node 12 fails, it first causes node 19 to exceed the load capacity failure, and then leads to the cascade failure of other nodes. The average weighted efficiency of the network after the cascading failure process is reduced by 0.0187 from the initial 0.4393. This means that node 12 is a "potential key node."

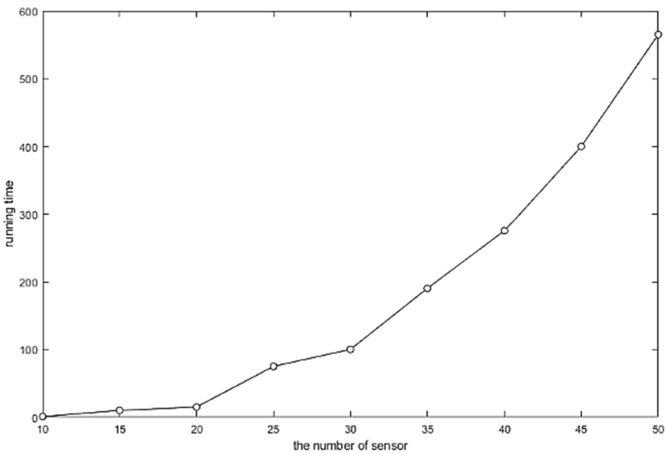

Fig. 9. Run time of evaluation for different size of networks.

The importance of the network with different number of nodes is evaluated. Its running time is shown in Fig. 9. Seen from Fig. 9, although it is inevitable that cascading failure will increase the running time, the runtime is still acceptable. If α is large enough, the failure of any node in the network will not trigger the cascade failure. At this time, the node importance evaluation result will be consistent with the evaluation result under the condition of not considering the cascade failure. Therefore, the evaluation method of this paper is compatible with the evaluation method that does not consider the cascade failure, and it is not contradictory.

5. Validity Test

In order to evaluate the effectiveness of the method, *SIR* is used to simulate the propagation process of node importance, and the scope of propagation is verified. The *SIR* model is the most classic model in the infectious disease model. The model divides the population within the spread of infectious diseases into three categories: S means susceptible, refers to those who are not sick, but lacks immunity, and is vulnerable to contact with susceptible patients. I means an infected person, a person infected with an infectious disease, which can be transmitted to the member of the S class; R means a person who is cured and who is immune to the disease and will not be infected again. Within each time step, only one node is in the S state, as a source of infection, and is transmitted to the susceptible person in the neighborhood with probability v ($v = 0.3$ in the text), and the infected person is healed with probability m ($m = 1$).

Suppose that at time t, the number of susceptible, infected, and recovered people in the network are $s(t)$, $i(t)$ and $r(t)$, respectively. The *SIR* basic model is expressed by the following differential equations:

$$\begin{cases} \dfrac{di}{dt} = \varepsilon si - \mu i \\ \dfrac{ds}{dt} = \varepsilon si \\ \dfrac{dr}{dt} = \mu i \end{cases} . \tag{8}$$

Under the above three basic assumptions, the block diagram of the process from illness to removal of susceptible individuals is shown in Fig. 10.

At time t, the sum of infected and removed persons in the network is $F(t)$. As time elapsed, the value of $F(t)$ increases, but after multiple rounds of operation, $F(t)$ will eventually reach one steady state. For different initial sources of infection, the

$$\boxed{S} \xrightarrow{\varepsilon si} \boxed{I} \xrightarrow{\mu i} \boxed{R}$$

Fig. 10. SIR transfer model.

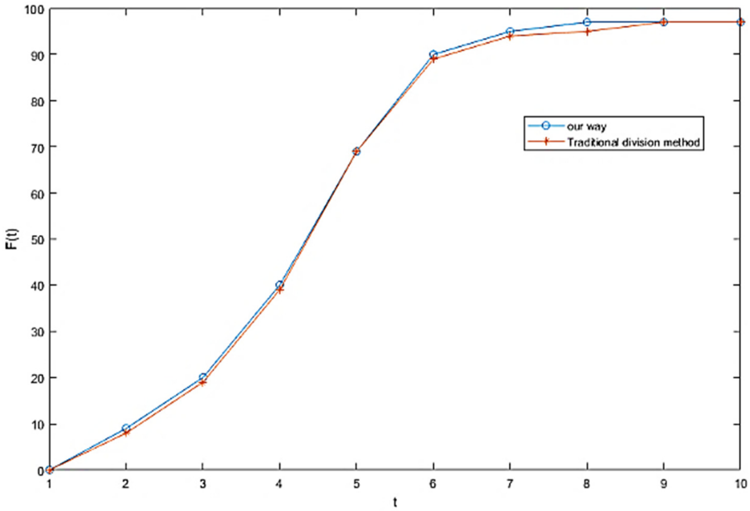

Fig. 11. (Color online) The SIR evaluation.

greater the $F(t)$ reaches a stable value, the more important the node is. Therefore, the $F(t)$ value can be used to evaluate the validity of key node decisions.

In Fig. 11, these two discounts correspond to our method and traditional method, respectively. The blue line is our method and the red line is the traditional method. Obviously, it can be seen that at the same time t (the number of running rounds), the value of $F(t)$ of the key nodes determined by our method is larger than that of the traditional method. In other words, our method is superior to the traditional method.

6. Conclusion

The cascading failure phenomenon in complex load networks cannot be ignored. Its essence is a related failure, and the related failure behavior in network security has always been a very difficult problem. This stems from our little knowledge of the relevant failure mechanisms in the network, especially the quantitative analysis methods. Evaluating the importance of nodes under the condition of cascading failures allows us to take an inside look of some neglected key nodes, which is of great significance for us to investigate the reliability and invulnerability of complex networks. This paper proposes a complex load network cascade failure model. The differences between our model and other models are shown as follows:

(1) An "overload function" is defined for each node. According to the value of the overload function, each node has three states, "normal", "overload" and "failure". When the load on the node is reduced, it is in an overload state. The nodes can also be restored to their normal state, which is more realistic, thus the bridge node will not be missed.

(2) Using node weight evolution replace topology evolution, node failure does not need to delete nodes and edges, but adjust its overload function value, thus the shortest path will automatically avoid the network problem and node failure performance degradation is reflected by the network's average weighted efficiency, making model operation easier.

Based on this model, the importance evaluation method and algorithm steps of complex load network nodes considering cascade failure are proposed. It is considered that the lower the average weighting efficiency of the network after the failure of the cascade failure triggered by the node is failure, the more important the node will be. The final experiment analysis shows that considering the cascade failure has an effective impact on the evaluation of the importance of complex network nodes, allowing us to find some "potential key nodes".

References

1. M. Obaidat and S. Misra, *Principles of Wireless Sensor Networks* (Cambridge University Press, 2014).
2. D.-G. Zhang, W.-B. Li, S. Liu and X.-D. Zhang, *J. Vibroeng.* **18**(1) (2016) 522.
3. D. Zhang *et al.*, *J. Electr. Eng. Technol.* **10**(6) (2015) 2384.
4. D.-G. Zhang, S. Zhou and Y. M. Tang, *Mobile Networks Appl.* **23**(34) (2017) 1.
5. P. Zhu, X. Song, L. Liu, Z. Wang and J. Han, *IEEE Access*, **6** (2018) 35292.
6. P. Zhu, J. Han, Y. Guo and F. Lombardi, *IEEE Network* **30**(6) (2016) 70.
7. Z. Zhou *et al.*, *IEEE Trans. Aerospace Electron. Syst.* **49**(4) (2013) 2146.
8. D.-G. Zhang *et al.*, *Comput. Math. Appl.* **64**(5) (2012) 1044.
9. R. R. Yin *et al.*, *Physica A* **409**(3) (2014) 8.
10. H. R. Liu *et al.*, *Chin. Phys. B.* **24**(5) (2015) 050506.
11. I. Milchtaich, *Int. J. Game Theory* **44**(3) (2015) 515.
12. P. Zhu, X. Wang, S. Li, Y. Guo and Z. Wang, *Appl. Math. Comput.* **359** (2019) 512.
13. P. Zhu, X. Wang, D. Jia, Y. Guo, S. Li and C. Chu, *Appl. Math. Comput.* **386** (2020) 125474.
14. M. T. Thai and P. M. Pardalos, *Handbook of Optimization in Complex Networks* (Springer, 2012).
15. S. V. Buldyrev *et al.*, *Nature* **464**(7291) (2010) 1025.
16. D. O. Leonardo and M. V. Srivishnu, *Struct. Safety* **31**(6) (2009) 157.
17. V. Cupac *et al.*, *Electr. Power Energy Syst.* **49**(49) (2013) 369.
18. Z. Li *et al.*, *Chaos* **29**(7) (2019) 073111.
19. R. Albert, H. Jeong and A.-L. Barabási, *Nature* **406**(6794) (2000) 378.
20. R. R. Albert and A. L. Barabasi, *Rev. Mod. Phys.* **74**(1) (2002) 46.
21. M. O. Ball, B. L. Golden and R. V. Vohra, *Oper. Res. Lett.* **8**(2) (1989) 73.
22. K. Malik, A. K. Mittal and S. K. Gupta, **9**(4) (1990) 223.
23. H. W. Corley and D. Y. Sha, *Oper. Res. Lett.* **4**(4) (1982) 157.
24. Y. Chen *et al.*, *High Technol. Lett.* **14**(1) (2004) 21.
25. P. Zhu *et al.*, *IEEE Trans. Circuits Syst. II. Exp. Briefs* **66**(1) (2019) 131.
26. J. Wu and Y.-J. Tan, Finding the most vital node by node contraction in communication networks, *Int. Conf. Communications, Circuits and Systems*, 27–30 May 2005, Hong Kong, China (IEEE, 2005).
27. A. E. Motter and Y. C. Lai, *Phys. Rev. E.* **66** (2002) 065102.

28. L. Zhao, K. Park and Y. C. Lai, *Phys. Rev. E-Stat. Nonlin. Soft Matter Phys.* **70**(3) (2004) 35.
29. R. Pastor-Satorras, A. Vázquez and A. Vespignani, *Phys. Rev. Lett.* **87**(25) (2001) 258.
30. R. Guimerà, A. Arenas, A. Díaz-Guilera and F. Giralt, *Phys. Rev. E-Stat. Nonlin. Soft Matter Phys.* **66**(2) (2002) 026704.
31. D. J. Watts, *Proc. Natl. Acad. Sci. USA* **9**(9) (2002) 5766.
32. P. Crucitti *et al.*, *Phys. Rev. E* **69**(4) (2004) 045104.