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A generalized gravity model for influential spreaders identification in complex networks

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ABSTRACT

How to identify influential spreaders in complex networks is still an open issue in network science. Many approaches from different perspectives have been proposed to identify vital nodes in complex networks. In these models, gravity model is an effective model to find vital nodes based on local information and path information. However, gravity model just uses degree of the node to judge local information, which is not precise. To address this issue, a generalized gravity model is proposed in this paper. Generalized gravity model measures local information from both local clustering coefficient and degree of each node, which is more comprehensive. Also, parameter α can be modified in different applications to get better performance. Generalized gravity model can degenerate into gravity model when $\alpha = 0$. Promising results from experiments on four real-world networks demonstrate the effectiveness of the proposed method.

1. Introduction

Massive complex systems in the world, including but not limiting to biology systems [1,2], social systems [3–5], and multi-agent systems [6,7] can be well described and presented in the form of complex networks. Network science is a subject for studying properties of complex networks. There are many hot topics in network science, like node similarity analysis [8], network division [9,10], self-similarity analysis [11,12], vulnerability analysis [13], resilience analysis [14,15], message passing [16–18], network evolution [19–22], uncertainty modeling [23–25], network cascading [26] and so on. Complex network theory is effective to present the components of a complex system and components' relationship. Properties of the complex network can be analyzed to get useful information. Methods and mathematical models in network science are widely used in massive applications under the real world, including but not limiting to time series analysis [27–29], decision making [30,31], image processing [32], social network analysis [33], traffic flow analysis [34]. We have witnessed the rapid development of network science in recent years [35–40].

Recently, influential spreaders identification problem is widely paid attention by many researchers [41,42]. Influential spreaders identification can be widely used in many fields and applications, like disease analysis [43,44], rumor analysis [45], knowledge graph [46], social computing [47,48] and so on. Nevertheless, how to find vital nodes in complex networks is still an open issue. There are many methods for finding influential spreaders in complex networks, like degree centrality (DC) [49], betweenness centrality (BC) [50], closeness centrality (CC) [51], PageRank [52,53], LeaderRank [54], eigenvector centrality (EC) [55], H-index [56] and so on. Mainstream of methods can fall into many categories. Some methods, like DC, local centrality (LC) [57] just use local information to judge which nodes are vital. Among these methods, local information dimensionality is presented as an effective model to rank influential nodes in complex networks with entropy function [58–60]. Local information dimensionality is also a special form of multi-local dimension [61]. These methods are easy to compute but they are not precise. Some methods use global information, like closeness centrality [51], betweenness centrality [50], Katz centrality [62] and so on. They are precise but they are time consuming, which will hinder their applications in some occasions. There are also many other methods to find vital nodes in a comprehensive perspective, considering both global and local information, like gravity formula based models [63,64], entropy models [65], evolutionary computation based methods [66] and other hybrid methods [67–71].

Gravity-formula is natural and has potential to represent the interaction between nodes or agents in the network. If a node or agent has more interaction with others, this node should be more important. Hence, there are many studies on how to use gravity-

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formula to identify influential spreaders. Gravity model is proposed by Ma et al. [72] as it uses local and path information to identify influential nodes. However, gravity model is time consuming in some large networks, which will limit its application. Li et al. solve this problem by introducing a truncation radius to gravity model [63], which greatly reduces the complexity of calculation. Liu et al. propose weighted gravity model [64], which considers self importance of the node compared to gravity model by introducing the eigenvector centrality of the node to gravity model. However, the eigenvector centrality may conflict with local and path information, which may influence the result of finding vital nodes.

The main contribution of this paper is to propose a generalized gravity model. The highlight of this paper is that the mass of each node is not its degree, but substituted by spreading ability of the node. The spreading ability of a node is decided by not only the degree of the node, but also the local clustering coefficient of the node. Assuming a node with the same degrees, if the local clustering coefficient of the node is higher, which corresponding to more edges among its neighbors, it is easier for information to propagate in its neighbors, not outer nodes, and this node is less influential. Hence, the local information is measured more precisely by spreading ability of the node compared with degrees of the node.

The remaining part of this paper is organized as follows. Preliminaries, including several centrality measures, Susceptible-Infected (SI) model and Kendall's tau coefficient will be briefly introduced in Section 2. In Section 3, generalized gravity model is proposed and presented. In Section 4, encouraging results of experiments on four real-world networks will be used to verify the effectiveness of the proposed method. Finally, conclusions will be given in Section 5.

2. Preliminaries

Preliminaries include centrality measures in complex networks, SI model and the Kendall's tau coefficient will be introduced in this section.

2.1. Centrality measures in complex networks

A network can be denoted as $G = (V, E)$, V and E are the set of nodes and edges respectively. Several centrality measures, degree centrality (DC) [49], betweenness centrality [50], closeness centrality [51], gravity model [63] and weighted gravity model [64] are presented as follows.

Definition 2.1. The DC of node v is defined as follows [49].

$$C_D(v) = k(v) \tag{1}$$

where $k(v)$ denotes the degree of node x.

Definition 2.2. The BC of node v is defined as follows [50].

$$C_B(v) = \frac{\sum\limits_{j \neq k, i \neq j, k} g_{j,k}(i)}{\sum\limits_{j \neq k, i \neq j, k} g_{k,j}} \tag{2}$$

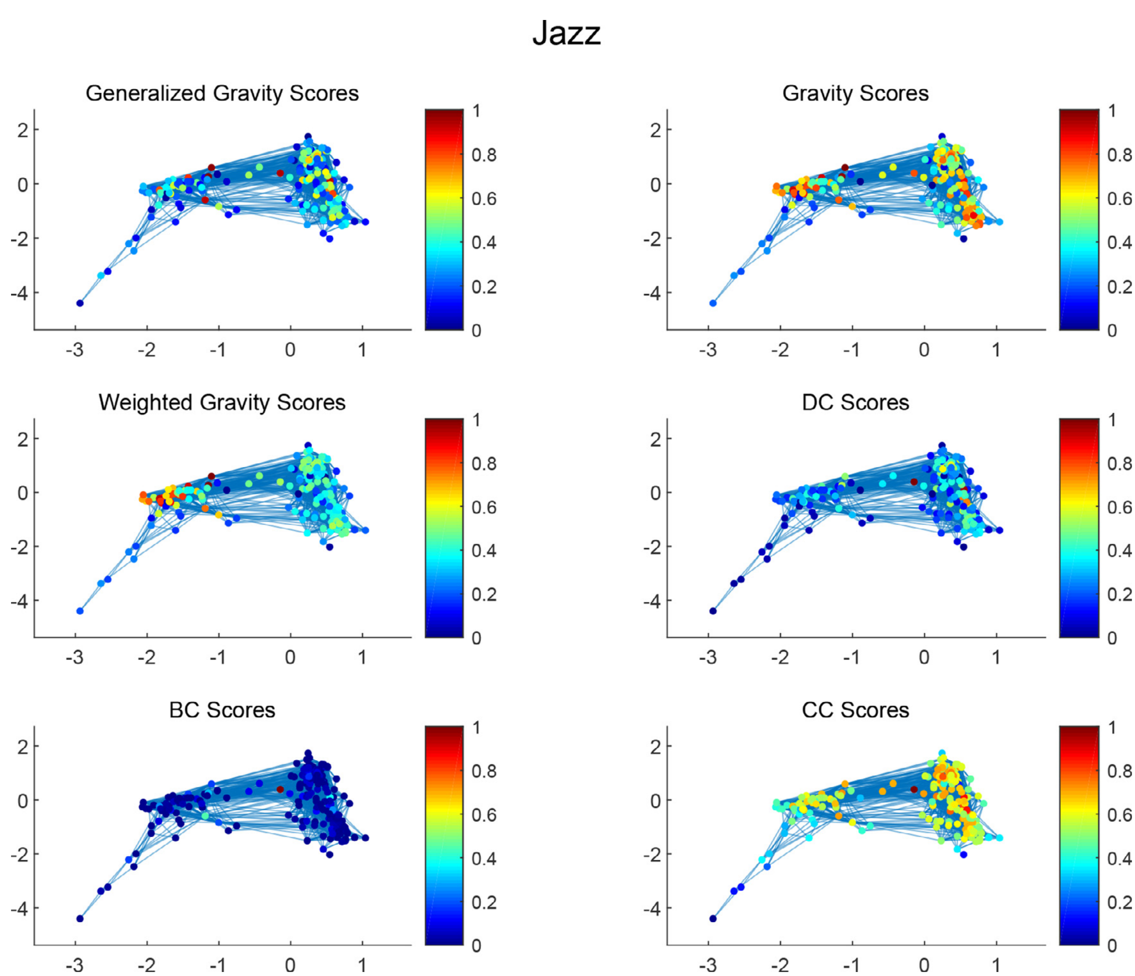

Fig. 1. This figure shows the centrality score of nodes in Jazz network.

where $g_{j,k}$ and $g_{j,k}(i)$ denotes the number of shortest paths between node j and k and the number of shortest paths between node j and k that go through node i.

Definition 2.3. The CC of node i is defined as follows [51].

$$C_C(v) = \frac{1}{\sum_{w}^{N} d_{vw}} \tag{3}$$

where N denotes the number of nodes in the network and d_{vw} denotes the topological distance between node v and node w.

Definition 2.4. The gravity model is used for influential spreaders identification. The gravity centrality (GC) is presented as follows [63].

$$C_G(i) = \sum_{j}^{N} \frac{k(i) \times k(j)}{d_{ij}^2} \tag{4}$$

However, gravity model is time-consuming. Li et al. solve this problem by introducing a truncation radius to gravity model [63].

$$C_G(i) = \sum_{d_{ij} \le R \& j} \frac{k(i) \times k(j)}{d_{ij}^2} \tag{5}$$

where the truncation radius $R = \frac{\langle d \rangle}{2}$, d is the average length of the shortest paths of the network. In this paper, gravity centrality is calculated by Eq. (5).

Table 1
Descriptions of parameters of SI model.

Parameter	Description of parameter
T	The simulation time of SI model
F(t)	The average number of infected nodes at time t
β	Probability of susceptible node infected by infected node
N	Number of independent experiments

Definition 2.5. The weighted gravity model is proposed by Liu et al. [64]. The weighted gravity centrality (WGC) can be calculated as following equations.

$$\begin{aligned} AX &= \lambda X \\ e_i &= X_i \\ C_{WG}(i) &= \sum_{d_{ij} \le R \& j} e_i \times \frac{k(i) \times k(j)}{d_{ij}^2} \end{aligned} \tag{6}$$

where λ and X the largest eigenvalue and normalized eigenvector respectively. e_i is the value of node i in X.

2.2. SI model

SI model is used to evaluate the performance of top rank-nodes selected by different centrality measures [73]. Every node has two states: susceptible or infected. Infected node keeps infected and infect its neighbors that are susceptible. Table 1 describes the parameters of the SI model.

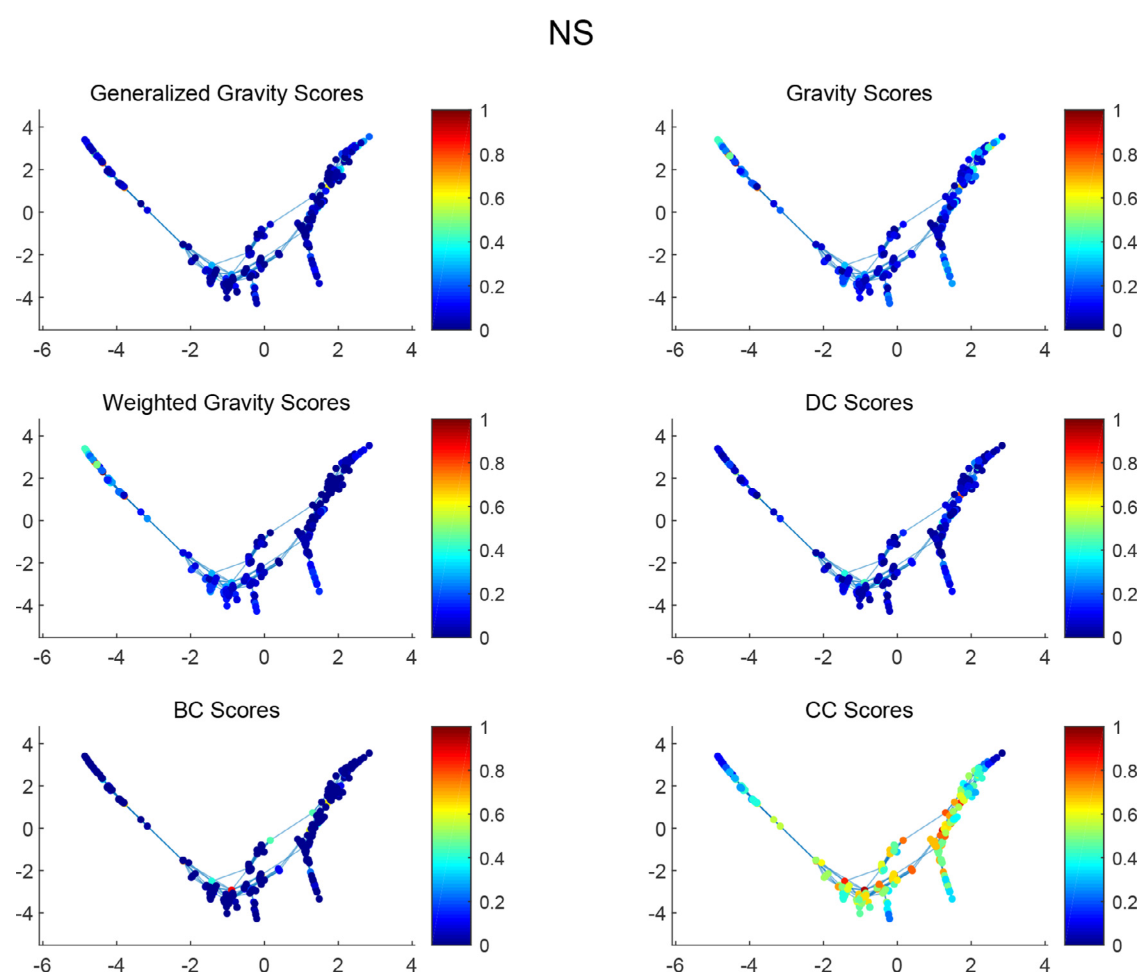

Fig. 2. This figure shows the centrality score of nodes in NS network.

2.3. The Kendall's tau coefficient

Kendall's tau coefficient is used to measure the correlation between centrality measures and infection ability [74].

Given two sets of sequences x and y. x_i and y_i are the ith value of sequence x and y respectively. Given a sequence pair (x_i, y_i), If $x_i > x_j$ and $y_i > y_j$ then (x_i, y_j) is recorded as a positive sequence pair otherwise negative sequence pair.

$$\tau = \frac{n^+ - n^-}{n(n-1)} \tag{7}$$

where n^+ and n^- denotes the number of positive sequence pairs and negative pairs respectively. If τ value is larger, the centrality measure is more accurate compared with the real spreading process. Centrality measure has the best performance when $\tau = 1$.

3. Proposed method

Gravity model uses degree of nodes as masses, which represents the local information of nodes, which is rough.

The local clustering coefficient of node v is defined as

$$C_v = \frac{2n_v}{k(v)(k(v)-1)} \tag{8}$$

where $k(v)$ denotes the degree of node v. n_v denotes the number of edges between neighbors of node v.

The spreading ability of node v is defined as

$$Sp(v) = e^{-\alpha C_v} \times k(v) \tag{9}$$

where $\alpha \geq 0$ and $k(v)$ is the degree of node v. α is a free parameter that can be modified flexibly in real applications.

The generalized gravity centrality (GGC) is defined as follows.

$$C_{GG}(i) = \sum_{d_{ij} \leq R \& j} \frac{Sp(i) \times Sp(j)}{d_{ij}^2} \tag{10}$$

when $\alpha = 0$, generalized gravity model will degenerate into gravity model. In this paper, we take $\alpha = 2$ for calculation.

In previous studies on gravity model, the local information is judged only by the degree of the node, which is not reasonable. Some studies show that the local clustering coefficient will also affect the spreading ability of the node [75]. In generalized gravity model, the spreading ability of the node is not only decided by the degree, but also the local clustering coefficient of the node, which is more reasonable. The mass of gravity model is substituted with the spreading ability of the node, which better considers the local information of the node.

4. Case studies

In this section, proposed method is compared with other centrality measures by three experiments on four real-world networks. Information of datasets is shown in Table 2. Jazz is a network

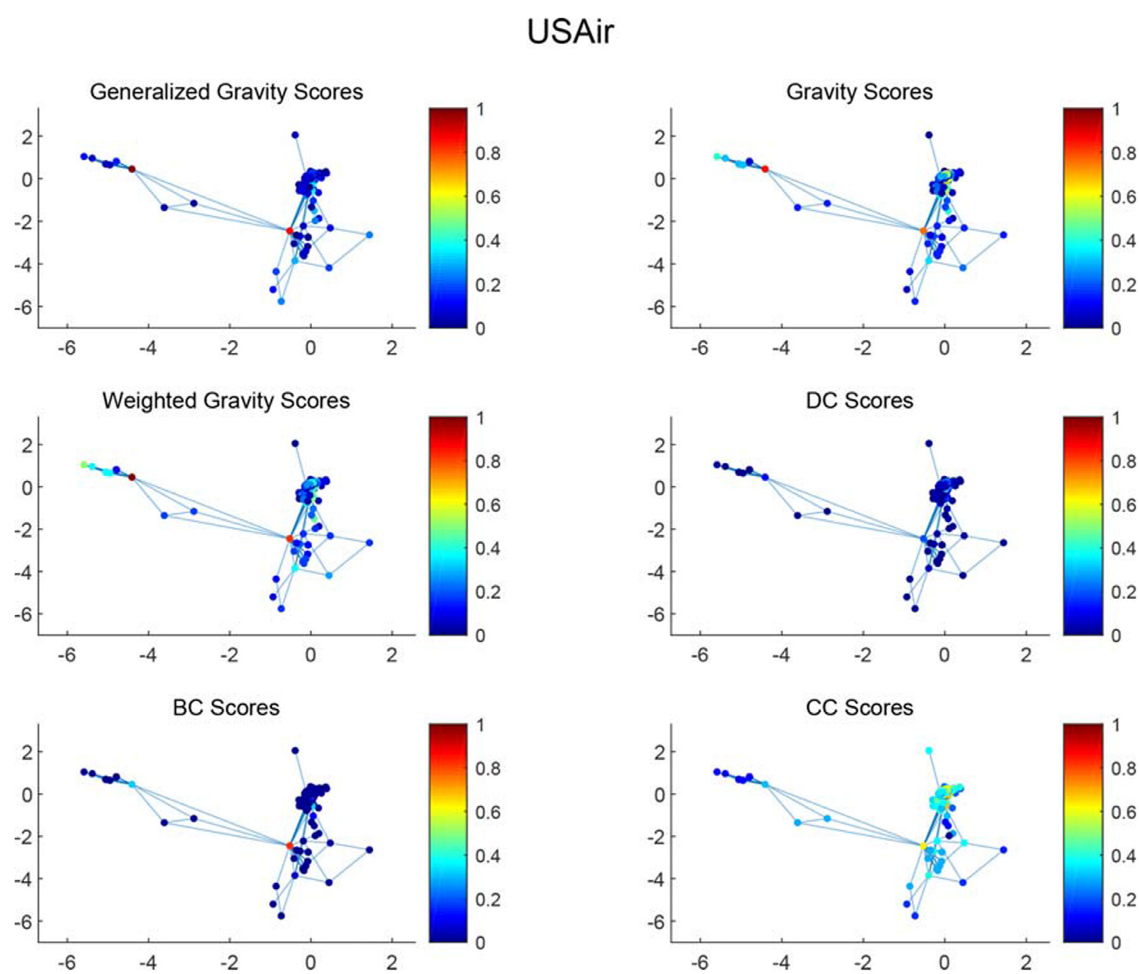

Fig. 3. This figure shows the centrality score of nodes in USAir network.

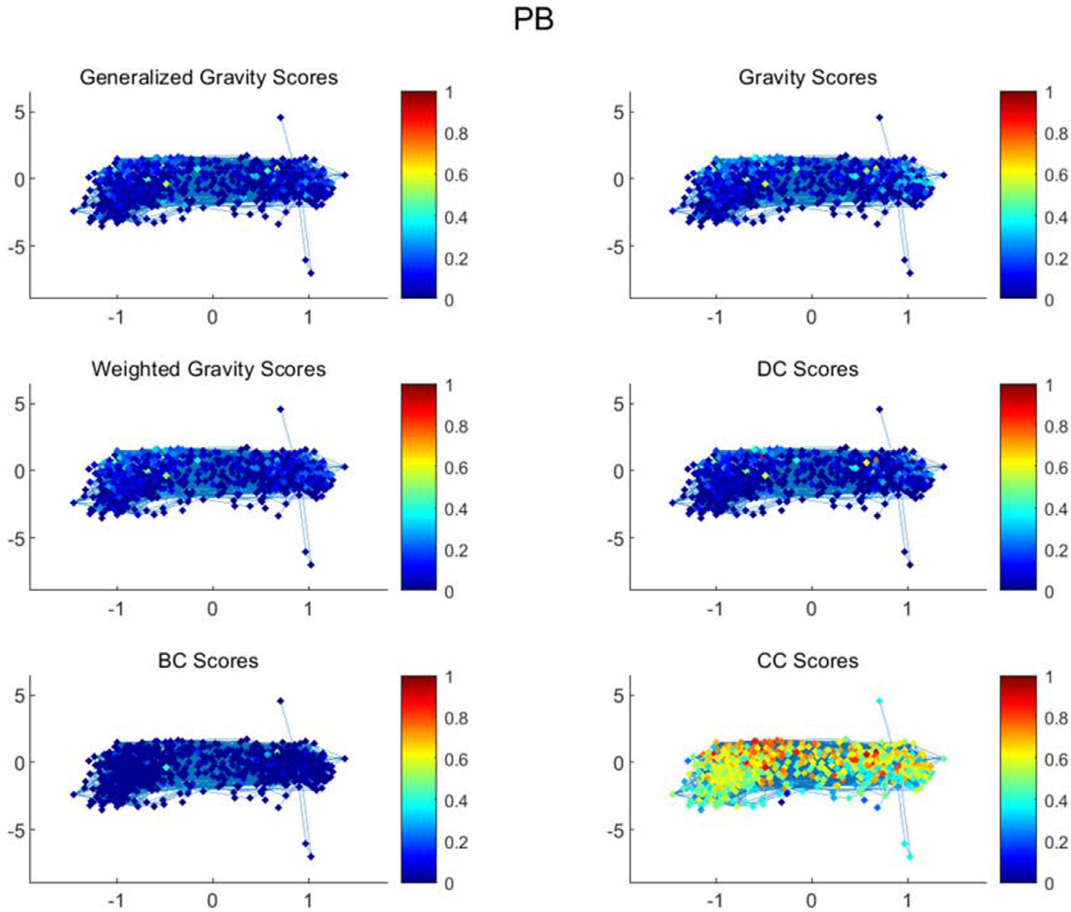

Fig. 4. This figure shows the centrality score of nodes in PB network.

Table 2
Information of datasets, n, m, $\langle k \rangle$ and $\langle d \rangle$ are number of nodes, edges, average degree and average shortest distance of the network respectively.

Network	n	m	$\langle k \rangle$	$\langle d \rangle$
Jazz [76]	198	2472	27.6970	2.2350
NS [77]	379	914	4.8232	6.0419
USAir	332	2126	12.8072	2.7381
PB [78]	1222	16714	27.3552	2.7375

that describes the relationship between Jazz musicians [76]. NS is a network of scientists that work together [77]. USAir is the aviation network of US transportation. PB is a network of political blogs [78]. All of the data is available and can be downloaded from https://github.com/MLIF/Network-Data. The code is available at https://github.com/Hanwen22Li/Generalized-gravity-model.

4.1. Centrality measures of different measures

In this experiment, different centrality measures, like DC, CC, BC, GC, WGC, GGC, are used to calculate the centrality scores of nodes in different networks. The calculation results are demonstrated in Figs. 1–4. The results are normalized. In the figures, if the color of the node is darker, the centrality score of the node is higher.

Generally, the values calculated by CC are large. GGC has similar distributions with GC and WGC. They all judge vital nodes based on local information and path information and calculate with gravity formula. So results of GGC are consistent with GC and WGC in general.

4.2. The comparisons of spreading ability

Figs. 5–6 demonstrate the spreading ability of three centrality measures, gravity centrality, weighted gravity centrality and generalized gravity centrality. In this experiment, the independent experiments are executed 100 times. The simulation time of SI model t is set to be 25. β is set to be 0.1. The top-10 ranked nodes by centrality measures are labeled as infected nodes in initial phase of the SI model in this experiment.

In Fig. 5, generalized gravity model is compared with gravity model. In Jazz network, when t is small, the number of infected nodes with GGC is higher than GC. When t is higher, the numbers of infected nodes of two methods converge to the certain value, because majority nodes of the network have been infected. In NS network, the infected nodes of two methods are increasing. However, the number of infected nodes of GGC is always higher than GC, which demonstrates the effectiveness of generalized gravity model. In USAir and PB model, the values of infected nodes by two methods are almost the same with t increases.

In Fig. 6, generalized gravity model is compared with weighted gravity model. In Jazz network, GGC has greater values than WC in the initial phase. In NS network, the value of infected nodes of GGC is greater than WGC in the whole process. In USAir network, when $t < 20$, the values of GGC are higher than WGC. In PB network, we can see that the values of infected nodes of two methods keep almost the same in the whole process. In overall, generalized gravity model performs better than weighted gravity model.

In a nutshell, we can conclude that generalized gravity model can find influential spreaders more effectively than gravity model and weighted gravity model. Generalized gravity model inherits

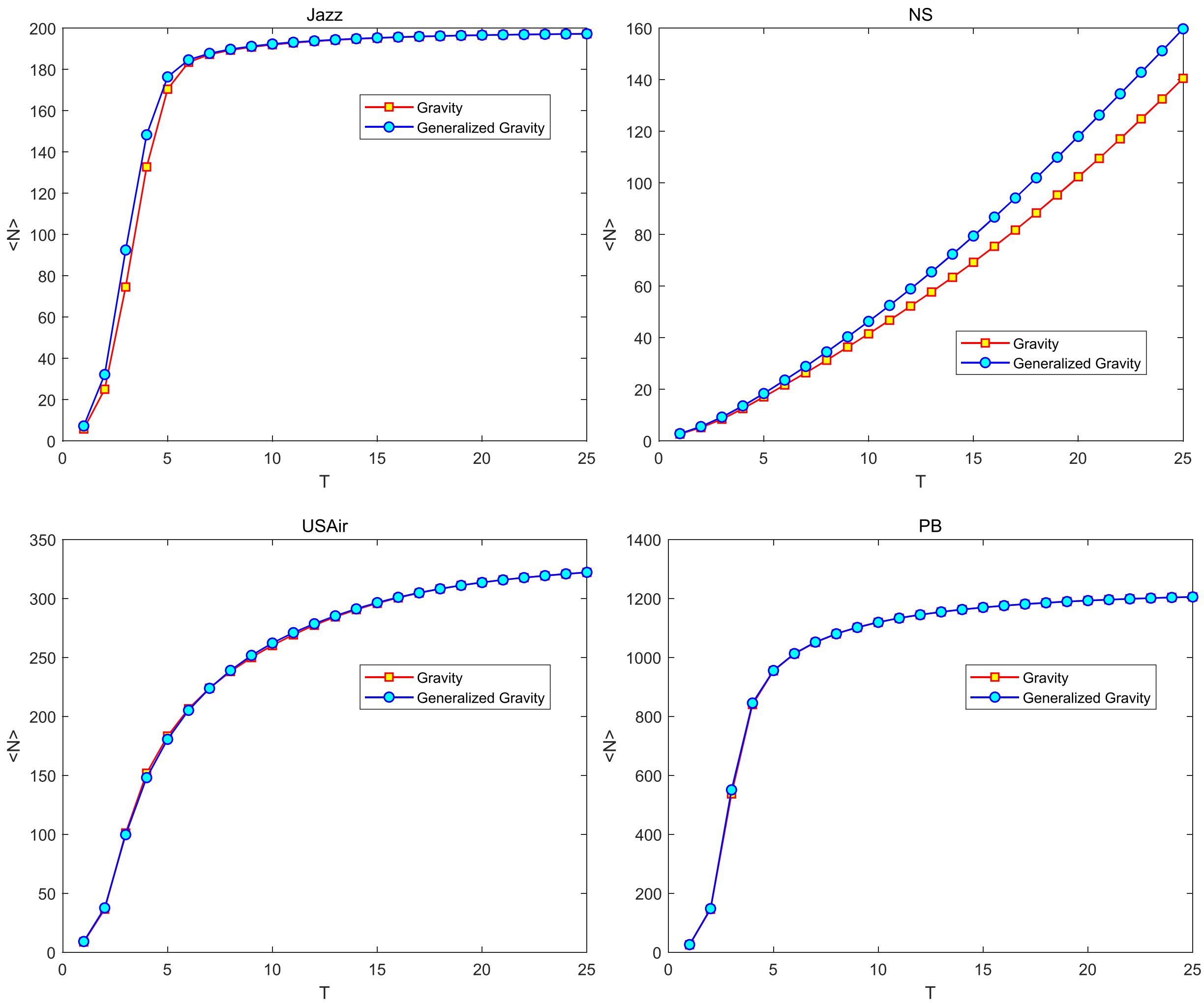

Fig. 5. Infection ability of GGC and GC in four different real-world networks.

some properties of gravity model as they both judge information from local information and path information. However, generalized gravity model judge local information by both cluster coefficient and degree of nodes, which is more precise.

Then we compare the proposed generalized gravity model with three classical centrality measures, CC, DC and BC. The results are shown in Figs. 7–9.

In Fig. 7, generalized gravity model is compared with degree centrality. Generalized gravity model is outperformed by DC in Jazz, NS and USAir networks. Only in PB network can the generalized gravity model be competitive with DC.

In Fig. 8, generalized gravity model is compared with closeness centrality. CC has better performance than GGC except in PB network.

In Fig. 9, generalized gravity model is compared with betweenness centrality. GGC has similar performance with BC in PB network. GGC has better performance than BC in Jazz network in the initial phase. GGC performs also slightly better than BC in USAir network when $t \leq 15$. GGC is only outperformed by BC in NS network.

It can be concluded that in four networks, GGC is better than BC for identifying influential spreaders and worse than CC and DC. However, GGC beats two state of the arts gravity-formula based models like GC and WGC. As of now, GGC is the best gravity-formula based approach and effective for influential spreaders identification.

4.3. The comparisons of centralities measures and SI model

τ value demonstrates the difference between SI model and centrality measures. In this experiment, the propagation time T is set to be 25. The infection probability β ranges from 0.1 to 1. The independent experiments have been done by 100 times. Results of experiments are shown in Figs. 10 and 11.

In Fig. 10, GGC is compared with GC. In Jazz network, when $t = 0.4, 0.5, 0.6, 0.7, 0.9$, the τ value of GGC is higher than GC. Hence, the performance of GGC is compatible with GC in Jazz network. In NS network, most of the values of GGC are higher than GC. In USAir and PB network, there are also five times that the value of GGC is higher than GC.

In Fig. 11, GGC is compared with WGC. In Jazz network, GGC is outperformed by WGC that there are only three times that the value of GGC is higher. In NS network, the values of GGC are higher than WGC in the whole process. In USAir network, when $t = 0.2$ and t from 0.4 to 0.9, the value of GGC is higher than WGC. In PB network, there are five times that the value of GGC is higher than WGC.

In all, it is rarely that generalized gravity model is outperformed by gravity model and weighted gravity model. However, generalized gravity model outperforms other two methods in NS network and is compatible with other two methods in other three networks, which shows the effectiveness of generalized gravity model.

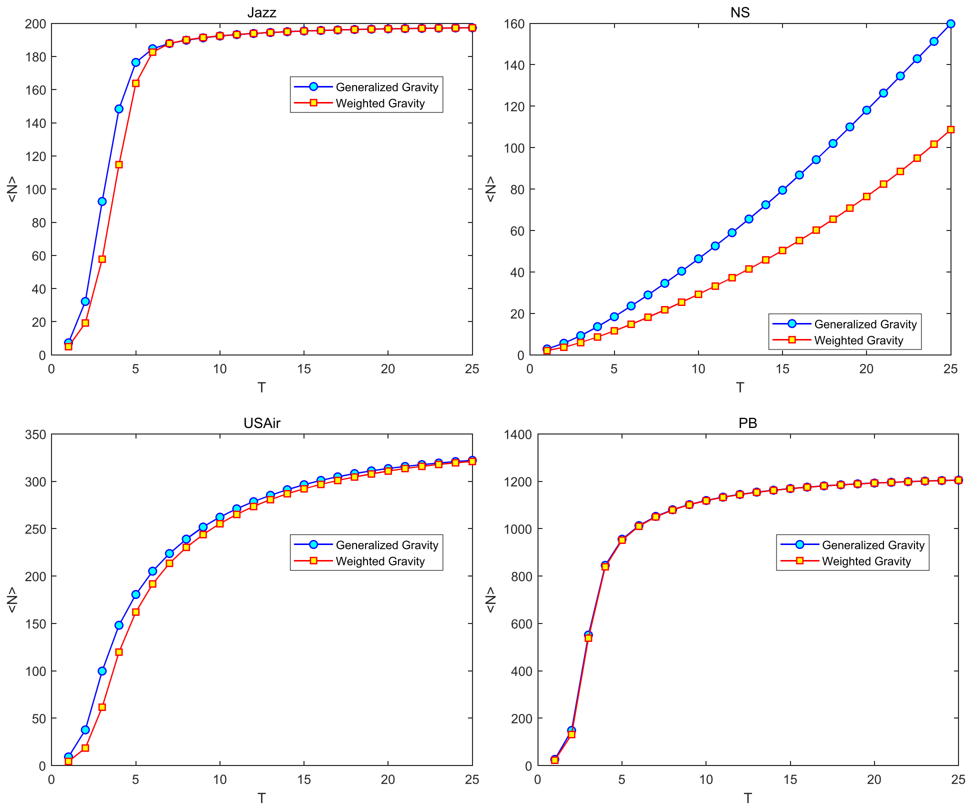

Fig. 6. Infection ability of GGC and WGC in four different real-world networks.

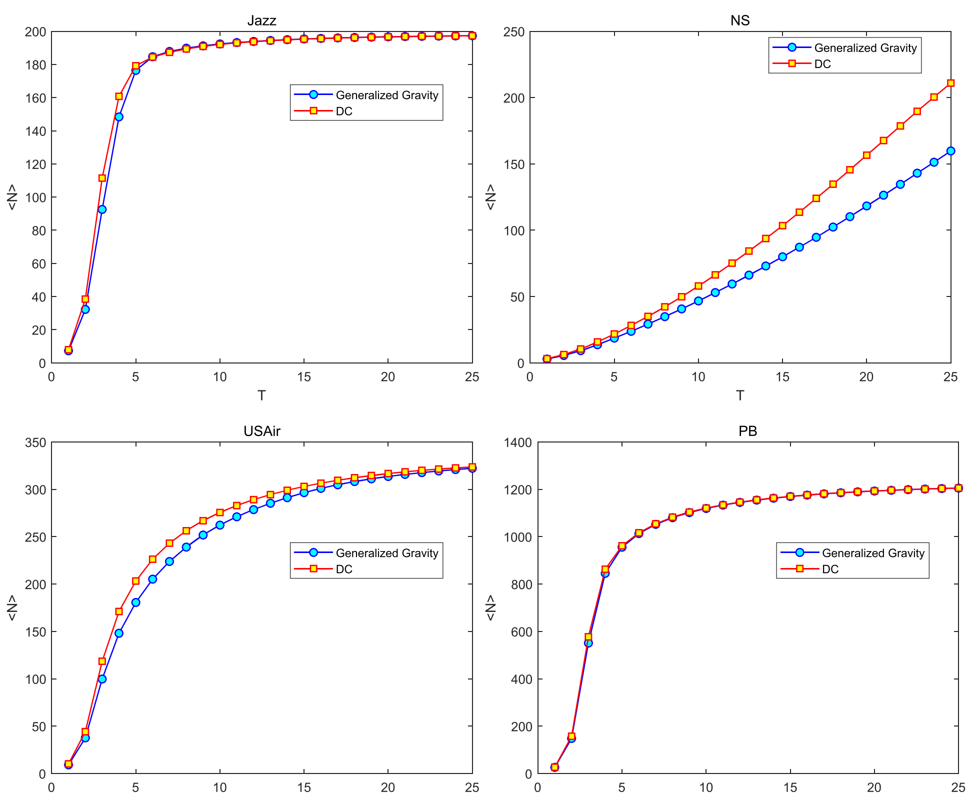

Fig. 7. Infection ability of GGC and DC in four different real-world networks.

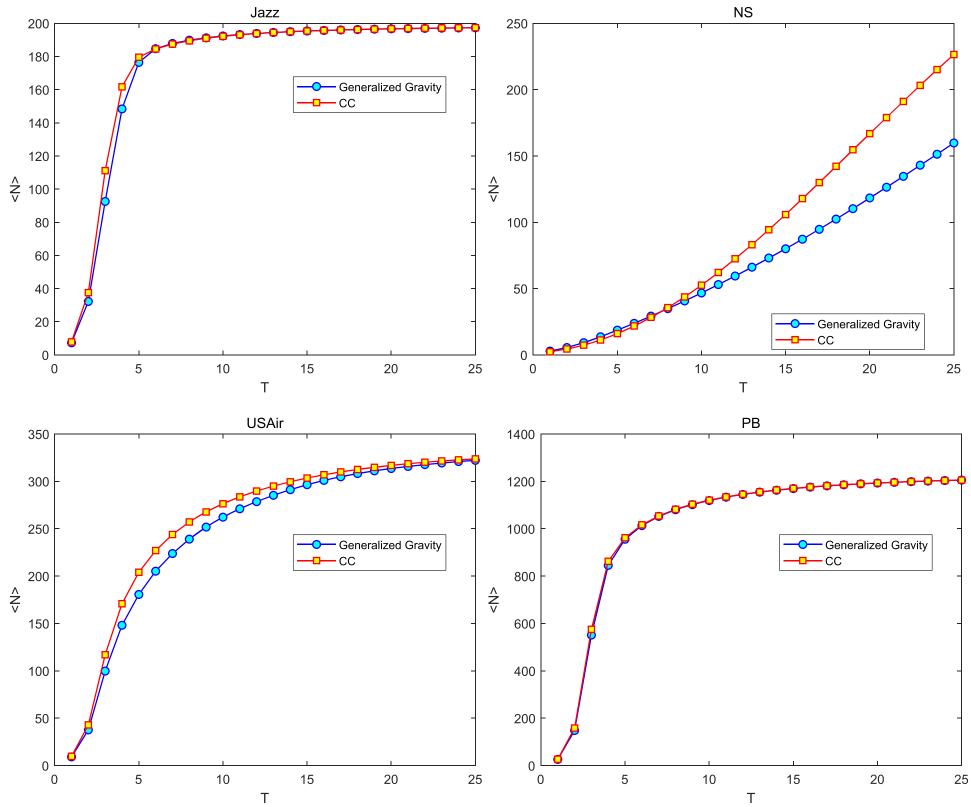

Fig. 8. Infection ability of GGC and CC in four different real-world networks.

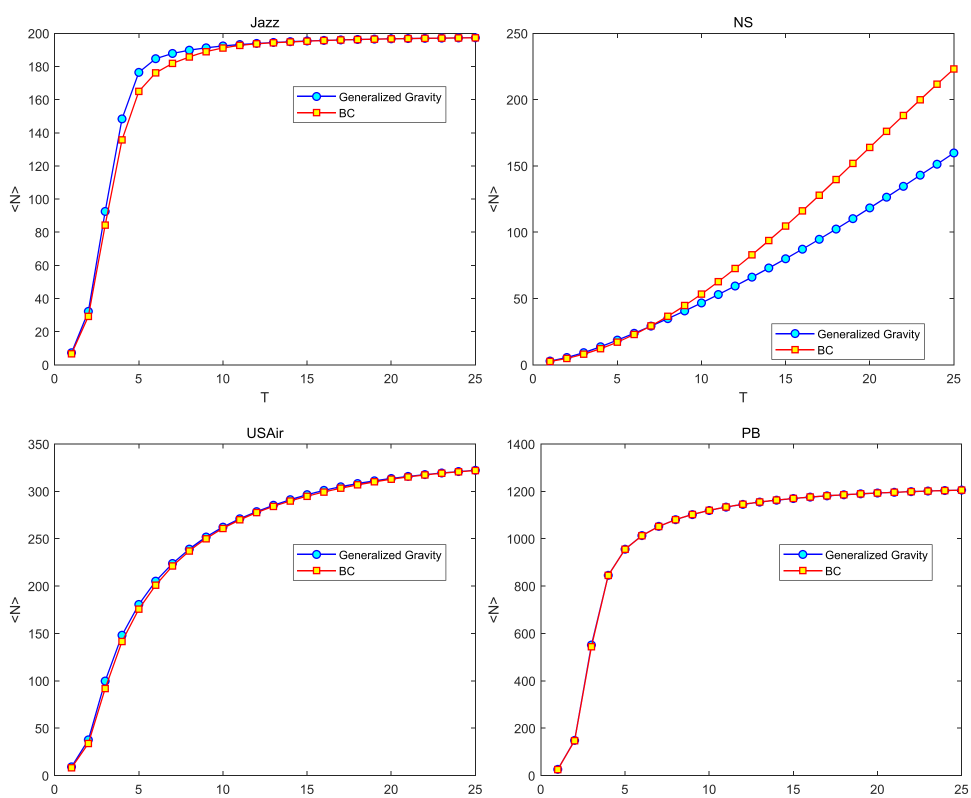

Fig. 9. Infection ability of GGC and BC in four different real-world networks.

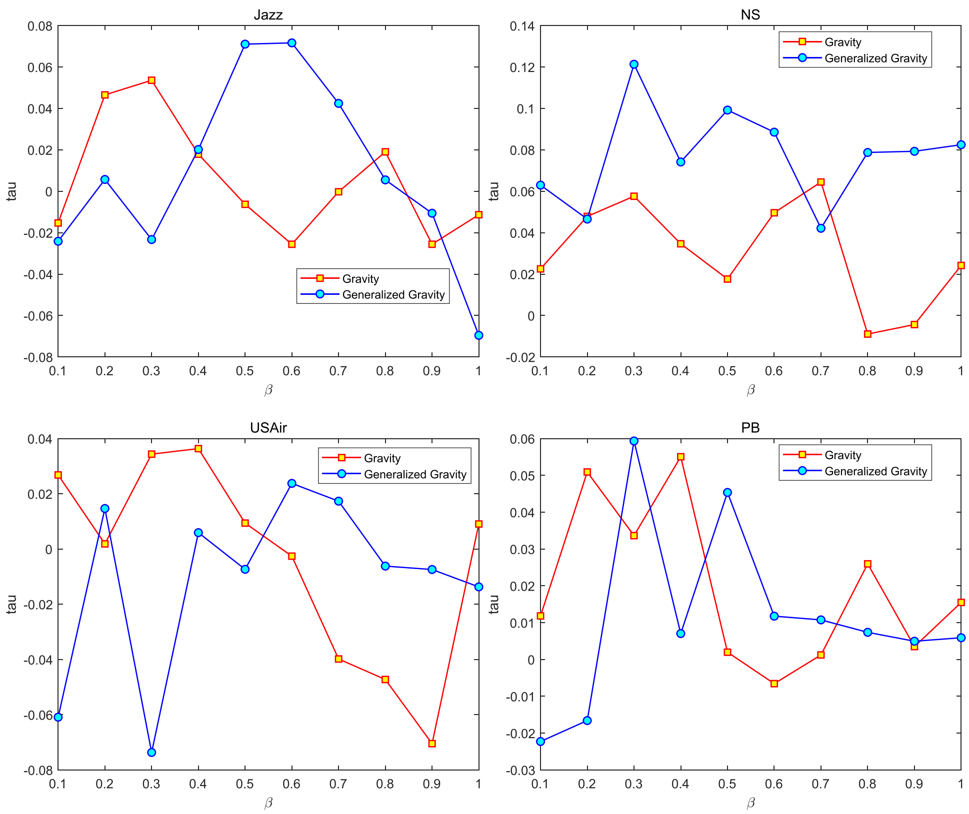

Fig. 10. The τ value of GGC and GC in four networks.

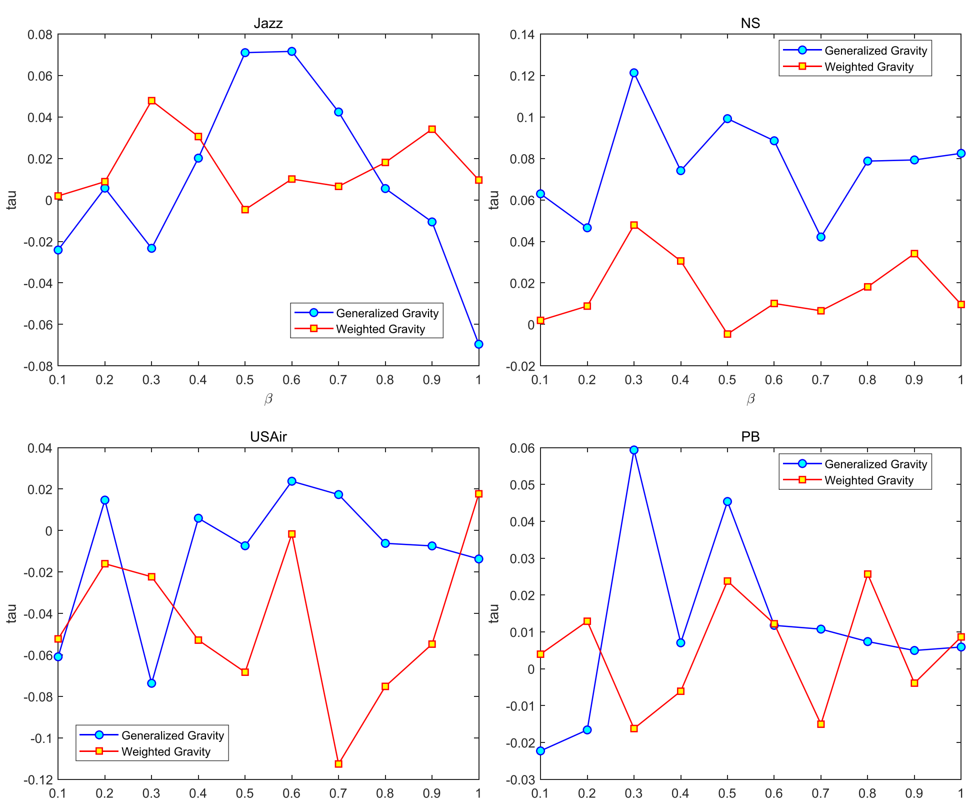

Fig. 11. The τ value of GGC and WGC in four networks.

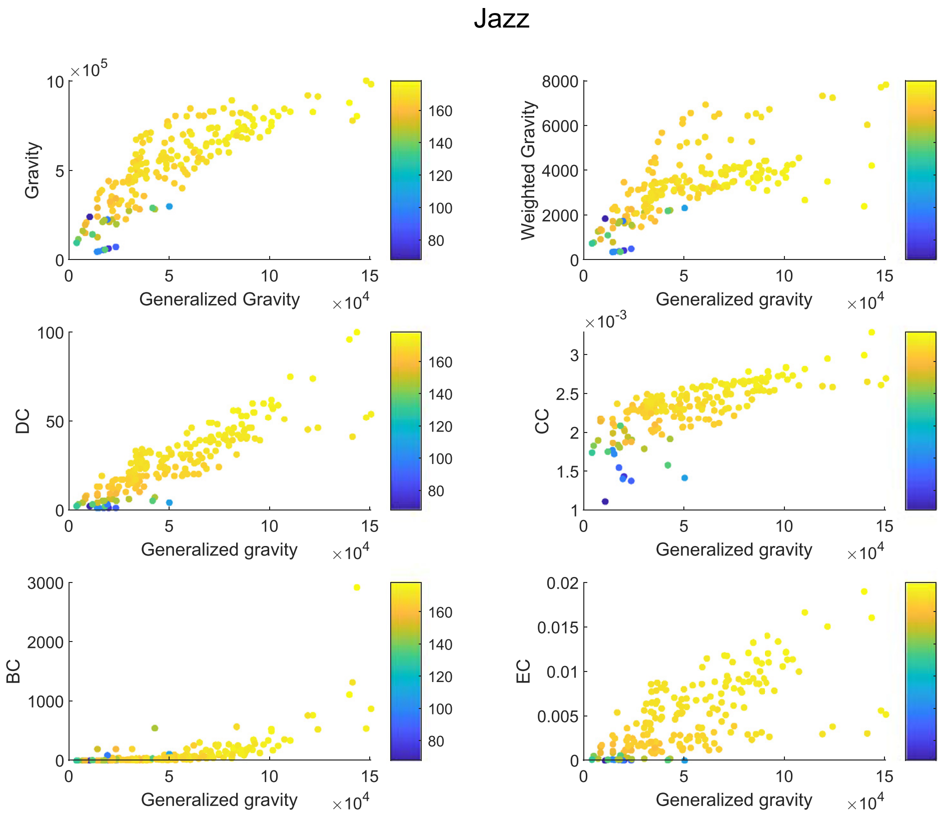

Fig. 12. The figure describes the relationship between generalized gravity model and other methods in Jazz networks. Generalized gravity model has good correlation with gravity and degree centrality.

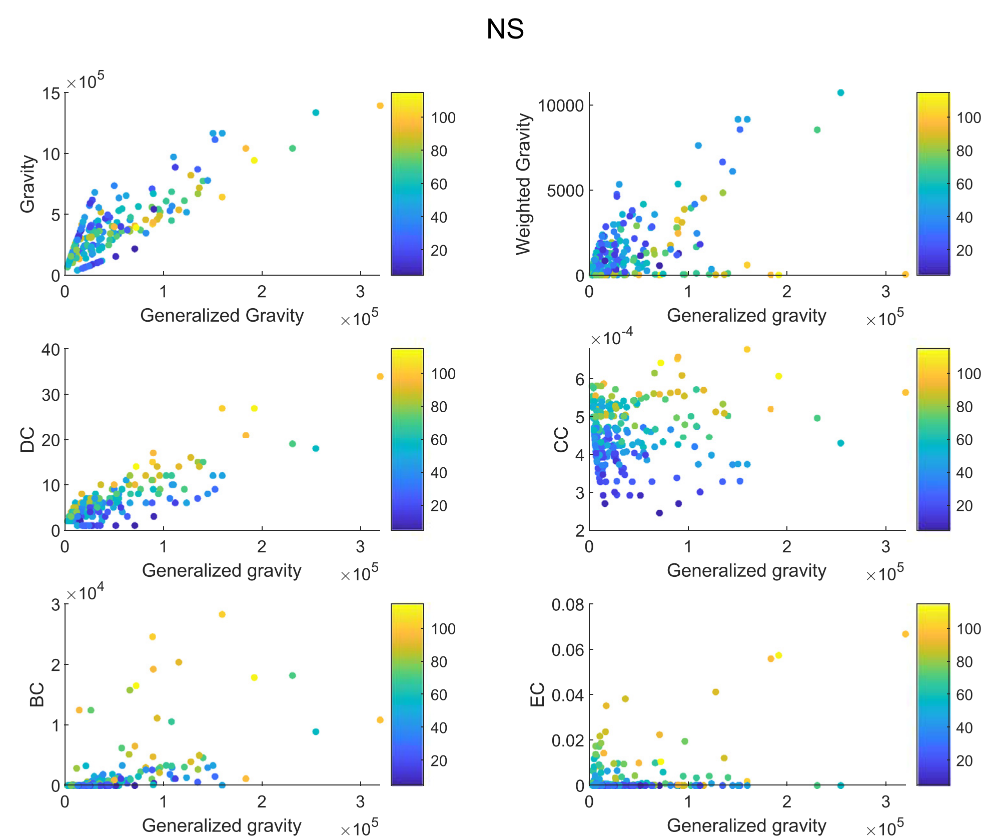

Fig. 13. The figure describes the relationship between generalized gravity model and other methods in NS networks. Generalized gravity model has good correlation with gravity centrality.

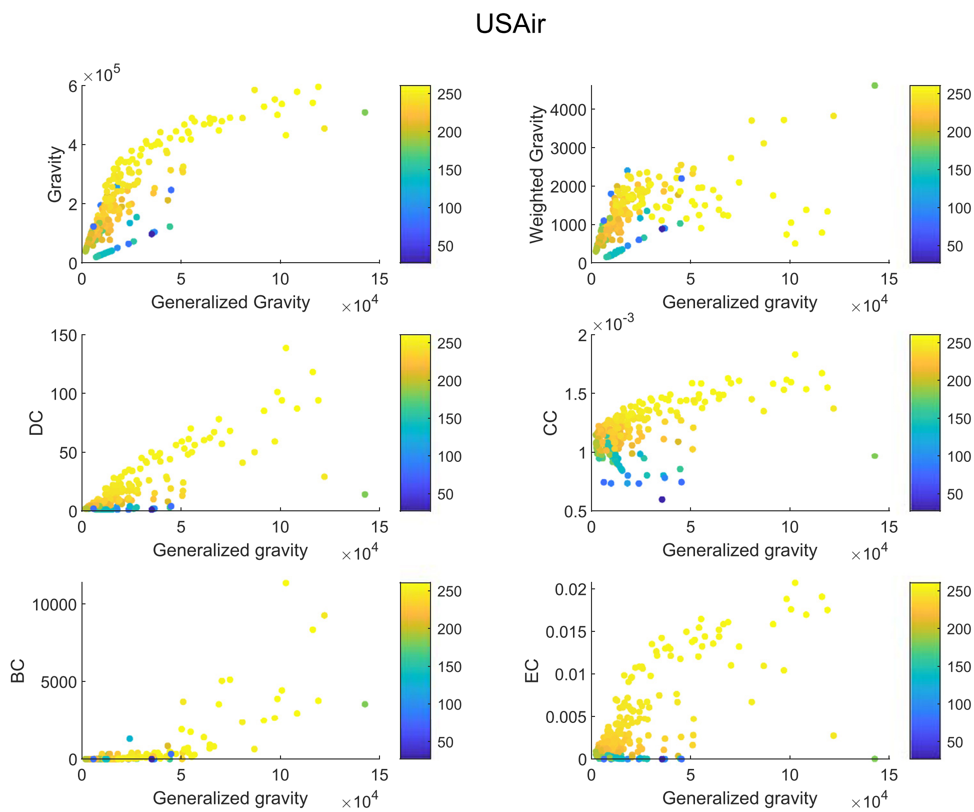

Fig. 14. The figure describes the relationship between generalized gravity model and other methods in USAir networks. Generalized gravity model has good correlation with degree centrality.

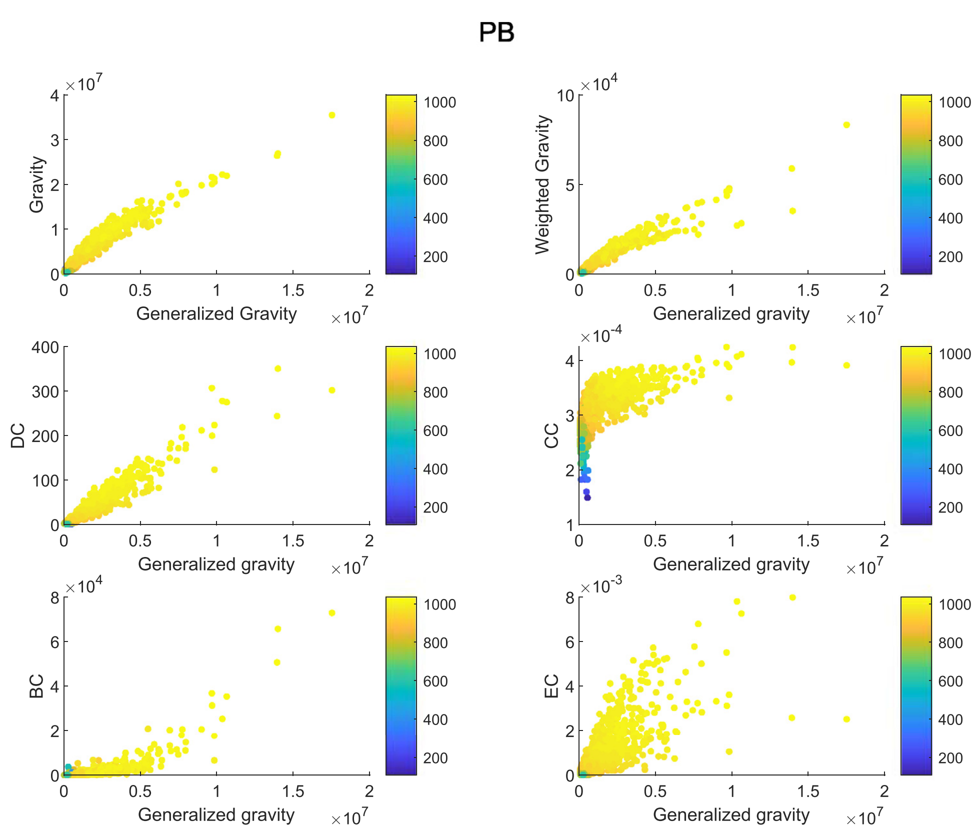

Fig. 15. The figure describes the relationship between generalized gravity model and other methods in PB networks. Generalized gravity model has good correlation with weighted gravity centrality, gravity centrality and degree centrality.

4.4. Correlations with other centrality measures

In this experiment, the generalized gravity centrality is compared with five other centrality measures, including DC, BC, CC, GC and WGC, which identify influential spreaders from different perspective. The points in the correlation graph means the values of centralities of points. The values of axis correspond to values obtained by different centrality measures. In this experiment, the color of points means the infectious ability of this node obtained by SI model in 25 steps, which is obtained by 100 independent experiments results when $\beta = 0.1$. The correlations between GGC and other centralities are shown in Figs. 12–15.

In a nutshell, the generalized gravity model has high correlation with gravity model, weighted gravity model and DC. Generalized gravity model find vital nodes by local and path information, which is inherited from gravity model. Weighted gravity model uses local and path information as well. Hence, generalized gravity model and weighted gravity model has high correlation. Also, DC uses local information, which is consistent with models based on gravity formula that also uses local information.

5. Conclusions

In this paper, a generalized gravity model is proposed for influential spreaders identification. The generalized gravity model can degenerate into gravity model when $\alpha = 0$. Compared with gravity model and weighted gravity model, generalized gravity model takes the local clustering coefficient into account, which better captures the local information of the node. The effectiveness of the proposed method is demonstrated by experiments on four real networks in the real world compared with gravity model and weighted gravity model. The proposed method has made a progress in making gravity-formula applicable in the realm of important nodes identification.

There are still some potential problems in the future. For example, how to use generalized gravity model in the weighted networks is still an open problem. Also, how to find proper parameter α quickly and accurately needs to be further investigated. We will also develop some other better methods based on gravity-formula to identify influential spreaders.

Declaration of Competing Interest

The authors declare that there is no conflict of interests.

CRediT authorship contribution statement

Hanwen Li: Writing - original draft, Investigation, Conceptualization, Methodology, Software, Validation. **Qiuyan Shang:** Methodology, Software, Validation. **Yong Deng:** Writing - review & editing, Supervision, Funding acquisition.

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