

# The 19 Rules of Inference:

1. Modus Ponens (M.P.)	$p \rightarrow q$ $p$ $\therefore q$
2. Modus Tollens (M.T.)	$p \rightarrow q$ $\sim q$ $\therefore \sim p$
3. Hypothetical Syllogism (H.S.)	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$
4. Disjunctive Syllogism (D.S.)	$p \vee q$ $\sim p$ $\therefore q$
5. Constructive Dilemma (C.D.)	$(p \rightarrow q) \cdot (r \rightarrow s)$ $p \vee r$ $\therefore q \vee s$
6. Absorption (Abs.)	$p \rightarrow q$ $\therefore p \rightarrow (p \cdot q)$
7. Simplification (Simp.)	$p \cdot q$ $\therefore p$
8. Conjunction (Conj.)	$p$ $q$ $\therefore p \cdot q$
9. Addition (Add.)	$p$ $\therefore p \vee q$

*Any of the following logically equivalent expressions can replace each other wherever they occur:*

10. De Morgan's Theorem (De M.)	$\sim (p \cdot q) \equiv (\sim p \vee \sim q)$ $\sim (p \vee q) \equiv (\sim p \cdot \sim q)$
11. Commutation (Com.)	$(p \vee q) \equiv (q \vee p)$ $(p \cdot q) \equiv (q \cdot p)$
12. Association (Assoc.)	$[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$ $[p \cdot (q \cdot r)] \equiv [(p \cdot q) \cdot r]$
13. Distribution (Dist)	$[p \cdot (q \vee r)] \equiv [(p \cdot q) \vee (p \cdot r)]$ $[p \vee (q \cdot r)] \equiv [(p \vee q) \cdot (p \vee r)]$
14. Double Negation (D.N.)	$p \equiv \sim \sim p$
15. Transposition (Trans.)	$(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$
16. Material Implication (M. Imp.)	$(p \rightarrow q) \equiv (\sim p \vee q)$
17. Material Equivalence (M. Equiv.)	$(p \equiv q) \equiv [(p \rightarrow q) \cdot (q \rightarrow p)]$ $(p \equiv q) \equiv [(p \cdot q) \vee (\sim p \cdot \sim q)]$
18. Exportation (Exp.)	$[(p \cdot q) \rightarrow r] \equiv [p \rightarrow (q \rightarrow r)]$
19. Tautology (Taut.)	$p \equiv (p \vee p)$ $p \equiv (p \cdot p)$