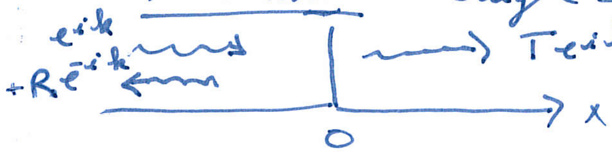
Transmission - Single Delta Function



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0 a\, \delta(x)\, \psi = E\psi$$

$$-\frac{\hbar^2}{2ma^2}\frac{d^2\psi}{d(x/a)^2} + V_0\, \delta(x/a)\,\psi = E\psi$$

$$\frac{d^2\psi}{dx'^2} - \frac{2ma^2 V_0}{\hbar^2}\delta(x')\,\psi = -\frac{2ma^2 E}{\hbar^2}\psi$$

let: $x' = x/a$

ν_0 = "strength" of the potential $\left(\nu_0 = \frac{2ma^2V_0}{\hbar^2}\right)$

k^2 = dimensionless energy $\left(k^2 = \frac{2ma^2E}{\hbar^2}\right)$

$x' < 0$

$\psi = e^{ikx'} + Re^{-ikx'}$

$x' > 0$

$\psi = Te^{ikx'}$

Match at $x' = 0$

(1) $\quad 1 + R = T$

$$\int_{-\epsilon}^{\epsilon}\frac{d^2\psi}{dx'^2}dx' - \nu_0\int_{-\epsilon}^{+\epsilon}\delta(x')\,\psi\, dx' = -k^2\int_{-\epsilon}^{+\epsilon}\psi\, dx$$

$\lim_{\epsilon\to 0}$

$$\left.\frac{d\psi}{dx}\right|_{\epsilon} - \left.\frac{d\psi}{dx}\right|_{-\epsilon} - \nu_0\,\psi(0) = 0$$

$$ikT - (ik - ikR) - \nu_0 T = 0$$

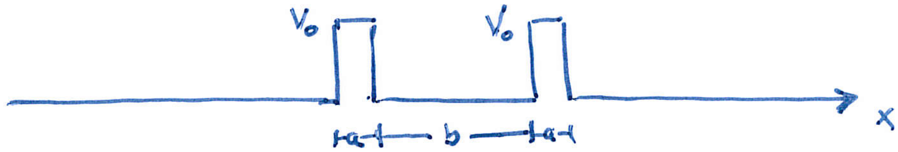
(2) $\quad T(ik - \nu_0) = ik(1-R)$

$$T(1 - \nu_0/ik) = 1 - (T-1)$$

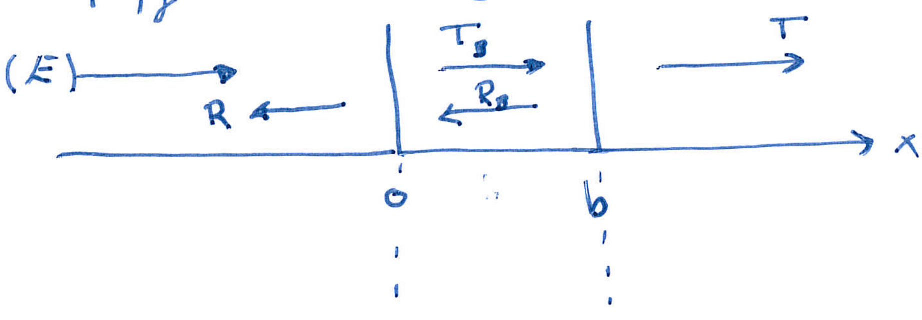
$$T(2 - \nu_0/ik) = 2$$

$$T = \frac{1}{(1 + i\nu_0/2k)} \Rightarrow \boxed{|T|^2 = \frac{1}{1 + (\nu_0/2k)^2}}$$

Resonant Tunneling



Simplify: Delta Functions



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0 b\left(\delta(x)+\delta(x-b)\right)\psi = E\psi \qquad \left(\begin{matrix}\text{1D NR}\\ \text{SE}\end{matrix}\right)$$

$$-\frac{\hbar^2}{2mb^2}\frac{d^2\psi}{d(x/b)^2} + V_0\left(\delta(x/b)+\delta(x/b-1)\right)\psi = E\psi \qquad \text{(change variables)}$$

⋮

$$\frac{d^2\psi}{dx'^2} - \nu_0\left(\delta(x')+\delta(x'-1)\right)\psi = -k^2\psi \qquad \text{(dimensionless)}$$

$$\frac{\hbar^2}{mb^2} \Rightarrow \frac{J^2\cdot s^2}{kg\cdot m^2} \Rightarrow \frac{J^2}{(kg\cdot m/s^2)\,m} \Rightarrow J$$

ν_0 = strength of the "potentials" - dimensionless $\left(\frac{2V_0 m b^2}{\hbar^2}\right)$

k^2 = energy of the electron - dimensionless $\left(\frac{2Emb^2}{\hbar^2}\right)$

$\underline{x' \le 0}$

$$\psi_< = e^{ikx'} + Re^{-ikx'}$$

$\underline{0 < x' < 1}$

$$\psi_B = T_B e^{ikx'} + R_B e^{-ikx'}$$

$\underline{x \geq 1}$

$$\psi_> = T e^{ikx'}$$

<u>Match at $x' = 0$</u>

$$\boxed{① \quad I + R = T_B + R_B}$$

<u>Match at $x' = 1$</u>

$$\boxed{② \quad T_B e^{ik} + R_B e^{-ik} = T e^{ik}}$$

<u>Integrate across the delta functions:</u>

$$\int_{-\epsilon}^{+\epsilon} \frac{d^2\psi}{dx'^2}\,dx' - N_o \int_{-\epsilon}^{+\epsilon} \psi\,\delta(x')\,dx' = -k^2 \int_{-\epsilon}^{+\epsilon} \psi\,dx'$$

$$\lim_{\epsilon \to 0} \quad \left.\frac{d\psi_B}{dx'}\right|_{+\epsilon} - \left.\frac{d\psi_<}{dx'}\right|_{-\epsilon} - N_o \psi_<(0) = 0$$

$$\boxed{③ \quad (ikT_B - ikR_B) - (ik - ikR) - N_o(I + R) = 0}$$

$$\int_{1-\epsilon}^{1+\epsilon} \frac{d^2\psi}{dx'^2}\,dx' - N_o \int_{1-\epsilon}^{1+\epsilon} \psi\,\delta(x'-1)\,dx' = -k^2 \int_{1-\epsilon}^{1+\epsilon} \psi\,dx'$$

$$\lim_{\epsilon \to 0} \quad \left.\frac{d\psi_>}{dx'}\right|_{1+\epsilon} - \left.\frac{d\psi_B}{dx'}\right|_{1-\epsilon} - N_o \psi_>(1) = 0$$

$$\boxed{④ \quad ikTe^{ik} - (ikT_B e^{ik} - ikR_B e^{-ik}) - N_o T e^{ik} = 0}$$

4- equations ; 4- unknowns (R, T_B, R_B, T)

⋮ 6 pages of matrix algebra!
⋮

$$|T|^2 = \left[(1+\beta_0'^2)^2 + \beta_0'^4 - 2\beta_0'^2(1-\beta_0'^2)\cos(2k)\right]^{-1}$$

$$\beta_0' = N_0/2k$$

Computer: Transfer Matrix

$$\begin{pmatrix} Te^{ik} \\ 0 \end{pmatrix} = \hat{d}_{21}\,\hat{P}_1\,\hat{d}_{10}\begin{pmatrix} 1 \\ R \end{pmatrix}$$

$$\hat{d}_{10} = \begin{pmatrix} (1-i\beta_0') & -i\beta_0' \\ i\beta_0' & (1+i\beta_0') \end{pmatrix} \Longrightarrow 2\times 2 \text{ matrix}$$

(match ψ's at delta function)
(discontinuity of slopes)

$$\hat{P}_1 = \begin{pmatrix} e^{ik} & 0 \\ 0 & e^{-ik} \end{pmatrix} \Longrightarrow \text{propagate between delta functions}$$

$$\hat{d}_{21} = \begin{pmatrix} (1-i\beta_1') & -i\beta_1' \\ i\beta_1' & (1+i\beta_1') \end{pmatrix} \Longrightarrow \beta_1' = N_1/2k$$

if $N_1 = N_0 \Longrightarrow \hat{d}_{21} = \hat{d}_{10}$

$$\hat{T} = \hat{d}_{21}\,\hat{P}_1\,\hat{d}_{10} \quad \text{(Transfer Matrix)}$$

$$\boxed{\begin{pmatrix} Te^{ik} \\ 0 \end{pmatrix} = \hat{T}\begin{pmatrix} 1 \\ R \end{pmatrix}}$$