I am submitting the assignment for:

☑ an individual project or

☐ a group project on behalf of all members of the group. It is hereby confirmed that the submission is authorized by all members of the group, and all members of the group are required to sign this declaration.

I/We declare that: (i) the assignment here submitted is original except for source material explicitly acknowledged/all members of the group have read and checked that all parts of the piece of work, irrespective of whether they are contributed by individual members or all members as a group, here submitted are original except for source material explicitly acknowledged; (ii) the piece of work, or a part of the piece of work has not been submitted for more than one purpose (e.g. to satisfy the requirements in two different courses) without declaration; and (iii) the submitted soft copy with details listed in the <Submission Details> is identical to the hard copy(ies), if any, which has(have) been / is(are) going to be submitted. I/We also acknowledge that I am/we are aware of the University's policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the University website http://www.cuhk.edu.hk/policy/academichonesty/.

In the case of a group project, we are aware that all members of the group should be held responsible and liable to disciplinary actions, irrespective of whether he/she has signed the declaration and whether he/she has contributed, directly or indirectly, to the problematic contents.

I/We also understand that assignments without a properly signed declaration by the student concerned and in the case of a group project, by all members of the group concerned, will not be graded by the teacher(s).

Signature(s)

24/3/2020
Date

Principe Jericho Bibat
Name(s)

1155144002
Student ID(s)

ENGG1120
Course code

Linear Algebra for Engineers
Course title

ENGG1120 HW4 ID:1155144002 PRINCIPE Jericho Bibat

4.1) $\cancel{S}\ W = \{(2,-1,3,4),(3,2,-2,1)\}$, $x = (5,8,-12,-5)$

$\therefore (5,8,-12,-5) = \alpha(2,-1,3,4) + \beta(3,2,-2,1) = (2\alpha+3\beta, -\alpha+2\beta, 3\alpha-2\beta, 4\alpha+\beta)$

$\therefore$ $\cancel{2\alpha+3\beta=5}$ $\cancel{-\alpha+2\beta=}$

$$\begin{cases} 2\alpha+3\beta = 5 & \text{— 1} \\ -\alpha+2\beta = 8 & \text{— 2} \\ 3\alpha-2\beta = -12 & \text{— 3} \\ 4\alpha+\beta = -5 & \text{— 4} \end{cases}$$

By 2,

$-\alpha+2\beta = 8$

$\alpha = 2\beta - 8$

Put $\alpha = 2\beta-8$ into 4,

$4(2\beta-8)+\beta = -5$

$\beta = 3$

Put $\beta = 3$ into 2,

$\alpha = -2$

$\because$ Both values of α and β satisfy 1, 2, 3, 4,

$\therefore x = (5,8,-12,-5) \in W$

4.2) $R = \{(2,-1,3,4,0),(1,1,2,2,-1),(3,-1,0,3,-2)\}$, $z = (1,1,5,3,1)$

$(1,1,5,3,1) = \alpha(2,-1,3,4,0) + \beta(1,1,2,2,-1) + \gamma(3,-1,0,3,-2)$

$= (2\alpha+\beta+3\gamma, -\alpha+\beta-\gamma, 3\alpha+2\beta, 4\alpha+2\beta+3\gamma, -\beta-2\gamma)$

$\therefore \cancel{2\alpha+\beta+3\gamma, -\alpha+\beta-\gamma, 3\alpha+2\beta, 4\alpha+2\beta+3\gamma, -\beta-2\gamma}$

$$\therefore \begin{cases} 2\alpha+\beta+3\gamma = 1 & \text{— 1} \\ -\alpha+\beta-\gamma = 1 & \text{— 2} \\ 3\alpha+2\beta = 5 & \text{— 3} \\ 4\alpha+2\beta+3\gamma = 3 & \text{— 4} \\ -\beta-2\gamma = 1 & \text{— 5} \end{cases}$$

From 5, $\beta = -1-2\gamma$

$\therefore$ From 3, $\alpha = \frac{7+4\gamma}{3}$

$\therefore$ From 2, $\gamma = \cancel{\frac{13+4\gamma}{9}}\ -1$

$\therefore$ From 5, $\beta = \cancel{\frac{17+8\gamma}{9}}\ 1$

$\therefore$ $\cancel{\alpha =}$ From 3, $\alpha = 1$

$\because$ The values of α, β, γ do not satisfy 1,

$\therefore z$ does not span R.

4.3) Let $S=\{(\alpha^2,0,1),(0,\alpha,2),(1,0,1)\}$

Consider $x(\alpha^2,0,1)+y(0,\alpha,2)+z(1,0,1)=(0,0,0)$

$\therefore (\alpha^2 x+z,\ \alpha y,\ x+2y+z)=(0,0,0)$

$$\therefore \begin{cases} \alpha^2 x + z = 0 & \text{—— ①} \\ \alpha y = 0 & \text{—— ②} \\ x+2y+z = 0 & \text{—— ③} \end{cases}$$

When $\alpha \neq 0$, $y=0$

$\therefore$ ~~By ③, $x+z=0$~~ By ①, $z=-\alpha^2 x$

$\therefore$ By ③, $x-\alpha^2 x=0$

$x(1-\alpha^2)=0$

$x(1+\alpha)(1-\alpha)=0$

$\therefore x=0$ or $\alpha=-1$ or $\alpha=1$

~~When $\alpha=-1$ or $=1$,~~

~~By ①, $z=-x$~~

$\therefore$ ~~S is~~ When $x=0$, ~~$z=0$~~

By ①, $z=0$ $\leftarrow$ $\because$ The vectors in S are linearly independant when $\alpha\neq 0, \neq -1, \neq 1$,

$\therefore$ S is a basis for $\mathbb{R}^3$ when $\alpha \neq 0 \neq -1 \neq 1$.

4.4) Let $S=\{(1,-1,1),(1,-3,1),(1,2,2)\}$

Consider $\alpha(1,-1,1)+\beta(1,-3,1)+\gamma(1,2,2)=(0,0,0)$

$$\therefore \begin{cases} \alpha+\beta+\gamma=0 & \text{—— ①} \\ -\alpha-3\beta+2\gamma=0 & \text{—— ②} \\ \alpha+\beta+2\gamma=0 & \text{—— ③} \end{cases}$$

With ③ − ①, $\gamma=0$

Take $\gamma=0$ into ② + ①, $\beta=0$

$\therefore$ By $\gamma=0$ and $\beta=0$, in ①, $\alpha=0$

$\therefore$ S in linearly independant.

Consider $\lambda(1,-1,1)+\mu(1,-3,1)+\sigma(1,2,2)=(x,y,z)$, where (x,y,z) is an arbitrary vector.

$$\therefore \begin{cases} \lambda+\mu+\sigma=x & \text{—— ①} \\ -\lambda-3\mu+2\sigma=y & \text{—— ②} \\ \lambda+\mu+2\sigma=z & \text{—— ③} \end{cases}$$

By ③ - $\cancel{④}$①, $\sigma = z - x$

With $\sigma = z - x$ into ② + ③, $\mu = -2x - \frac{y}{2} + 2z$

With μ an σ in ①, $\lambda = 4x + \frac{y}{2} - 3z$

$\therefore (x, y, z) = (4x + \frac{y}{2} - 3z)(1,-1,1) + (-2x - \frac{y}{2} + 2z)(1,-3,1) + (z-x)(1,2,2)$

$\therefore$ S is a basis for R^3.

~~4.5) Let $S = \{(1,-1,1),(1,-3,1),(1,2,2)\}$~~

~~$S=$~~

4.5)
$$\begin{cases} x_1 + 3x_2 + 3x_3 - x_4 + 2x_5 = 0 \\ x_1 + 2x_2 + 2x_3 - 2x_4 + 2x_5 = 0 \\ x_1 + x_2 + x_3 - 3x_4 + 2x_5 = 0 \end{cases}$$

$$= \left[\begin{array}{ccccc|c} 1 & 3 & 3 & -1 & 2 & 0 \\ 1 & 2 & 2 & -2 & 2 & 0 \\ 1 & 1 & 1 & -3 & 2 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} 1 & 1 & 1 & -3 & 2 & 0 \\ 0 & \cancel{1} & \cancel{1} & -1 & 0 & 0 \\ 0 & 2 & 2 & -2 & 0 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} 1 & 1 & 1 & -3 & 2 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ The basis of the solution space $= \{(1,1,1),(3,2,1)\}$

The dimension $= \cancel{3}$ 2.

4.6) Let $S = \{(2,2,1,0),(1,2,0,2),(0,1,2,2),(2,0,2,1)\}$

Let $\{v_1, v_2, v_3, v_4\}$ be ~~an~~ the orthogonal basis.

$v_1 = (2,2,1,0)$

$v_2 = \cancel{(2,2,1,0)}\ (1,2,0,2) - \frac{(2,2,1,0)\cdot(1,2,0,2)}{\|(1,2,0,2)\|^2}(2,2,1,0) = \cancel{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}},\ (-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, 2)$

~~$v_3 = (0, 1/2, 2) - \frac{(1,2,0,2)\cdot(0,1,2,2)}{\|(1,2,0,2)\|^2}\|(2,2,1,0)\|^2$~~

$v_3 = (0,1,2,2) - \frac{(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, 2)\cdot(0,1,2,2)}{\|(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, 2)\|^2}(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, 2) - \frac{(2,2,1,0)\cdot(0,1,2,2)}{\|(2,2,1,0)\|^2}(2,2,1,0) = (-\frac{2}{3}, -\frac{1}{3}, 2, \frac{2}{3})$

$v_4 = (2,0,2,1) - \frac{(2,2,1,0)\cdot\cancel{(0,1,2,2)}(2,0,2,1)}{\|(2,2,1,0)\|^2}(2,2,1,0) - \frac{(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, 2)\cdot(2,0,2,1)}{\|(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, 2)\|^2}(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, 2)$
$- \frac{(-\frac{2}{3}, -\frac{1}{3}, 2, \frac{2}{3})\cdot(2,0,2,1)}{\|(-\frac{2}{3}, -\frac{1}{3}, 2, \frac{2}{3})\|^2}(-\frac{2}{3}, -\frac{1}{3}, 2, \frac{2}{3}) = (\frac{10}{9}, -\frac{10}{9}, 0, -\frac{13}{9})$

$\therefore$ The orthogonal basis is $\{(2,2,1,0), (-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, 2), (-\frac{2}{3}, -\frac{1}{3}, 2, \frac{2}{3}), (\frac{10}{9}, -\frac{10}{9}, 0, -\frac{13}{9})\}$

4.7)(a) $q_1 = (1,1,1)$

$$q_2 = (-1,0,-2) - \frac{(1,1,1)\cdot(-1,0,-2)}{\|(1,1,1)\|^2}\cdot(1,1,1) = (0,1,-1)$$

$\therefore \{q_1, q_2\} = \{(1,1,1), (0,1,-1)\}$

(b)