

Let N_i be the indicator random variable for the event that the i^{th} couple are sitting together.

$$\therefore P(N_i) = \frac{2}{9}$$

$$E[N_i] = \frac{2}{9}$$

$$E[N] = 5 \cdot \frac{2}{9} = \frac{10}{9}$$

$$\text{Var}[N_i] = \frac{2}{9} \left(1 - \frac{2}{9}\right) = \frac{14}{81}$$

$$\therefore \text{Cov}[N_i, N_{i+1}] = \cancel{E[N_i, N_{i+1}]} E[N_i = 1, N_{i+1} = 1] - E[N_i] E[N_{i+1}]$$

$$E[N_i = 1, N_{i+1} = 1] = P(N_{i+1} = 1 \mid N_i = 1) P(N_i = 1)$$

$$= \frac{7}{81} \cdot \frac{2}{9}$$

$$= \frac{1}{18}$$

$$\therefore \text{Var}[N] = 5 \cdot \frac{14}{81} + 5 \cdot (5-1) \left(\frac{1}{18} - \left(\frac{2}{9}\right)^2 \right)$$
$$= \frac{80}{81}$$