

Let  $\theta$  be the probability that a die of a certain type is chosen.

$$\therefore P(\theta=1=2=3) = \frac{1}{3} \text{ (prior)}$$

$$\begin{aligned} \therefore P_{\theta|X}(\theta=1 | X=1,2) &\propto P_{X|\theta}(X=1,2 | \theta=1) P_{\theta}(\theta=1) \\ &\propto \left(\frac{1}{2} \cdot \frac{1}{4}\right) \frac{1}{3} \\ &\propto \frac{1}{24} \end{aligned}$$

$$\begin{aligned} P_{\theta|X}(\theta=2 | X=1,2) &\propto P_{X|\theta}(X=1,2 | \theta=2) P_{\theta}(\theta=2) \\ &\propto \frac{1}{24} \end{aligned}$$

$$\begin{aligned} P_{\theta|X}(\theta=3 | X=1,2) &\propto P_{X|\theta}(X=1,2 | \theta=3) P_{\theta}(\theta=3) \\ &\propto \left(\frac{1}{4} \cdot \frac{1}{4}\right) \frac{1}{3} \\ &\propto \frac{1}{48} \end{aligned}$$

The normalization constant,

$$\begin{aligned} P(X=1,2) &= \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{2}{3} + \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{3} \\ &= \frac{5}{48} \end{aligned}$$

$$\begin{aligned} \therefore P_{\theta|X}(\theta=1 | X=1,2) &= \frac{\frac{1}{24}}{\frac{5}{48}} \\ &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} P_{\theta|X}(\theta=2 | X=1,2) &= \frac{\frac{1}{24}}{\frac{5}{48}} \\ &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} P_{\theta|X}(\theta=3 | X=1,2) &= \frac{\frac{1}{48}}{\frac{5}{48}} \\ &= \frac{1}{5} \end{aligned}$$