ENGG2780 Final 1155144002

1a) $(10\% \cdot 90\%) + (90\% \cdot 10\%) + (10\% \cdot 10\%)$ ← False positive error = 10%

$= 0.09 + 0.09 + 0.01$

$= 0.19$

b) $\binom{3}{2}(0.1 \cdot 0.1 \cdot 0.9) + 0.1^3$

$= 0.000027$

2a) $\bar{X} = (552+343+301+326)/4 = 380.5$

$S^2 = [(552-380.5)^2 + (343-380.5)^2 + (301-380.5)^2 + (326-380.5)^2] / 3$

$= 40109/3$

~~b) $P(t(3) \leq 97.5\%) \approx$~~

b) $P(t(3) \leq z) = 97.5\%$

~~$z \approx$~~ Using an online calculator,

$z \approx 3.182$

$\therefore$ 95% confidence interval for μ,

$(380.5 - (3.182 \cdot 40109/3)/2, 380.5 + (3.182 \cdot 40109/3)/2)$

$= (-20890.63967, 21651.63967)$

c) Using online calculator,

$z_- \approx 0.216$ $(P\,\chi^2(3) \leq z_-) \approx 2.5\%$

$z_+ \approx 9.35$ $P(\chi^2(3) \geq z_+) \approx 2.5\%$

$\therefore$ 95% confidence interval for σ,

$\left(\sqrt{\frac{3 \cdot 40109/3}{\cancel{0.216}\ 9.35}}, \sqrt{\frac{(3 \cdot 40109/3)}{\cancel{9.35}\ 0.216}}\right)$

$= (65.496, 430.9174)$

$\therefore$ 95% C.I. for ~~Var~~ variance σ^2,

$(65.496^2, 430.9174^2)$

$= (4289.73, 185689.81)$

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3a) Null hypothesis = ~~$\frac{1}{2}$~~ Binomial$(256,\frac{1}{2})$

$\therefore P(\text{Binomial}(256,\frac{1}{2}) > 256\cdot h) = 5\%$

Using online calculator,

~~$P_{H_0}(+)$ $h \approx 115$~~

$256\cdot h \approx 142$

$h = 0.5546875$

$\therefore$ ~~0.55~~ ~~At~~ A fraction of 0.5546875 heads or tails is needed for 95% confidence of bias.

~~3b)~~ Assuming 0=tail and 1=head,

3b) $P(\text{Binomial}(256,\frac{1}{2}) < 116) \approx 0.07522$

$\therefore$ It cannot be concluded with 95% confidence.

4a) Null = Normal(0,1)

Alt. = Normal(0,2)

By Neyman-Pearson Lemma, $+$ when $x>t$, $-$ when $x<t$

$\therefore P_{H_0}(+) = P(\text{Normal}(0,1) \geq t) = 10\%$

Using online calculator,

$t \approx 1.282$

~~$\therefore P_{H_1}(-) = P(\text{Normal}(0,2) < 1.282)$~~

~~$= 0.7392$~~

$\therefore$ To obtain false positive error 10%, we choose $t = 1.282$.

4b) Using online calculator

$P_{H_1}(-) = P(\text{Normal}(0,2) < 1.282)$

$= 0.7392$

5a) $F = \beta M + N$

$\beta = \frac{Cov[M,N]}{Var[M]}$

$\therefore \quad = \frac{11}{23}$

$E[F] = \frac{11}{23} \cdot E[M] + E[N]$

$30 = \frac{11}{23} \cdot 35 + E[N]$

$\therefore E[N] = 13.26$

$Var[F] = \frac{11}{23} Var[M] + Var[N]$

$13 = \frac{11}{23} \cdot 23 + Var[N]$

$\therefore Var[N] = 2$

~~b) $\bar{Z} = \bar{F} - \bar{M}$~~

~~$= 5$~~

b) Null: $\bar{F} = \bar{M}$

Alt: $\bar{F} < M$

~~$\therefore P(\frac{11}{23} Normal(35, 23) + Normal(13.26, 2) > 30\,35)$~~

$\therefore P(\frac{11}{23} Normal(35, 23) + Normal(13.26, 2) < 35)$

$= P(Normal(48.26, \sqrt{15}) < 35)$

$= 0.0003$

$\therefore$ There is ~99% confidence that $F < M$.

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