ENGG2760A Finals 1155144002 PRINCIPE Jericho Bibat

1a) No. When $P(A) \neq 1$, then $P(A) = P(A|B) + P(A|B^c) \neq 1$.

b) Yes.

$$P(A \cap B | A \cup B) \leq P(A|B)$$

$$\frac{P(A \cap B \cap (A \cup B))}{P(A \cup B)} \leq \frac{P(A \cap B)}{P(B)}$$

$$\frac{\cancel{P(A \cap B)}}{P(A \cup B)} \leq \frac{\cancel{P(A \cap B)}}{P(B)}$$

$$P(B) \leq P(A \cup B)$$

$\therefore$ It is true.

2a) When Event E has exactly 4 independant random numbers > 0.7, its probability is

$$P(E) = \binom{10}{4} \cdot 0.3^4 \cdot 0.7^6$$

$$= 0.2001\ldots$$

(b) $n_i = \text{Uniform}(0,1)$

$$\therefore E[n_i] = \int_0^1 x \cdot 1 \, dx$$

$$= \frac{x^2}{2}\Big|_0^1$$

$$= \frac{1}{2}$$

$$E[n_i^2] = \int_0^1 x^2 \cdot 1 \, dx$$

$$= \frac{x^3}{3}\Big|_0^1$$

$$= \frac{1}{3}$$

~~$\therefore$ Var Var n_i~~

$$\therefore Var[n_i] = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12}$$

$$\therefore E[n] = 50 \cdot E[n_i] = 25$$

$$Var[n] = 50 \cdot \frac{1}{12} = \frac{25}{6}$$ $\leftarrow \therefore$ By CLT,

$$\therefore P(20 \leq x \leq 25) = P(x \leq 25) - P(x < 20)$$

~~$\approx P(25 + N \cdot \sqrt{\tfrac{25}{6}} \leq 25) - P$~~

$$\approx P\left(25 + N \cdot \sqrt{\tfrac{25}{6}} \leq 25\right) - P\left(25 + N\sqrt{\tfrac{25}{6}} \leq 20\right)$$

$$= P(N \leq 0) - P(N < -2.449\ldots)$$

$$\approx 0.4928$$

3a) ~~A = Exponential(2), B = Exponential(1)~~

$$\cancel{\therefore P(B \mid 2\text{years}) = \frac{P(2\text{years} \mid B)\,P(B)}{P(2\text{years} \mid B)\,P(B) + P(2\text{years} \mid A)\,P(A)}}$$

From

$$\cancel{= \frac{\left(1-\left(1-e^{-(1)(2)}\right)\right)\frac{1}{4}}{\left(1-\left(1-e^{-(1)(2)}\right)\right)\frac{1}{4} + \left(1-\left(1-e^{-(2)(2)}\right)\right)\frac{3}{4}}}$$

$$\cancel{=}$$

4a) $f_T(t) = \begin{cases} \frac{1}{t^2} & t \geq 1 \\ 0 & o/w \end{cases}$

$$\therefore F_T(t) = P(T \leq t) = \int_1^t f_T(x)\,dx$$
$$= \int_1^t \frac{1}{x^2}\,dx$$
$$= -\frac{1}{x}\Big|_1^t$$
$$= 1 - \frac{1}{t}$$

(b) $\because V = \frac{1}{T}$

$$\therefore f_V(v) = \begin{cases} v^2 & v \geq 1 \\ 0 & o/w \end{cases}$$
$$\therefore E[V] = \int_1^v x\,f(x)\,dx$$
$$= \int_1^v x^3\,dx$$
$$= \frac{x^4}{4}\Big|_1^v$$
$$= \frac{v^4}{4} - \frac{1}{4}$$

5(a) $\therefore P(\overset{S}{\cancel{E}}) = 2 \cdot \frac{10}{20} \cdot \frac{9}{19} \cdot \frac{8}{18} \cdot \frac{7}{17}$

$= 0.086687\ldots$

Event S occurs ~~others~~ when the team are of same gender.

Scanned with CamScanner

b) ~~Es~~ From (a),

$P(\text{Both genders}) = 1 - P(S)$

$= 0.913313$

$\therefore P(\text{All teams of mixed genders}) = P(\text{both genders})^5$

$= 0.6354746$

$> 50\%$

$\therefore$ It is true.

3a) $A = \text{Exponential}\,\cancel{(4)}\left(\frac{1}{2}\right)$, $B = \text{Exponential}(1)$

$$\therefore P(\text{From } B \mid \geq 2\text{ years}) = \frac{\cancel{P(\geq 2\text{ years} \mid \text{From } B)}\ P(\geq 2\text{ years} \mid \text{From } B)}{P(\geq 2\text{years} \mid \text{From } B) + P(\geq 2\text{years} \mid \text{From } A)}$$

$$= \frac{\left(1-\left(1-e^{-(1)(2)}\right)\right)\cdot\frac{1}{4}}{\left(1-\left(1-e^{-(1)(2)}\right)\right)\frac{1}{4} + \left(1-\left(1-e^{-\frac{1}{2}(2)}\right)\right)\frac{3}{4}}$$

$= 0.10923\ldots$

b) $P(B > A) = \int_A^\infty (1)e^{-(1)t}\,dt$, CDF of $A = \int_0^\infty \frac{1}{2}e^{-(\frac{1}{2})(t)}\,dt$

$= \int_A^\infty e^{-t}$ $\qquad = -\frac{1}{4}e^{-\frac{1}{2}t}\Big|_0^\infty$

$= -e^{-t}\Big|_A^\infty$ $\qquad =$

$= e^{-A}$

$= e^{-1-e^{-\lambda}}$