ENGG1120 HW3 ~~115~~ ID no.: 1155144002 Principe Jericho Bibat

Q3. (a)

$$\tilde{A} = \left(\begin{array}{ccc|c} k & 1 & 2 & 1 \\ 2 & 1 & k & -7 \\ k & 0 & 1 & 3 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} k & 1 & 2 & 1 \\ 0 & \frac{k-2}{k} & \frac{k^2-4}{k} & -\frac{7k+2}{k} \\ 0 & -1 & -1 & 2 \end{array}\right)$$

$$= \left(\begin{array}{ccc|c} k & 1 & 2 & 1 \\ 0 & \frac{k-2}{k} & \frac{k^2-4}{k} & -\frac{7k+2}{k} \\ 0 & 0 & k+1 & -\frac{5k+6}{k-2} \end{array}\right)$$

~~∴ The domain of k ≠ is all real values except~~

∴ The given system has a unique solution for all real values of k except $\{-1, 0, 2\}$

$\cancel{|A| = k \cdot \frac{k-2}{k} \cdot k+1}$

If A is invertible, $\det(A) \neq 0$

$$\therefore |A| = k \cdot \frac{k-2}{k} \cdot (k+1) \cancel{+ \frac{k^2-4}{k} + 2 - 2 \cdot \frac{k-2}{k} \cdot k \cdot \frac{k^2-4}{k} - (k+1)}$$

$$= k^2 - k - 2 \cancel{+ \frac{k^2-4}{k} + 2 / \frac{2k^2 4}{k} \; 2k^2 + 4 - k - 1}$$

$$\cancel{= -2k + \frac{k(k-2)}{k} + 3} = (k+1)(k-2)$$

$$\cancel{= -k+1}$$

∴ Matrix A is invertible for all real values of k except $\cancel{\{-1, 0, \ldots, 2\}}$ $\{-1, 2\}$.

b) The given system has infinitely many solutions / is inconsistent when $\det(A) = 0$.

∴ From (a),

$\det(A) = 0$ when $k = -1$ or $= 2$

when $k = -1$,

$$\tilde{A} = \left(\begin{array}{ccc|c} -1 & 1 & 2 & 1 \\ 0 & 3 & 5 & -5 \\ 0 & 0 & 0 & \frac{1}{3} \end{array}\right)$$

∴ no real solutions when $k = -1$.

When $k = 2$,

$$\tilde{A} = \left(\begin{array}{ccc|c} 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & -7 \\ 2 & 0 & 1 & 3 \end{array}\right) = \left(\begin{array}{ccc|c} 2 & 0 & 1 & 3 \\ 2 & 1 & 2 & -7 \\ 0 & 0 & 0 & 48 \end{array}\right)$$

∴ no real solutions when $k = 2$

∴ There is no real solutions when the system has infinite solutions.

∴ The system is inconsistent when $k = -1$ or $= 2$.

Q3.2a) $\tilde{A} = \left(\begin{array}{ccc|c} 1 & k & 1 & k \\ k & 1 & 1 & 2 \\ -3 & 0 & -1 & -2 \end{array}\right)$

$\because$ The linear system has a unique solution when $\det(A) \neq 0$

$\therefore |A| = -1 + (-3k) - (-3) - (-k^2)$

$= k^2 - 3k + 2$

$= (k-2)(k-1)$

~~$\therefore (k-2)(k-1) \neq 0 \to k \neq 2$ or~~

$\therefore (k-2)(k-1) \neq 0 \to k \neq 2$ or $\neq 1$

$\therefore$ The system has a unique solution when $\{k \in \mathbb{R} \setminus \{1, 2\}\}$.

By Cramer's rule,

$x_n = \frac{\det(A_n)}{\det(A)}$

$\therefore x_1 = \frac{\det(A_1)}{\det(A)}$

$= \frac{\begin{vmatrix} k & k & 1 \\ 2 & 1 & 1 \\ -2 & 0 & -1 \end{vmatrix}}{(k-2)(k-1)}$

$= \frac{-k - 2k - (-2) - (-2k)}{(k-2)(k-1)}$

$= \frac{-(k-2)}{(k-2)(k-1)}$

$= -\frac{1}{k-1}$

$x_2 = \frac{\det(A_2)}{\det(A)}$

$= \frac{\begin{vmatrix} 1 & k & 1 \\ k & 2 & 1 \\ -3 & -2 & -1 \end{vmatrix}}{(k-2)(k-1)}$

$= \frac{-2 - 3k - 2k - (-6) - (-2) - (-k^2)}{(k-2)(k-1)}$

$= \frac{k^2 - 5k + 6}{(k-2)(k-1)}$

$= \frac{(k-2)(k-3)}{(k-2)(k-1)}$

$= \frac{k-3}{k-1}$

$x_3 = \frac{\det(A_3)}{\det(A)}$

$= \frac{\begin{vmatrix} 1 & k & k \\ k & 1 & 2 \\ -3 & 0 & -2 \end{vmatrix}}{\det(A)}$

$= \frac{-2 - 6k - (-3k) - (-2k^2)}{(k-2)(k-1)}$

$= \frac{2k^2 - 3k - 2}{(k-2)(k-1)}$

$= \frac{(2k+1)(k-2)}{(k-2)(k-1)}$

$= \frac{2k+1}{k-1}$

$\therefore$ The system has the solution $x_1 = -\frac{1}{k-1}$, $x_2 = \frac{k-3}{k-1}$, $x_3 = \frac{2k+1}{k-1}$, when the system has a unique solution where the possible value of k is all real numbers except 1 and 2.

b) The given system is inconsistent / has infinitely many solutions when $\det(A)=0$

From (a), $\det(A)=(k-1)(k-2)=0$

$\therefore k=1$ or $=2$

When $k=1$,

$$\tilde{A}=\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ -3 & 0 & -1 & -2 \end{array}\right)$$

$$=\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -3 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

~~$\therefore$ The~~

$\therefore$ The system is inconsistent when $k=1$.

When $k=2$,

$$\tilde{A}=\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & 1 & 1 & 2 \\ -3 & 0 & -1 & -2 \end{array}\right)$$

$$=\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & 6 & 2 & 4 \end{array}\right)$$

$$=\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\therefore$ The system has infinitely many solutions when $k=2$.

When $k=2$, ~~Let the solution of the system when $k=2$ be z.~~

$$\tilde{A}=\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -3 & -1 & -2 \end{array}\right)$$

$$\begin{cases} x_1+2x_2+x_3=2 & \text{——— ①} \\ -3x_2-x_3=-2 & \text{——— ②} \end{cases}$$

②, $-3x_2-x_3=-2$

$$x_2=\frac{2-x_3}{3}$$

~~Sub ① into ②~~,

Sub ② into ①,

$$x_1+2x_2+x_3=2$$

$$x_1=2-x_3-2\left(\frac{2-x_3}{3}\right)$$

$$=\frac{2-x_3}{3}$$

$\therefore x_1=\begin{pmatrix}\frac{2-x_3}{3}\\ \frac{2-x_3}{3}\\ x_3\end{pmatrix}=\begin{pmatrix}\frac{2}{3}\\ \frac{2}{3}\\ 0\end{pmatrix}+\begin{pmatrix}-\frac{x_3}{3}\\ -\frac{x_3}{3}\\ x_3\end{pmatrix}$, where x_3 is a free variable.

$\therefore$ When $k\neq 1$ or $\neq 2$, the solution is $x_1=-\frac{1}{k-1}$, $x_2=\frac{k-3}{k-1}$, $x_3=\frac{2k+1}{k-1}$.

When $k=2$, the solution is $x_1=\begin{pmatrix}\frac{2}{3}\\ \frac{2}{3}\\ 0\end{pmatrix}+\begin{pmatrix}-\frac{x_3}{3}\\ -\frac{x_3}{3}\\ x_3\end{pmatrix}$, where x_3 is a free variable.

When $k=1$, there is no solution.