ENGG2780A Mid-term 1155144002

~~1a) Prior V = Uniform(0, 1/2) = 2 Uniform(0,1) = 2·Beta(1,1)~~

~~∴ 2·B(2,1) = 2·1/2 = 1~~

~~∴ Posterior V = f(θ) = θ~~

1a) ~~PRIOR = Uniform(0, 1/2) = 2·Uniform(0,1) = 2·Beta(1,1)~~

~~Posterior = Beta(2,1)~~

f

2a) Possible values of $\bar{X}$: 4, 3.5, 3, 2.5, 2, 1

$\bar{X}=4 : \left(\frac{500}{1000}\right)^2 = \frac{1}{4}$

$\bar{X}=3.5 : 2\left(\frac{500}{1000}\cdot\cancel{\frac{2500}{}}\frac{250}{1000}\right) = \cancel{\frac{1}{8}}\frac{1}{4}$

$\bar{X}=3 : \left(\frac{250}{1000}\right)^2 = \frac{1}{16}$

$\bar{X}=2.5 : \left(\frac{500}{1000}\cdot\frac{250}{1000}\right)\cdot 2 = \cancel{\frac{1}{8}}\frac{1}{4}$

$\bar{X}=2 : 2\left(\frac{250}{1000}\right)^2 = \cancel{\frac{1}{16}}\frac{1}{8}$

$\bar{X}=1 : \left(\frac{250}{1000}\right)^2 = \frac{1}{16}$

b) ~~E[X̄] = PMF~~

$\mu = E[\text{No. of biscuits}] = 4\cdot\frac{1}{2} + 3\cdot\frac{1}{4} + 1\cdot\frac{1}{4}$

$= 3$

∴ There is $\frac{1}{16}$ probability that $\mu = \bar{X}$.

3) $\bar{X} = \frac{83+103+93+93}{4} = 93$

$S^2 = \frac{100+100+0+0}{3} = \frac{200}{3}$

$S = \sqrt{\frac{200}{3}}$

By using online calculator for $t(3)$ R.V.,

$Z \approx 3.18$ for 95% probability.

$\therefore P(\bar{X} - Z_+\frac{S}{\sqrt{n}} \leq \mu \leq \bar{X} - Z_-\frac{S}{\sqrt{n}}) = P(93-12.98 \leq \mu \leq 93+12.98)$

$\therefore$ 95%-confidence interval is $[80.02, 105.98]$

4a) PMF of $\theta = f_{\theta|X}(\theta|X) \propto f_{X|\theta}(X|\theta) f_\theta(\theta)$

MLE is found when $f_{X|\theta}(X|\theta)$ is maximized.

$\therefore f_{X|\theta}(X|\theta) = \frac{1}{3}\theta \cdot \frac{1}{3}\theta \cdot (1-\theta)$

$= \frac{1}{9}\theta^2 - \frac{1}{9}\theta^3$

$\therefore \frac{d}{d\theta}(\frac{1}{9}\theta^2 - \frac{1}{9}\theta^3) = 0$

$(2-3\theta)\theta = 0$ (online calculator used)

$\therefore \theta = 0$ or $\theta = \frac{2}{3}$

$\because f_{X|\theta}(X|\theta) = 0$ when $\theta = 0$,

$\therefore$ MLE of $\hat{\theta} = \frac{2}{3}$.

b) $E[\theta] =$