基于圣维南方程组动量守恒公式，忽略侧向动量项q与风F_f、局部损失项F_e：

运动波： 陡峭山区，惯性项与附加比降都可忽略

扩散波： 河漫滩，惯性项不能忽略

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\left(\frac{\partial h}{\partial x} + S_f\right) = 0 \tag{1}$$

其中 $h = y + z$, y是水深、z是河床高度，河床比降$\frac{\partial z}{\partial x} = -S_0$。

$$\frac{\partial h}{\partial x} = \frac{\partial y}{\partial x} - S_0, \tag{2}$$

1 等价形式

1.1 q 的形式

q为单宽流量（$q = \frac{Q}{B}, A = By$），因此$q = Vy$

$$\frac{\partial qB}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2 B^2}{By}\right) + gBy\left(\frac{\partial h}{\partial x} + S_f\right) = 0 \tag{3}$$

$$\underbrace{\frac{\partial q}{\partial t}}_{\text{局部惯性}} + \underbrace{\frac{\partial}{\partial x}\left(\frac{q^2}{y}\right)}_{\text{对流惯性}} + \underbrace{gy\frac{\partial h}{\partial x}}_{\text{压强/水面坡度}} + \underbrace{gyS_f}_{\text{摩阻}} = 0 \tag{4}$$

1.2 V 的形式

$$\frac{\partial Q}{\partial t} = \frac{\partial AV}{\partial t} = A\frac{\partial V}{\partial t} + V\frac{\partial A}{\partial t} \tag{5}$$

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) = \frac{\partial}{\partial x}(AV^2) = \underbrace{V^2\frac{\partial A}{\partial x}}_{\text{产出自 } A} + \underbrace{2AV\frac{\partial V}{\partial x}}_{\text{产出自 } V} \tag{6}$$

结合连续性方程，

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \frac{\partial A}{\partial t} = -\left(V\frac{\partial A}{\partial x} + A\frac{\partial V}{\partial x}\right) \tag{7}$$

综上可解得：

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\left(\frac{\partial h}{\partial x} + S_f\right) = 0 \tag{8}$$

Chow 2003, Eq. 9.1.37。

3 扩散波

式 3 忽略对流惯性项，

$$
\begin{aligned}
\frac{\partial q}{\partial t} + gy\frac{\partial h}{\partial x} + gyS_f = 0 \\
\frac{\partial q}{\partial t} + gy\left(\frac{\partial h}{\partial x} + S_f\right) = 0
\end{aligned}
\tag{9}
$$

根据曼宁公式$V = \frac{\sqrt{S_f}}{n} R^{\frac{2}{3}}, S_f = \frac{n^2}{R^{\frac{4}{3}}} \frac{q^2}{y^2}$

将

$$
\frac{q^{n+1} - q^n}{\Delta t} = -gy^n \left(S_h^n + S_f^{n+1}\right) \tag{10}
$$

其中，上标n，$n+1$代表时刻。

$$
S_f^{n+1} \approx \gamma^n q^{n+1}, \qquad \gamma^n := \frac{n^2 q^n}{R^{4/3}(y^n)^2} \quad \text{（冻结系数线性化）} \tag{11}
$$

$$
\boxed{q^{n+1} = \frac{q^n - gy^n \Delta t S_h^n}{1 + g\Delta t y^n \gamma^n} = \frac{q^n - gy^n \Delta t S_h^n}{1 + g\Delta t \frac{n^2 q^n}{R^{4/3} y^n}}} \tag{12}
$$

```julia
qnew = ( q - g0*depth*δt*slope_inst ) /
       ( 1 + g0*δt*0.03^2*q / ( R^(4/3)*depth ) )
```