Homework 11

problems 2, 4, 6, 7, 10, 15 ONLY are due on Thursday November 29, 2018, 9:00am EST,
but all problems will be included in the solution sheet

Include all intermediate steps of the computations in your answers. If the answer is readily available on the web (e.g., on wikipedia), then credit is only given for the intermediate steps.

1. Given is a random sample $X_1, X_2, \ldots, X_n$ from a Bernoulli distribution with parameter p. One considers the estimators $T_1 = \frac{1}{n}(X_1 + \cdots + X_n)$ and $T_2 = \min\{X_1, ..., X_n\}$ of p.
   (a) Calculate the bias of T_1 and of T_2.
   (b) Calculate $\mathrm{MSE}(T_1)$ and $\mathrm{MSE}(T_2)$.
   (c) Which estimator would you prefer when $n = 2$?
2. Problem 7.59 from our text.
3. Problem 7.5 from our text.
4. Problem 7.1 from our text.
5. Consider the following probability density function with an unknown parameter θ:
$$f(x) = \begin{cases} \theta^2 x e^{-\theta x} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$
   Let $X_1, \ldots, X_n$ be a random sample from this probability density function.
   (a) Calculate the bias of the estimator $\Theta_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{X_i}$ for θ.
   (b) Calculate the maximum likelihood estimator Θ_2 for θ.
   (c) Which of these two estimators is better? Explain.
6. Kindergarten places in New York City are very competitive; the acceptance rate at several public schools is lower than that of Columbia University. For one particular school, children are required to do a test with a school-appointed psychologist, and following is a random sample of 10 test scores:

101
107
117
120
115
107
112
109
101
111.

(a) Compute the sample mean.

(b) Compute the sample standard deviation.

(c) Assuming the test scores have a normal distribution, construct a 95% confidence interval for the population mean.

(d) Interpret the confidence interval from part (c).

(e) Those children with a score above 118 proceed to a second assessment round. Based on the above sample, construct a two-sided 60% confidence interval for the proportion of four-year olds *unable* to go to the second round.

7. (Continued from hw09.) Recent newsreports about fires caused by tampered electrical circuits in NYC have put the inspection unit on high alert. On a recent day, city inspector Edward Lectrician inspected 11 circuits and recorded the following information about the electrical current drawn (in ampere):

$$
\begin{array}{r}
10 \\
9 \\
0 \\
20 \\
0 \\
22 \\
0 \\
12 \\
3 \\
1 \\
0.
\end{array}
$$

We assume this is a random sample.

(a) Assuming that these currents are a sample from the normal distribution, construct a 95% two-sided confidence interval for the population mean.

(b) The standard wiring in many apartments heats up at draws of more than 15 ampere. Construct an approximate 70% one-sided confidence bound on the proportion of circuits in the city that heat up. As part of this problem, you need to determine if the city would be interested in a lower or upper bound. You may assume that the large-sample intervals from class are appropriate without checking any conditions.

8. For a random sample of size 100 from the normal distribution with unknown mean μ and unknown variance σ^2, we find that $\bar{x} = 10$ and $s^2 = 36$. Compute a 90% two-sided confidence interval for the population variance σ^2.

9. For three years you have worked on a simulation program for the valuation of so-called Sudric options. The price of such an option is the (unknown) population mean μ of some random variable X with known variance $\sigma^2 = 1$.

The input to your program is a number n. Your program produces a random sample $x_1, \ldots, x_n$ of size n and outputs the sample mean $\bar{x}$ as the estimate for the option price. The valuation error is $|\bar{x} - \mu|$.

Find n so that you are approximately 98% confident that the error $|\bar{x} - \mu|$ will not exceed \$0.05.

10. Problem 7.9 from our text.
11. Problem 7.11 from our text.
12. Problem 7.23 and 7.24 from our text.
13. Problem 7.30 from our text.
14. Problem 7.45 from our text.
15. Problem 7.48 from our text.