Design and Experimental Validation of a Monopod Robot with 3-DoF Morphable Inertial Tail for Somersault

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***Abstract*—The dynamics equations.**

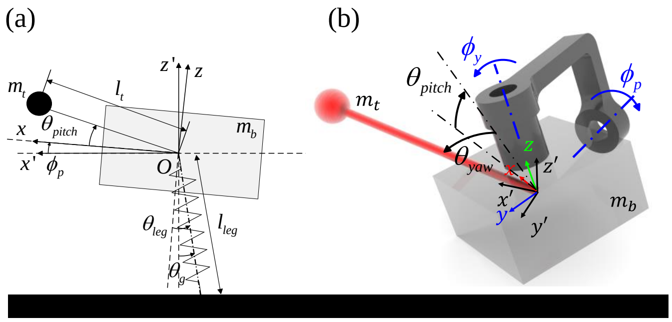

Fig. 1. Simplified robot model in: (a) stance phase; (b) flight phase.

TABLE I
EQUATION PARAMETERS

x,y,z	Body's position in the world frame
l_t	Tail length
θ_{pitch}	Tail swing angle in the body pitch direction
θ_{yaw}	Tail swing angle in the body yaw direction
$\theta_{m,1}$,$\theta_{m,2}$	Tail TOS motors' swing angle
ϕ_p	Body pitch rotation angle
ϕ_y	Body twist angle
I_{xx},I_{yy},I_{zz}	Body inertia in the body fram $O-xyz$
$I_{m,t,s}$,$I_{m,t,l}$,$I_{m,h}$	Motor Inertia of the tail TOS motors, the tail TMIS motor, and the hip joint motor
F_s	Leg spring force
τ_h	Hip joint torque
$\tau_{t,s,p}$	Tail swing torque in the pitch direction
$\tau_{t,s,y}$	Tail swing torque in the yaw direction
$\tau_{t,l}$	Tail extension/retraction torque

I. STANCE PHASE

$q \in \mathbb{R}^5$ in the stance phase. And $q = [l_{leg} \quad \theta_g \quad l_t \quad \theta_{leg} \quad \theta_{pitch}]^T$.

Tail positions in the world frame:

$$\begin{aligned} x_t^{'} &= x + l_t \cos(\theta_{pitch} + \phi_p) \\ &= l_{leg} \sin\theta_g + l_t \cos(\theta_{pitch} + \theta_{leg} - \theta_g), \\ z_t^{'} &= z + l_t \sin(\theta_{pitch} + \phi_p) \\ &= l_{leg} \cos\theta_g + l_t \sin(\theta_{pitch} + \theta_{leg} - \theta_g), \end{aligned} \tag{1}$$

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where $\theta_{pitch} = \frac{\theta_{m,1}}{R_{t,s}} = \frac{\theta_{m,2}}{R_{t,s}}$, and $R_{t,s}$ is the tail TOS motors' gearbox ratio (25:1).

Tail velocities in the world frame:

$$\begin{aligned} \dot{x}_t^{'} &= \dot{l}_{leg} \sin\theta_g + l_{leg} \cos\theta_g \dot{\theta}_g + \dot{l}_t \cos(\theta_{pitch} + \theta_{leg} - \theta_g) \\ &\quad - l_t \sin(\theta_{pitch} + \theta_{leg} - \theta_g)(\dot{\theta}_{pitch} + \dot{\theta}_{leg} - \dot{\theta}_g), \\ \dot{z}_t^{'} &= \dot{l}_{leg} \cos\theta_g - l_{leg} \sin\theta_g \dot{\theta}_g + \dot{l}_t \sin(\theta_{pitch} + \theta_{leg} - \theta_g) \\ &\quad + l_t \cos(\theta_{pitch} + \theta_{leg} - \theta_g)(\dot{\theta}_{pitch} + \dot{\theta}_{leg} - \dot{\theta}_g). \end{aligned} \tag{2}$$

The Kinetic energy and Potential energy:

$$\begin{aligned} T &= \frac{1}{2} m_b (\dot{x}^2 + \dot{z}^2) + \frac{1}{2} I_{yy}^{'} \dot{\phi_p}^2 + \frac{1}{2} m_t \left[\dot{x_t^{'}}^2 + \dot{z_t^{'}}^2 \right] + \\ &\quad 2 \cdot \frac{1}{2} I_{m,t,s} (R_{t,s} \dot{\theta}_{pitch})^2 + \frac{1}{2} I_{m,h} (R_h \dot{\theta}_{leg})^2 \\ V &= m_b g z + m_t g z_t^{'}, \end{aligned} \tag{3}$$

where R_h is the hip motor's gearbox ratio (49:1).

We derive the dynamcis equations based on:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \quad , i = 1, 2, 3, 4, 5. \tag{4}$$

Here $\boldsymbol{\tau} = [F_s \quad 0 \quad \tau_{t,l} \quad \tau_h \quad \tau_{t,s,p}]^T$. The dynamic equation matrix is listed in the file "stance-MCG.txt".

II. FLIGHT PHASE (WITH CONSTRAINTS)

$q \in \mathbb{R}^7$ in the flight phase. And $q = [\phi_p \quad \phi_y \quad \theta_{pitch} \quad \theta_{yaw} \quad l_t \quad x \quad z \quad]^T$.

Tail positions in the body frame:

$$\begin{aligned} x_t &= l_t \cos\theta_{pitch} \cos\theta_{yaw}, \\ y_t &= l_t \sin\theta_{yaw}, \\ z_t &= l_t \sin\theta_{pitch} \cos\theta_{yaw}. \end{aligned} \tag{5}$$

Tail positions in the world frame:

$$\begin{aligned} x_t^{'} &= x - l_t \cos\phi_p \sin\phi_y \sin\theta_{yaw} - l_t \cos\theta_{yaw} \sin\phi_p \\ &\quad \sin\theta_{pitch} + l_t \cos\phi_p \cos\phi_y \cos\theta_{yaw} \cos\theta_{pitch}, \\ y_t^{'} &= l_t \cos\phi_y \sin\theta_{yaw} + l_t \cos\theta_{yaw} \cos\theta_{pitch} \sin\phi_y, \\ z_t^{'} &= z + l_t \cos\phi_p \cos\theta_{yaw} \sin\theta_{pitch} - l_t \sin\phi_p \sin\phi_y \\ &\quad \sin\theta_{yaw} + l_t \cos\phi_y \cos\theta_{yaw} \cos\theta_{pitch} \sin\phi_p \end{aligned} \tag{6}$$

We have:

$$\dot{x}_t^{'} = \dot{x} - \dot{l}_t \cos\phi_p \sin\phi_y \sin\theta_{yaw} - \dot{l}_t \cos\theta_{yaw} \sin\phi_p \sin\theta_{pitch} + \dot{l}_t \cos\phi_p \cos\phi_y \cos\theta_{yaw} \cos\theta_{pitch} - \dot{\phi}_y l_t \cos\phi_p \cos\phi_y \sin\theta_{yaw} - \dot{\theta}_{yaw} l_t \cos\phi_p \cos\theta_{yaw} \sin\phi_y - \dot{\phi}_p l_t \cos\phi_p \cos\theta_{yaw} \sin\theta_{pitch} - \dot{\theta}_{pitch} l_t \cos\theta_{yaw} \cos\theta_{pitch} \sin\phi_p + \dot{\phi}_p l_t \sin\phi_p \sin\phi_y \sin\theta_{yaw} + \dot{\theta}_{yaw} l_t \sin\phi_p \sin\theta_{yaw} \sin\theta_{pitch} - \dot{\phi}_p l_t \cos\phi_y \cos\theta_{yaw} \cos\theta_{pitch} \sin\phi_p - \dot{\phi}_y l_t \cos\phi_p \cos\theta_{yaw} \cos\theta_{pitch} \sin\phi_y - \dot{\theta}_{yaw} l_t \cos\phi_p \cos\phi_y \cos\theta_{pitch} \sin\theta_{yaw} - \dot{\theta}_{pitch} l_t \cos\phi_p \cos\phi_y \cos\theta_{yaw} \sin\theta_{pitch}, \tag{7}$$

$$\begin{aligned}
\dot{y}_t^{'} &= \dot{l}_t \cos\phi_y \sin\theta_{yaw} + \dot{l}_t \cos\theta_{yaw} \cos\theta_{pitch} \sin\phi_y + \dot{\theta}_{yaw} l_t \cos\phi_y \cos\theta_{yaw} - \dot{\phi}_y l_t \sin\phi_y \sin\theta_{yaw} + \dot{\phi}_y l_t \cos\phi_y \cos\theta_{yaw} \cos\theta_{pitch} - \dot{\theta}_{yaw} l_t \cos\theta_{pitch} \sin\phi_y \sin\theta_{yaw} - \dot{\theta}_{pitch} l_t \cos\theta_{yaw} \sin\phi_y \sin\theta_{pitch}, \\
\dot{z}_t^{'} &= \dot{z} + \dot{l}_t \cos\phi_p \cos\theta_{yaw} \sin\theta_{pitch} - \dot{l}_t \sin\phi_p \sin\phi_y \sin\theta_{yaw} + \dot{l}_t \cos\phi_y \cos\theta_{yaw} \cos\theta_{pitch} \sin\phi_p + \dot{\theta}_{pitch} l_t \cos\phi_p \cos\theta_{yaw} \cos\theta_{pitch} - \dot{\phi}_p l_t \cos\phi_p \sin\phi_y \sin\theta_{yaw} - \dot{\phi}_y l_t \cos\phi_y \sin\phi_p \sin\theta_{yaw} - \dot{\theta}_{yaw} l_t \cos\theta_{yaw} \sin\phi_p \sin\phi_y - \dot{\phi}_p l_t \cos\theta_{yaw} \sin\phi_p \sin\theta_{pitch} - \dot{\theta}_{yaw} l_t \cos\phi_p \sin\theta_{yaw} \sin\theta_{pitch} + \dot{\phi}_p l_t \cos\phi_p \cos\phi_y \cos\theta_{yaw} \cos\theta_{pitch} - \dot{\phi}_y l_t \cos\theta_{yaw} \cos\theta_{pitch} \sin\phi_p \sin\phi_y - \dot{\theta}_{yaw} l_t \cos\phi_y \cos\theta_{pitch} \sin\phi_p \sin\theta_{yaw} - \dot{\theta}_{pitch} l_t \cos\phi_y \cos\theta_{yaw} \sin\phi_p \sin\theta_{pitch},
\end{aligned} \tag{8}$$

Assuming we neglects the tail's inertia about its own COM. The system kinetic energy and potentail energy:

$$\begin{aligned}
T &= \frac{1}{2} m_b (\dot{x}^2 + \dot{z}^2) + \frac{1}{2}[I_{xx}^{'} \omega_x^2 + I_{yy}^{'} \omega_y^2 + I_{zz}^{'} \omega_z^2 + 2I_{xy}^{'} \omega_x \omega_y + 2I_{xz}^{'} \omega_x \omega_z + 2I_{yz}^{'} \omega_y \omega_z] + \frac{1}{2} m_t \left[\dot{x}_t^{'2} + \dot{y}_t^{'2} + \dot{z}_t^{'2} \right] + \frac{1}{2} I_{m,t,s} (R_{t,s} \dot{\theta}_1)^2 + \frac{1}{2} I_{m,t,s} (R_{t,s} \dot{\theta}_2)^2 + \frac{1}{2} I_{m,t,l} (R_{t,l} \dot{\theta}_m)^2, \\
V &= m_b g z + m_t g z_t^{'}.
\end{aligned} \tag{9}$$

Here $I_{xx}^{'}$,$I_{yy}^{'}$, $I_{zz}^{'}$, $I_{xx}^{'}$, $I_{yy}^{'}$, $I_{zz}^{'}$ are the robot inertia parameters in the world frame. As noted in the paper, θ_1 is the angle measured counterclockwise along the vector $\vec{\boldsymbol{a}}$ from the Oxy - plane to the OAB - plane and θ_2 is the angle measured clockwise along the $\vec{\boldsymbol{d}}$ from the Oxy - plane to the ODC - plane. θ_m is the angle from the vector $\vec{\boldsymbol{p}}$ to one of the two bars not adjacent to the drive joints in the tail TMIS's last loop. We have:

$$\begin{aligned}
I_{xx}^{'} &= I_{xx} \cos{\phi_p}^2 \cos{\phi_y}^2 + I_{yy} \cos{\phi_p}^2 \sin{\phi_y}^2 + I_{zz} \sin{\phi_p}^2, \\
I_{yy}^{'} &= I_{yy} + I_{xx} \sin{\phi_y}^2 - I_{yy} \sin{\phi_y}^2, \\
I_{zz}^{'} &= I_{zz} \cos{\phi_p}^2 + I_{xx} \cos{\phi_y}^2 \sin{\phi_p}^2 + I_{yy} \sin{\phi_p}^2 \sin{\phi_y}^2, \\
I_{xy}^{'} &= \cos\phi_p \cos\phi_y \sin\phi_y (I_{xx} - I_{yy}), \\
I_{xz}^{'} &= \cos\phi_p \sin\phi_p (I_{xx} - I_{zz} - I_{xx} \sin{\phi_y}^2 + I_{yy} \sin{\phi_y}^2), \\
I_{yz}^{'} &= \cos\phi_y \sin\phi_p \sin\phi_y (I_{xx} - I_{yy}), \\
\dot{\theta}_1 &= \dot{\theta}_{pitch} - \frac{\sec^2(\theta_{yaw})\dot{\theta}_{yaw}}{tan(67.25°)} / \sqrt{1 - (\frac{tan(\theta_{yaw})}{tan(67.25°)})^2}, \\
\dot{\theta}_2 &= \dot{\theta}_{pitch} + \frac{\sec^2(\theta_{yaw})\dot{\theta}_{yaw}}{tan(67.25°)} / \sqrt{1 - (\frac{tan(\theta_{yaw})}{tan(67.25°)})^2}, \\
\dot{\theta}_m &= (-553.1346 \cdot l_t^2 + 305.4870 \cdot l_t - 45.7513) \cdot \dot{l}_t.
\end{aligned} \tag{10}$$

Here the relationship between $\dot{\theta}_m$ and $\dot{l}_t$ is simplified by the polynomial curve fitting. $R_{t,l}$ is the tail TMIS gearbox ratio (20:1). $\omega_x^{'} = -\dot{\phi}_y \sin\phi_p$, $\omega_y^{'} = -\dot{\phi}_p$, and $\omega_x^{'} = \dot{\phi}_y \cos\phi_p$ are the robot body's angular velocity in the world frame.

We derive the dynamcis equations based on:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \quad , i = 1,2,3,4,5,6,7. \tag{11}$$

Here $\boldsymbol{\tau} = [0 \quad 0 \quad \tau_{t,s,p} \quad \tau_{t,s,p} \quad \tau_{t,l} \quad 0 \quad 0]^T$. The dynamic equation matrix is listed in the file "flight-MCG.txt".