Hopping Robot Document

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1 Introduction

2 Dynamic Equation of 3D Model in Flight Phase

In this section, we are going to introduce the 3D model of our hopping robot. At this stage, we only focus on the 3D dynamics in flight phase.

2.1 Model

Fig.1 shows the model of the 3D system. Note that there is no leg in the model because the massless leg will not contribute to the dynamics of the system. To make the derivation simple, we decomposed the flight phase dynamics into two parts: "internal dynamics" and "external dynamics". "Internal dynamics" refers to the dynamics of the system observed in frame 0. Frame 0 is a downward(-z direction) accelerating frame, or a transnational non-inertia frame. In this frame, there is no relative velocity among all particles. Besides, unlike rotating frame, it won't generate pseudo force(centrifugal force and Coriolis force). Hence one can simply consider the system being put in outer space where no gravity acts on the system. In such situation, only joint angles are variables while center of mass of the system is stationary. While "external dynamics" refers to the dynamics of frame 0, which only includes motions in x y z direction and under constant gravity in z-direction.

2.2 Equations

2.2.1 Recursive Newton-Euler method

For "internal dynamics", we apply recursive Newton-Euler method to get the analytic form of joint torques first

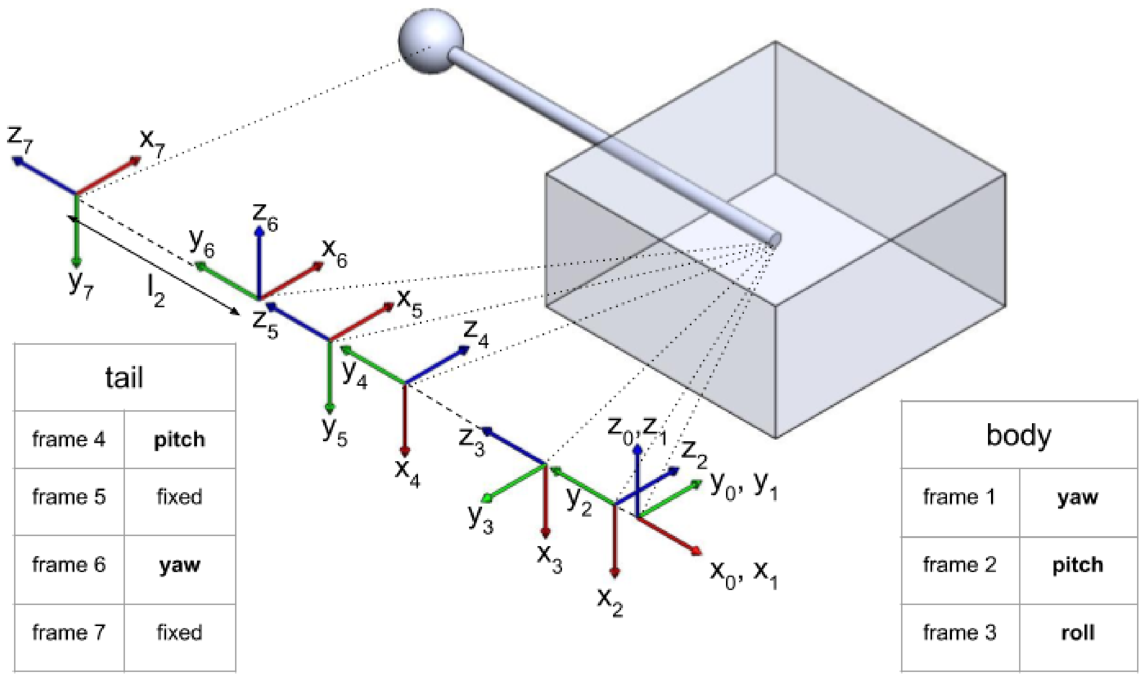

Figure 1: 3D model in flight phase. Frmae 5 is a fix frame, hence there are only 6DOFs.

Outward iteration: i from 0 to 6

$$
\begin{aligned}
{}^{i+1}\omega_{i+1} &= {}^{i+1}R_i\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} \\
{}^{i+1}\dot{\omega}_{i+1} &= {}^{i+1}R_i\,{}^{i}\dot{\omega}_i + {}^{i+1}R_i\,{}^{i}\omega_i \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} \\
{}^{i+1}\dot{v}_{i+1} &= {}^{i+1}R_i({}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times ({}^{i}\omega_i \times {}^{i}P_{i+1})) + {}^{i}\dot{v}_i \\
{}^{i+1}\dot{v}_{C_{i+1}} &= {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{C_{i+1}} + {}^{i+1}\omega_{i+1} \times ({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_{i+1}}) + {}^{i+1}\dot{v}_{i+1} \\
{}^{i+1}F_{i+1} &= m_{i+1}\,{}^{i+1}\dot{v}_{C_{i+1}} \\
{}^{i+1}N_{i+1} &= {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\omega_{i+1}
\end{aligned}
\tag{1}
$$

Inward iteration: i from 7 to 1

$$
\begin{aligned}
{}^{i}f_i &= {}^{i}R_{i+1}\,{}^{i+1}f_{i+1} + {}^{i}F_i \\
{}^{i}n_i &= {}^{i}N_i + {}^{i}R_{i+1}\,{}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_i + {}^{i}P_{i+1} \times {}^{i}R_{i+1}\,{}^{i+1}f_{i+1} \\
\tau_i &= {}^{i}n_i^T\,{}^{i}\hat{Z}_i
\end{aligned}
\tag{2}
$$

[!] missing description of the variables in above equations.

[!] missing the assignment of the first element in the iteration

i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	$-\pi/2$	0	$\pi/2+\theta_2$	0
3	$-\pi/2$	0	θ_3	0
4	$\pi/2$	0	θ_4	0
5	$-\pi/2$	0	$-\pi/2$	0
6	$\pi/2$	0	θ_6	0
7	$-\pi/2$	0	0	d_7+l_2

Figure 2: DH table of the linkage.

2.2.2 Dynamic Equation Formulation

After getting the joint torques(τ) from recursive Newton-Euler method, we can get the elements of the matrices: $M(q)_{6\times 6}, C(q,\dot{q})_{6\times 6}$. Since we are free from external contact, we don't have relative motion due to the effect of gravity. As a result, we can consider $G(q)_{6\times 1}$ as a zero vector.

$$\begin{aligned} M_{i,j} &= \frac{\partial}{\partial \ddot{\theta}_j}\tau_i \\ C_{i,j} &= \frac{1}{2}\sum_{k=1}^{6}\Big(\frac{\partial M_{i,j}}{\partial \theta_k} + \frac{\partial M_{i,k}}{\partial \theta_j} - \frac{\partial M_{k,j}}{\partial \theta_i}\Big)\dot{\theta}_k \\ G_i &= 0 \end{aligned} \tag{3}$$

$M_{1,1} =$
$S_2(I_{zz}S_2+m_2S_4(d_7+l_2)^2(S_2S_4-C_2C_3C_4)+m_2C_4S_6(d_7+l_2)^2(C_4S_2S_6-C_2C_6S_3+C_2C_3S_4S_6))+C_2(S_3(I_{yy}C_2S_3-m_2C_6(d_7+l_2)^2(C_4S_2S_6-C_2C_6S_3+C_2C_3S_4S_6))+C_3(I_{xx}C_2C_3-m_2C_4(d_7+l_2)^2(S_2S_4-C_2C_3C_4)+m_2S_4S_6(d_7+l_2)^2(C_4S_2S_6-C_2C_6S_3+C_2C_3S_4S_6)))$

$\mathrm{M}_{1,2} =$
$C_2(C_3(I_{xx}S_3+m_2C_4^2S_3(d_7+l_2)^2+m_2S_4S_6(d_7+l_2)^2(C_3C_6+S_3S_4S_6))-S_3(I_{yy}C_3+m_2C_6(d_7+l_2)^2(C_3C_6+S_3S_4S_6)))+m_2C_4C_6S_2(d_7+l_2)^2(C_3S_6-C_6S_3S_4)$

$\mathrm{M}_{1,3} =$
$S_2(I_{zz}+m_2S_4^2(d_7+l_2)^2+m_2C_4^2S_6^2(d_7+l_2)^2)-m_2C_2C_4C_6(d_7+l_2)^2(S_3S_6+C_3C_6S_4)$

$\mathrm{M}_{1,4} = m_2C_6(d_7+l_2)^2(C_4S_2S_6-C_2C_6S_3+C_2C_3S_4S_6)$