3 International Parity Conditions

International trade or exchange of goods and services across borders gives rise to international settlement with payments being made in different currencies. Discrepancies may arise as a consequence when the settlement is executed in one currency as against the other currency. Moreover, economic conditions and changes in economic conditions in different countries may take effect on the value of goods measured in different currencies and the relative values and opportunity costs of these currencies. International parities are concerned with the relationships between the values of two or more currencies and the respective economic conditions in these countries, and the way in which these relationships respond to the changing economic conditions in these countries. International parities are important since they establish relative currency values and their evolution in terms of economic circumstances, and cross border arbitrage may be possible when they are violated.

This chapter studies three international parity conditions: purchasing power parity (PPP), covered interest rate parity (CIRP), and uncovered interest rate parity (UIRP) or the international Fisher effect (IFE). The relationships between these parities are also examined and discussed. PPP is concerned with the relative values or the exchange rate of two currencies and the prices in the two countries. CIRP identifies the relationships between the spot exchange rate, the forward exchange rate, and the interest rates in the two countries. IFE establishes the relationship between changes in the exchange rate and the interest rate differential in the two countries, exploiting the relationship between the interest rate differentials and the inflation differentials.

3.1 Purchasing Power Parity

Purchasing power parity is a theory about exchange rate determination based on a plain idea that the two currencies involved in the calculation of the exchange rate have the same purchasing power for the same good sold in the two countries. Simply put, it is the law of one good, one price. In international finance, PPP means that the same goods or basket of goods should sell at the same price in different countries when measured in a common currency, in absence of transactions costs. The idea of PPP can be traced back to the medieval times, though the theory

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is more formally contributed to Cassel (1918, 1922), developed during a period that endured noticeable inflation in the world for the first time and the inflation rate as well varied noticeably between major trading nations. Therefore, PPP has been associated with inflation and inflation differentials from the start, being thrust in to a theory by real world issues. Cassel (1918) states that "The rate of exchange between two countries is primarily determined by the quotient between the internal purchasing power against goods of the money of each country.", and remarks that "The general inflation which has taken place during the war has lowered this purchasing power in all countries, though in a very different degree, and the rates of exchanges should accordingly be expected to deviate from their old parity in proportion to the inflation of each country." A testimony is then reached: "At every moment the real parity between two countries is represented by this quotient between the purchasing power of the money in the one country and the other. I propose to call this parity 'the purchasing power parity'."

There are two versions of PPP. One is absolute PPP and the other is relative PPP. The former studies the exchange rate for the two currencies in terms of the absolute prices for the same basket of goods in the two countries, and the latter examines how the exchange rate changes over time in response to changes in the price levels in the two countries.

3.1.1 Absolute Purchasing Power Parity

Absolute PPP is best described by the law of one good, one price. It is the application of the law of one good, one price in international finance which states that the same goods or basket of goods should sell at the same price in different countries when measured in a common currency, in absence of transactions costs, i.e.:

$$P_h = S \times P_f \tag{3.1}$$

where P_h is the price for a good or basket of goods in the domestic or home country, P_f is the price for the same good or basket of goods in the foreign country, and S is the exchange rate expressed as the units of the home currency per foreign currency unit. Equation (3.1) indicates that the exchange rate is the ratio of the prices for the same goods in the two countries:

$$S = \frac{P_h}{P_f} \tag{3.2}$$

For example, if the price of a basket of apples is \$3 in the US and the price of a basket of same apples is £2 in the UK, then according to PPP, the direct quote of the exchange rate in the US must be S = P_h/P_f = \$3/£2 = \$1.5/£, and the direct quote of the exchange rate in the UK must be S = P_h/P_f = £2/\$3 = £0.6667/\$. Suppose the exchange rate departs from \$1.5/£, one might be able to make profit by buying apples in the UK (US) and transporting the apples to the US (UK) for sale.

Let us examine what may happen, assuming that the basket of apples still sells at £2 in the UK and \$3 in the US, but the exchange rate is \$1.6/£ instead. Measured in the US dollar as the common currency, the price of the basket of apples is \$3 in the US but \$3.2 (£2×\$1.6/£) in the UK; and measured in the British pound as the common currency, the price of the basket of apples is £2 in the UK but £1.875 (\$3/\$1.6/£) in the US. The law of one good one price, or absolute PPP, is violated, and consequently, there might be arbitrage opportunities for cross border trade activities. In this case, the pound is over valued and the dollar is under valued, or the same apples are cheaper in the US than in the UK. Therefore, it is profitable to export the apples from the US to the UK, e.g.:

Step1: Buy 10 baskets of apples for \$30 in the US;
Step 2: Transport the apples to the UK for sale for £20;
Step 3: Exchange the pound for the dollar at the exchange rate of \$1.6/£, resulting in \$32.

The profit from this trade activity is \$2 or the profit margin is 6.7% [(32-30)/30], ignoring the transportation cost and other transaction costs. Obviously, there can be no profit if the transportation cost is equal to or higher than \$2 per 10 baskets.

We can infer two implications from the above example. Absolute PPP is a *sufficient* condition for no arbitrage in international trade and finance, but it is not a *necessary* condition. Upholding of absolute PPP guarantees the elimination of arbitrage, while a violation of absolute PPP may or may not give rise to arbitrage opportunities, depending on the level of associated costs in the international trading process. Let examine a second example:

Suppose that the price of a Mini Cooper is €18,000 in France and S = €1.4/£, then the price of the same Mini Cooper in the UK should be $P_f = P_h/S$ = €18,000/(€1.4/£) =£12,857. We ask three questions. (a) If instead the Mini Cooper is sold for £14,250 in the UK, what may happen? (b) If HM Customs and Excise of the UK levies an import duty of 20% on manufactured goods, does arbitrage still exist? (c) If the importer incurs a 15% cost to cover transportation, administrative and other fees, does arbitrage still exist? We discuss these three questions in the following.

(a) It is cheaper to buy the car in France. Therefore, it is profitable to import the cars from France and sale them in the UK, e.g.:

Step1: Buy 100 Mini Cooper cars in France for 100×€18,000 = €1,800,000;

Step 2: Transport the cars to the UK for sale for 100×£14,250 = £1,425,000;

Step 3: Exchange the pound for the euro at the exchange rate of €1.4/£, resulting in €1,995,000.

The profit from this trade activity is €195,000 and the profit margin is 10.83% [(1,995,000-1,800,000)/1,800,000]. Arbitrage will take place. The price of Mini Cooper will rise in France and fall in the UK until the arbitrage opportunity is wiped out.

(b) Now the cost for a Mini Cooper in the UK through this exporting channel will be €18,000/€1.4/£×120% = £15,428.57, higher than £14,250, the price of a Mini Cooper currently prevailing in the UK. So, it is not profitable to import Mini Cooper from France in this case.

(c) Now the cost for a Mini Cooper in the UK through this exporting channel will be €18,000/€1.4/£×115% = £14,785.71, higher than £14,250, the price of a Mini Cooper currently prevailing in the UK. So, it is not profitable to import Mini Cooper from France in this case.

From the above example, it is observed that transaction costs, such as transportation, taxes and tariffs, commission charges, prevent seemingly existed arbitrage opportunities from materialising. Therefore, there would be no arbitrage opportunities while in the meantime the law of one price does not hold in two countries, if the countries are geographically distant, and/or are not engaged in or committed to free trade. The prices for the same goods remain distorted in the two countries, since it is not profitable to exploit the difference.

Having studied PPP theory and examined the above cases, it might be interesting to present the well publicised big Mac standard of *The Economist* in the following, and reflect on what the figures in Table 3.1 suggest. In addition to the Big Mac index, *The Economist* has been compiling a Starbucks "tall latte" index since 2004. Both the Big Mac index and the Starbucks index are based on the theory of PPP and serve to provide a simple, intuitive vindication of PPP. More serious is the Penn World Table assembled by the economists at the University of Pennsylvania (*cf.* Heston *et al.* 2006). The latest version of the Penn World Table, PWT6.2, provides PPP and national income accounts converted to international prices for 188 countries for some or all of the years 1950-2004. Their work can be traced back to 1978 (Kravis *et al.* 1978) and has evolved into the contemporary time (Summers and Heston 2005).

However, research does not rely on evidence from one or two examples, but from many that can be representative. In the case of PPP, it means that a substantial number of goods should be included in the comparison of the relative prices of goods in the two countries in concern, in relation to the exchange rate. In this regard, research on PPP usually adopts the weighted average of the prices for a basket of goods, in the form of the consumer price index (CPI) or other kinds of price indices, rather than the prices of individual goods, in empirical investigations at specific times or during certain time periods. Equation (3.2) then becomes:

Table 3.1. The Big Mac Standard of The Economist

A feast of burgernomics
The Big Mac index

	Big Mac prices		Implied PPP* of the dollar	Actual dollar exchange rate Jan 31st	Under (-)/over (+) valuation against the dollar, %
	In local currency	in dollars			
United States†	$3.22	3.22			
Argentina	Peso 8.25	2.65	2.56	3.11	-18
Australia	A$3.45	2.67	1.07	1.29	-17
Brazil	Real 6.4	3.01	1.99	2.13	-6
Britain	£1.99	3.90	1.62‡	1.96‡	+21
Canada	C$3.63	3.08	1.13	1.18	-4
Chile	Peso 1,670	3.07	519	544	-5
China	Yuan 11.0	1.41	3.42	7.77	-56
Colombia	Peso 6,900	3.06	2,143	2,254	-5
Costa Rica	Colones 1,130	2.18	351	519	-32
Czech Republic	Koruna 52.1	2.41	16.2	21.6	-25
Denmark	DKr27.75	4.84	8.62	5.74	+50
Egypt	Pound 9.09	1.60	2.82	5.70	-50
Estonia	Kroon 30	2.49	9.32	12.0	-23
Euro area§	€2.94	3.82	1.10**	1.30**	+19
Hong Kong	HK$12.0	1.54	3.73	7.81	-52
Hungary	Forint 590	3.00	183	197	-7
Iceland	Kronur 509	7.44	158	68.4	+131
Indonesia	Rupiah 15,900	1.75	4,938	9,100	-46
Japan	¥280	2.31	87.0	121	-28
Latvia	Lats 1.35	2.52	0.42	0.54	-22
Lithuania	Litas 6.50	2.45	2.02	2.66	-24
Malaysia	Ringgit 5.50	1.57	1.71	3.50	-51
Mexico	Peso 29.0	2.66	9.01	10.9	-17
New Zealand	NZ$4.60	3.16	1.43	1.45	-2
Norway	Kroner 41.5	6.63	12.9	6.26	+106
Pakistan	Rupee 140	2.31	43.5	60.7	-28
Paraguay	Guarani 10,000	1.90	3,106	5,250	-41
Peru	New Sol 9.50	2.97	2.95	3.20	-8
Philippines	Peso 85.0	1.74	26.4	48.9	-46
Poland	Zloty 6.90	2.29	2.14	3.01	-29
Russia	Rouble 49.0	1.85	15.2	26.5	-43
Saudi Arabia	Riyal 9.00	2.40	2.80	3.75	-25
Singapore	S$ 3.60	2.34	1.12	1.54	-27
Slovakia	Crown 57.98	2.13	18.0	27.2	-34
South Africa	Rand 15.5	2.14	4.81	7.25	-34
South Korea	Won 2,900	3.08	901	942	-4
Sri Lanka	Rupee 190	1.75	59.0	109	-46
Sweden	SKr32.0	4.59	9.94	6.97	+43
Switzerland	SFr6.30	5.05	1.96	1.25	+57
Taiwan	NT$75.0	2.28	23.3	32.9	-29
Thailand	Baht 62.0	1.78	19.3	34.7	-45
Turkey	Lire 4.55	3.22	1.41	1.41	nil
UAE	Dirhams 10.0	2.72	3.11	3.67	-15
Ukraine	Hryvnia 9.00	1.71	2.80	5.27	-47
Uruguay	Peso 55.0	2.17	17.1	25.3	-33
Venezuela	Bolivar 6,800	1.58	2,112	4,307	-51

Sources: McDonald's; *The Economist*

*Purchasing-power parity: local price divided by price in United States
†Average of New York, Atlanta, Chicago and San Francisco ‡Dollars per pound
§Weighted average of prices in euro area **Dollars per euro

Source: The Economist

$$S_t = \frac{P_{h,t}}{P_{f,t}} = \frac{\sum_{i=1}^{N_h} w_h^i p_{h,t}^i}{\sum_{i=1}^{N_f} w_f^i p_{f,t}^i} \tag{3.3}$$

where S_t is the exchange rate at t, $P_{h,t}$ is the price level at t in the home country, $P_{f,t}$ is the price level at t in the foreign country, $p_{h,t}^i$ is the price for the i^{th} good at time t in the home country, w_h^i is the weight of the i^{th} good in the price index in the home country, N_h is total number of goods included in the price index in the home country, $p_{f,t}^i$ is the price for the i^{th} good at time t in the foreign country, w_f^i is the weight of the i^{th} good in the price index in the foreign country, and N_f is total number of goods included in the price index in the foreign country. Ideally, $N_h = N_f$ and $w_h^i = w_f^i$ for same, if not identical, goods for verifying the parity conditions, i.e., the weight for the same good in the price index is the same and the number and types of goods included in the price index are the same in the all countries under investigation. But these are usually not the case and, consequently, a divergence from equation (3.3) is not necessarily a rejection of PPP. Issues related to empirical tests of PPP will be dealt with in Section 3.1.4. It is worthwhile pointing out that a logarithm version of equation (3.3):

$$s_t = p_{h,t} - p_{f,t} \tag{3.4}$$

is typically used in empirical research, where $s_t = Ln(S_t)$, $p_{h,t} = Ln(P_{h,t})$ and $p_{f,t} = Ln(P_{f,t})$.

3.1.2 Real Exchange Rates

Having studied absolute PPP, we can progress to introduce the concept of the real exchange rate. The real exchange rate is the exchange rate adjusted by the price levels in the two countries, defined as follows:

$$Q = S \times \frac{P_f}{P_h} \tag{3.5}$$

where Q is real exchange rate. Real exchange rates are one if absolute PPP holds. Home or domestic currencies are over valued if $Q < 1$ and are under valued if $Q > 1$. Foreign currencies are under valued if $Q < 1$ and are over valued if $Q > 1$.

A logarithm version of the real exchange rate is:

$$q = s - (p_h - p_f) \tag{3.6}$$

where $q = Ln(Q)$, $s = Ln(S)$, $p_h = Ln(P_h)$ and $p_f = Ln(P_f)$. The real exchange rate in logarithms is zero when absolute PPP holds. Home or domestic currencies are over valued if q is negative and are under valued if q is positive. Foreign currencies are under valued if q is negative and are over valued if q is positive.

The real exchange rate is a useful concept and simple means to gauge the strength of currencies. For example, in the above Mini case, the real exchange rate of the euro vis-à-vis the pound in France is:

$$Q = S \times \frac{P_f}{P_h} = €1.4/£ \times \frac{£950}{€1200} = 1.1083 > 1 \cdot$$

So, the euro is under valued and the pound is over valued if Mini Cooper cars are the only goods traded between France and the UK. From the view of the UK as the domestic country, the real exchange rate of the pound against the euro in the UK is:

$$Q = S \times \frac{P_f}{P_h} = £0.7143/€ \times \frac{€1200}{£950} = 0.9023 < 1,$$

which indicates again that the pound is over valued and the euro is under valued.

Since the real exchange rate is the exchange rate adjusted by the price levels of the two countries, it is absolute PPP in a different expression. Tests for the validity of absolute PPP are equivalent to testing whether the real exchange rate is one. Many empirical studies adopt this line of enquiry.

3.1.3 Relative Purchasing Power Parity

Unless it is in a one-good case where absolute PPP can be exactly checked, examinations of PPP involve aggregate price levels in the two countries. Aggregate price levels are merely index numbers that provide a good indication of changes in price levels over time but have little meaning about their absolute value at a specific time. For example, a statement such as “the US CPI is 135 in the fourth quarter of 1998” itself does not tell us whether the price level is high a low. So, aggregate price levels must have a base year, at which the price level is set to be 100 and against which the size of increase or decrease can be measured. e.g., if the base year is 1990 in the above case, then a CPI of 135 means that the price level at that time is 35 percent higher than that in 1990. That is, in the fourth quarter of 1998, one can use \$135 to purchase goods that are only worth \$100 in 1990. In other words, the purchasing power of $135 in the fourth quarter of 1998 is equal to the purchasing power of $100 in 1990.

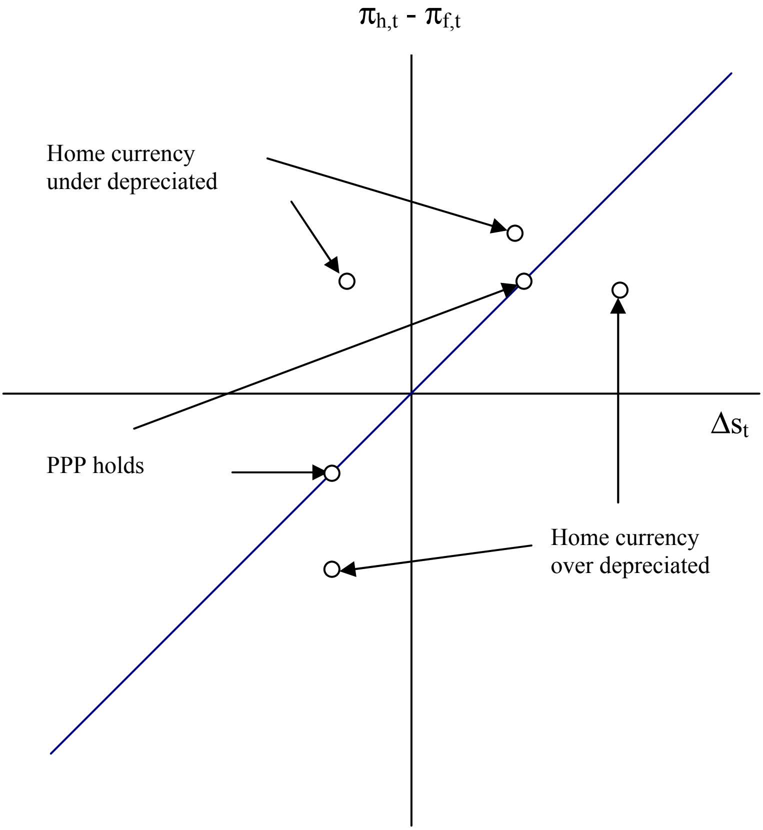

Figure 3.1. Graphical illustration of relative PPP

With the above considerations, it is logical to introduce a relative version of PPP. Relative PPP examines the relationship between changes in exchange rates and changes in the aggregate price levels in the two countries involved. Taking log differences of the absolute PPP equation yields:

$$\Delta s_t \approx \Delta p_{h,t} - \Delta p_{f,t} = \pi_{h,t} - \pi_{f,t} \tag{3.7}$$

where $\Delta s_t = \ln(S_t) - \ln(S_{t-1})$ is the (percentage) change in exchange rates in the period t-1 to t; $\pi_{h,t} = \Delta p_{h,t} = \ln(P_{h,t}) - \ln(P_{h,t-1})$ is the (percentage) change in the price levels, or the inflation rate, in the domestic country in the same period; and $\pi_{f,t} = \Delta p_{f,t} = \ln(P_{f,t}) - \ln(P_{f,t-1})$ is the (percentage) change in the price levels, or the inflation rate, in the foreign country in the same period. Price indices, usually consumer price indices (CPI), are used for the measurement of inflation.

Relative PPP establishes an evolution path for exchange rate changes, which is a consequence of relative price developments in the two countries. Suppose absolute PPP holds at time *t*-1 and relative PPP holds in the period *t*-1 to *t*, then absolute PPP holds at time *t* also. The size of the change in the exchange rate reflects the relative purchasing power gain of one of the two currencies, or the relative purchasing power loss of the other currency, to the right extent when relative PPP holds. However, if absolute PPP does not hold at time *t*-1 but relative PPP holds in the period *t*-1 to *t*, then absolute PPP does not hold at time *t* either, even if the exchange rate has adjusted to reflect correctly the relative purchasing power gain or loss during this period. This exposes a problem with relative PPP: it is possible that absolute PPP or PPP in its original form does not hold at any times while relative PPP holds at all times. One of the reasons for adopt relative PPP is that absolute PPP is difficult, if not impossible, to be tested and verified in empirical research. But test results from relative PPP cannot substitute those for absolute PPP. Confirmation of relative PPP does not necessarily lead to the conclusion that the two currencies have the right purchasing power with the given exchange rate – parity may or may not exist.

Relative PPP indicates that: (a) the domestic currency will depreciate if inflation in the domestic country is higher than that in the foreign country, and (b) the domestic currency will depreciate to the extent equal to the inflation differential between the two countries. For example, if inflation in the US was 2% and inflation in Japan was 0% in 2000, the yen should have appreciated by 2% vis-à-vis the dollar in the same period, according to relative PPP.

Figure 3.1 is a graphical illustration of relative PPP. Points on the 45 degree line indicate that changes in the exchange rate are equal to the two countries' inflation differentials and relative PPP holds. A point to the right of the 45 degree line means that the home currency is over depreciated. It can be that an increase in the exchange rate is greater than the inflation differential between the home country and the foreign country, e.g., $\Delta s_t = 2\%$, $\pi_{h,t} = 3\%$ and $\pi_{f,t} = 2\%$; or an increase in the exchange rate while the domestic inflation is lower than the foreign inflation, e.g., $\Delta s_t = 1\%$, $\pi_{h,t} = 1\%$ and $\pi_{f,t} = 2\%$; and so on. A point to the left of the 45 degree line suggests that the home currency does not depreciate enough to reflect the inflation differential between the two countries in the period.

3.1.4 Factors Contributing to the Departure from PPP

PPP may or may not hold under varied circumstances and with a number of contributing factors playing different roles. These factors and their roles in contributing to the departure from PPP under varied circumstances have been subject to extensive scrutiny ever since international trade became an importance and integrated part of the world's economic activity. This section provides intuitive explanations of, and a brief summary on, the roles these factors play and the ways in which these factors play their roles.

Construction of Price Indices

PPP is based on the concept of comparing identical baskets of goods in two economies. An important problem facing researchers in this aspect is that different countries usually attach different weights to different goods and services when constructing their price indices. This means that it is difficult to compare the goods in the two baskets - it is difficult to match the attributes of the goods in the two baskets and it is probably only by chance to allocate the same weights to the same goods in the baskets. This issue is most probably significant in testing for PPP between developed economies and developing economies that have vastly different consumption patterns. People in developing countries usually spend a high proportion of their income on basics such as food and clothing while these sorts of consumption take up a much smaller proportion of people's expenditure in developed economies. But, we should note that it is the difference in the attributes of goods and weightings that primarily give rise to international trade, along with taking advantages in relative prices. Consequently, testing for PPP was, is and will always be difficult.

Transportation Costs and Trade impediments

Transportation costs and restrictions on trade may be substantial enough to prevent some goods and services from being traded between countries. Studies, such as Frenkel (1981), note that PPP holds better when the countries concerned are geographically close and trade linkages are high, which can partly be explained by transportation costs and the existence of other trade impediments such as tariffs. Nonetheless, since transportation costs and trade barriers do not change dramatically over time they are not factors explaining sufficiently for the failure of relative PPP.

Relative Prices of Non-tradable Goods and the Balassa-Samuelson Effect

The existence in all countries of non-traded goods and services whose prices are not linked internationally allows systematic deviations even from relative PPP. Because the price of a non-tradable is determined entirely by its domestic supply and demand, changes in these may cause the domestic price of a broad commodity basket to change relative to the foreign price of the same basket. Other things kept unchanged, a rise in the price of a country's non-tradables will raise its price level relative to foreign price levels, or the purchasing power of any given currency will fall in countries where the prices of non-tradables rise.

The Balassa-Samuelson effect (Balassa 1964; Samuelson 1964), also known as the Balassa-Samuelson hypothesis and Harrod-Balassa-Samuelson effect (Harrod 1933; Balassa 1964; Samuelson 1964; Samuelson 1994) offers explanations to departure from PPP. It suggests that price levels in wealthier countries are systematically higher than those in poorer countries, because productivity varies more amongst countries in traded goods sectors than in non-traded goods sectors, which is also the underlying assumption of the hypothesis. While prices equalise between countries for tradable goods and wage rates equalise in individual countries between tradable goods sectors and non-tradable goods sectors owing to the homogenous labour market, the prices of non-tradables are higher in countries with higher productivity in traded goods sectors. Consequently, the overall price or price index in the rich countries is higher than that in poor countries. The hypothesis can be illustrated mathematically as follows, with the simplified assumption that labour is the only factor of production.

There are traded goods and non-traded goods sectors in both countries. The price of traded goods is equalised across countries through international arbitrage. Consider a two-country two-goods case where one good is tradable and the other is non-tradable. International arbitrage means:

$$P_T = SP_T^* \tag{3.8}$$

where P_T is the price of the traded good in the domestic country, P_T^* is the price of the traded good in the foreign country, and S is the exchange rate. The wage rate is equal to the price multiplied by the marginal product of labour in both tradables sector and non-tradables sector:

$$w_T = P_T MPL_T = P_{NT} MPL_{NT} = w_{NT} \tag{3.9}$$
$$w_T^* = P_T^* MPL_T^* = P_{NT}^* MPL_{NT}^* = w_{NT}^* \tag{3.10}$$

where w_T is the wage rate in the tradables sector in the domestic country, w_{NT} is the wage rate in the non-tradables sector in the domestic country, MPL_T is the marginal product of labour in the tradables sector in the domestic country and MPL_{NT} is the marginal product of labour in the non-tradables sector in the domestic country. Superscript * denotes the foreign country variable. From equation (3.9) and equation (3.10), we have:

$$\frac{P_{NT}}{P_T} = \frac{MPL_T}{MPL_{NT}} \tag{3.11}$$
$$\frac{P_{NT}^*}{P_T^*} = \frac{MPL_T^*}{MPL_{NT}^*} \tag{3.12}$$

Suppose $\dfrac{MPL_T}{MPL_{NT}} > \dfrac{MPL_T^*}{MPL_{NT}^*}$, then $\dfrac{P_{NT}}{P_T} > \dfrac{P_{NT}^*}{P_T^*}$. It follows that $P_{NT} > SP_{NT}^*$. From this analysis, the assumption that the productivity difference in terms of the marginal product of labour is small between countries in the non-traded good can be relaxed. The Balassa-Samuelson effect can be observed as long as the ratio of the marginal product of labour in the tradables sector to that in the non-tradables sector differs between two countries. The price of non-tradables is higher in the country with a higher ratio of the marginal product of labour in the tradables sector to that in the non-tradables sector, leading to an overall higher price index in that country.

Empirical observations of the Balassa-Samuelson effect are documented in a survey by Rogoff (1996). Further evidence shows that tradable goods prices are nowhere as dissimilar internationally as those of non-traded goods. Consequently the overall higher price index in rich countries is mainly due to the fact that non-tradable goods prices are higher in developed than developing countries. Ricci and MacDonald (2005) investigate the impact of the distribution sector on the real exchange rate, controlling for the Balassa-Samuelson effect, as well as other macro variables. Their main results indicate that an increase in the productivity and competitiveness of the distribution sector with respect to foreign countries leads to an appreciation of the real exchange rate, similarly to what a relative increase in the domestic productivity of tradables does, which contrasts with the result when the distribution sector is consigned to the non-tradables sector. With their use of the services from the distribution sector in the tradables sector however, their results seem to coincide, not to contrast, with the expected results.

3.1.5 Empirical Tests and Evidence on PPP

There is a huge body of literature on the study of the validity of PPP. These studies feature different sample periods, different currencies, different specifications, and different estimation methods. The majority of these studies are backed up by contemporarily popular econometric models. This section attempts to present, analyse and explain the empirical evidence in and verdict on PPP in an intuitive way, avoiding technical econometric jargons whenever possible.

With regard to the coverage of recent empirical research, while the currencies investigated differ, the majority of studies focus on bilateral exchange rates of industrialised countries against the US dollar. Frequently examined sample periods are the era of flexible exchange rates that began in the 1970s, though there are studies where the sample period covers two centuries. The price indices used are also different, with the most commonly employed for research being the consumer price index (CPI), followed by the wholesale price index (WPI), producer price index (PPI) and the gross domestic product (GDP) deflator.

Results and findings from the empirical literature are broadly mixed. In the following, we briefly review some of the representative studies in the area, which traces the evolution of the research, covering static tests of PPP based on linear regressions of exchange rates on relative prices, unit root tests on the stationarity of real exchange rates, cointegration tests on the comovement between the nominal exchange rate and relative prices, and panel data tests on the convergence to PPP using cross country data.

Regression Tests of PPP

In the mid 1970s, the conventional tests of PPP involved the use of regression analysis, that is, to test the coefficient restrictions imposed by PPP. Frenkel (1978) runs regression[1] for the monthly dollar-pound, franc-dollar and franc-pound exchange rates over the period of February 1921 to May 1925 during which exchange rates were flexible. His results are generally supportive of the PPP hypothesis in both the absolute and relative versions. However, Frenkel (1981) uses data from the recent float for the dollar-pound, dollar-franc and dollar-mark exchange rates and finds that PPP is not supported by the data. Similar results are reported by Krugman (1978) for both the inter-war and the recent float. Applying the long-horizon regression approach and considering 60 bilateral inter-country relations for 21 OECD countries, Serletis and Gogas (2004) test the PPP hypothesis during the recent floating exchange rate period and provide weak evidence in favour of PPP.

Stationarity of Real Exchange Rates

In the early 1980s, with the development of econometric techniques, a further test of PPP shifts attention to examine the time series properties of real exchange rates. Studies at this stage take into account the non-stationarity of variables. A means of testing for PPP in a framework that allows for non-stationarity is to define the real exchange rate and to test if this series is non-stationary. If the null hypothesis of a unit root or non-stationarity cannot be rejected, then the real exchange rate contains a unit root and does not revert to its mean value, indicating, consequently, that PPP does not hold in the long-run. In determining whether a variable follows a non-stationary process, the unit root test is one of the commonly employed econometric methods. With this method, many previous studies have reported that non-stationarity cannot be rejected for the real exchange rate series in the floating period, for example, Adler and Lehmann (1983), Darby (1983), Hakkio (1984), Meese and Rogoff (1988) and Baillie and McMahon (1989). Consequently, these studies reject the notion of long-run PPP, implying that shocks have a permanent effect on the level of the real exchange rate, while changes are unpredictable, and that there is little or no tendency for nominal exchange rates and prices to adjust in such a way as to promote PPP. Some recent studies also testify the non-stationarity of real exchange rates, and therefore the rejection of PPP, including

[1] The regression equation is $s_t = \alpha + \beta(p_{h,t} - p_{f,t}) + v_t$, to test the restrictions $\alpha = 0$ and $\beta = 1$. PPP in the form of equation (3.4) is confirmed when $\alpha = 0$ and $\beta = 1$.

Borsic and Beko (2006), among others; whereas Choi (2004), Nusair (2004), Narayan (2005) and Murray and Papell (2005) report mixed results accompanied by complications. Yabu (2004) explores the implications of transportation costs. A standard model with transportation costs implies that real exchange rates should follow a band threshold model where the process is a random walk within the bands and mean-reverting outside them. Because of technological improvements, these bands should narrow over time. No such evidence is found in his study supported by the data.

While most of the empirical evidence does not reject the unit root hypothesis for real exchange rates for most countries in the post Bretton Woods era and reject the notion of long-run PPP, there are nevertheless a few studies that are in favour of PPP, especially those carried out in the 00s of this century employing new model specifications and designs. For example, using monthly data between September 1975 and May 1981, Cumby and Obstfeld (1984) reject the non-stationarity hypothesis for the real exchange rate between the US dollar and the Canadian dollar. Abuaf and Jorion (1990) examine the real exchange rates of 10 industrialised countries for the recent floating period and the period between 1900 and 1972 and find mean-reversion in the real exchange rate. More recently, Holmes and Wang (2005) investigate the possibility that the adjustment toward long-run relative PPP is dependent upon the nature of deviations from PPP that are experienced. They employ a new cointegration test that tests for an asymmetric adjustment towards parity with respect to positive and negative deviations of the real exchange rate from its equilibrium value. Using a sample of ten African economies with data taken from the post Bretton Woods floating exchange rate era, long-run PPP holds in eight of these cases if an explicit distinction is made between positive and negative deviations. In a further study, Holmes and Wang (2006) scrutinise the asymmetric adjustment process toward parity for a sample of nine Asian economies during the post-Bretton Woods floating exchange rate era.. They find that long-run purchasing power parity is most likely to hold with respect to positive deviations only in most case in the sample. Imbs *et al.* (2005) show the importance of a dynamic aggregation bias in accounting for the PPP puzzle. Established time series and panel methods are claimed to fail to control for heterogeneous dynamics in the components of the aggregate real exchange rate. Using Eurostat data, they find that the estimated persistence of real exchange rates falls dramatically when heterogeneity is taken into account. Its half-life may fall to as low as eleven months. Lean and Smyth (2007) apply univariate and panel Lagrange Multiplier unit root tests with structural breaks to real exchange rates for 15 Asian countries and find evidence of PPP for two thirds in the sample. Wu *et al.* (2004) find that real exchange rates among Pacific Basin countries are stationary, but subject to a one-time structural break. Assaf (2008) also claims that his research has provided new evidence on the stationarity of bilateral real exchange rates, after allowing for regime changes. Akinboade and Makina (2006) offer the similar results that the number of cases for stationary real exchange rates has increased once structural breaks have been taken into account. Applying a kind of nonlinear unit root test to the bilateral real exchange rates of both European and other industrial countries with the French franc and German mark, the euro after

1998, as well as the US dollar as numeraire currencies, Zhou *et al.* (2008) document the results that provide stronger support for PPP than any earlier studies of bilateral PPP for industrial countries and suggest that PPP tends to hold well within the EU even before the adoption of the euro, and the evidence for PPP becomes more significant for both EU and non-EU countries when the sample period is extended to the euro era. Examining real effective exchange rate data for 90 developed and developing countries in the post-Bretton Woods period, Cashin and McDermott (2004) also lend support for PPP. Using long span real exchange rates of industrialised countries, Papell and Prodan (2006) investigate two alternative versions of PPP. One is reversion to a constant mean in the spirit of Cassel and the other is reversion to a constant trend in the spirit of Balassa and Samuelson and both account for structural changes. They find evidence of PPP for 14 countries applying the unit root tests in the presence of restricted structural change, compared with nine countries with conventional unit root tests. Kanas (2006) revisits the evidence on PPP in the twentieth century, allowing for Markov regime switching in regression. It is alleged that there are periods over which the real exchange rate is stationary and PPP holds and periods over which the real exchange rate is non-stationary and PPP does not hold for most countries. Further it is found that the probability of the real exchange rate being stationary is less than 50% for most countries. Cashin and McDermott (2006) test for PPP using real effective exchange rate data for 90 developed and developing countries in the post Bretton Woods period. They claim that support for PPP is found, with the majority of countries experiencing finite deviations of real exchange rates from parity. The speed of parity reversion is found to be much faster for developed countries than for developing countries; and fairly plainly, to be considerably faster for countries with flexible nominal exchange rate regimes than countries with fixed nominal exchange rate regimes. Similar results have been obtained by Sollis (2005) that offer a certain degree of support to PPP, utilising univariate smooth transition models. The empirical study of Alba and Park (2005) also supports the validity of PPP with the case of Turkey

However, doubts were raised at this stage of the debate over the power of standard tests. Given the phenomenal volatility of floating exchange rates, it can be very hard to distinguish between slow mean reverting and a non-stationary the real exchange rate, especially for the post Bretton Woods data, since the current floating exchange rate period is too short to assess accurately the validity of PPP. Froot and Rogoff (1994) calculated that, if PPP deviations damp sufficiently slow, suppose that the half life of PPP deviations is three years, then it may require 76 years for one to be able to reject reliably the existence of a unit root in real exchange rates. With a longer half life, even longer data would be required. Put it differently, PPP deviations can be slow to reverse, and conventional econometric techniques have low power to identify stationary but persistent dynamics. Therefore, it is argued that the reason that most previous studies fail to reject a unit root in real exchange rate time series is probably to the poor power of the tests employed rather than the evidence against long-run PPP. Lopez *et al.* (2005) testify that whether PPP holds in the long-run is still controversial. They argue that the results are quite sensitive to the lag selection in unit root tests, and the number of the cases in support of PPP falls significantly with their superior lag selection methods.

One of the approaches to address the low power problem is to expand the sample period. Frankel (1986), using 116 years (1869-1984) of annual data for dollar-pound real exchange rate, is able to reject the unit root hypothesis for the whole period between 1869 and 1984, but is unable to reject the hypothesis using data from 1945 to 1984. He finds that PPP deviations have an annual decay rate of 14 percent and a half life of 4.6 years. The long-horizon data sets are also employed by several studies during the 1990s, with a variety of different approaches, and almost invariably tend to find evidence of mean reversion in the real exchange rate. Diebold *et al.* (1991), for example, look at the data during the Gold Standard era, with shortest data spanning 74 years and longest 123 years, and are able to reject the unit root hypothesis by adopting the fraction integration model. They find that PPP holds in the long-run for each of the currencies and the typical half life of a shock to parity is approximately three years. Similarly, non-stationarity is rejected by Grilli and Kaminsky (1991) for the time period between 1885 and 1986 and by Glen (1992) for the period spanning 1900 and 1987. Employing two centuries' annual exchange rate data, Lothian and Taylor (1996) find strong evidence of mean reversion with an estimated half life being 4.7 years for the dollar-pound exchange rate and 2.5 years for the franc-pound exchange rate. Hence, based on studies using long historical data sets, Lothian (1998) concludes that real exchange rates contain economically important mean-reverting components and that, as a result, PPP is still a useful approximation. Moreover, Lothian (1998) shows that the difficulty in finding evidence of PPP with the US dollar as the numerator currency is primarily the result of the 1979-1982 period during which the US dollar first strongly depreciated and then strongly appreciated.

Although the above long time span based studies do find the mean reversion of real exchange rate, they have ignored changes in exchange rate regimes. It is not clear whether the findings based on long time period data confirm simply the presence of parity reversion in the pre modern floating period or show its presence over the recent float as well. In addition, it is also argued that tests of unit roots in the real exchange rate preclude the coefficients, that is, the construction of the real exchange rate implicitly restricts the coefficients corresponding to the domestic and foreign price levels as -1 and 1. Due to measurement errors as well as trade barriers, the exchange rate may not move one by one with price levels as implied by PPP. In this regard, the recently developed cointegration methodology offers a more appropriate econometric test means for this kind of relationship.

Comovement Between the Nominal Exchange Rate and Relative Price Levels

If two or more variables, such as the exchange rate and the corresponding price levels, are cointegrated, then in the long-run these variables will settle down together in a unique way, without wandering away far apart. Therefore the implication of long-run PPP is generally interpreted as the comovement of the nominal exchange rate and the relative price levels between the two countries in concern over time.

The early application of the cointegration approach to testing for PPP is based on the Engle and Granger (1987) two-step procedure. Taylor (1988), Enders (1988), Mark (1990) and Patel (1990), among others, carried out investigations in this framework for the recent floating period; whereas Kim (1990) employs data sets spanning most of the 20th century. In general, the empirical tests adopting the Engle-Granger two-step procedure generally fail to observe long-run tendencies for nominal exchange rates and relative prices to settle down on an equilibrium track.

The above Engle-Granger procedure based tests have been criticised by a number of more recent studies, for example, MacDonald (1993), Cheung and Lai (1993), Cochrane and DeFina (1995), who have argued that the failure to find a cointegration relationship between the exchange rate and relative prices may be due to the econometric method used, rather than the absence of a long-run relationship. These studies advocate the use of then newly emerged multivariate cointegration methodology of Johansen (1988) and Johansen and Juselius (1990), commonly known as the Johansen procedure. This maximum likelihood based approach allows testing for PPP in a trivariate framework and avoids some drawbacks of the Engle-Granger two-step regression procedure.

Adopting the Johansen procedure, MacDonald (1993) tests for a long-run relationship between exchange rates and relative prices for five bilateral US dollar exchange rates against the Canadian dollar, the French franc, the German mark, the Japanese yen and the British pound, and also tests for the proportionality of the exchange rate with respect to relative prices, using post Bretton Woods data from January 1974 to June 1990. He classifies the distinction between what he calls the weak-form PPP and the strong-from PPP. The weak-form PPP requires deviations from a linear combination of exchange rates and national price levels be stationary; while the strong-form PPP additionally requires the degree one homogeneity of the exchange rate with respect to the relative prices in the two countries. He reports that the weak-form PPP receives robust support from the data, whilst the strong-form PPP is given practically no support. In other words, there is a long-run relationship between a number of bilateral US dollar exchange rates and their corresponding relative prices, but the proportionality of the exchange rate to the relative prices fail to be established. Similar research is also followed by Cheung and Lai (1993), Cochrane and DeFina (1995), Kugler and Lenz (1993), Pippenger (1993) and Jacobson and Nessen (2004), among others. Overall, their evidence is supportive of the week-form PPP, i.e., comovement between the exchange rate and the respective relative prices; but is less in favour of PPP in the strong-form, i.e., comovement *and* proportionality. They argue that given the transportation cost, tariffs, and cross-country differences in the construction of price indices, PPP may be consistent with the above findings. Chowdhry *et al.* (2005) claim to provide a novel method for extracting a proxy for realised pure price inflation from stock returns and find strong support for relative PPP in the short-term using the extracted inflation measures. Using theoretically well motivated nonlinear models for artificially created real exchange rates, Paya and Peel (2007) investigate the properties of two alternative cointegration procedures of Johansen and Saikkonen. The latter procedure appears to outperform the former in terms of finding the "true" cointegrating coefficients, with which they generate "new" real exchange rates that exhibit, in most cases, much lower half-life shocks than the ones predicted by the Rogoff (1996) puzzle. Cheung *et al.* (2004) show that nominal exchange rate adjustment, not price adjustment, is the key engine governing the speed of PPP convergence. However, nominal exchange rates are found to converge much more slowly than prices. With the reversion being driven primarily by nominal exchange rates, then real exchange rates also revert at a slower rate than prices.

A few other studies attempt to investigate the validity of PPP under special circumstances such as high inflation and the European Monetary System (EMS). Frenkel (1981) has argued that for countries experiencing high money supply and variable rates of inflation, short-term deviations from PPP will occur, but price movements and nominal exchange rate movements will offset each other over time so that long-run PPP is likely to hold. If, however, the economy suffers real shocks, long-run PPP will not hold. Empirical tests for high inflation can also be found in McNown and Wallace (1989), and Mahdavi and Zhou (1994). These studies generally find some but not clear cut evidence of long-run PPP. They suggest that PPP may hold over a range of inflationary experience, and it is likely to hold more consistently when the inflation rate is very high. The evidence for the EMS, which is a system of managed floating exchange rates, is analysed by Chen (1995), Cheung *et al.* (1995), Chowdhury and Sdogati (1993) to assess how long-run PPP is affected by the EMS exchange rate arrangement. In general, the results of these studies support the view that currency realignments of the EMS have been effective in maintaining PPP among its member countries.

Cross Country Panel Data Approach

An alternative way to circumvent the low power of many traditional tests has been the use of panel data. A number of competing studies have emerged, arguing that the standard unit root and cointegration tests have low power against stationary alternatives in small samples and suggesting that failure to support the long-run PPP in early studies may result from this shortcoming. Although a few studies have turned to long period time series, these long period samples are available for only a few currencies. More importantly, exchange rate data spanning a very long time period suffer changes in exchange rate regimes. As Mussa (1986) has pointed out, real exchange rates behave very differently under different exchange rate regimes. If there are different parameters governing fixed versus floating exchange rates, test credibility may be heavily reduced by the inclusion of fixed rate period.

In this framework, Frankel and Rose (1996) examine deviations from PPP using a panel of data for 150 countries. The panel shows strong evidence of mean-reversion similar to that observed in long-run time series. The deviations from PPP have a half-life of approximately four years. The panel approach is also adopted by Jorion and Sweeney (1996), Mark (1995), Wu (1996), Oh (1996), Papell and Theodoridis (1998), Lothian (1997), Hoarau (2007) and Narayan (2008). All have reported similar findings. Nonetheless, concerns have been gradually raised about the panel approach's potential bias in producing stationarity, with the newer findings being less supportive of PPP. For example, O'Connell (1998) points out a potentially important problem with this approach. He shows that the standard practice of calculating all real exchange rates relative to the US dollar leads to cross sectional dependence in time series panel data. Adjusting for this problem appears to make it more difficult to reject the null of non-stationarity. Taylor and Sarno (1998) also point out that there may exist bias, sometimes substantial bias, towards stationarity in such tests. More recently, Banerjee *et al.* (2005) question the unit root test results of PPP in previous studies adopting panel methods. They offer the reasons why PPP usually holds when tested for in panel data but usually does not hold in univariate analysis. They challenge the usual explanation for this mismatch that panel tests for unit roots are more powerful than their univariate counterparts. They claim that cross-section cointegration relationships would tie the units of the panel together, which tends to make the test results appear stationary. Using simulations, they show that if this important underlying assumption of panel unit root tests is violated, the empirical size of the tests is substantially higher than the nominal level, and the null hypothesis of a unit root is rejected too often even when it is true. Subsequently, they warn against the "automatic" use of panel methods for testing for unit roots in macroeconomic time series, in addition to testing for PPP. Recent empirical results also tend to confirm the above raised concerns and offered explanations. For example, Harris *et al.* (2005) investigate PPP for a group of 17 countries using a panel based test of stationarity that allows for arbitrary cross-sectional dependence. They use monthly data of real exchange rates of 17 countries vis-à-vis the US dollar between January 1973 and December 1998. They treat the short-run time series dynamics non-parametrically to avoid the need to fit separate, and potentially mis-specified, models for the individual series. It is documented that significant evidence is found against the PPP null hypothesis even when allowance is made for structural breaks. Osterholm *et al.* (2007) show that the strong PPP hypothesis is rejected in favour of weak PPP with heterogeneous cointegrating vectors.

In summary, the above review and analysis show that, although great efforts have been made over the past decades on empirical tests of PPP, the results and findings are mixed. There are a number of factors causing departures from PPP, each of which is discussed in the previous sub-section[2]. The findings and conclusions in empirical studies are attempts to take these factors into account while verifying PPP, which lead to rejection or validation of PPP and rather often, controversial verdict on PPP.

3.2 Interest Rate Parities

Interest rate parities are concerned with expected exchange rate changes and the interest rate differential between the two involved countries or currency zones during a certain period. The expected exchange rate change can be *covered* by entering a forward contract for the foreign exchange transaction at a future time. Subsequently, it is called covered interest rate parity (CIRP), which identifies the

[2] For details see, for example, Krugman and Obstfeld (1994).

relationships between the spot exchange rate, the forward exchange rate, and the interest rates in the two countries in a time period. Without a forward contract on future foreign exchange transactions, the expected change in exchange rates is *uncovered*, and then an appropriate relationship between expected exchange rate changes and the interest rate differential between the two countries during a certain period is justified and established by uncovered interest rate parity (UIRP), which is usually associated with the international Fisher effect (IFE) to be discussed in the next section.

3.2.1 Covered Interest Rate Parity

CIRP states that the forward premium must be equal to the two countries' interest rate differential, otherwise there exist exploitable profitable arbitrage opportunities. Prior to proving this statement, which is in fact a reasonable approximation, we present an exact relationship between the spot exchange rate, the forward exchange rate, and the interest rates in the two involved countries in a certain period.

Define $F_{0,1}$ the forward exchange rate contracted now and to be delivered in the next period, e.g., in 30 days, S_0 the currently prevailing spot exchange rate, r_h the interest rate in the home country, and r_f the interest rate in the foreign country during the period. Then the following relationship (parity) between the spot exchange rate, forward exchange rate, and the interest rates in the two countries must hold to eliminate any arbitrage opportunities:

$$\frac{F_{0,1}}{S_0} = \frac{1+r_h}{1+r_f} \tag{3.13}$$

The rationale of equation (3.13) can be illustrated by the following example.

Suppose you have €1,000,000 to invest for one year. You can

> either invest in the euroland at $r_€$ (r_h). The future value of your investment in one year = €1,000,000×(1 + $r_€$);
>
> or convert your € for £ at the spot rate, invest in the UK at $r_£$ (r_f) and enter into a forward contract simultaneously to sell £ in one year. The future value of this investment in one year = €1,000,000×($F_{0,1}/S_0$)× (1 + $r_£$).

The two investments must have the same future value; otherwise arbitrage exists, so ($F_{0,1}/S_0$)×(1 + $r_£$) must equal (1 + $r_€$), or equivalently ($F_{0,1}/S_0$) = (1 + $r_€$)/(1 + $r_£$) = (1 + r_h)/(1 + r_f). This verifies equation (3.13). Deducting 1 from both sides of equation (3.13) leads to another expression of the relationship:

$$\frac{F_{0,1} - S_0}{S_0} = \frac{r_h - r_f}{1 + r_f} \approx r_h - r_f \tag{3.14}$$

It can be observed that, exactly speaking, the forward premium is equal to the interest rate differential adjusted by a factor of $(1 + r_f)$. Only when r_f is fairly small can the adjustment be comfortably ignored. Now let us progress to the approximate, but commonly used, version of CIRP. Taking logarithms of both sides of equation (3.13) yields:

$$f_{0,1} - s_0 = r_h - r_f \tag{3.15}$$

where $f_{0,1} = \ln(F_{0,1})$ and $s_0 = \ln(S_0)$. Further define the (approximate) forward premium $p_{0,1} = f_{0,1} - s_0$, equation (3.15) becomes:

$$p_{0,1} = r_h - r_f \tag{3.15'}$$

Notice that it has involved approximation in the process from equation (3.13) to equation (3.15). We should use equation (3.13) if interest rates are not small. However, equation (3.15) and equation (3.15') present the relationship as a commonly adopted CIRP statement that the forward premium must be equal to the two countries' interest rate differential to eliminate any arbitrage opportunities.

The relationship revealed by CIRP is graphically illustrated in Figure 3.2. Any Points on the 45 degree line indicate that CIRP holds. As a result, no arbitrage opportunities exist and it makes no difference whether one chooses to invest in the home country or the foreign country. A point to the left of the 45 degree line is where the forward premium is smaller than the interest rate differential between the home country and the foreign country. So the benefit of exploiting the forward premium is more than offset by the benefit from the interest rate differential. For example, if the forward premium is 1% and the interest rate differential is 2% during a given period. In this case, the home currency has depreciated but has not depreciated enough to the extent dictated by the interest rate differential. The gain from the foreign exchange transaction (1%) is smaller than the opportunity cost of going abroad (2%). A point to the right of the 45 degree line suggests that the forward premium is greater than the interest rate differential between the home country and the foreign country. Subsequently, it is profitable to exploit the arbitrage opportunity by investing in the foreign country, which involves converting the home currency into the foreign currency at the spot exchange rate prevailing at the beginning of the period, investing in the foreign country and earning interest at the rate of r_f in the period, and converting the foreign return back into the home currency at the end of the period through the use of a forward contracted at the beginning of the period. Notice that the home country and the foreign country are relative terms and one does not necessarily stay in the home country. In the following, we examine some examples to apply the CIRP theory and to exploit the arbitrage opportunities exposed by the violation of CIRP step by step.

Exchange rate and interest rate information in the US and the euroland is given as follows:

The \$/€ spot rate	$S_0 = 1.1237$
One year \$/€ forward rate	$F_{0,360} = 1.1128$
US discount rate	2.50% pa
Euroland discount rate	3.75% pa

Let us examine and answer these questions: (a) Do arbitrage opportunities exist? (b) How will you invest to exploit the arbitrage opportunities?

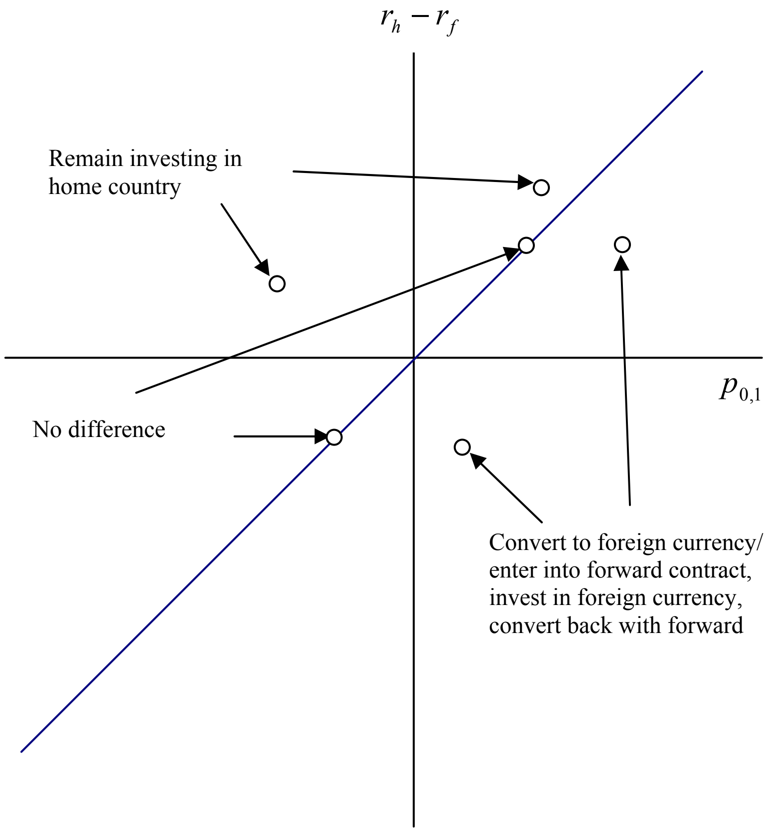

Figure 3.2. Graphical illustration of CIRP

Answers to part (a) are to check whether CIRP holds in this case, or whether the forward premium is equal to the interest rate differential. We adopt the exact version of CIRP and apply equation (3.4) to the above exchange rate and interest rate figures.

> The forward premium (exact) $(F_{0,1} - S_0)/S_0$ = (1.1128-1.1237) / 1.1237= -0.0097;
> The interest rate differential (exact) $(r_h - r_f)/(1 + r_f)$ = -0.01205.

From the result and referring to Figure 3.2, it can be concluded that:

> As forward premium ≠ interest rate differential, arbitrage opportunities exist.
> As forward premium > interest rate differential, it would be more profitable to invest in €. That is, if you have money in the US dollar to invest, you will convert the dollar to the euro at the beginning of the period, enter into a forward contract simultaneously to sell the euro at the end of the period, invest in the euro, and convert the euro back to the dollar with the forward contract at the end of the period. If you have money in the euro to invest, you simply invest in the euro.

Following the above analysis, we know that there are arbitrage opportunities and it is more profitable to invest in the euroland in this case. We present two procedures for the part (b) question below showing how to exploit the opportunities posed by the infringement of CIRP, one with the investor's own fund, and the other with borrowed fund, to make the investment.

Procedures for (b.1)
Suppose the investor has $1,000,000 and is based in the US, then the procedure to exploit the arbitrage opportunities can be:

> Step 1(a): Convert the fund in $ to € at the spot exchange rate S_0 = $1.1237/€, which gives $1,000,000/$1.1237/€ = €889917.2;
>
> Step 1(b): Enter into a forward contract simultaneously to sell € at the forward exchange rate $F_{0,360}$ = $1.1128/€, the size of the contract, €923289.1, is decided below;
>
> Step 2: Invest the amount of € 889917.2 for one year in the euroland at the rate of $r_f = r_€ = 3.75\%$. At the end of one year, the investment return is €889917.2×(1+0.0375) = €923289.1, all of which is to be converted back into $. So €923289.1 is also the size of the forward contract in step 1(b);

Step 3: Convert €923289.1 to \$ with the forward contract in step 1(b), resulting in €923289.1×\$1.1128/€ = \$1,027,436

If the investor does not engage in the above foreign exchange transaction activities and invest in the US for one year at the rate of $r_h = r_{\$} = 2.50\%$ instead, then the investment return is \$1,000,000×(1+0.025) = \$1,025,000. Let us call the former the foreign strategy and the latter the home strategy. The arbitrage profit is the difference between the investment returns of the foreign strategy and the home strategy, which is \$1027436-\$1025000 = \$2,436. As discussed earlier, the home country and the foreign country are relative terms. So if the investor has €1,000,000, then she or he adopts the "home strategy" and remains investing in the euroland. It can be concluded that it is the euroland that is a place to make higher returns in this case.

Procedures for (b.2)
Suppose the investor does not use her or his own fund and borrows \$1,000,000, then the procedure to exploit the arbitrage opportunities can be:

Step 1: Borrow \$1,000,000 at the cost of $r_h = r_{\$} = 2.50\%$;

Step 2(a): Convert the fund in \$ to € at the spot exchange rate S_0 = \$1.1237/€, leading to \$1,000,000/\$1.1237/€ = €889917.2;

Step 2(b): Enter into a forward contract simultaneously to sell € at the forward exchange rate $F_{0,360}$ = \$1.1128/€, the size of the contract, €923289.1, is decided below;

Step 3: Invest the amount of € 889917.2 for one year in the euroland at the rate of $r_f = r_€ = 3.75\%$. At the end of one year, the investment return is €889917.2×(1+0.0375) = €923289.1, all of which is to be converted back into \$. So €923289.1 is also the size of the forward contract in step 2(b);

Step 4: Convert €923289.1 to \$ with the forward contract in step 2(b), resulting in €923289.1×\$1.1128/€ = \$1,027,436;

Step 5: Repay the lender at the future value of the borrowing (principal plus interest), which is \$1,000,000×(1+0.025) = \$1,025,000.

After repaying the lender the principal of the borrowing and the interest incurred in one year, there is an amount of \$2,436 = \$1027436-\$1025000 left with the investor or arbitrageur. The result is the same as in part (b1) with the investor's own fund. However, it will make a difference when transaction costs are considered and included, e.g., banks' lending rate is usually higher than the borrowing rate. Therefore, there may be situations when there are arbitrage opportunities using one's own funds, but such arbitrage opportunities will not materialise using borrowed funds.

3.2.2 CIRP and Arbitrage in the Presence of Transaction Costs

Transaction costs reduce arbitrage opportunities or prevent arbitrage opportunities from materialising. These include bid-ask spreads in foreign exchange rates and in case of using borrowed funds, the difference in lending and borrowing rates.

For example, if in the previous case it is assumed that the lending rate is 3.00% per annum and the borrowing rate is 2.00% per annum in the US, the lending rate is 4.25% per annum and the borrowing rate is 3.25% per annum in the euroland, and a zero bid-ask spread is still assumed for exchange rates, as summarised below, is it possible to make arbitrage profit using borrowed US dollar funds?

The \$/€ spot rate	$S_0 = 1.1237$
One year \$/€ forward rate	$F_{0,360} = 1.1128$
US borrowing rate	2.00% pa
US lending rate	3.00% pa
Euroland borrowing rate	3.25% pa
Euroland lending rate	4.25% pa

We repeat the above procedure with the interest rates being replaced their respective borrowing and lending rates.

Step 1: Borrow \$1,000,000 at the cost of $r_{\$,lending} = 3.00\%$ (bank's lending rates are borrowers' borrowing rates);

Step 2(a): Convert the fund in \$ to € at the spot exchange rate S_0 = \$1.1237/€, leading to \$1,000,000/\$1.1237/€ = €889917.2;

Step 2(b): Enter into a forward contract simultaneously to sell € at the forward exchange rate $F_{0,360}$ = \$1.1128/€, the size of the contract, €918839.5, is decided below;

Step 3: Invest the amount of € 889917.2 for one year in the euroland at the euro borrowing rate of $r_{€,borrowing} = 3.25\%$. At the end of one year, the investment return is €889917.2×(1+0.0325) = €918839.5, all of which is to be converted back into \$. So €918839.5 is also the size of the forward contract in step 2(b);

Step 4: Convert €918839.5 to \$ with the forward contract in step 2(b), resulting in €918839.5×\$1.1128/€ = \$1,022,485;

Step 5: Repay the lender at the future value of the borrowing (principal plus interest) at a cost of $r_{\$,lending} = 3.00\%$, which is \$1,000,000×(1+0.03) = \$1,030,000.

It is obvious that the future value of the borrowing, which is \$1,030,000, is greater than the investment return from the "arbitrage" process, which is \$1,022,485, so there exist no exploitable arbitrage opportunities due to the presence of the transaction cost in the form of differential lending and borrowing interest rates. However, it is still more profitable to adopt the "foreign" strategy if one uses her or his own US dollar funds, illustrated as follows.

Step 1(a): Convert the fund in \$ to € at the spot exchange rate S_0 = \$1.1237/€, which gives \$1,000,000/\$1.1237/€ = €889917.2;

Step 1(b): Enter into a forward contract simultaneously to sell € at the forward exchange rate $F_{0,360}$ = \$1.1128/€, the size of the contract, €918839.5, is decided below;

Step 2: Invest the amount of € 889917.2 for one year in the euroland at the euro borrowing rate of $r_{€,borrowing} = 3.25\%$. At the end of one year, the investment return is €889917.2×(1+0.0325) = €918839.5, all of which is to be converted back into \$. So €918839.5 is also the size of the forward contract in step 1(b);

Step 3: Convert €918839.5 to \$ with the forward contract in step 1(b), resulting in €918839.5×\$1.1128/€ = \$1,022,485;

If the investor does not engage in the above foreign exchange transaction activities and invest in the US for one year at the US rate of $r_{\$,borrowing} = 2.00\%$ instead, then the investment return is \$1,000,000×(1+0.02) = \$1,020,000. Therefore, there is an arbitrage profit of \$2,480 = \$1,022,485 - \$1,020,000 from the arbitrage activity. When other kinds of transaction costs are taken into consideration, such as commonly observed bid-ask spreads in exchange rates, arbitrage opportunities will be further reduced.

3.2.3 Uncovered Interest Rate Parity

UIRP states that there is a relationship between the expected change in the spot exchange rate and the interest rate differential between the two countries and the expected change in the spot exchange rate is equal to the two countries' interest rate differential. There are no profitable opportunities if:

$$F_{0,1} = E_0\{S_1\} \tag{3.16}$$

while CIRP holds, where $E_0\{S_1\}$ is the expectations, formed at time 0, of the future spot exchange rate at time 1. Therefore, in relation to equation (3.13), the following relationship (parity) must hold:

$$\frac{E_0\{S_1\}}{S_0} = \frac{1+r_h}{1+r_f} \tag{3.17}$$

UIRP implies that the forward exchange rate is an unbiased predictor of the future spot exchange rate. Deducting one from both sides of equation (3.17) yields:

$$\frac{E_0\{S_1\}-S_0}{S_0} = \frac{r_h - r_f}{1+r_f} \approx r_h - r_f \tag{3.18}$$

The above equation states that the expected (percentage) change in the spot exchange rate, exactly speaking, is equal to the two countries' interest differential adjusted by a factor of $(1+r_f)$. The factor of $(1+r_f)$ can only be ignored when r_f is fairly small. We can work out an approximate, but commonly used, version of equation (3.17), similar to that for CIRP, by taking logarithms on both sides of equation (3.17):

$$E_0\{\Delta s_1\} = E_0\{s_1\} - s_0 = r_h - r_f \tag{3.19}$$

It has involved approximation in the process from equation (3.17) to equation (3.19), similar to the CIRP case. Although equation (3.19) is an approximation, it presents the relationship as a commonly adopted UIRP statement suggesting that the expected change in the spot exchange rate be equal to the two countries' interest rate differential. There can be profitable opportunities if UIRP does not hold while the expectations about future spot exchange rates are right. But one may make no profits or even make a loss, if UIRP does not hold *and* the expectations about future spot exchange rates are wrong. Unlike CIRP, UIRP is *not* covered by a forward contract and, subsequently, the expected outcome is not guaranteed.

Figure 3.3 illustrates graphically the relationship established by UIRP. Points on the 45 degree line indicate that UIRP holds. As a result, it is not expected to make any difference whether one chooses to invest in the home country or the foreign country. Any points to the right of the 45 degree line mean that the expected depreciation of the home currency is greater than the interest rate differential between the home country and the foreign country. If the expectations are proved correct, then it is profitable to invest in the foreign country, which involves converting the home currency into the foreign currency at the beginning of the period at the prevailing spot exchange rate at the time, investing in the foreign country and earning interest at the rate of r_f in the period, and converting the foreign return back into the home currency at the future spot exchange rate prevailing at the end of the period. The outcome would be the same as that in the CIRP case when the expectations are correct, i.e., $S_1 = E_0\{S_1\} = F_{0,1}$. Any points to the left of the 45 degree line indicate that the expected depreciation of the home currency is smaller than the interest rate differential between the home country and the foreign country. Consequently, the benefit of exploiting the expected changes in the exchange rate is more than offset by the benefit from the interest rate differential if the expectations are proved correct. The cases can be that the expected depreciation of the home currency is smaller than the right extent suggested by the interest rate differential between the home country and the foreign country; or the expected appreciation of the home currency is larger than the interest rate differential between the foreign country and the home country.

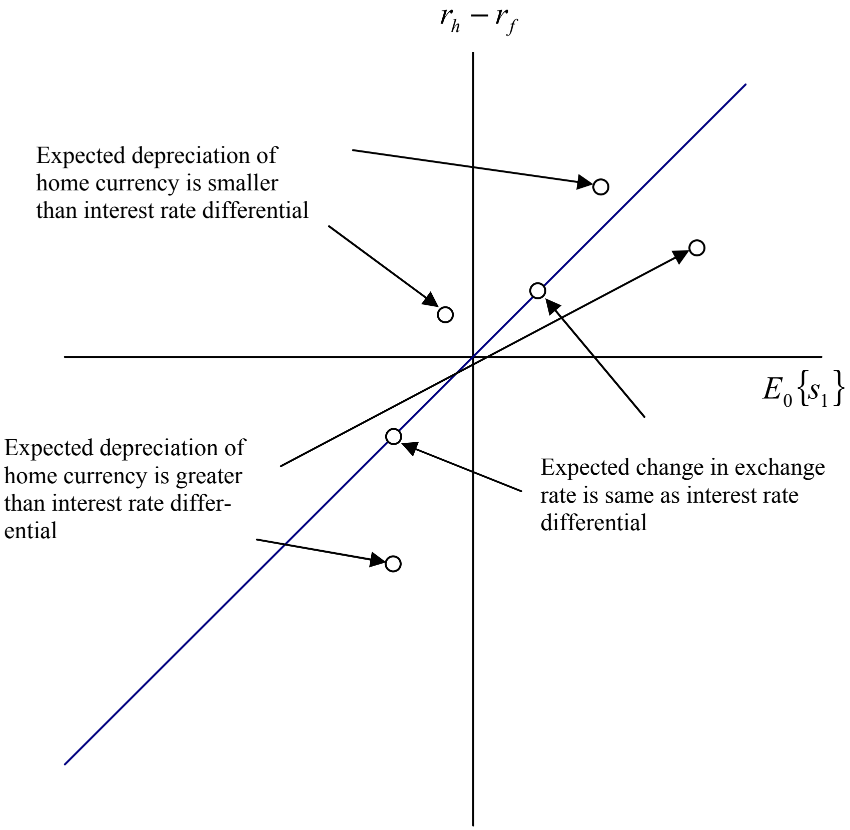

Figure 3.3. Graphical illustration of UIRP

We conclude this part with a simple example. Suppose that the UK interest rate is and will remain 3.25% per annum for the next 12 months, the US interest rate is and will remain 2.50% per annum for the next 12 months, and the current spot exchange rate of the US dollar vis-à-vis the British pound is \$1.6036/£. (a) What is the expected spot exchange rate in 12 months time? (b) If the UK interest rate is expected to rise in six months by a quarter to two quarter points, what will the expected spot exchange rate be? UIRP is assumed to holds.

The answer to part (a) is an application of UIRP and equation (3.12), which gives:

$$E_0\{S_1\} = S_0 \times \frac{1+r_h}{1+r_f} = \$1.6305/£ \times \frac{1+0.0250}{1+0.0325} = \$1.5587/£ \cdot$$

The answer to part (b) is qualitative. Since the UK interest rate will be higher than 3.25 per annum overall in the next twelve months, the interest rate differential between the US and the UK will be smaller than that used in the above calculation. Therefore, the expected spot exchange rate will be smaller than \$1.5587/£ as obtained in part (a).

3.3 International Fisher Effect

Prior to studying the international Fisher effect (IFE), we first review the Fisher effect, which is concerned with the relationship between the real interest rate, the nominal interest rate, and inflation, in a domestic economic setting.

Denote a as the real interest rate, r as the nominal interest rate, and $E\{\pi\}$ the expected inflation rate in a certain period (t-1, t], then:

$$1+r = (1+a) \times [1+E\{\pi\}] \tag{3.20}$$

An approximation can be made from the above equation:

$$\begin{aligned} 1+r &= (1+a) \times [1+E\{\pi\}] \\ &= 1+a+E\{\pi\}+a \times E\{\pi\} \\ &\approx 1+a+E\{\pi\} \end{aligned} \tag{3.20'}$$

Deducting one from both sides of equation (3.20') results in:

$$r \approx a + E\{\pi\} \tag{3.21}$$

Equation (3.21) is a mathematical expression of the Fisher effect, which states that that the nominal interest rate is the sum of the real interest rate and the inflation expectations in the period. The statement is indeed an approximation of the effect of expected inflation on the nominal interest rate, given the real interest rate. The error in the approximation is small when the real interest rate and inflation are low. The Fisher effect suggests that changes in the nominal interest rate reflect the revised inflation expectations; and revised inflation expectations have an impact on the level of the nominal interest rate.

Applying the Fisher effect to two concerned countries leads to the international Fisher effect. This involves combining the two countries' Fisher effects with exchange rate expectations and PPP, assuming real interest rates are equalised across countries. The Fisher effect for the two countries, one of which is home and the other is foreign, is presented as:

$$r_h \approx a_h + E\{\pi_h\} \tag{3.22a}$$

$$r_f \approx a_f + E\{\pi_f\} \tag{3.22b}$$

A version of PPP, which involves exchange rate expectations, is given as:

$$E\{\Delta s_t\} = E\{\pi_h\} - E\{\pi_f\} \tag{3.23}$$

where Δs_t is the (percentage) change in the exchange rate during (t-1, t], the same period as for the prevailing inflation. Combining the above equation (3.22) and equation (3.23) leads to:

$$\begin{aligned} E\{\Delta s_t\} &= E\{\pi_h\} - E\{\pi_f\} \\ &= (r_h - a_h) - (r_f - a_f) \\ &= r_h - r_f \end{aligned} \tag{3.24}$$

assuming real interest rates are equalised across countries, i.e., $a_h = a_f$.

IFE suggests that the expected change in exchange rates be equal to the interest rate differential between the two countries, which is a statement of UIRP derived under different circumstances. IFE pays attention to expected inflation and is derived through analysing the effect of expected inflation and the inflation differential between the two countries on expected changes in exchange rates. Expected inflation is a channel to cause nominal interest rates to change, the differential in the expected inflation of the two countries is a channel to result in the interest rate differential between the two countries, leading to expected relative changes in purchasing power and subsequent adjustment in exchange rate expectations. In contrast, UIRP relies on the forward exchange rate as an unbiased predictor of the future spot exchange rate and the validity of CIRP. A graphical illustration of IFE would be similar to Figure 3.3 for UIRP.

3.4 Links Between the Parities: a Summary

The relationships between the spot exchange rate, the forward exchange rate of one currency vis-à-vis the other, and the interest rates and inflation in the two countries give rise to the above examined parities and parity conditions. These relationships are about how changes (expected and *ex ante*) in one or two variables result in changes in other variables. Relevant parities apply when changes or expected changes in relevant variables are examined. That is, the relevance of a specific parity depends on the circumstances, under which the relationships are inspected, and the theme of the issue to be investigated. Figure 3.4 shows the links between different parities and the interactions between various factors.

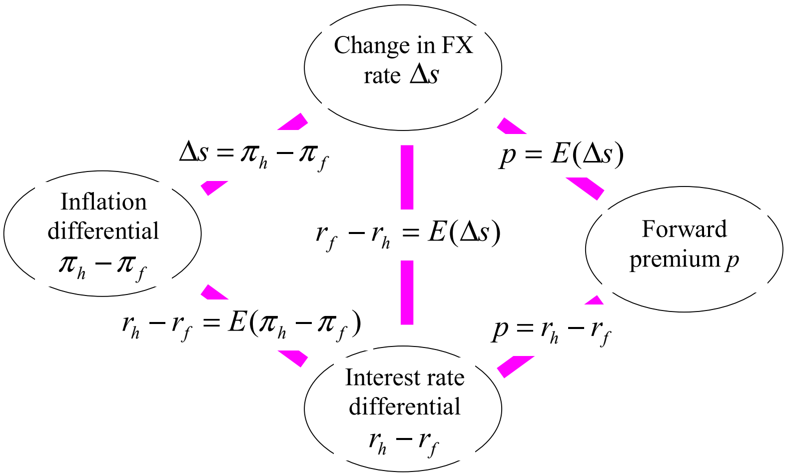

Figure 3.4. Links between parities and relationships between factors

The factors involved in these links are expected and realised changes in the spot exchange rate, the forward premium, the expected and realised inflation differential, and the interest rate differential. The upper left branch of the graph is PPP in the relative form, which states that changes in the spot exchange rate between two currencies in a given period is equal to the inflation differential in these two countries in the same period. The lower right branch is CIRP, which states that the forward premium that involves the spot and forward exchange rates for two currencies in a certain period is equal to the interest rate differential in the two countries in the same period. The lower left branch of the graph presents the Fisher effect in two countries, showing that interest rate differentials are equal to inflation differentials in the two countries in concern. This, through the channel of the upper left branch, establishes IFE as a relationship between expected changes in the exchange rate and the interest rate differential, indicating that the expected changes in the exchange rate is equal to the interest rate differential in the two countries under consideration. The upper right branch illustrates the hypothesis that forward exchange rate be an unbiased predictor of the future spot exchange rate, in the form that the expected change in the spot exchange rate in a given period is equal to the forward premium in the same period for the two currencies involved. Combining the upper right and lower branches of the graph, the link between expected changes in the spot exchange rate and interest rate differentials is derived as UIRP, which states that the expected change in the spot exchange rate in a certain period is equal to the interest rate differential between the two countries in concern in the same period. In the middle of the graph is a direct link between the expected changes in the spot exchange rate and the interest rate differential, which is the IFE through the two left branches and UCIP through the two right branches in the graph.

Figure 3.4 not only presents the links between various parities but also indicates the roles of each of the factors or variable and the way in which they adjust and make corresponding comovements, as suggested by relevant parities. Nevertheless, these parities are in essence theories and hypotheses, and their validity is subject to empirical enquiries under various and relevant circumstances as discussed earlier in this chapter. One should not rely on them in making policy or investment decisions without cautious prior analysis.