# Violations of Uncovered Interest Rate Parity and International Exchange Rate Dependences


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## Abstract

The uncovered interest rate parity puzzle questions the economic relation existing between short term interest rate differentials and exchange rates. One would indeed expect that the differential of interest rates between two countries should be offset by an opposite evolution of the exchange rate between them, hence ruling out any limited risk profit opportunities. However, it has been shown empirically that this relation is not holding and accordingly has led, over the past two decades, to the reinforcement of a well-known trading strategy in financial markets, namely the currency carry trade. This paper investigates how highly leveraged, mass speculator behaviour affects the dependence structure of currency returns. We propose a rigorous statistical modelling approach using two complementary techniques in order to demonstrate that speculative carry trade volumes are informative in both the covariance and tail dependence of high and low interest rate currency returns, whereas the price based factors previously suggested in the literature hold little explanatory power. We add a new feature to the understanding of the link between the UIP condition and the carry trade strategy, specifically attributed to the large joint exchange rate movements in high and low risk environments.



*Keywords:* Forward Premium Puzzle, Speculative Trading Volumes, Multivariate Tail Dependence, Mixture Copula Models, Currency Carry Trade, Covariance Regressions


## 1. Introduction

Understanding the behaviour of currency markets has been an active area of research for the past few decades. Much of the literature has focused on the marginal behaviours of exchange rates and carry trade portfolio returns resulting from the established violations of uncovered interest rate parity. However, there have been fewer studies investigating the joint exchange rate behaviours for the currencies as a function of their interest rate differentials. One of the most robust puzzles in financial econometrics still to be satisfactorily explained is indeed the uncovered interest rate parity puzzle and the associated excess average returns of currency carry trade strategies. Such investment strategies are popular approaches which involve constructing portfolios by selling low interest rate currencies in order to buy high interest rate currencies, thus profiting from the interest rate differentials. When such portfolios are highly leveraged this can result in sizeable profits. The presence of such opportunities, pointed out by Hansen and Hodrick (1980); Fama (1984); Backus et al. (2001) and more recently by Lustig and Verdelhan (2007); Brunnermeier et al. (2008); Burnside et al. (2011); Christiansen et al. (2011); Lustig et al. (2011); Menkhoff et al. (2012), violates the fundamental relationship of uncovered interest rate parity (UIP). The UIP refers to the parity condition in which exposure to foreign exchange risk, with unanticipated changes in exchange rates, is uninhibited and therefore if


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one assumes rational risk-neutral investors, then changes in the exchange rates should offset the potential to profit from the interest rate differentials between high interest rate (investment) currencies and low interest rate (funding) currencies. The existence of a working UIP relationship is related to two primary assumptions, which are capital mobility and perfect substitutability of domestic and foreign assets. When UIP holds, then given foreign exchange market equilibrium, the interest rate parity condition implies that the expected return on domestic assets will equal the exchange rate-adjusted expected return on foreign currency assets. Therefore, no arbitrage opportunities should arise in practice; however such opportunities are routinely observed and exploited by large volume trading strategies.

On the other hand, an established strand of the microeconomic literature emphasizes the price variability and volume relationship (Tauchen and Pitts (1983), Gallant et al. (1992), Ané and Geman (2000), Hasbrouck and Seppi (2001), Bernhardt and Taub (2008), He and Velu (2014)) by demonstrating that commonalities among asset volumes stemming from speculative trading strategies, such as the carry trade for foreign exchange markets, subsequently lead to commonalities among asset returns. In this paper, we build on the existing literature by studying how speculative inflows and outflows capture or relate to stochastic features in the joint behaviour of the currency exchanges relative to their respective level of short term interest rates and the resulting speculative appeal. We aim to explore to what extent one can attribute either significant joint depreciations or appreciations in the value of the high or the low interest rate currencies to speculative flows. We postulate that such analyses should also benefit from consideration not only of the marginal behaviours of the processes under study, in this case the exchange rates of currencies in a portfolio, but also a rigorous analysis of the joint dependence features of such relationships. Therefore, we investigate such joint relationships in light of the UIP condition.

To achieve this, we thoroughly study the dynamic of high interest rate and low interest rate currencies from two complementary perspectives. The first step of our analysis consists in assessing the impact of speculative inflows and outflows upon the marginal characteristics of each exchange rate as well as the non-extremal dependences among them using a covariance regression model. In our model, the conditional covariance regression is reinterpreted as a random effects model recently proposed by Hoff and Niu (2011), aiding the estimation via an Expectation-Maximisation approach. This regression framework allows us to differentiate the effects of speculative interest on the cross-sectional exchange rates conditional mean from those upon the conditional covariance matrix.

In a second step we complete this analysis by conditioning our impact estimation only on joint extreme movements in the high and low interest rate currency baskets and demonstrate how this could be explained by the currency purchases and sales resulting from the carry trade. We indeed argue that the analysis of the international exchange rates dependence structure is better informed when the covariance analysis is completed by jointly modelling the multivariate behaviour of the marginal processes of currency baskets accounting for potential multivariate extremes, and multivariate tail dependence features, whilst still incorporating heavy-tailed relationships studied in marginal processes. To model accurately the potentially complex extremal multivariate dependence features, the statistical models considered should be sufficiently flexible to accommodate a variety of potential tail dependence relationships. In this regard, we consider the mixture copula models comprised of different members of the Archimedean copula family, which admit different degrees of asymmetric upper and lower tail dependence. We thus extract a measure of the tail dependences within a basket of high interest rate currencies that we compare with another basket only composed of low interest rate currencies and finally consider to what extent their respective and joint dynamics could be explained by the behaviour of the speculators present in the market.

Among the outcomes of our study, we demonstrate that speculative inflows and outflows resulting from the carry trade are systematically impacting the tail dependences and the conditional covariances among currencies from the high and the low interest rate countries. We also emphasize that this influence on the tail behaviours is not only occurring




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during stress periods but also within low volatility market conditions associated with no stress. As a consequence of our findings we believe that such stochastic non-linear relationships among exchange rates and speculator interventions should then be taken into consideration for the monetary policy of central banks, since any unilateral decision to modify short term interest rates could unevenly or asymmetrically impact the country's exchange rate with their commercial partners as far as they pertain or not to the same interest rate level basket. Therefore, our analysis provides interesting insight into the seemingly complex understanding of how speculative strategies can impact the marginal and joint dynamics of macro-economic variables such as exchange rates through a microeconomic perspective based on the speculative volumes and variability commonalities. Furthermore, our results should also find a resonance with currency portfolio modelling as the dynamic dependence among international exchange rates should therefore be altered to account for the relation between volumes and the covariance of portfolio components, while also including the diverse tail dependence features associated to sets of currencies with similar interest rate differentials.

The paper is structured as follows. In the second section, we review the concept of UIP and the associated literature. The third section is devoted to methods where we describe the covariance regression and the copula models considered for our two stage analysis of the speculator impact on the currency price commonalities. In the same section, we also describe the model we retain to measure the upper and lower tail dependences within the high interest rate and low interest rate baskets. The fourth section starts with a detailed description of the data utilised in the analysis and discusses the results we obtained through our covariance regression model. Section five completes our analysis by examining the mixture copula joint tail behaviours over time and by demonstrating how the speculative flows relate to the extremal joint dependences among the high and low interest rate currency baskets. We also extend in this section the analysis of the time-varying multivariate tail dependence present in carry trade baskets using 34 currencies.

## 2. The Uncovered Interest Rate Parity Puzzle

### *2.1. UIP Definitions*

The Uncovered interest rate parity (UIP) condition is directly linked to the arbitrage relation existing between the spot and the forward prices of a given currency, namely the Covered interest rate parity (CIP) condition.

**Definition 2.1.** *Covered Interest Rate Parity (CIP)*
*This relation states that the price of a forward rate can be expressed as follows:*

$$F_t^T = e^{(r_t - r_t^f)(T-t)} S_t, \tag{2.1}$$

*where $F_t^T$ and $S_t$ denote respectively the forward and the spot prices at time $t$ for a currency exchange rate. While $r_t$ and $r_t^f$ represent the local risk free rate*[1] *and the foreign risk free rate. We denote by $T$ the maturity of the forward contract considered. The CIP condition states that one should not be able to make a risk free profit by selling a forward contract and replicating its payoff through the spot market.*

It is worth emphasizing that under the hypothesis of an absence of arbitrage opportunities, the CIP relation is directly resulting from the replication of the forward contract payoff using a self financed strategy. Moreover, the validity of this arbitrage relation has been demonstrated empirically in the currency market by Juhl et al. (2006); Akram et al. (2008) when one considers daily data. The highly unusual period following the onset of the financial crisis in August 2007 saw large deviations from CIP due to the funding constraints of arbitrageurs and uncertainty

[1]We mean by local risk free rate the interest rate prevailing in the reference country, which would be for instance the dollar for an American investor.




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about counterparty risk, see Coffey et al. (2009) for a thorough analysis of this period, though this was an exceptional case and typically CIP holds. In this paper our interest lies rather with the associated concept of Uncovered Interest Rate Parity (UIP) conditions.

**Definition 2.2.** *Uncovered Interest Rate Parity (UIP)*
*If we assume that the forward price is a martingale under the risk neutral probability $\mathbb{Q}$, Musiela and Rutkowski (2011), then the fair value of the forward contract at time $t$ equals:*

$$E_{\mathbb{Q}}[S_T|\mathcal{F}_t] = F_t^T, \tag{2.2}$$

*where $\mathcal{F}_t$ is the filtration associated to the stochastic process $S_t$. Replacing the expression (Equation 2.2) in the relation (Equation 2.1) leads to the UIP equation:*

$$E_{\mathbb{Q}}\left[\frac{S_T}{S_t}\middle|\mathcal{F}_t\right] = \frac{F_t^T}{S_t} = e^{(r_t - r_t^f)(T-t)}. \tag{2.3}$$

*The UIP equation states that under the risk neutral probability the expected variation of the exchange rate $S_t$ should equal the differential of interest rate between the two countries. Thus if an investor borrows $S_t e^{-r_t^f(T-t)}$, converts it into foreign currency and invests it in the foreign risk free bond and finally converts it back. The profit or the loss resulting at the maturity date $T$ should be equal to $S_t e^{(r_t - r_t^f)(T-t)}$ which was the price paid initially for the forward contract under the hypothesis of absence of arbitrage opportunities.*

Unlike the CIP condition, where we hedge the exchange rate risk by selling a forward contract, it is not true that UIP regularly holds in practice. Numerous empirical studies Fama (1984); Hansen and Hodrick (1980); Engel (1996); Lustig and Verdelhan (2007) have previously demonstrated the failure of UIP, i.e. that investors can actually earn profits by borrowing in a country with a lower interest rate, exchanging for foreign currency, and investing in a foreign country with a higher interest rate, whilst allowing for any losses (or gains) from exchanging back to their domestic currency at maturity. Therefore, it is believed that trading strategies that aim to exploit the interest rate differentials may be profitable on average. This is notably the case for the currency carry trade which is thus the simple investment strategy of selling a high interest rate currency forward and then buying a low interest rate currency forward. The idea is that the interest rate returns will outweigh any potential adverse moves in the exchange rate. Historically the Japanese yen and Swiss franc have been used as "funding currencies", since they have maintained very low interest rates for a long period. The currencies of developing nations, such as the South African rand and Brazilian real have been typically used as "investment currencies". Whilst this sounds like an easy money making strategy there is of course a downside risk. This risk comes in the form of currency crashes in periods of global FX volatility and liquidity shortages. A prime example of this is the sharp yen carry trade reversal in 2007.

### *2.2. A Review of the Literature*

Among the justifications of the UIP violation phenomenon, Fama (1984) initially proposed a time varying risk premium within the forward rate relative to the associated spot rate. Thus, the exchange rates are evolving in order to compensate for different risk exposures during recession and crisis periods. This approach based on common factor models in currency returns has been also adopted by Lustig et al. (2011) who employ two linearly independent factors in order to explain the variability in the cross section of the international exchange rates. The first factor is a level factor, named "dollar risk factor" or $DOL$, which is essentially the average relative value change of a foreign currency basket against the dollar. The second factor represents the market induced rate of appreciation or depreciation of currencies as a function of their interest rate differential with the reference currency, in our case the USD, it is named in the literature the High-Minus-Low risk factor or $HML_{FX}$. It has been shown by Lustig et al. (2011) that one can typically observe over time that higher interest rate currencies have a tendency to load more on this slope factor than low interest rate currencies. The $HML_{FX}$ factor is found to have good explanatory power when characterizing the intertemporal presence of the cross-sectional variation on average exchange rates among high and low interest rate currencies. An associated trend study to that of Lustig et al. (2011) is the study of Menkhoff et al. (2012) who consider a related approach still based on the standard factors asset pricing model but where the $HML_{FX}$ factor is replaced by the innovations in global foreign exchange volatility. The authors demonstrate that high interest rate currencies tend to be negatively related to the innovations in global foreign exchange volatility, which is considered as a proxy for unexpected changes in the foreign exchange market volatility. It is shown that global foreign exchange volatility is a pervasive risk factor in the cross section of foreign exchange excess returns and that its explanatory power extends to several other test assets. Liquidity is also shown to be a useful risk factor in understanding cross-sectional variations in exchange rates, but to a lesser extent than global foreign exchange volatility.

Another hypothesis that seeks to explain violations of the UIP in foreign exchange markets was proposed by Farhi and Gabaix (2008), which consists in justifying the UIP puzzle through the inclusion of a mean reverting risk premium. According to their model, a risky country, which is more sensitive to economic extreme events, represents a high risk of currency depreciation and has thus to propose, in order to compensate this risk, a higher interest rate. Then, when the risk premium reverts to the mean, their exchange rate appreciates while they still have a high interest rate which thus replicates the forward rate premium puzzle. Furthermore, Weitzman (2007) demonstrates through a Bayesian approach that the uncertainty about the variance of the future growth rates combined with a thin-tailed prior distribution would generate the fat-tailed distribution required to solve the forward premium puzzle.

Unlike the studies we undertake in this paper, these previous analyses seek to explain the UIP through analysis of marginal tail behaviour in currency foreign exchange rates as well as average linear dependence relationships with common factors. One of our contributions to these analyses is to incorporate a third element not yet explored, namely the joint extremal dependences between currency returns that we show also have strong explanatory power when understanding the currencies dynamic, even having accounted for these other factors such as $DOL$ and $HML_{FX}$. In the same vein as our approach, the causality relation between the interest rate differential and the currency shocks can be presented the other way around, as detailed in Brunnermeier et al. (2008). In this article, the authors indeed assume that the currency carry trade mechanically attracts investors and more specifically speculators who accordingly increase the probability of a market crash. Simultaneous tail events among currencies would thus be caused by speculators' need to unwind their positions when they get closer to funding constraints, which in turn puts pressure on the value of their marked to market portfolio, hence the liquidity spiral described in Brunnermeier and Pedersen (2009) which leads to the currency log-returns extremal dependences.

We also add to this liquidity crisis theory by first showing that it is more appropriate to consider the extremal dependence that arises through a measure of intertemporal tail dependence, or extremal dependence that characterizes the inter-temporal joint propensity for co-currency appreciations or depreciations instead of using cross sectional skewness behaviour. Furthermore, we empirically corroborate and complete the conclusions of Brunnermeier et al. (2008) with a quantitative and thorough measurement, through a two stage approach, of the direct effect of speculative flows upon international exchange rate marginal and joint distributions. We also point out that speculative volumes are not only influencing foreign exchange rates during stressed or risky periods, but also when lower risk environments are prevailing.

## 3. Methods: Modelling Exchange Rate Baskets - Trends, Covariance, Copula Dependence Structures

In this section, we outline the modelling frameworks that we apply to the study of the funding and investment currency baskets in performing our dynamic analysis. Throughout this paper we focus on the joint currency behaviours and hence the majority of this section is dedicated to our approaches to the joint modelling performed.

Firstly, we consider a specialised framework of covariance regression, which allows us to decompose the correlation and variance terms in the inter-temporal movements in currency exchange rates over time in terms of important macro-economic factors such as currency based market factors, carry trade currency based factors as well as volume based factors.

Having understood the mean and covariance based dynamics of the currency baskets, we then extend this study to a complete analysis of high-order dependence structures. To achieve this, we consider an approach which involves first fitting marginal models to each currency return time series to capture the marginal dynamics, then based on the residuals from these models we study the joint stochastic structures, as proposed in the inference for margins (IFM) approach of Joe and Xu (1996) for which statistical properties are mentioned in the online technical appendix. To perform the analysis of the marginals we considered a range of models, discussed in detail in Ames et al. (2015), which involved different GARCH models, similar to those proposed in Alexander and Lazar (2006); Bontemps and Meddahi (2012). Since the focus of this paper is on joint dynamics, we adopted the recommendations for the currency returns data analysed in Ames et al. (2015), which demonstrated best performance for a class of locally stationary models based on a 125 day sliding window structure with a 124 day overlap which made an assumption of local stationarity in the returns time series. To fit each marginal model in the sequence of sliding windows, a log generalized gamma model was adopted for the local returns series structure. Such models and the properties are discussed in detail in the online technical appendix accompanying this paper, see Appendix Section 3. To capture the joint stochastic structures we consider mixture copula based models that allow one to obtain decompositions of higher order dependence relationships. In particular, we consider dependence relationships in the extremes of co-currency movement appreciations or depreciations in the funding and investment baskets. Furthermore, we again demonstrate how to decompose these extremal dependence features of the baskets of currency exchange rates according to contributions from relevant macro-economic factors such as currency based market factors, carry trade currency based factors as well as volume based factors. This allows us to decompose which factors most influence the central linear trend, the covariance relationships and the extremal non-linear dependence relationships between the baskets of currencies over time.

### *3.1. Covariance Regressions for Currency Baskets*

To perform the study of the covariance relationships we formulate our covariance regression model as a special type of random-effects model, see Hoff and Niu (2011), for observed data $\boldsymbol{y_1}, \ldots \boldsymbol{y_n}$ ($d$-dimensional basket daily log returns for a time block of length $n$ days).

The first step is to perform the mean-regression, via in our case a standard linear regression model, with interactions that are described in Section 4. This will allow us to obtain zero-mean residuals $\boldsymbol{e_i}$, given by $\boldsymbol{e_i} = \boldsymbol{y_i} - \boldsymbol{\beta x_i}$, where $\boldsymbol{\beta}$ is the vector of regression loadings and the covariate vector is denoted by $\boldsymbol{x_i}$, which contains daily realizations of observable relevant macro-economic factors such as currency based market factors, carry trade currency based factors as well as weekly volume based factors.

Having defined the residuals with de-trended mean, we perform the covariance regression of these residuals on the factors, using the random-effects representation of the following form

$$
\begin{aligned}
&\boldsymbol{e_i} = \gamma_i \times \boldsymbol{B x_i} + \boldsymbol{\epsilon_i} \ , \\
&\mathbb{E}[\boldsymbol{\epsilon_i}] = 0 \quad , \quad Cov(\boldsymbol{\epsilon_i}) = \boldsymbol{\Psi} \ , \\
&\mathbb{E}[\gamma_i] = 0 \quad , \quad Var[\gamma_i] = 1 \quad , \quad \mathbb{E}[\gamma_i \times \boldsymbol{\epsilon_i}] = 0.
\end{aligned}
$$

The resulting covariance matrix for $\boldsymbol{e_i}$, conditional on $\boldsymbol{x_i}$ is then given by,

$$
\begin{aligned}
\mathbb{E}[(\boldsymbol{e_i})(\boldsymbol{e_i})^T|\boldsymbol{x_i}] &= \mathbb{E}[\gamma_i^2 \boldsymbol{B x_i x_i^T B^T} + \gamma_i(\boldsymbol{B x_i \epsilon_i^T} + \boldsymbol{\epsilon_i x_i^T B^T}) + \boldsymbol{\epsilon_i \epsilon_i^T}|\boldsymbol{x_i}] \\
&= \boldsymbol{B x_i x_i^T B^T} + \boldsymbol{\Psi}.
\end{aligned}
\tag{3.1}
$$

We may now perform the estimation of the coefficients in this covariance regression model via the Expectation Maximization (EM) algorithm. This random-effects model allows maximum likelihood parameter estimation of $\boldsymbol{B}$ and $\boldsymbol{\Psi}$ via the EM algorithm. We proceed by iteratively maximising the complete data log-likelihood of $\boldsymbol{E} = \boldsymbol{e_1}, \ldots, \boldsymbol{e_n}$ denoted $l(\boldsymbol{B}, \boldsymbol{\Psi}) = \log p(\boldsymbol{E}|\boldsymbol{B}, \boldsymbol{\Psi}, \boldsymbol{X}, \boldsymbol{\gamma})$, which is obtained from the multivariate normal density given by:

$$
-2l(\boldsymbol{B}, \boldsymbol{\Psi}) = np\log(2\pi) + n\log|\boldsymbol{\Psi}| + \sum_{i=1}^{n} (\boldsymbol{e_i} - \gamma_i \boldsymbol{B x_i})^T \boldsymbol{\Psi}^{-1} (\boldsymbol{e_i} - \gamma_i \boldsymbol{B x_i}). \tag{3.2}
$$

We note that the conditional distribution of the random effects given the data and covariates is then conveniently given by a normal distribution in each element according to $\{\gamma_i|\boldsymbol{E}, \boldsymbol{X}, \boldsymbol{\Psi}, \boldsymbol{B}\} = \mathcal{N}(m_i, v_i)$ with mean $m_i = v_i(\boldsymbol{e_i^T} \boldsymbol{\Psi}^{-1} \boldsymbol{B x_i})$ and variance $v_i = (1 + \boldsymbol{x_i^T B^T} \boldsymbol{\Psi}^{-1} \boldsymbol{B x_i})^{-1}$, see details in Hoff and Niu (2011). The advantage of this random effects specification of the covariance regression is that taking the conditional expectation of the complete data log likelihood, with respect to the conditional distribution of the random effect parameters $\gamma_i$, one obtains a closed form expression for the Expectation E-step. In addition, expressions for the maximization step (m-step) are also attainable in closed form. The stages of the expectation and maximisation are then summarised under the algorithm as follows:

- Initialize the parameter matrices $\widehat{\boldsymbol{\Psi}}$ and $\widehat{\boldsymbol{B}}$.
- Calculate the conditional estimators:

$$
\begin{aligned}
v_i &= \mathbb{V}\text{ar}\left[\Gamma_i |\, \widehat{\Psi}, \widehat{B}, \boldsymbol{e_i}\right] = (1 + \boldsymbol{x_i^T B^T} \boldsymbol{\Psi}^{-1} \boldsymbol{B x_i})^{-1} \ , \\
m_i &= \mathbb{E}\left[\Gamma_i |\, \widehat{\Psi}, \widehat{B}, \boldsymbol{e_i}\right] = v_i(\boldsymbol{e_i^T} \boldsymbol{\Psi}^{-1} \boldsymbol{B x_i}) \ .
\end{aligned}
\tag{3.3}
$$

- Construct new matrices $\widetilde{\boldsymbol{X}}$ and $\widetilde{\boldsymbol{E}}$ based on the data $\boldsymbol{y}_{1:n}$ and covariates $\boldsymbol{x}_{1:n}$.
- Evaluate the updated model parameters via the following least squares solutions for updated $\widehat{\boldsymbol{\Psi}}$ and $\widehat{\boldsymbol{B}}$ according to:

$$
\begin{aligned}
\widehat{\boldsymbol{B}} &= \widetilde{\boldsymbol{E}}^T \widetilde{\boldsymbol{X}} \left(\widetilde{\boldsymbol{X}}^T \widetilde{\boldsymbol{X}}\right)^{-1} \ , \\
\widehat{\boldsymbol{\Psi}} &= \frac{1}{n} \left(\widetilde{\boldsymbol{E}} - \widetilde{\boldsymbol{X}} \widehat{\boldsymbol{B}}\right)^T \left(\widetilde{\boldsymbol{E}} - \widetilde{\boldsymbol{X}} \widehat{\boldsymbol{B}}\right) \ ,
\end{aligned}
\tag{3.4}
$$

  where matrix $\widetilde{\boldsymbol{E}}$ is the $2n \times 1$ matrix given by $\left(\boldsymbol{E}^T, 0 \times \boldsymbol{E}^T\right)^T$ and $\widetilde{\boldsymbol{X}}$ is a $2n \times d$ matrix with $i$-th row given by $m_i \boldsymbol{x}_i$ and whose $(n+i)$-th is $\sqrt{v}_i \boldsymbol{x}_i$.

- repeat the above procedure until convergence.

We will demonstrate the results of applying this covariance regression model in Section 4.3, in order to examine the contribution of the explanatory factors to the covariance of the log returns of the currencies in the baskets. We define the following useful summary measure in Equation 3.5, which shows the proportion of covariation in the covariance regression attributed to these factors relative to the total second order explanatory power of the covariance regression. This measure focuses on the covariance explained when the covariate regression design matrix $\boldsymbol{X}$ takes its median value, denoted $\boldsymbol{X}_{(0.5)}$.

$$\text{Non-Baseline Variance Percentage} = 100 \times \frac{trace(\boldsymbol{B}\boldsymbol{X}_{(0.5)}\boldsymbol{X}_{(0.5)}^T\boldsymbol{B}^T)}{trace(\boldsymbol{B}\boldsymbol{X}_{(0.5)}\boldsymbol{X}_{(0.5)}^T\boldsymbol{B}^T) + trace(\boldsymbol{\Psi})} \ . \tag{3.5}$$

Furthermore, a confidence interval approximation for model parameters can be provided by Wald intervals as demonstrated in the in Section 6 of the online appendix.

*3.2. Higher Order and Extremal Dependence Relationships for Currency Baskets*

The study of the extremal relationships between joint appreciations and depreciations in currency baskets used in the carry trade is of primary importance to understanding upside and downside risks associated with such investment strategies and therefore also directly of relevance to understanding the risk borne by investors when attempting to invest in a manner that violates the UIP discussed in Section 2.

We discuss the notion of extremal dependence in this paper through the concept of tail dependence, which we parametrically model via copula distributions. In particular, we consider the Clayton-Frank-Gumbel mixture, where the Frank component allows for periods of no tail dependence within the basket as well as negative dependence. To be clear, we fit these copula models to each of the long and short baskets separately, dynamically over time. The number of currencies in the short basket (funding currencies) and the long basket (investment currencies) is 3 and 4 respectively. For the extended set of 34 currencies considered in Section 5.4 this varies across the time period considered from a minimum of 2 up to a maximum of 6, depending upon the number of currencies available in the dataset.

In this section we consider two stages, firstly the estimation of suitable heavy tailed marginal models for the currency exchange rates (relative to USD), followed by the estimation of the dependence structure of the multivariate model composed of multiple exchange rates in currency baskets for long and short positions. To achieve this we consider a copula model framework where we utilise the well-known result of Sklar's theorem (see Theorem 3.1), as it allows one to separate the multivariate distribution into its marginal distributions and the dependence structure between the margins known as a copula distribution, which is unique for continuous marginal models.

**Theorem 3.1.** *Sklar's Theorem (1959)*
*Consider a d-dimensional cdf $F$ with marginals $F_1, \dots, F_d$. There exists a copula $C$, s.t.*

$$F(x_1, ..., x_d) = C(F_1(x_1), \dots, F_d(x_d)), \tag{3.6}$$

*for all $x_i \in (-\infty, \infty), i \in 1, \dots, d$. Furthermore, if $F_i$ is continuous for all $i = 1, \dots, d$ then $C$ is unique; otherwise $C$ is uniquely determined only on $RanF_1 \times \cdots \times RanF_d$, where $RanF_i$ denotes the range of the cdf $F_i$.*

This result demonstrates how copula models therefore provide a mechanism to model the marginal behaviour of each currency and then separately to focus on developing hypotheses regarding the possible dependence structures between the log returns of the forward exchange rates of the currencies in the baskets, which can be tested through parametrisation of a model via a copula and then a process of model selection.

*3.3. Archimedean Mixture Copula Models*

There are many possible copula models that could be considered in the modelling of the multivariate dependence features of the currency baskets. Previously in the exchange rate modelling literature Patton (2006) considered a symmetrized Joe-Clayton copula in order to capture the asymmetric dependence structure between the Deutsche mark and the yen. The author demonstrated the time-varying nature of the dependence, noting that the mark-dollar and yen-dollar exchange rates were more correlated when they were depreciating against the dollar than when they were appreciating. A flexible time-varying copula model is introduced by Dias and Embrechts (2010) using the Fisher information to specify the dynamic correlation. The authors find a significantly time-varying correlation between euro/US dollar and yen/US dollar, dependent on the past return realizations. Creal et al. (2013) further introduced a a generalized autoregressive score model to facilitate time-varying distributions using the lagged score of the density as the forcing variable in the dynamic. However, the UIP puzzle introduced in Section 2 has not been explored in the literature from a copula modelling approach. The intention of this analysis was to work with models that have well understood tail dependence features and are relatively parsimonious with regard to the number of parameters specifying the copula and the resulting tail dependence.

In order to add additional flexibility in the possible dependence features one can study for the currency baskets, we decided to utilize mixtures of copula models. In this regard we have the advantage that we can consider asymmetric dependence relationships in the upper tails and the lower tails in the multivariate model. In addition we can perform a type of model selection purely by incorporating into the estimation the mixture weights associated with each dependence hypothesis. That is the data can be utilised to decide the strength of each dependence feature as interpreted directly through the estimated mixture weight attributed to the feature encoded in the particular mixture component from the Archimedean family.

In particular, we have noted that mixture copulae (see Definition 3.2) can be used to model asymmetric tail dependence, i.e. by combining the one-parameter families discussed in Table 1 in the online appendix or indeed by any combination of copulae. This is possible since a linear convex combination of two copulae is itself a copula, see Nelsen (2006).

**Definition 3.2.** *Mixture Copula*
*A mixture copula is a linear weighted combination of copulae of the form:*

$$C_M(\mathbf{u};\mathbf{\Theta}) = \sum_{i=1}^{N} w_i C_i(\mathbf{u};\theta_{\mathbf{i}}), \tag{3.7}$$

*where* $0 \leq w_i \leq 1 \ \ \forall i = 1, ..., N$ *and* $\sum_{i=1}^{N} w_i = 1$.

Thus we can combine a copula with lower tail dependence, a copula with positive or negative dependence and a copula with upper tail dependence to produce a more flexible copula capable of modelling the multivariate log returns of forward exchange rates of a basket of currencies. For this reason in this analysis we will use the Clayton-Frank-Gumbel (C-F-G) mixture model.

**Remark 3.3.** *We note that the tail dependence of a mixture copula can be obtained as the linear weighted combination of the tail dependence of each component in the mixture weighted by the appropriate mixture weight, as discussed in for example Nelsen (2006) and Peters et al. (2012).*

Archimedean copula models are completely specified by their generator function $\psi$, where a function $\psi$ is said to generate an Archimedean copula if it satisfies the properties in Definition 3.4.

**Definition 3.4.** *Archimedean Generator*
*An Archimedean generator is a continuous, decreasing function* $\psi : [0, \infty) \to [0, 1]$ *which satisfies the following conditions:*

1. $\psi(0) = 1$,

2. $\psi(\infty) = lim_{t\to\infty}\psi(t) = 0$,

3. *$\psi$ is strictly decreasing on* $[0, inf\{t : \psi(t) = 0\}]$.

**Definition 3.5.** *Archimedean Copula*

*A d-dimensional copula C is called Archimedean if for some generator $\psi$ it can be represented as:*

$$C(\mathbf{u}) = \psi\{\psi^{-1}(u_1) + \cdots + \psi^{-1}(u_d)\} = \psi\{t(\mathbf{u})\}, \quad \forall \mathbf{u} \in [0,1]^d, \tag{3.8}$$

*where* $\psi^{-1} : [0,1] \to [0,\infty)$ *is the inverse generator with* $\psi^{-1}(0) = inf\{t : \psi(t) = 0\}$.

Note the shorthand notation $t(\mathbf{u}) = \psi^{-1}(u_1) + \cdots + \psi^{-1}(u_d)$ that will be used throughout this section.

### *3.3.1. Finding Archimedean Copula Densities*

For likelihood based estimation and inference it will be beneficial to obtain formulas for computing the copula densities.

**Definition 3.6.** *Archimedean Copula Density*

*McNeil and Nešlehová (2009) prove that an Archimedean copula C admits a density c if and only if $\psi^{(d-1)}$ exists and is absolutely continuous on $(0,\infty)$. When this condition is satisfied, the copula density c is given by*

$$c(\mathbf{u}) \quad = \quad \frac{\partial^d C(u_1,\ldots,u_d)}{\partial u_1 \ldots \partial u_d} \quad = \quad \psi^{(d)}\{t(\mathbf{u})\}\prod_{j=1}^{d}(\psi^{-1})'(u_j) \quad , \quad \mathbf{u} \in (0,1)^d. \tag{3.9}$$

Hence to evaluate pointwise the likelihood one can see from Equation 3.9 that it is required to compute high dimensional derivatives of a composite function. In order to achieve this one may utilise a specific multivariate chain rule result widely known as the Faà di Bruno's Formula, see Faa di Bruno (1857) and discussions in for example Constantine and Savits (1996) and Roman (1980).

In our mixture copula model we consider mixture copula components from the one parameter Archimedean copula families. A one parameter Archimedean copula is one in which the generator $\psi$ has a single parameter that characterizes the strength of the multivariate concordance structures of the resulting copula sub-family, see a detailed description in Cruz et al. (2015). In this section we describe three of the one parameter multivariate Archimedean family copula models which have become popular model choices and are widely used for estimation due to their directly interpretable dependence (concordance) features. We select the Clayton, Frank and Gumbel models for our mixture model components since they each contain differing tail dependence characteristics. Clayton provides lower tail dependence whilst Gumbel provides upper tail dependence. The Frank copula also provides dependence in the unit cube with elliptical contours with semi-major axis oriented at either $\pi/4$ or $3\pi/4$ depending on the sign of the copula parameter in the estimation. Therefore, the Frank model component will allow us to capture parsimoniously potential negative dependence relationships between the currencies in the basket under study. Formulas for these copulae, as well as their respective generators, inverse generators and the d-th derivatives of their generators (required for the density evaluation) are given in Table 1 in the online appendix. The explicit formulas for the d-th derivatives for all of the copulae in Table 1 in the online appendix are derived in Hofert et al. (2012).

### *3.4. Joint Tail Exposure in the Carry Trade via Extremal Tail Dependence Coefficient*

In order to fully understand the tail risks of joint exchange rate movements present when one invests in a carry trade strategy one must look at four extremal measures of dependence: both downside extremal tail exposures and both upside extremal tail exposures, that occur within the long and the short baskets that comprise the strategy. The downside tail exposure can be seen as the crash risk of the basket, i.e. the risk that one will suffer large joint losses from each of the currencies in the basket. These losses would be the result of joint appreciations of the currencies one is short in the low interest rate basket and/or joint depreciations of the currencies one is long in the high interest rate basket. The downside tail exposures are thus characterised by the conditional probabilities that one or more currencies in the long/short basket depreciates/appreciates beyond an extreme threshold given that the remaining currencies in the long/short basket depreciate/appreciate beyond this threshold.

The upside tail exposure is the risk that one will earn large joint profits from each of the currencies in the basket. These profits would be the result of joint depreciations of the currencies one is short in the low interest rate basket and/or joint appreciations of the currencies one is long in the high interest rate basket. The upside tail exposures are thus characterised by the conditional probabilities that one or more currencies in the short/long basket depreciates/appreciates beyond an extreme threshold given that the remaining currencies in the short/long basket depreciate/appreciate beyond this threshold.

We can formalise this notion of the dependence behaviour in the extremes of the multivariate distribution through the concept of tail dependence. The tail dependence coefficient is defined as the conditional probability that a random vector exceeds a certain threshold (in the limit as the threshold increases) given that the remaining components of the random vector in the joint distribution have exceeded this threshold. This concept had previously been considered in the bivariate setting and extended to the multivariate setting in for instance De Luca and Rivieccio (2012) and further utilised in the analysis of currency baskets in Ames et al. (2015). Now one may accurately interpret the tail dependence present between sub-vector partitions of the multivariate random vector of currency exchange rates in either basket with regard to joint tail dependence behaviours. In the context of the applications we consider in this paper, this allows us to examine the probability that any subvector of the log return forward exchange rates for the basket of currencies will exceed a certain threshold given that the log return forward exchange rates for the remaining currencies in the basket have exceeded this threshold. The interpretation of such results is then directly relevant to assessing the chance of large adverse movements in multiple currencies which could potentially increase the risk associated with currency carry trade strategies significantly, compared to risk measures which only consider the marginal behaviour in each individual currency. We thus consider these tail upside and downside exposures as features that can show that even though average profits may be made from the violation of UIP, it comes at significant joint tail exposure. Next we formally define the upper and lower tail dependence, as well as the explicit generalised multivariate expressions for Archimedean copulae in Equations (3.10) and (3.11).

**Definition 3.7.** *Generalized Upper Tail Dependence Coefficient*
*Let $X = (X_1, ..., X_d)^T$ be a $d$ dimensional random vector with marginal distribution functions $F_1, ..., F_d$. The coefficient of upper tail dependence is defined as:*

$$\begin{aligned}\lambda_u^{1,\dots,h|h+1,\dots,d} &= \lim_{\nu \to 1-} P\left(X_1 > F^{-1}(\nu), ..., X_h > F^{-1}(\nu) | X_{h+1} > F^{-1}(\nu), ..., X_d > F^{-1}(\nu)\right) \\ &= \lim_{t \to 0^+} \frac{\sum_{i=1}^{d} \left( \binom{d}{d-i} i(-1)^i \left[ \psi^{-1'}(it) \right] \right)}{\sum_{i=1}^{d-h} \left( \binom{d-h}{d-h-i} i(-1)^i \left[ \psi^{-1'}(it) \right] \right)}.\end{aligned} \tag{3.10}$$

**Definition 3.8.** *Generalized Lower Tail Dependence Coefficient*
*Let $X = (X_1, ..., X_d)^T$ be a $d$ dimensional random vector with marginal distribution functions $F_1, ..., F_d$. The coefficient of lower tail dependence is defined as:*

$$\begin{aligned}\lambda_l^{1,\dots,h|h+1,\dots,d} &= \lim_{\nu \to 0+} P\left(X_1 < F^{-1}(\nu), ..., X_h < F^{-1}(\nu) | X_{h+1} < F^{-1}(\nu), ..., X_d < F^{-1}(\nu)\right) \\ &= \lim_{t \to \infty} \frac{d}{d-h} \frac{\psi^{-1'}(dt)}{\psi^{-1'}((d-h)t)}.\end{aligned} \tag{3.11}$$

We use as our exchange rates the number of units of foreign currency per one dollar. Therefore the downside exposures are the upper tail dependence in the high interest rate basket and the lower tail dependence in the low interest rate basket. The upside exposures are the lower tail dependence in the high interest rate basket and the upper tail dependence in the low interest rate basket.

## 4. Exploring Intertemporal Cross-Sectional Volatility-Volume Relations

There has been a growing interest in the literature to study the effects of different factors, empirical features, macro and microeconomic factors on the mean and volatility dynamics of individual currency exchange rates. For instance, in Christiansen et al. (2012) they study return volatility of exchange rates with respect to different functions of macro economic variables such as: equity market variables and risk factors such as the dividend price ratio, the earnings price ratio, equity market returns for leverage effects, Fama and French (1993) risk factors; interest rates, spreads and bond market factors such as T-bill rates, term spreads and other factors related to term structure forward rates discussed in Cochrane and Piazzesi (2002); foreign exchange rate variables and risk factors such as the average forward discount for capturing counter cyclical FX risk premia, the $DOL$ risk factor and the $HML_{FX}$ carry factors of Lustig and Verdelhan (2007); Lustig et al. (2011); liquidity and credit risk factors such as yield spreads between BAA and AAA rated bonds (i.e. default spreads), TED spreads for LIBOR rate and T-Bill rates discussed in funding liquidity in Brunnermeier and Pedersen (2009) as well as aggregate measures of bid-ask spreads in foreign exchange markets such as those discussed in Menkhoff et al. (2012); and macro-economic variables. In this study of Christiansen et al. (2012) they concentrate on explaining carefully individual exchange rates through Bayesian model averaging, however they neglect to study the joint relationships between multiple exchange rates and the influence of these factors proposed.

In this analysis, we intend to generalize these types of studies to focus on joint behaviours in multiple exchange rates and we focus on a few important factors principally related to the exchange rate market dynamics. In particular, we highlight the importance of factors based on speculative order flows in influencing the joint appreciation and depreciation dynamics of baskets of multiple exchange rates. This is interesting since there is mounting evidence that speculative order flows in the markets have a substantial impact on the dynamics of certain financial assets. An extended literature is documenting the empirical relation between the trading activity and its impact upon the asset drift (Singleton (2013), Hong and Yogo (2012)) or price innovations relative to a benchmark (Henderson et al. (2015)) or even the volatility (Ané and Geman (2000), Gallant et al. (1992)) of a given asset. The challenge with such findings is that speculator behaviours consist of an evolutionary process, which is naturally a function of the current market conditions, but also and significantly of the economic environment, which could be more or less prone to the growth of speculative inflows for instance to satisfy the hedging needs from the non-financial sphere.

Following this literature trend we set out to demonstrate the existence of a dual relation between high and low interest rate differential currency baskets and the associated dependences, by comparing them with the amount of speculative inflows and outflows on the available funding and investing currency futures. While numerous authors have emphasized the relation between the volume traded on a specific asset and its volatility (Ané and Geman (2000), Gallant et al. (1992)) we propose hereafter to focus more particularly on the speculative flows which allows us to broaden the analysis and in so doing investigate the relation between the non-commercial traders net positions, commonly considered as speculators, and the dependence between financial assets. Hence, we assume that setting up a carry position in the currency market will synchronously impact all the currency prices and thus materialise into higher price dependences and consequently a less sparse log-returns covariance. While several articles, such as Hasbrouck and Seppi (2001), Bernhardt and Taub (2008) and He and Velu (2014) investigate the relation between the volume commonalities and the price commonalities none of them have focused on the speculative volumes nor have they studied the currency markets. Moreover, our analysis contributes to the literature as we propose in this section a new approach, namely the covariance regression, to study this relation between volume and asset prices, while in the following section we will investigate this causal relation for extremal return commonalities.

Through different price and volume based factors we thus explore the effect of the speculators behaviour on the first two moments of the cross-sectional currency returns. Firstly, we describe the data we used for the empirical studies. Then, after studying the informational content of the speculative volume time series, we apply a mean regression of the individual currency returns. Finally, we perform a covariance regression of the multivariate basket returns given the explanatory factors.

### *4.1. Exchange Rate Multivariate Data Description and Currency Portfolio Construction*

We consider for our empirical analysis a set of eight major developed markets, as in Brunnermeier et al. (2008), namely Australia (AUD), Canada (CAD), Japan (JPY), New Zealand (NZD), Norway (NOK), Switzerland (CHF), United Kingdom (GBP), and the euro area (EUR). We also extend this dataset to include the following developing countries: Singapore (SGD), Taiwan (TWD), India (INR), Mexico (MXN), South Africa (ZAR), Brazil (BRL) and Turkey (TRY). We have considered daily settlement prices for each currency exchange rate as well as the daily settlement price for the associated 1 month forward contract. The daily time series analysed range from 04/01/1999 to 29/01/2014.

We indeed considered the point of view of an American investor as this is generally the hypothesis retained in the literature (see Brunnermeier et al. (2008); Menkhoff et al. (2012)). However the same analysis could be carried out from any other investor standpoint as the phenomena we will describe does not only depend on a specific currency but on two or more sets of currencies. These sets of currencies correspond to the high interest rate currencies which are used to obtain the highest return (named the "investment currencies") and the low interest rate currencies which allow for borrowing at a low cost the amount of money necessary for this investment (named the "financing currencies"). Following the approach proposed by Brunnermeier et al. (2008), we consider the historical level of interest rate differential in order to distinguish between the financing and the investing currencies, which accordingly leads to fixed high and low interest rate portfolio components for the duration of the time period we analyse.

For our analysis of the speculative flows effects we rely on the data provided by the CFTC[2] which indeed provides the Commitments of Traders, a weekly report showing the open positions, long and short, on the currency future contracts traded on the Chicago Mercantile Exchange and breaks them down into commercial, non-commercial and non-reportable positions. The group of the non-commercial investors also qualified as speculative traders are not holding the futures positions until expiry and will in general be more nimble and prone to build carry positions in the market. On the contrary, the commercial traders are using the futures market to hedge an existing exposure on the physical underlying.

We note that the volume data provided by the CFTC for the Norwegian Krone are not long enough for a robust analysis which constrained us to exclude this currency from our analysis. We indeed retained for our volume analysis a period of time which spans 27/06/2006 to 29/01/2014. The start point corresponding to the date when the New Zealand Dollar (NZD) contract started to be liquid enough to be included in our analysis[3]. Furthermore, given that the CFTC data we need to run our regression analysis are available on a weekly basis, we build the corresponding weekly returns based on the daily settlement prices we have at our disposal. This construction is provided in Section 7.

[2]The US Commodity Futures Trading Commission.

[3]Before this date the NZD contract open interest was mostly equal to zero, which means that no positions were during this time opened on the future markets, thus justifying that we exclude from our volume analysis the data until this date.

Finally, in section 5.4 we extend the tail dependence estimations to a much larger dataset of daily time series ranging from 02/01/1989 to 29/01/2014 comprised of the following 34 currencies: Australia (AUD), Brazil (BRL), Canada (CAD), Croatia (HRK), Cyprus (CYP), Czech Republic (CZK), Egypt (EGP), Euro area (EUR), Greece (GRD), Hungary (HUF), Iceland (ISK), India (INR), Indonesia (IDR), Israel (ILS), Japan (JPY), Malaysia (MYR), Mexico (MXN), New Zealand (NZD), Norway (NOK), Philippines (PHP), Poland (PLN), Russia (RUB), Singapore (SGD), Slovakia (SKK), Slovenia (SIT), South Africa (ZAR), South Korea (KRW), Sweden (SEK), Switzerland (CHF), Taiwan (TWD), Thailand (THB), Turkey (TRY), Ukraine (UAH) and the United Kingdom (GBP).

*4.2. Currency Mean Dynamic Decomposition*

In order to understand, relative to the price based information flows, that the speculative trading volumes are distinctly influencing the international exchange rates, we first assess in this section if the cross-sectional ratios of speculator net positions on the market open interest have a significant impact on the mean dynamic of individual currencies once we have accounted for the variability explained by the common price based FX market factors described in Lustig and Verdelhan (2007), namely the dollar factor $DOL$ and the carry high minus low factor $HML_{FX}$. Before doing that, we need nevertheless to justify the use of the CFTC non-commercial open positions as a proxy of the carry trade speculative positions.

As a matter of fact, these time series provided by the CFTC do not distinguish the open positions resulting from the carry trade or the other potential motivations for the speculator. For instance, the open position will definitely be impacted by the setting up of a dollar position or a relative value trade between the European currencies and the others. In order to discern the common factors impacting the individual currencies percentage of non-commercial traders among futures open positions we run a principal component analysis on the net speculative positions published by the CFTC and found that 49.5% of the variance associated to this set of currencies, is explained by the first principal component. Moreover, the currency loadings associated to this first factor are interestingly (almost) monotonically decreasing according to their respective average differential of interest rates with the US Dollar. We should also notice that the associated eigenvector (see Figure 1) displays a negative sign for the principal financing currencies such as JPY and CHF whereas the investing currencies, such as NZD and AUD shows a positive relation with the first component. These results confirm that a large part of the net speculative positions in futures is directly following from the carry trade strategy.

Then to achieve our analysis of the currency conditional mean dynamic, we consider a regression of each of the individual currency returns time series, for currencies utilised in the carry trade, onto explanatory risk factors given by the time series of the covariates $DOL$ and $HML_{FX}$, as well as the percentages of the net speculative position relative to open interest, which we term $SPEC$, for all the available currencies. As mentioned earlier, we perform this regression on weekly data, considering the period spanning from 27/06/2006 to 29/01/2014 and excluding as a consequence the Norwegian krone from our analysis. One can note that even though speculative volumes seem to contribute to the variability of a couple of currency returns, it turns out that their contribution to the explanatory power of the regression model remains ancillary. We indeed notice that the $R^2$ is very marginally improved once we include the speculative open interest covariates. This statement leads us to the conclusion that the dollar and the high minus low factors proposed by Lustig and Verdelhan (2007) clearly prevail over the speculative volumes variables as far as it concerns the mean dynamic of the cross-sectional currency returns. This assertion corroborates the microeconomic literature regarding the variability-volume theory, wherein it has been demonstrated that volumes traded in equity markets mainly relate to the variance of the very same equities and not their price or even their return dynamics.

|  | $AUD$ | $CAD$ | $CHF$ | $EUR$ | $GBP$ | $JPY$ | $NZD$ |
|---|---|---|---|---|---|---|---|
| $Constant$ | 0.0000 | 0.0020 | -0.0008 | -0.0009 | 0.0009 | 0.0011 | -0.0007 |
|  | (0.2563) | (0.3761) | (0.4437) | (0.9754) | (0.0607) | (0.3478) | (0.5458) |
| $DOL$ | -0.5099** | -0.3130** | -0.3466** | -0.3834** | -0.3187** | -0.0585** | -0.5225** |
|  | (0.0000) | (0.0042) | (0.0000) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $HML_{FX}$ | 0.3087** | 0.2028** | -0.5919** | -0.2876** | -0.1095** | -0.5727** | 0.3142** |
|  | (0.0000) | (0.0000) | (0.0006) | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $AUD$ | -0.0014 | 0.0019 | -0.0027 | 0.0022 | -0.0002 | 0.0013 | 0.0004 |
|  | (0.2644) | (0.5863) | (0.9212) | (0.4238) | (0.3659) | (0.1154) | (0.8255) |
| $CAD$ | 0.0009 | -0.0056** | -0.0014 | 0.0003 | 0.0015 | 0.0006 | 0.0018 |
|  | (0.8632) | (0.7607) | (0.3368) | (0.5140) | (0.0059) | (0.4032) | (0.2946) |
| $CHF$ | 0.0004 | 0.0047 | 0.0003 | -0.0011 | -0.0022 | 0.0017 | -0.0014 |
|  | (0.5611) | (0.4709) | (0.3871) | (0.7944) | (0.0651) | (0.8821) | (0.4708) |
| $EUR$ | -0.0013 | -0.0021 | -0.0001 | -0.0007 | 0.0056* | -0.0022 | 0.0002 |
|  | (0.7564) | (0.4784) | (0.0323) | (0.4956) | (0.3903) | (0.9633) | (0.9390) |
| $GBP$ | 0.0013 | 0.0055** | 0.0012 | -0.0037 | -0.0041 | 0.0038 | -0.0007 |
|  | (0.0652) | (0.1688) | (0.1275) | (0.4929) | (0.0239) | (0.5135) | (0.7691) |
| $JPY$ | -0.0041** | -0.0012 | 0.0018 | 0.0006 | 0.0054** | -0.0052* | 0.0010 |
|  | (0.7649) | (0.0309) | (0.0219) | (0.0050) | (0.5046) | (0.3306) | (0.5773) |
| $NZD$ | 0.0004 | -0.0026 | 0.0040* | 0.0011 | -0.0011 | -0.0049 | -0.0008 |
|  | (0.5605) | (0.0882) | (0.5904) | (0.8047) | (0.2717) | (0.0448) | (0.7120) |
| $NOK$ | - | - | - | - | - | - | - |
|  | - | - | - | - | - | - | - |
| $R^2$ ($DOL$, $HML_{FX}$) | 92% | 68% | 80% | 81% | 60% | 57% | 90% |
| $R^2$ ($DOL$, $HML_{FX}$, $SPEC$) | 92% | 69% | 81% | 81% | 61% | 59% | 90% |

Table 1: Regression of the individual currencies returns on the $DOL$ index, $HML_{FX}$ index and the $SPEC$ ratio (the ratio of each currency future speculative net position to the total future open interest, as provided by the CFTC), as well as cross relations among them. The open interest data provided by the CFTC as well as the computed $DOL$ and $HML_{FX}$ indexes are weekly data while the respective tail dependence measurement corresponds to the average value over each week. The period of time considered for this analysis spans from June, 27th, 2006 to January, 29th, 2014 and corresponds to the longest overlapping sample for all the currencies considered and available. Numbers in parentheses show Newey and West (1987) HAC p-values. All the possible cross effects among the currencies are not significantly contributing to the regression (HAC p-value below 5%)

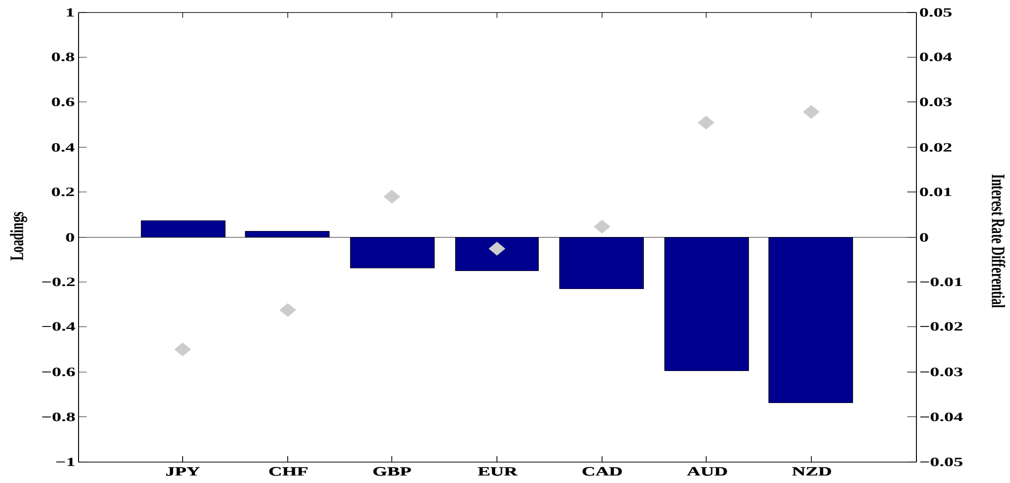


Figure 1: Loadings of the First Principal Component of Developed Countries Speculative Percentage.The bars (left axis) represent the loadings values on the speculative percentages first principal component while the grey diamonds (right axis) depicts the level of interest rate differential with the 1 month US interest rates.

### *4.3. A Covariance Regression Model Considering $DOL$, $HML_{FX}$ and $SPEC$ Factors*

While in the previous section we considered regression on the mean structure looking at whether the market price factor $DOL$, carry factor $HML_{FX}$ and speculative volume factors $SPEC$ provided statistically significant explanatory power in describing the trend in the returns dynamic of individual currencies and currency baskets constructed from ordering of interest rate differentials. In this section, we extend these mean-regressions of carry trade basket returns to study how these factors load directly on the regression against the covariance and correlation structure of the assets in the currency baskets. This will tell us the proportion of covariance, between the currencies in each basket, that can be explained by the $DOL$, $HML_{FX}$ and $SPEC$ factors. We will investigate two sets of covariates for each of the high and low interest rate baskets. Firstly, we will examine the power of the $DOL$ and $HML_{FX}$ factors in explaining the covariance. Secondly we will analyse what explanatory power is contributed by the $SPEC$ factors and the first order cross terms between the $SPEC$ factors.

To perform this study we formulate our covariance regression model as a special type of random-effects model, see Hoff and Niu (2011), for observed data $\boldsymbol{y_1}, \ldots \boldsymbol{y_n}$ ($d$-dimensional high and low interest rate basket daily log returns for a time block of length $n$). The first step is the mean-regression, as seen in the previous section, to produce zero-mean residuals $\boldsymbol{e_i}$, given by $\boldsymbol{e_i} = \boldsymbol{y_i} - \boldsymbol{\beta x_i}$, where the covariate vector is constructed as $\boldsymbol{x_i} = [DOL(i),\ HML_{FX}(i)]^T$ or $\boldsymbol{x_i} = [DOL(i),\ HML_{FX}(i),\ SPEC(i),\ SPEC(i) \times SPEC(i)]^T$. Having defined the residuals with de-trended mean, we perform the covariance regression of these residuals on the factors, using the random-effects representation presented in Section 3.1 of this manuscript, which involved characterization of the covariance matrix for $\boldsymbol{e_i}$, conditional on $\boldsymbol{x_i}$ given by,

$$\mathbb{E}[(\boldsymbol{e_i})(\boldsymbol{e_i})^T|\boldsymbol{x_i}] = \boldsymbol{B x_i x_i^T B^T} + \boldsymbol{\Psi}. \tag{4.1}$$

We may now perform the estimation of the coefficient matrices $\boldsymbol{B}$ and $\boldsymbol{\Psi}$ in this covariance regression model via the Expectation Maximization (EM) algorithm presented in Section 3.1. The interest for our empirical work of this covariance regression model is to examine the contribution of the explanatory factors to the conditional covariance matrix of the currency log returns relative to the associated interest rate differentials, and thus the basket each currency belongs to.

We plot the summary measure described in Equation 3.5, which shows the proportion of covariation in the covariance regression attributed to these factors relative to the total second order explanatory power of the covariance regression on each 125 week sliding window. This measure focuses on the covariance explained when $\boldsymbol{X}$ takes its median value, denoted $\boldsymbol{X}_{(0.5)}$. Figure 2 shows this result for both the high interest rate basket and the low interest rate basket. We can see that the explanatory power of the two factors, $DOL$ and $HML_{FX}$, is time-varying, but that there is very little power in explaining the linear 2nd order co-movements of currencies in either the high or low interest rate baskets as captured by variance and correlation structures intertemporally. Further strengthening our hypothesis that one must look at co-currency movements using flexible models that capture appropriate concordance/discordance relationships such as tail dependence. The large increase in explanatory power by using the full $DOL + HML_{FX} + SPEC + SPEC \times SPEC$ model over the two factor $DOL + HML_{FX}$ model can be seen for both the high interest rate basket and the low interest rate basket in Figure 3. We observe that the explanatory power of the model incorporating SPEC and its crosses is several orders of magnitude greater than the two factors $DOL + HML_{FX}$ model.

It is worth highlighting that for each sliding window we estimate the regression relationships with covariate vector $\boldsymbol{x}_t = [1, x_{1,t}, \ldots, x_{q,t}]$ given by the factors discussed and the resulting covariance matrix parameter vectors defined by row vectors of the $(p \times q)$ matrix $\boldsymbol{B}$ which is given by $\{\boldsymbol{b}_1, \ldots, \boldsymbol{b}_p\}$ vectors, which then result in the regression models for each element $j$ of the covariance matrix of the currency basket being given by[4]:

$$\begin{aligned}
\mathbb{V}\mathrm{ar}\left[\left(\boldsymbol{y}_j - \boldsymbol{\beta}\boldsymbol{x}_t\right)\middle|\, \boldsymbol{x}_t\right] &= \Psi_{j,j} + \boldsymbol{b}_j \boldsymbol{x}_t \boldsymbol{x}_t^T \boldsymbol{b}_j^T \\
&= \Psi_{j,j} + \sum_{s=1}^{q} b_{j,s} x_{s,t} x_{s,t}^T b_{j,s}^T \,, \\
\mathbb{C}\mathrm{ov}\left[\left(\boldsymbol{y}_j - \boldsymbol{\beta}\boldsymbol{x}_t\right)\left(\boldsymbol{y}_k - \boldsymbol{\beta}\boldsymbol{x}_t\right)\middle|\, \boldsymbol{x}_t\right] &= \Psi_{j,k} + \boldsymbol{b}_j \boldsymbol{x}_t \boldsymbol{x}_t^T \boldsymbol{b}_k^T \\
&= \Psi_{j,k} + \sum_{s=1}^{q} b_{j,s} x_{s,t} x_{s,t}^T b_{k,s}^T .
\end{aligned} \tag{4.2}$$

We perform this estimation on a daily sliding window, whereby for each sliding window period we obtain point estimators for the $\boldsymbol{B}$ matrix parameters. Therefore as we slide the window we get different realizations based on the data fits for the estimated parameter relationships. We summarise these by constructing box plots of the parameter estimates for the full model containing $DOL + HML_{FX} + SPEC + SPEC \times SPEC$ covariates for both the high interest rate basket, which can be seen in Figure 4, and also for the low interest basket which is provided in Figure 5. In addition, for each sliding window we can test the statistical significance of the estimated coefficient where the null hypothesis would be that the parameter is zero versus an alternative that it is non-zero. A description of the test statistic is provided in Section 6 in the online appendix. The results of the test on each sliding window are indicated by adjusting the width of each box so that it is equal to the proportion of the sliding windows for which this parameter was significant, i.e. its confidence intervals did not cross zero. For details of the calculation of the confidence intervals for the parameters see Section 6 in the online appendix and more generally Hoff and Niu (2011).

To aid in the interpretation of this analysis we partition the results for the $\boldsymbol{B}$ matrix parameter estimation according to the loadings on each currency for a given factor, for instance in the high interest rate analysis we have four currencies considered so for each factor such as $DOL, HML_{FX}, SPEC$ and $SPEC \times SPEC$ we have four box plots, one for the parameter estimate time-evolving loadings for each currency in the order GBP, AUD, CAD and NZD. Then we have the analogous structure for the low interest rate basket with in this case three box plots per factor due to the fact

[4]In our case the dimensions of the matrix $\boldsymbol{B}$ are linked to the portfolio under scrutiny. In the high interest rate basket covariance regression, we analyse the detrended returns variability of the GBP, the AUD, the CAD and the NZD relative to 12 covariates (which are the $DOL$, the $HML_{FX}$, the four associated currency $SPEC$ factors and six cross $SPEC \times SPEC$ factors) which leads to $(4 \times 12)$ matrix $\boldsymbol{B}$. Likewise, in the low interest rate basket the dimensions of $\boldsymbol{B}$ are $(3 \times 8)$ as we focus on the variability of the EUR, the JPY and the CHF relative to 8 covariates (which are the $DOL$, the $HML_{FX}$, the three associated currency $SPEC$ factors and three cross $SPEC \times SPEC$ factors).

that we only have EUR, JPY and CHF in this basket.

We can then interpret these factor loadings as the proportion of change we would expect in the covariance relationships between each currency in the basket given a unit change in the factor. The utility of the covariance regression model lies in the additional variability $\mathbf{B x_i x_i^T B^T}$, which is randomly added to the baseline variability, $\boldsymbol{\Psi}$, of the detrended data. Thus, for the low interest rate basket we should interpret the set of vectors $\{\mathbf{b_{CHF}}, \mathbf{b_{JPY}}, \mathbf{b_{EUR}}\}$ associated to each currency as how the additional variability or heteroskedasticity is manifested across the covariates. In other words the components of each vector, for instance $\mathbf{b_{CHF}} = [\mathrm{b_{CHF,DOL}}, \mathrm{b_{CHF,HML_{FX}}}, \ldots]$, correspond to the sensitivities of the Swiss franc detrended returns variability to the set of factors $\{DOL, HML_{FX}, \ldots\}$. We should accordingly associate a high norm of the vector $\mathbf{b_{CHF}}$ to a high heteroskedasticity in the Swiss franc detrended returns. Another interesting aspect of this analysis is the cross analysis of the vectors $\{\mathbf{b_{CHF}}, \mathbf{b_{JPY}}, \mathbf{b_{EUR}}\}$, since the pair of significantly different from zero vectors $\{\mathbf{b_{CHF}}, \mathbf{b_{JPY}}\}$ means that the Swiss franc and the Japanese yen become more correlated when their variances increase. We could even augment the granularity of our analysis and interpret the highly significant pairs of vector components $\{b_{CHF,DOL}, b_{JPY,DOL}\}$ and $\{b_{CHF,HML_{FX}}, b_{JPY,HML_{FX}}\}$ as the significant effect of the $DOL$ and $HML_{FX}$ factors upon the covariance between the Swiss franc and the Japanese yen. Said differently, when the $DOL$ and the $HML_{FX}$ factors change, the covariance of the Swiss franc and the Japanese yen consequently picks up. Furthermore, we see features which indicate that for the joint currency covariance structure the $DOL$ factor is significant on all three currencies and loads more substantially on the variance of the EUR compared to the JPY and CHF, indicating the heightened sensitivity of the volatility of the EUR to the $DOL$ factor than the other currencies. This last reasoning from the covariance regression is particularly relevant for the analysis of the impact of the speculative positions we undertake in this manuscript. In Figure 5, the width of the boxes associated to the Swiss franc speculative interest covariate shows a significant increase in the covariance between the Swiss franc and the Japanese yen returns when the speculative interests on the Swiss franc increase. Whereas the width of the boxes associated to the Euro speculative covariate show that when the Euro speculative interests increase, a significant increase of the correlation between the Euro and the Swiss franc is noticeable. These results demonstrate that speculator interventions on low interest rate currencies systematically influence the covariances among these currencies and that volume based information provides complementary indication about asset price dynamics and dependences.

Likewise, it is apparent in the high interest rate basket results that the $DOL$ and the $HML_{FX}$ factors load significantly on the covariance relationships for the GBP, more substantially than any of the relationships for the other currencies in this basket. This is especially the case for the $HML_{FX}$ factor and in the majority of cases they are statistically significant loadings at 5% significance. Moreover, there seems to be asymmetry in the factor loadings for the speculative open interest of one currency exchange on another currency exchange. For instance, whilst the impact of the speculative open interest of the GBP $SPEC$ is significant predominantly in all fits for the GBP, AUD and CAD exchange rates (not for the NZD) we see that this speculative open interest factor loads much more substantially historically on the CAD than it does on the AUD or GBP. Conversely, when we look at the speculative open interest on the CAD we see that an asymmetry arises since the most dominant loading of this speculative open interest is on the GBP, CAD and NZD and it is largely statistical significant in the majority of fitted time periods.

Another very interesting point worth noting from this covariance regression is the relation between the GBP variance and the speculative volumes. We can indeed notice that for each $SPEC$ covariate the average width of the first box, which corresponds to the statistical significance of each covariate effect upon the GBP variance, is conspicuously higher than for the other currencies. Finally, the last two covariate results displayed in Figure 4 shed light on the relation existing between the AUD and NZD speculator inflows correlation and the covariance between the GBP and NZD exchange rates while the same relation shows up between the CAD and NZD speculator inflows synchronicity and the

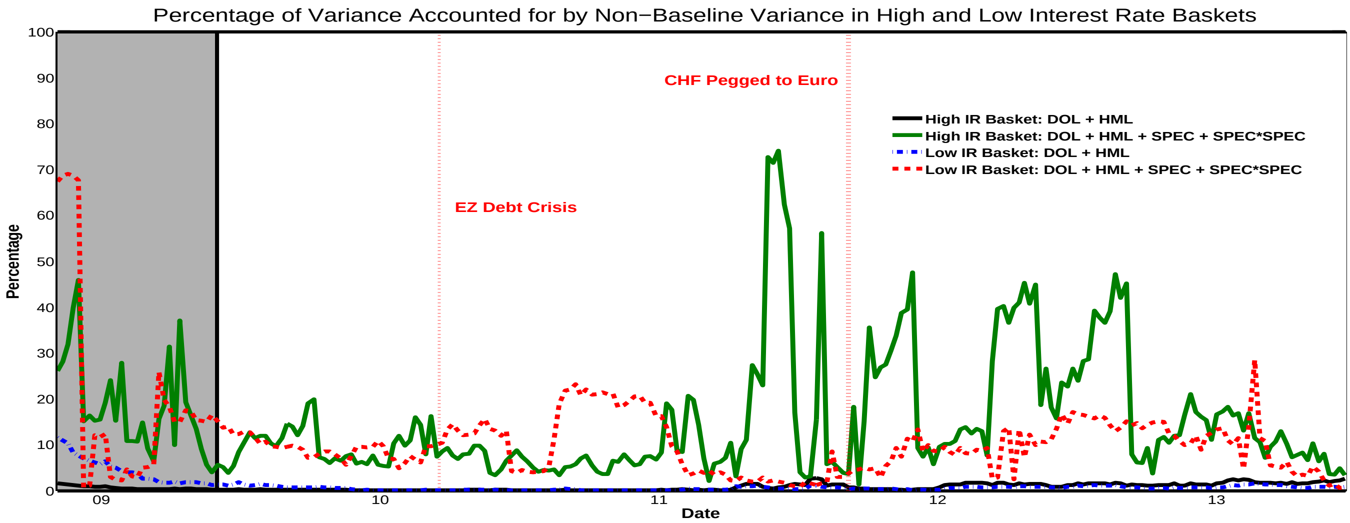


Figure 2: High IR and Low IR Basket. $DOL + HML_{FX}$ vs $DOL + HML_{FX} + SPEC + SPEC \times SPEC$. 125 week lookback periods.

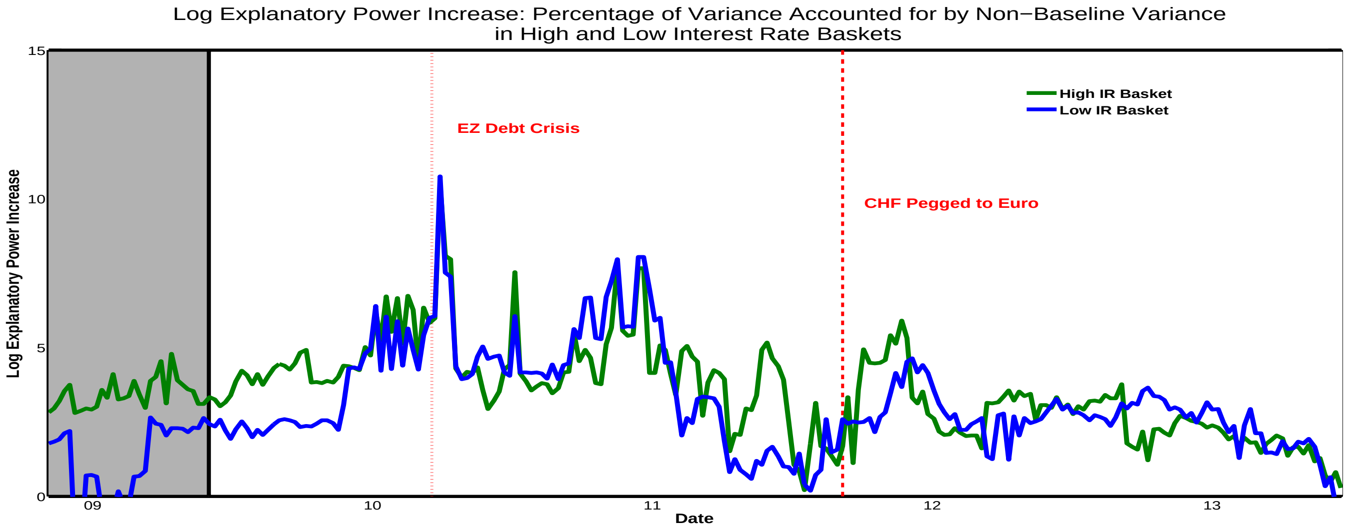


Figure 3: Log Explanatory Power Increase: High IR and Low IR Basket. $DOL+HML_{FX}$ vs $DOL+HML_{FX}+SPEC+SPEC \times SPEC$. 125 week lookback periods.

correlation between GBP, AUD and NZD exchange rates. These interesting outcomes substantiate our assertion about the impact of the speculative interest changes upon the individual variability and the dependence structure among the high interest rate currencies as well as the low interest rate currencies.

## 5. Interest Rate Differentials, Carry Trade and Tail Dependence Features in the Foreign Exchange Market

In this section, we focus mainly on the interpretation of the dynamic copula mixture estimation of exchange rate time series ranked relative to their respective level of local interest rate. The mixture copula parameters estimated thereby provide, through the combination of the copula mixture components (Definition 3.2) and the associated upper or lower tail dependence expressions (Equations (3.10) and (3.11)), a parametric estimation of the upper or lower tail dependences, which quantify the level of upper or lower extremal dependence among the high interest rate and low interest rate sets of currencies. More precisely, our resulting estimation of the respective upper and lower tail dependences characterizing each basket of currencies enlightens us about the complex non-linear relations existing

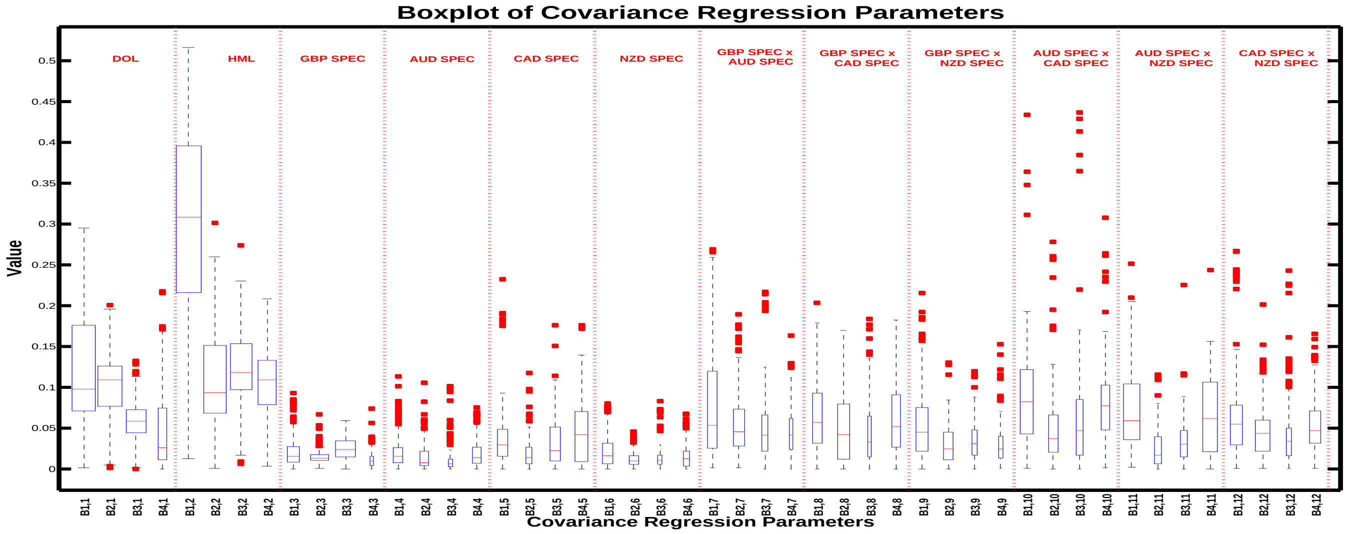


Figure 4: High IR Basket Parameter Boxplot: $DOL + HML_{FX} + SPEC + SPEC \times SPEC$. 125 week lookback periods. The 4 currencies in the high interest basket are ordered as (GBP; AUD; CAD; NZD). The width of each box is equal to the proportion of the sliding windows for which this parameter was significant, i.e. 95% confidence intervals not crossing zero.

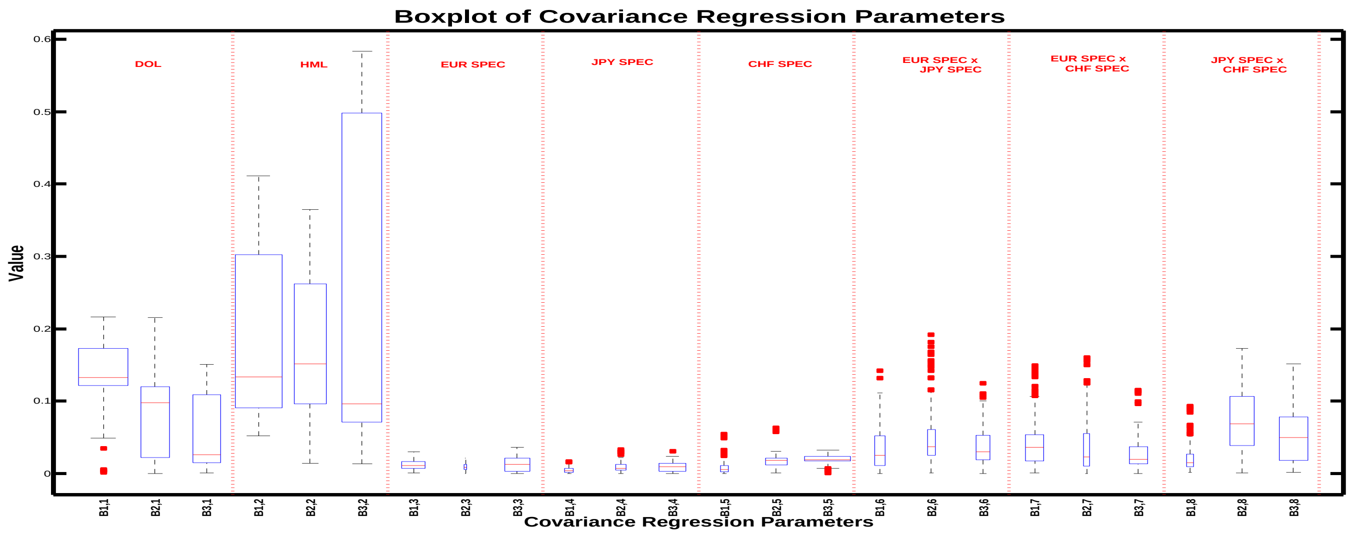


Figure 5: Low IR Basket Parameter Boxplot: $DOL+HML_{FX}+SPEC+SPEC\times SPEC$. 125 week lookback periods. The 3 currencies in the low interest basket are ordered as (EUR; JPY; CHF). The width of each box is equal to the proportion of the sliding windows for which this parameter was significant, i.e. 95% confidence intervals not crossing zero.

between currencies, which remain totally imperceptible when one only considers either marginal characteristics of individual exchange rates or any linear central measure of dependence, such as covariance or correlation.

### *5.1. Skewness of Cross-Sectional Currency Returns: Pre and Post-Crisis Analysis*

Standard linear measurements of association or concordance fail to provide any measurement of the asymmetric extreme relations exhibited by exchange rates. Following the approach proposed in Brunnermeier et al. (2008), where the authors compare the individual skewnesses of a set of currencies once they have been ranked as a function of their interest rate differential. We plotted the very same chart representing the skewness and interest rate differentials[5]. for developed countries, but we also extend the analysis by considering a combination of developed and developing countries[6]. We furthermore divided the data samples in two, before and after the 2008 financial crisis.

We can notice that whatever the basket of exchange rates under scrutiny, (Figure 6 or Figure 7), between 01/01/1999 and 29/01/2014 the skewnesses of the highest interest rate countries were clearly positive, which means that the depreciations of these currencies were asymmetrically more important than their appreciations. We can likewise observe that the currencies with the lowest interest rate differentials display a negative skewness, which shows a significant asymmetry with higher interest rate differential currencies. That being said, Figures 6 and 7, as well as the associated Table 2 show contradictory information between 30/06/2009 (which corresponds to the end of the most recent recession according to the NBER statistics[7]) and 29/01/2014 given that both the high and the low interest rate currency marginal distributions are suggesting a positive skewness over this period of time. The evolution of the rolling high and low interest rate exchange rates cross-sectional average skewness across time (Figure 8) confirms our finding and shows since the end of the financial crisis a different average asymmetry dynamic of the respective currency marginal distributions. It seems indeed that the cross-sectional average skewness of the two sets of currencies are, over the past five years, showing noticeable synchronicity, which was not necessarily the case formerly. Furthermore, as we can see in Figure 9, the low interest rate basket components recently display significant positive skewness, which reflects the European debt crisis and the monetary policy decisions made by the Japanese and Swiss central banks during this period.

Hence, as far as the speculator impact on market prices is concerned, the linear cross-sectional relation between marginal skewness and the interest rates differential pointed out by Brunnermeier et al. (2008) seems to be non-stationary over time and thus puts into question the cross-sectional relation between carry trade speculative flows and the currency dynamics described in the same article. We indeed argue that even though a peculiar event could marginally impact a specific currency, the construction of the carry trade portfolio by speculators should on the contrary simultaneously affect several currencies as a function of the associated interest rate differential. That being

[5]Please note that in our analysis, the data used corresponds to the inverse of the exchange rates considered in the article of Brunnermeier et al. (2008). We indeed retain for our investigation the amount of foreign currency per unit of dollar, whereas Brunnermeier et al. (2008) consider the amount of dollars per foreign currency. Thus, a decrease in the exchange rates means in our case a depreciation of the dollar relative to the foreign currency, while an increase in the exchange rate represents an appreciation of the dollar. The slope of the regression of the skewness on the interest rate differentials is accordingly of opposite sign in our analysis.

[6]To choose our currencies, we indeed consider all the developed and developing currencies available and look at the average interest rates differential with the US local interest rates over a time periods spanning from the 01/01/1999 to the 29/01/2014 and eventually rank them. Then, we retain the five currencies the most often present among the five highest interest rates differentials (namely TRY, BRL, ZAR, INR, MXN) and the five currencies the most often present among the five lowest interest rates differentials (namely JPY, TWD, CHF, SGD, EUR).

[7]The National Bureau of Economic Research dates economic recessions in the USA, by defining them as a persistent decline in several economic variables such as real GDP, real income, employment, industrial production, and wholesale-retail sales. For more information you can access these data on the NBERs website (http://www.nber.org/cycles.html)

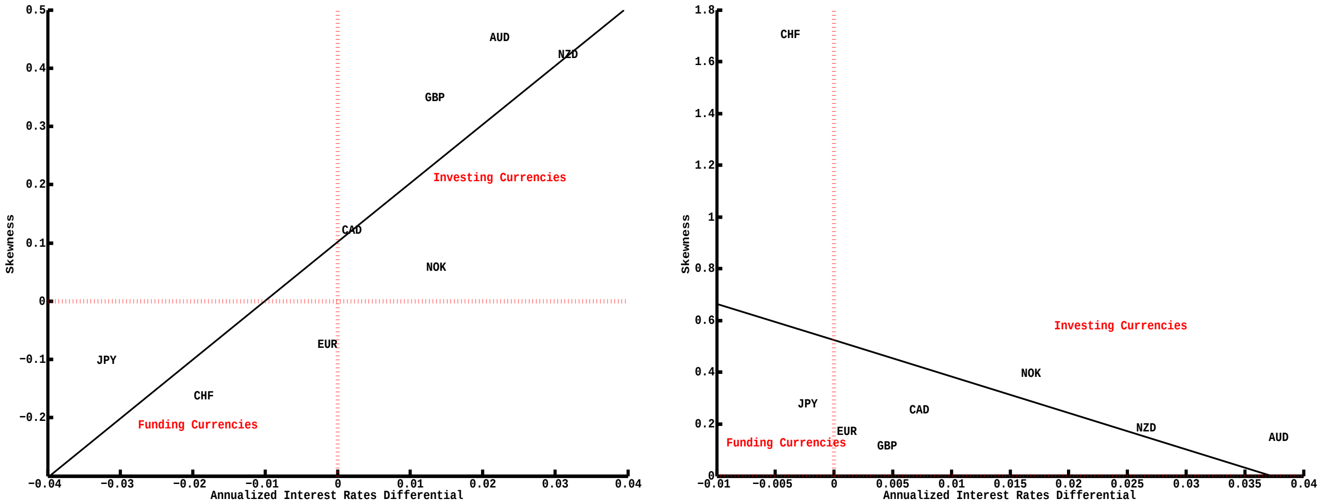


Figure 6: Developed Countries: Skewness vs Interest Rate Differential.

**Left Subplot:** Before the 2008 Financial Crisis. **Right Subplot:** After the 2008 Financial Crisis.

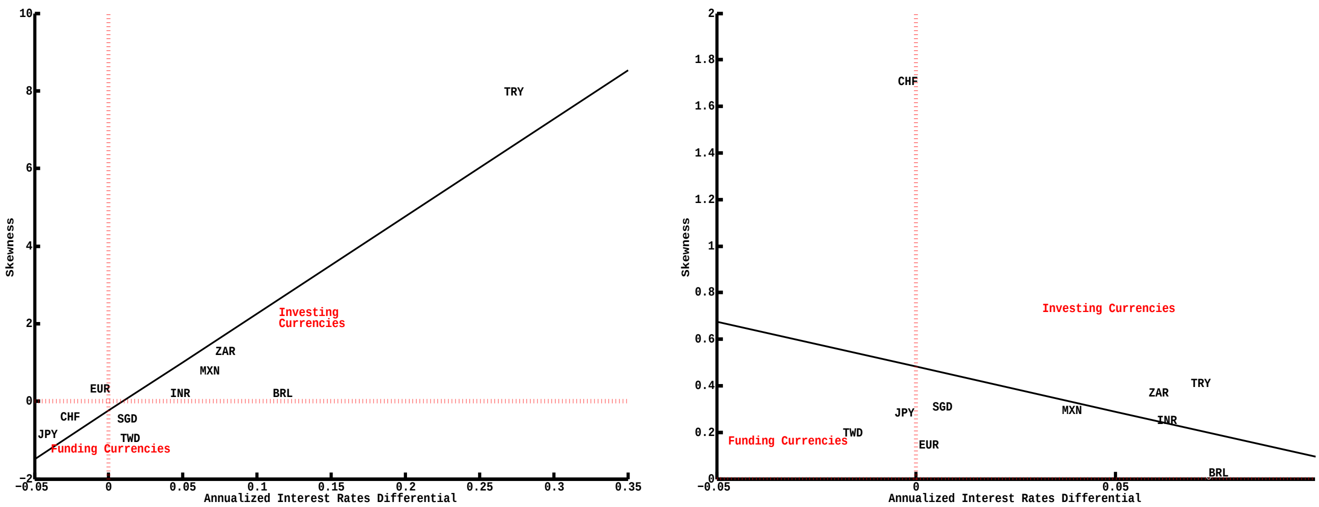


Figure 7: Developed and Developing Countries: Skewness vs Interest Rate Differential.

**Left Subplot:** Before the 2008 Financial Crisis. **Right Subplot:** After the 2008 Financial Crisis.

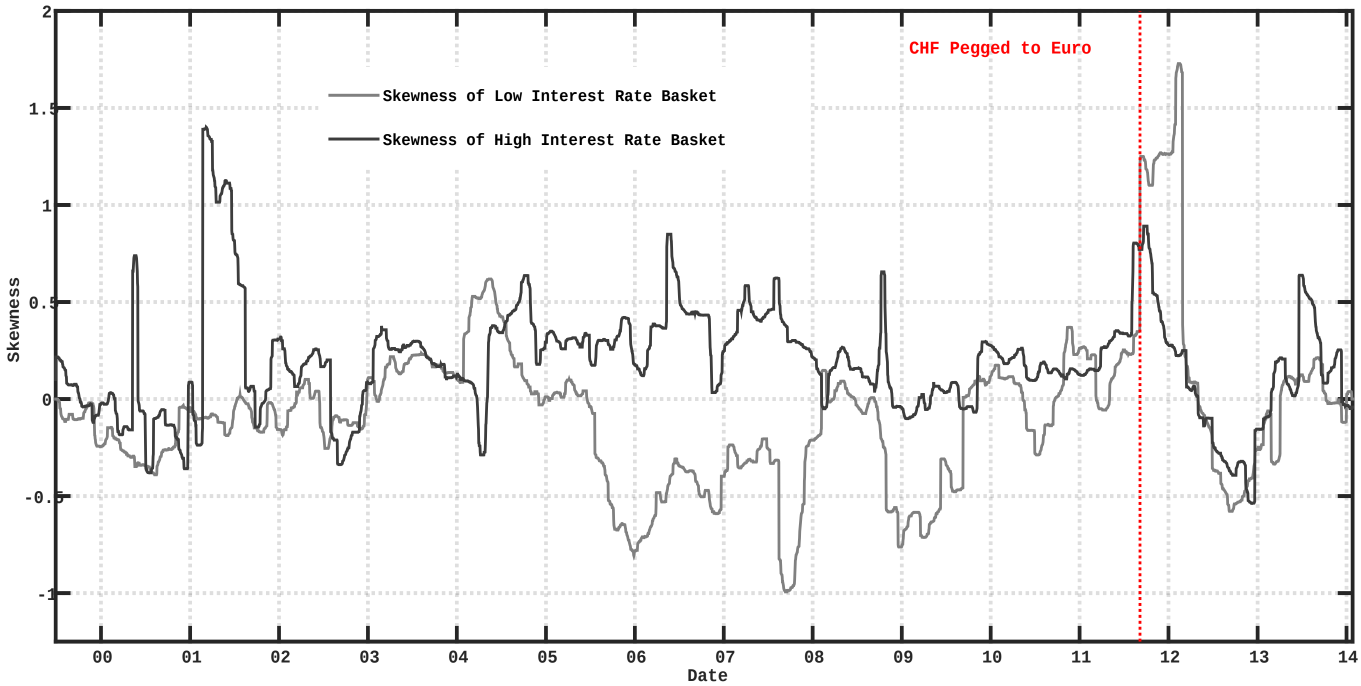


Figure 8: 6-month rolling marginal skewness of high interest rate developed countries (averaged over each marginal currency: GBP, AUD, CAD, NOK, NZD) compared to rolling marginal skewness of low interest rate developed countries (averaged over each marginal currency: JPY, CHF, EUR).

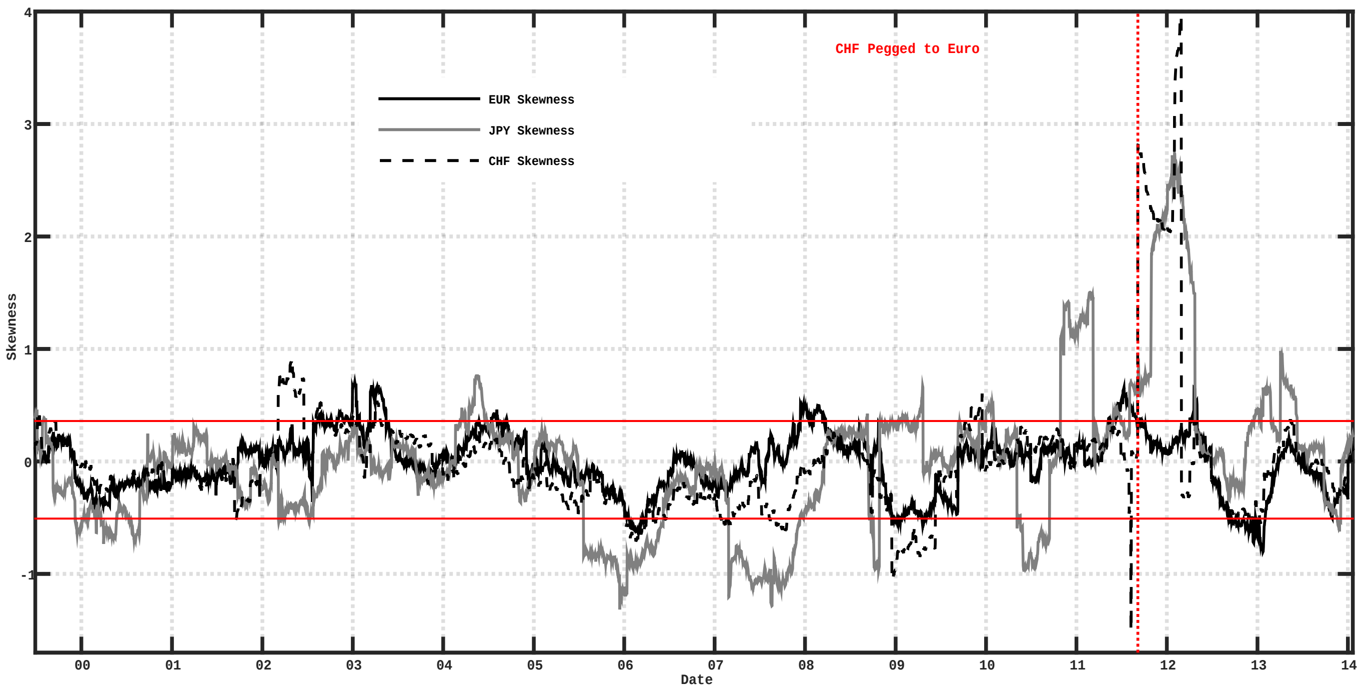


Figure 9: 6-month rolling marginal skewness of low interest rate developed countries (namely JPY, CHF, EUR) with upper and lower confidence intervals.

said, such confined events can still impact the carry trade performance and thus lead to position unwinding, which should again simultaneously impact the currencies composing the carry portfolio. Said differently, the features of the marginal distributions, such as the individual skewnesses retained by Brunnermeier et al. (2008), are not necessarily presuming the joint distribution characteristics, such as high and low interest rate tail dependences. Furthermore, we assert that by selling the funding currencies and buying the investing currencies, speculators should asymmetrically influence the upper and lower extremal currency joint behaviour. In such a context, we now compare the dynamic of the respective upper and lower tail dependences characterizing the high and the low interest rate currency baskets. To ensure that the speculative flows influence the extreme joint behaviour of the exchange rates we need first to understand how theoretically the building and the unwinding of a dynamic carry trade strategy is impacting the high and low interest rate currencies. As detailed in Section 2, to benefit from the UIP violations a speculator will buy the high interest rate currencies while selling the low interest rate currencies relative to a reference currency, which is in our case the US dollar. As a result, when the international exchange rates system receives speculative inflows we should perceive an increase of the low interest rates basket upper tail dependence (proof of significant sales of the basket currencies against the US dollar) while the high interest rates currencies will simultaneously display an increasing lower tail dependence (proof of significant purchases of the basket currencies against the US dollar). We assume that at the same time, no carry trade position will be unwound which is represented by a low upper tail dependence within the high interest rate basket combined with the converse in the funding basket, i.e. a low lower tail dependence of the low interest rates currencies. Conversely, when the international exchange rates system faces speculative outflows, high interest currencies will be simultaneously sold in order to buy low interest rate currencies closing existing carry trade positions[8]. The outcome of this financial operation is naturally an increase of the high interest rate basket upper tail dependence simultaneously with an increase of the low interest rate lower tail dependence. Provided that investors are closing their positions we assume that no carry trade inflows are taking place at this point, hence we should notice a decrease of the high interest rate basket lower tail dependence as well as a decrease of the low interest rate upper tail dependence or at least observe low levels for these two dependence measures. In order to validate our hypothesis, we need now to detect when market participants are increasing or reducing their speculative positions in the market. We propose two different methods in order to demonstrate the direct relation existing between the tail dependences and the carry trade or the differential of local interest rates with the US dollar.

### *5.2. Extremal Carry Trade Behaviour and Average Currency Volatility*

The financial literature about the carry trade states that carry positions will be sensitive to the risk aversion and the level of volatility in the foreign exchange markets (Menkhoff et al. (2012), Brunnermeier et al. (2008), Farhi and Gabaix (2008)). We could thus postulate, that speculators tend to close their carry position when the foreign exchange market volatility is increasing or at a high level (embodying an increasing uncertainty and thus investor risk aversion), whereas they will build their carry positions when the foreign exchange market volatility is decreasing or at a low level. This first assumption about speculator behaviour improves the ability to detect the potential relation between tail dependence and the propensity of a speculator to build or unwind carry trades. According to our previous investigation on the respective basket average skewness, we split our data set into two very distinct sub-periods. First, we estimate the low and high interest rate basket tail dependence on the pre-financial crisis period, which runs from 01/01/1999 to 30/06/2009, according to the NBER recession statistics, while the post-financial crisis period runs from the 01/07/2009 to the 29/01/2014. Then, we individually regressed the tail dependence time series upon the average foreign exchange market volatility as shown in Equation 5.1. The results of this regression in Table 3 demonstrate

[8]We assume that no reverse carry trade positions are permitted in this economy, even though this would not dampen our conclusions.

| | Before Crisis (29-Jul-1999 / 30-Jun-2009) | | After Crisis (01-Jul-2009 / 29-Jan-2014) | |
|---|---|---|---|---|
| | Developed Countries | Developing and Developed Countries | Developed Countries | Developing and Developed Countries |
| Intersect | 0.102 | -0.246 | 0.524 | 0.482 |
| Slope | 10.104 | 25.090 | -14.018 | -3.853 |
| $R^2$ | 0.749 | 0.811 | 0.151 | 0.082 |
| t-stat | 4.231 | 5.855 | -1.034 | -0.843 |
| P-value | 0.005 | $3.8 * 10^{-4}$ | 0.341 | 0.424 |

Table 2: Cross-sectional regression of the skewness on the interest rates differential for developed (AUD, CAD, JPY, NZD, NOK, CHF, GBP and EUR) and developing countries (SGD, TWD, INR, MXN, ZAR, BRL and TRY).

that in accordance with our assumptions, the high interest rate lower tail dependence is negatively sensitive to the average volatility of foreign exchange markets, except for the developed countries before the financial crisis. We should also notice that the relation is statistically significant. Likewise, the upper tail dependence of the low interest rate basket displays a negative significant relation with the average volatility in currency markets. Contrary to this, the low interest rate lower tail dependence is positively reacting to market volatility. We note that during the pre-crisis period the high interest rate upper tail dependence and the low interest rate lower tail dependence regression coefficients were less significant, which could be explained by the fact that a large part of this sample was characterized by a low volatility environment during which the carry trade strategies performed very well (see Lustig and Verdelhan (2007)) and thus a well-known time of heightened carry trade construction which means that the most important sensitivities to consider during this period of time were the low interest rate upper tail dependence and the high interest rate lower tail dependences, both of which corroborate our hypothesis.

If we consider now the post-crisis period, we notice that in such a high volatility environment the high interest rate upper tail dependence is significantly positively related to the market volatility and the high interest rate lower tail dependence is conversely significantly negatively affected by volatility changes. These results are definitely validating our model and the associated hypothesis about the impact of carry trade speculative flows upon the extreme joint behaviour of international exchange rates relative to their level of short-term interest rates. It is also particularly interesting to notice that during the post-financial crisis the low interest rate upper and lower tail dependences remain significantly sensitive to the level of volatility in the foreign exchange markets and thus, according to our hypothesis, to the carry trade flows. This statistical stability has to be weighed against the switching behaviour of the average skewness identified in the previous section.

$$\hat{\lambda}^i_{j,t} = \beta^{i,j}\sigma^{FX}_t + \epsilon^i_{j,t} \quad , \quad i = \{H, L\}\,, \, j = \{u, l\}. \tag{5.1}$$

where $\sigma^{FX}_t = \frac{1}{N}\sum_{n=1}^{N}|r_{n,t}|$ and $r_{n,t}$ is the log return of currency $n$ on day $t$.

*5.3. Extremal Carry Trade Behaviour and Currency Speculative Open Positions*

To validate the assertion about the influence of carry trade speculative flows on currency extremal joint behaviour, we propose in this second model to consider the same covariates as for the covariance regression model in section 4.3, namely the $DOL$ and the $HML_{FX}$ factors combined with the $SPEC$ factor. We indeed provided in section 4.3 the

| | Before Crisis (29-Jul-1999 / 30-Jun-2009) | | After Crisis (01-Jul-2009 / 29-Jan-2014) | |
|---|---|---|---|---|
| | Upper TD | Lower TD | Upper TD | Lower TD |
| Low Int. Rates Developed Countries | -1.448 | 0.411 | -1.771 | 2.067 |
| | (0.000) | (0.259) | (0.000) | (0.000) |
| High Int. Rates Developed Countries | 0.147 | 1.072 | 0.657 | -1.344 |
| | (0.699) | (0.000) | (0.099) | (0.000) |
| High Int. Rates All Countries | 0.563 | -2.036 | 1.874 | -1.410 |
| | (0.145) | (0.000) | (0.000) | (0.001) |

Table 3: Regression of the high interest rate upper and lower tail dependences time series ($\hat{\lambda}^H_{u,t}$, $\hat{\lambda}^H_{l,t}$) and the low interest rate upper and lower tail dependences time series ($\hat{\lambda}^L_{u,t}$, $\hat{\lambda}^L_{l,t}$) on the average volatility for developed (AUD, CAD, JPY, NZD, NOK, CHF, GBP and EUR) and developing countries (SGD, TWD, INR, MXN, ZAR, BRL and TRY).

evidence that speculative order flows in the markets have a substantial impact on the variance and the covariance dynamics of international exchange rates. To complete this analysis we demonstrate in this section the existence of the previously stated dual relation between on one hand the high and low interest rate currency baskets upper and lower tail dependences, and on the other hand the amount of speculative inflows and outflows associated to these funding and investing currencies futures. In this section, our empirical study consists thus in investigating the relation between the non-commercial traders net position (long − short) and our extreme environment dependence measure, namely the tail dependence. The base idea is to assume that while speculators set up or unwind a carry position in the currency market, this will synchronously impact all the currency prices increasing accordingly certain tail dependences among high and low interest rates currencies. Furthermore, we should observe a synchronous change in the net open position of the speculators. As a first example, we can see from Figure 10 that there is a negative relationship between the net position of speculators on the Swiss franc (one of the main financing currencies) and the upper tail dependence associated to the low interest rates basket[9].

In order to verify this assertion we model the four tail dependences as a function of the $SPEC$ factor, i.e. the ratio of the non-commercial net positions at the end of each week divided by the total number of futures contracts still open in the market at the end of each week. This will then act as a factor to help explain how much of the currency extremal dependence can be explained by the speculative positions. The first problem we have to deal with is the homogeneous impact that the dollar can have on the common behaviour of our currency open positions.

When the dollar index, defined as a basket of currencies against the dollar, increases the tail dependences could potentially be modified too. To extract the linear effects associated to this component we follow the analysis carried out in Lustig and Verdelhan (2007), who demonstrated the effect of the dollar index through a principal component analysis in which they interpret the first principle component as the dollar index.

To achieve this, we first extend the monthly PCA analysis of Lustig and Verdelhan (2007) to a daily frequency, motivating the construction of daily $DOL$ and $HML_{FX}$ factors, which we then use to compute the weekly factors.

[9]Since 1999 the Swiss franc has indeed always been one of the lowest interest rate currency relative to the US Dollar and thus always used by the speculator as a financing currency.

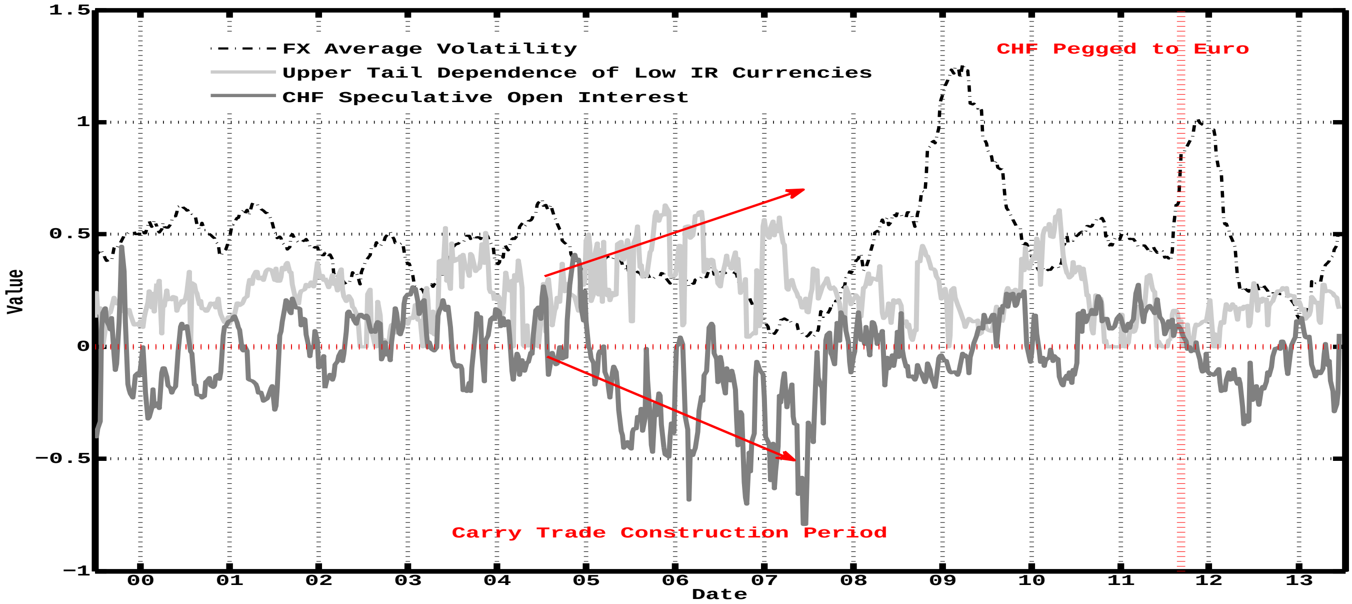


Figure 10: 6-month rolling upper tail dependence of low interest rate developed countries (namely JPY, CHF, EUR) compared to net open position of the Swiss franc future contract traded on the CME. The black line corresponds to the average 6-month rolling historical volatility computed from the low interest rate basket.

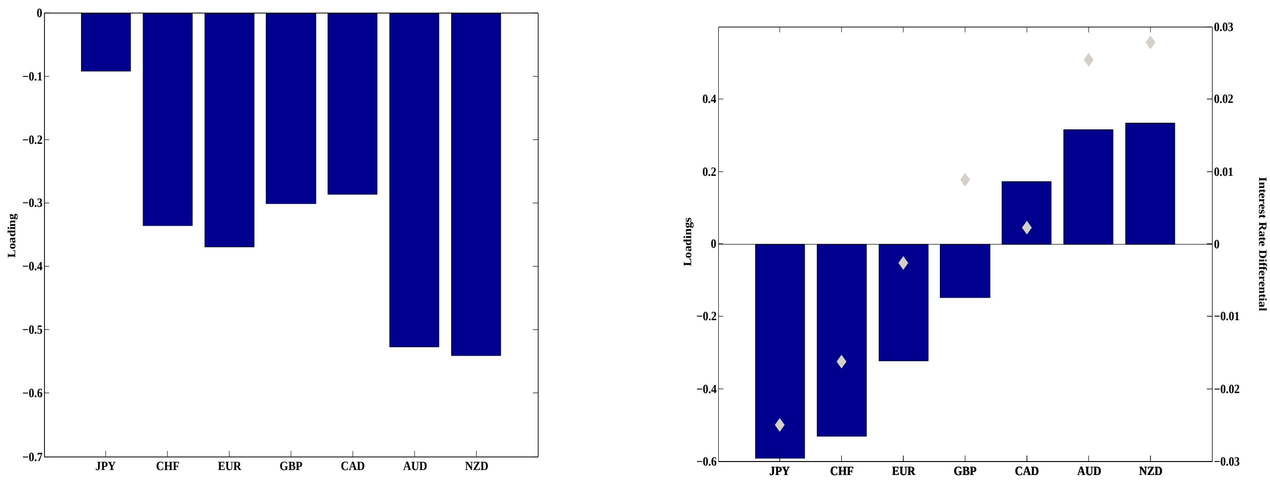


Figure 11: First and second eigenvectors of the developed countries currency returns covariance matrix. **Left Subplot:** Loadings of the First Principal Component of Developed Countries Currency Returns; **Right Subplot:** Loadings of the Second Principal Component of Developed Countries Currency Returns. The bars (left axis) represent the loadings values on the returns second principal component while the grey diamonds (right axis) depicts the level of interest rate differential with the one month US interest rate.

To construct these factors at the daily frequency, we must calculate the daily carry returns via an interpolation on the 1 month forward curve using the following market price data: overnight rate, one week rate, two week rate, three week rate and the one month rate (as used in the rest of the paper). The details of this interpolation procedure can be seen in Section 7 in the online appendix, along with an example of an interpolated curve for one particular day. From this interpolated forward curve we can construct a daily time series of carry returns for each of the 7 currencies via a mark to market of the forward contract that would be held if one was continuously rolling one month forward contracts at the end of each month, as in Lustig and Verdelhan (2007). These individual currency carry returns can then be used to compute the covariance matrix that is used for the principal component analysis.

Instead of applying the principal component analysis to a set of portfolios, we used directly the seven currencies for which we have at our disposal the open interest published by the CFTC. We can see in Figure 11, that our daily analysis replicates the results of Lustig and Verdelhan (2007), where all the currencies are negatively impacted by the first component (see the left subplot of Figure 11) which represents the dollar effect ($DOL$), whereas the second component is (almost) monotonically increasing with the rate differential (see the right subplot of Figure 11) which is analysed as the high minus low effect ($HML_{FX}$). We find that over 76% of the variation of the daily carry returns can be explained in the first two principal components. It is worth emphasizing that these two projections of the currency returns to linear combinations of the currency returns with the largest unconditional variances are not necessarily related to the conditional variance model we described in Section 4. In the remainder of this section we consider the tail dependence regression and thus use the first two principal components time series as independent variables for our regression to cancel out the effect of these two price effects related to $DOL$ and $HML_{FX}$.

More formally, we regress the upper and lower tail dependences of the high and the low interest rate sets of currencies (respectively $\hat{\lambda}_u^H, \hat{\lambda}_l^H, \hat{\lambda}_u^L, \hat{\lambda}_l^L$) on the $DOL$ and the $HML_{FX}$ factors as well as the ratios of the speculators net position with the total open interest associated to each of the N currencies and denoted by $SPEC$ [10]. We then further allow cross terms between the $SPEC$ factors:

$$\hat{\lambda}^i_{j,t} = \underbrace{\beta^{i,j}_{DOL} DOL_t + \beta^{i,j}_{HML_{FX}} HML_{FX_t}}_{\text{Dollar and Carry Factors}} + \underbrace{\sum_{k=1}^{N} \beta^{i,j}_k SPEC^k_t + \sum_{k=1}^{N} \sum_{l>k} \beta^{i,j}_{k,l} SPEC^k_t \times SPEC^l_t}_{\text{Speculative Volume Factors}} + \epsilon^{i,j}_t \quad , \tag{5.2}$$

where $i = \{H, L\}$ and $j = \{u, l\}$.

The first observation to make on the results displayed in Table 4 is that the speculator activity contributes significantly to the explanatory power of the tail dependences regression whereas the $DOL$ and $HML_{FX}$ factors variances and covariance do not systematically explain with significance this currency extremal joint behaviour. Furthermore, we should also highlight that the $R^2$ is noticeably increased once we add the variables related to the speculators positions in the market.

As for our graphical analysis of the Swiss franc speculative interests we can see in Table 4 that this common financing currency is significantly related to the upper and the lower tail dependences of the low interest rates basket and that the sign associated is also corroborating our hypothesis. As a matter of fact, when the Swiss franc is mainly sold by the speculators, who are building their carry portfolio, the upper tail dependence of the low interest rate currencies is increasing. Conversely, when the Swiss franc is mainly bought by the speculators, who are unwinding their carry portfolio, the lower tail dependence of the low interest rates basket tends to increase.

Interestingly enough, the Australian dollar is playing exactly the same role for the high interest rate basket. This

[10] The Commitments of Traders report provides since the 20th of June 2006 the commercial and non-commercial positions for the following currencies against the dollar: AUD, CAD, CHF, EUR, GBP, JPY, NZD.

| | $\hat{\lambda}_u^H$ | | | $\hat{\lambda}_u^L$ | | | $\hat{\lambda}_l^H$ | | | $\hat{\lambda}_l^L$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $Constant$ | 0.096 | 0.057 | 0.080 | 0.370 | 0.337 | 0.270 | 0.350 | 0.433 | 0.339 | 0.121 | 0.086 | 0.178 |
| | (0.122) | (0.435) | (0.268) | (0.000)** | (0.000)** | (0.000)** | (0.000)** | (0.000)** | (0.000)** | (0.045)* | (0.190) | (0.012)** |
| $DOL$ | 0.083 | -0.023 | -0.162 | -0.319 | -0.264 | -0.107 | 0.143 | 0.134 | 0.014 | 0.249 | -0.006 | -0.061 |
| | (0.707) | (0.913) | (0.317) | (0.089) | (0.079) | (0.418) | (0.321) | (0.390) | (0.920) | (0.223) | (0.972) | (0.707) |
| $HML_{FX}$ | 0.303 | 0.030 | -0.032 | -0.205 | 0.056 | -0.172 | -0.020 | 0.135 | -0.058 | 0.410 | 0.155 | -0.041 |
| | (0.380) | (0.922) | (0.921) | (0.445) | (0.807) | (0.398) | (0.945) | (0.609) | (0.785) | (0.227) | (0.590) | (0.859) |
| $\sigma_{DOL}$ | -8.696 | -2.343 | 0.625 | -8.733 | -7.866 | -8.883 | 3.166 | -0.335 | 1.640 | -2.268 | -1.606 | 2.691 |
| | (0.051) | (0.553) | (0.863) | (0.000)** | (0.001)** | (0.000)** | (0.324) | (0.920) | (0.503) | (0.609) | (0.664) | (0.510) |
| $\sigma_{HML_{FX}}$ | 26.700 | 23.177 | 16.159 | -8.918 | -10.300 | 4.468 | -0.246 | -1.715 | -2.267 | 31.495 | 31.478 | 16.269 |
| | (0.003)** | (0.009)** | (0.058) | (0.098) | (0.186) | (0.516) | (0.976) | (0.869) | (0.794) | (0.001)** | (0.000)** | (0.065) |
| $\sigma_{DOL,HML_{FX}}$ | -258.500 | -13.838 | -201.897 | -1088.660 | -958.776 | -710.730 | 577.011 | 239.629 | 333.003 | 1103.721 | 868.169 | 776.107 |
| | (0.488) | (0.966) | (0.576) | (0.000)** | (0.000)** | (0.004)** | (0.072) | (0.378) | (0.173) | (0.008)** | (0.007)** | (0.023)* |
| AUD | | -0.129 | -0.173 | | 0.089 | -0.049 | | 0.083 | 0.272 | | -0.083 | -0.105 |
| | | (0.017)* | (0.059) | | (0.052) | (0.403) | | (0.066) | (0.000)** | | (0.154) | (0.160) |
| CAD | | 0.126 | 0.281 | | 0.025 | -0.017 | | -0.053 | 0.114 | | 0.054 | -0.013 |
| | | (0.008)** | (0.011)* | | (0.663) | (0.804) | | (0.160) | (0.204) | | (0.236) | (0.899) |
| CHF | | -0.023 | -0.014 | | -0.136 | -0.130 | | 0.173 | 0.036 | | 0.186 | 0.219 |
| | | (0.679) | (0.900) | | (0.002)** | (0.028)* | | (0.000)** | (0.687) | | (0.001)** | (0.027)* |
| EUR | | 0.154 | 0.232 | | 0.129 | 0.149 | | -0.084 | -0.109 | | 0.002 | -0.031 |
| | | (0.014)* | (0.042)* | | (0.012)* | (0.049)* | | (0.101) | (0.125) | | (0.975) | (0.812) |
| GBP | | 0.046 | 0.133 | | -0.162 | 0.035 | | -0.070 | -0.290 | | -0.011 | -0.124 |
| | | (0.404) | (0.203) | | (0.004)** | (0.637) | | (0.214) | (0.006)** | | (0.855) | (0.214) |
| JPY | | -0.023 | -0.053 | | -0.040 | 0.016 | | -0.128 | -0.137 | | -0.054 | -0.109 |
| | | (0.631) | (0.534) | | (0.370) | (0.771) | | (0.002)** | (0.009)** | | (0.278) | (0.084) |
| NZD | | 0.008 | -0.020 | | -0.027 | -0.064 | | -0.104 | 0.025 | | 0.122 | 0.129 |
| | | (0.878) | (0.745) | | (0.379) | (0.141) | | (0.007)** | (0.675) | | (0.017)* | (0.024)* |
| NOK | | - | - | | - | - | | - | - | | - | - |
| | | - | - | | - | - | | - | - | | - | - |
| CROSS | | | Cross1 | | | Cross2 | | | Cross3 | | | Cross4 |
| $R^2$ | 14.5% | 27.1% | 39.4% | 15.6% | 30.9% | 53.7% | 5.2% | 23.6% | 46.0% | 11.7% | 31.7% | 48.5% |

Table 4: Regression of the high and the low interest rate respective tail dependences on the $DOL$ index, $HML_{FX}$ index, $DOL$ index volatility, $HML_{FX}$ index volatility, $DOL$ and $HML_{FX}$ indices covariance and the $SPEC$ ratio (the ratio of each currency future speculative net positions to the total future open interest, as provided by the CFTC) as well as cross relations among them. The open interest data provided by the CFTC as well as the computed $DOL$ and $HML_{FX}$ indexes are weekly data while the respective tail dependence measurement corresponds to the average value over each week. The period of time considered for this analysis spans from June 20th 2006 to January 28th 2014 and corresponds to the longest overlapping sample for all the currencies considered and available. Numbers in parentheses show Newey and West (1987) HAC p-values. Cross1 corresponds to all the possible cross effects among which the following are statistically significant (below 5%): AUD/EUR, EUR/GBP, EUR/NZD. For Cross2 the following cross effects are statistically significant (below 5%): AUD/JPY, CAD/EUR, EUR/GBP. For Cross3 the following cross effects are statistically significant (below 5%): AUD/GBP, EUR/GBP, GBP/JPY. For Cross4 the following statistically crosses effects are statistically significant (below 5%): AUD/EUR, AUD/GBP, CHF/EUR, EUR/JPY, JPY/NZD.

typical investing currency is indeed significantly contributing to the upper and the lower tail dependences of the high interest rate basket. Furthermore, the two signs associated to the regression coefficients of the Australian currency also validate our theory, since the purchase of Australian dollar, following from the construction of a carry trade position, mainly by speculators is leading to an increase in the lower tail dependence among the high interest rate currencies. On the contrary, when speculators tend to sell the Australian currency in order to reduce their carry trade exposure, we observe an increase in the upper tail dependence among the high interest rate currencies.

Finally, we notice that the speculator positions on the Euro, the British pound and the New Zealand dollar also have informative power about the extremal joint behaviour of the international exchange rates.

### *5.4. Results: Extended Dataset of 34 Currencies*

In this section, we present an extended carry trade analysis using 34 currencies and show that upper and lower tail dependences, when also considering developing countries, display the same features as for the developed countries we have retained until now. In order to differentiate the "financing currencies" from the "investment currencies", we start by classifying each currency relative to its differential of risk free rate with the US dollar. We demonstrated in Equation 2.3 that the differential of interest rates between two countries can be estimated through the ratio of the forward contract price and the spot price, see Juhl et al. (2006) who show this holds empirically on a daily basis. Accordingly, instead of considering the differential of risk free rates between the reference and the foreign countries, we build our respective baskets of currencies with respect to the ratio of the forward and the spot prices for each currency. On a daily basis we compute this ratio for each of the $n$ currencies (available in the dataset on that day) and then build five baskets. The first basket gathers the $n/5$ currencies with the highest positive differential of interest rate with the US dollar. These currencies are thus representing the "investment" currencies, through which we invest the money to benefit from the currency carry trade. The last basket will gather the $n/5$ currencies with the highest negative differential (or at least the lowest differential) of interest rate. These currencies are thus representing the "financing" currencies, through which we borrow the money to build the currency carry trade. Conditional upon this classification we investigate the joint distribution of each group of currencies to understand the impact of the currency carry trade, embodied by the differential of interest rates, on currencies returns.

We briefly mention the model estimation, however since it was routine, we refer the reader to Ames et al. (2015) where more details are provided. We considered the marginal exchange rate log-returns for each currency on which we fit a log Generalized Gamma distribution (l.g.g.d.), under assumptions of local stationarity over a 6-month sliding window. Then we utilised each of the l.g.g.d. marginal distribution fits for a given day's set of currencies in the high interest rate and low interest rate baskets to analyse the joint multivariate features. To achieve this for each of the currencies, the exchange rate log-return data was transformed via the l.g.g.d. marginal model's distribution function. Thus producing pseudo data which is approximately marginally uniform [0, 1]. Finally the mixture Clayton-Frank-Gumbel copula (denoted C-F-G ), is fitted each day to the same sliding window of 6 months pseudodata for both the high interest rate and low interest rate baskets. Then according to the definition of the upper and lower tail dependence provided in Equations (3.10) and (3.11) we compute the level of tail dependence present in the respective currency baskets for this given period of time. Perhaps the most interesting and revealing representation of the tail dependence characteristics of the currency baskets can be seen in Figures 12 and 13. Here we can see that there are indeed periods of heightened upper and lower tail dependence in the high interest rate and the low interest rate baskets. In understanding this analysis we note that Figures 12 and 13 show the probability that one currency in the basket will have a move above/below a certain extreme threshold given that the other currencies have had a move beyond this threshold. There is a noticeable increase in upper tail dependence in the high interest rate basket at times of global market volatility. Specifically, during late 2007, i.e. the global financial crisis, there is a sharp peak in upper tail dependence. Preceding this, there is an extended period of heightened lower tail dependence from 2004 to 2007, which could tie in with the building of the leveraged carry trade portfolio positions, as demonstrated earlier in our study for the developed countries. This period of carry trade construction is also very noticeable in the low interest rate basket through the very high levels of upper tail dependence.

## 6. Conclusion

This article sheds light on an important topic that has been discussed for a long time in the economics and financial literature which is the uncovered interest rate parity (UIP) puzzle. This market phenomenon is particularly interesting from a theoretical standpoint as well as for the understanding of financial market mechanisms. It has been demonstrated that the currency markets were indeed not respecting empirically a fundamental relation in finance which connects the currency exchange rates and the interest rates associated with two different countries. This phenomenon has given birth to a famous and widespread strategy, namely the carry trade, which appeals to a growing population of speculators. The main contribution of our paper is to tackle the problem from a microeconomic perspective and to investigate how mass speculator behaviour affects the price discovery process of exchange rates. To this end, we proposed a rigorous statistical modelling approach using two complementary techniques in order to capture a range of causalities between the differential of interest rates and the exchange rate joint dynamics. Our two-stage approach allows us to capture the effects of the speculative carry trade volumes upon the individual currency log-return distributions, as well as the joint features such as the dependence structures prevailing between baskets of exchange rates. Having gained an understanding of the dependence structure among high and low interest rate currencies, it was then our main emphasis to demonstrate and assess the effects of the speculators' behaviour on it. Our distinction between the currency covariance dynamics and the higher order and extremal dependence behaviours provided us with two different angles of analysis, which reinforced our conviction that the carry trade definitely induces a broad and multi-level alteration of the exchange rates joint behaviour. We furthermore completed the recent literature by proving that beyond their admitted role during stress periods, speculative volumes also affect the dependence structure during lower risk periods.

In addition to this within basket temporal analysis, from the perspective of undertaking a currency carry trade strategy we would need to consider the relative relationships between the temporal dependence features of the high interest rate and low interest rate currency baskets. We demonstrate several interesting features from our model fits relating to asymmetries between the high and low interest rate baskets over time, especially during periods of high volatility in global markets. We indeed found substantial evidence to support arguments for time varying behaviours in the structural dependence hypotheses posed about the currency baskets, as captured by the level of tail dependence assessed from the copula mixture model we considered, which also displayed interesting asymmetries between the high and low interest rate baskets over time.

To conclude, the combination of the covariance regression model and the copula based probabilistic modelling approaches allows us to demonstrate that besides the intrinsic risk associated to each particular high and low interest rate currency, another factor constitutes a determining source of risk, which turns out to be the level of speculative interests prevailing in the market coupled with the induced sensitivity to market risk. It was demonstrated in our analysis that both upper and lower tail dependence features consequently displayed significant association and asymmetries with each other between the high and low interest rate baskets during periods of relative financial stability versus periods of heightened market volatility.

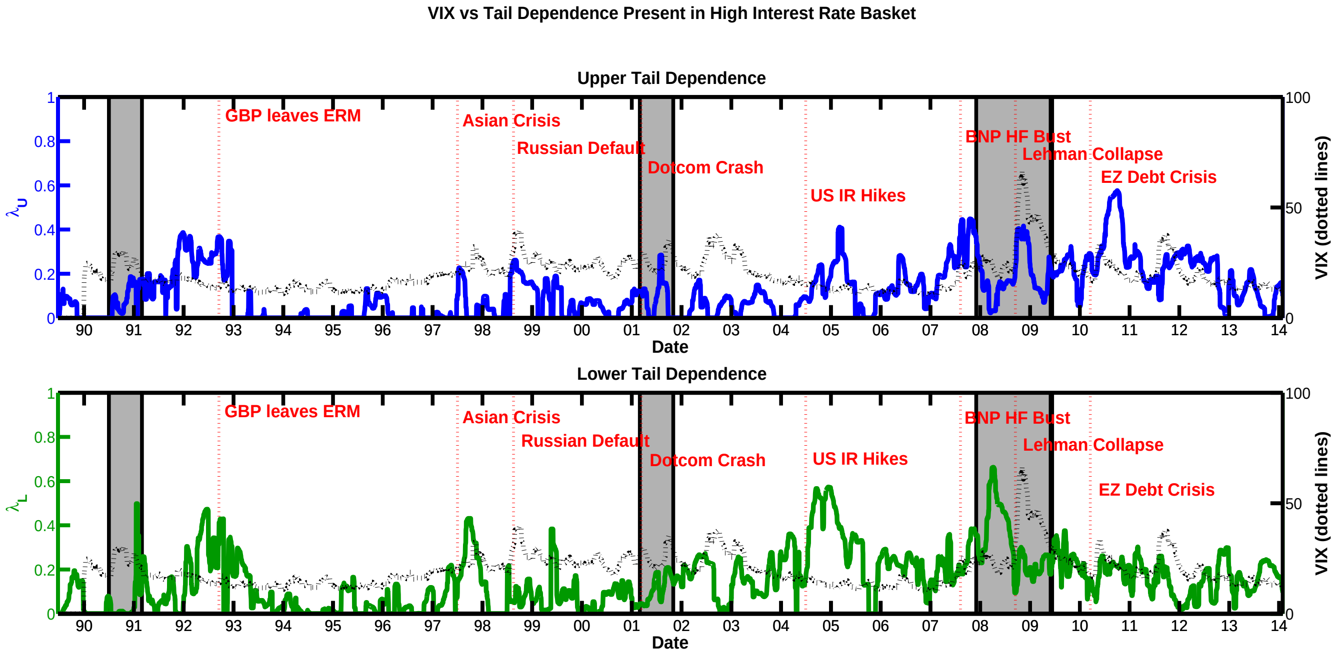


Figure 12: Comparison of Volatility Index (VIX) with upper and lower tail dependence of the high interest rate basket. US NBER recession periods are represented by the shaded grey zones. Some key crisis dates across the time period are labelled.

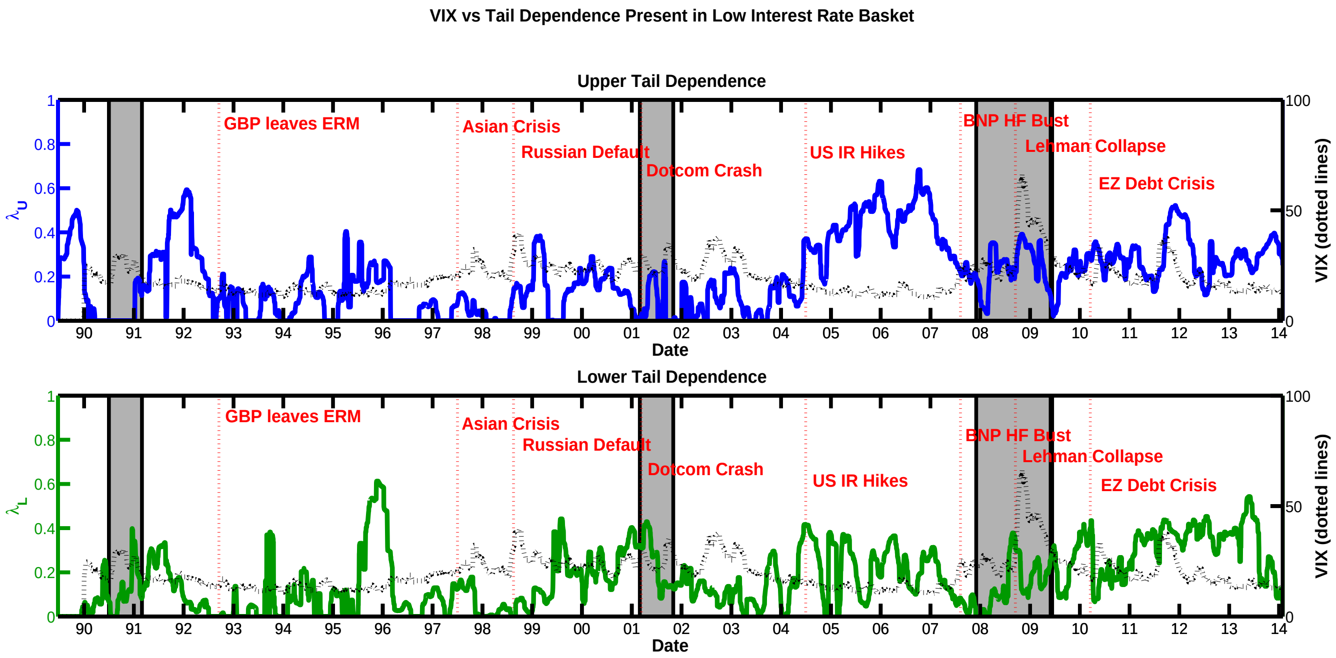


Figure 13: Comparison of Volatility Index (VIX) with upper and lower tail dependence of the low interest rate basket. US NBER recession periods are represented by the shaded grey zones. Some key crisis dates across the time period are labelled.

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