Stochastic Differentail Equations

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Generic SDE

A generic SDE takes the form of

$$dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dB_t$$

Example

If we are looking at the Heath-Jarrow-Morton Model framework we can start with a generic SDE for maturity T.

$$df(t, T) = \mu(t, T)dt + \sigma_f(t, T)dW_t$$

We can then relate to the bond pricing equation

$$dP(t, T) = r_t P(t, T)dt + \sigma_P(t, T)P(t, T)dW_t$$

Then use the forward dynamics of a zero coupon bond

$$f(t, T) = -\frac{d}{dT}ln(P(t, T))$$

Then find differentials

$$df(t, T) = -\frac{d}{dT}dln(P(t, T))$$

Then from there if we use Ito's lemma for finding the differential of the log pricing equation we get

$$dlnP(t, T) = \frac{1}{P(t, T)} - \frac{1}{2} \cdot \frac{1}{P(t, T)^2}dP(t, T)^2$$