November 4th, 2021

Weekly Meeting Notes

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Overview:

- The 2Y - 10Y is a common recession indicators
- Hebbian Learning can help bridge the gap between the brain and neural networks
- Convex optimization is a common mathematical problem that comes up everywhere in markets

News: 2Y - 10Y Yield Curve Inversion

The idea behind it

- The yield curve tracks the preferences of interest rates for different tenors amongst lenders
- A healthy yield curve should have a positive slope which implies that lenders think that the government can pay back more money in the future
- An unhealthy yield curve would be inverted which means that lenders would prefer
- If the yield curve inverted
  - That means that the 2Y interest rate is higher than the 10Y interest
  - It means that investors have a preference to have their money
  - Historically this has been seen as a recession indicator
- Recently the difference between the 2Y and 10Y flattened by 13 basis points

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Quant: Convexity and Applications

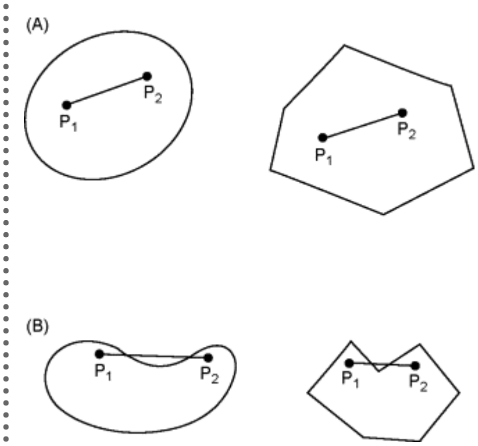
Convex sets and functions have properties that make various mathematical problems, most notably optimization, easy to solve. Many such optimization problems appear in finance and are often solved using methods that rely on the convexity of some set and/or function. In general, convex functions and sets allow for polynomial-time optimization, where the problem of optimization is NP-hard in general.

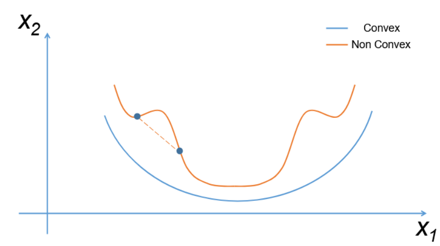
Definition of Convex:

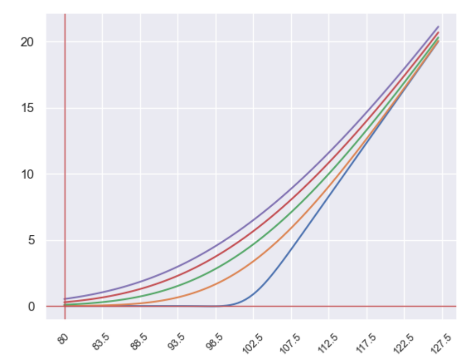
- For a convex set S, if x and y are in S, then any linear combination of x and y are also in S
  - For x,y ∈ S and α ∈ [0, 1), we have αx + (1-α)y ∈ S
- For a convex function f, for any real numbers x and y, the secant line connecting f(x) and f(y) lies above the function between x and y
  - For real numbers x, y and α ∈ [0, 1), f(αx + (1-α)y) ≤ αf(x) + (1-α)f(y)
  - Equivalently, the region above f is a convex set
  - Strictly convex for f(αx + (1-α)y) < αf(x) + (1-α)f(y)
- Additional properties:
  - A convex function lies above all its tangent lines
  - If a function is convex and differentiable, its second derivative is nonzero (or for a multivariable function, its Hessian is positive semi-definite, or has non-negative eigenvalues)
- Jensen's Inequality: for a convex function f and random variable X, f(E[X]) ≤ E[f(X)]
  - Function of the expected value is less than the expected value of the function

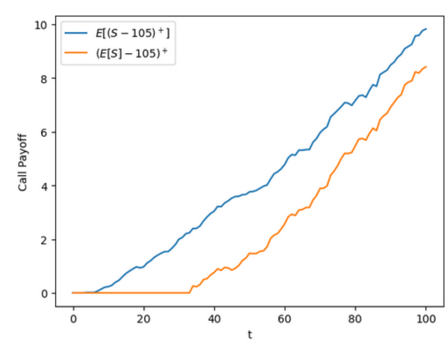
Examples of Convexity:

- Discounting (time value of money) results in convexity with respect to time and interests rates (from e^{-rt} term)
  - E.g. bond convexity - second derivative of bond price w.r.t. interest rates, divided by price of bond; used to obtain more accurate estimates of interest rate sensitivity
- Derivatives payoff functions are often convex with respect to stock price
  - Jensen's inequality appears in derivatives pricing
- Stock portfolio variance is a convex function of the weights (and weight constraints form a convex set)
  - Hessian given by sample covariance
  - Sample covariance = X^TX, where each column is the return of a stock minus its mean return (deviation from the mean)
- Utility functions are often assumed to be concave (negative is convex)









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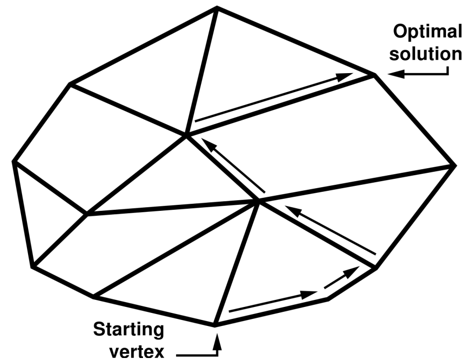
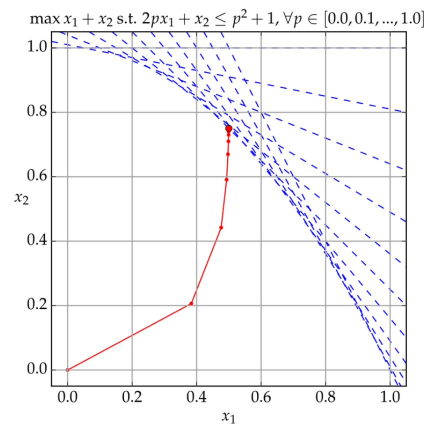
Applications

- Gradient Descent - good for unconstrained problems
  - When objective function is convex, guaranteed to converge to a global minimum
- Least-Squares
- Linear Programming - linear objective function and linear constraints
  - Convex function, convex constraint set
  - Simplex method - travel within constraint set toward optimal solution; usually very efficient O(n) time, in very rare cases may be $O(2^n)$
  - E.g. cash flow matching, network flow problems, etc.
- Interior point methods - can be used to solve other problems with convex objective functions and convex constraint sets
  - Begin inside the constraint set, follow a path towards the optimal solution





<u>Articles:</u>

Wikipedia: Convex Function (link here https://en.wikipedia.org/wiki/Convex_function)

Fmin: Convex function (here)

Harel Jacobson: Turbocharging derivatives - variance, convexity, and everything in between (here)

LibreTexts: Maximization by The simplex method (here)

Wikipedia: Interior-point method (here)

Quant: Hebbian Learning

The human brain remains the biggest inspiration to the entire field of AI. But, there is a mismatch between neural network design and the human brain.

- Hebb's Rule:
  - Cells that fire together, wire together
  - Foundation of synaptic plasticity
  - Strong synaptic connections constitute basics of long-term memory

- Backpropagation algorithm:
  - Relies on top-down knowledge distribution model
  - Not how the brain works
  - Equivalent of in order to form a neural connection, the brain would have to predict future events

Hebb's Rule for Neural Networks

- IBM proposes:
  - Change of synapse strength during the learning process is proportional to the activity of the presynaptic cell and to a function of the activity of the postsynaptic cell
  - Given an image input, unsupervised training models generate vector of training currents
  - Activations are used to update weights of lower layers using synaptic mechanism
  - Training of neural net is completed using traditional supervised learning and SGD-based optimizations

Advantages:

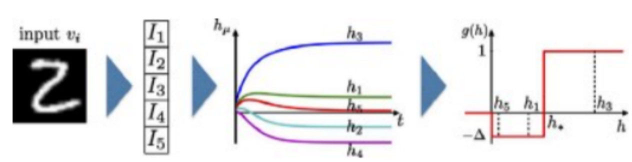
- 1) The first part of the training is completely unsupervised and does not require large volumes of labeled data

- 2) The weights of the lower hidden layers are inferred based only on local activity without requiring expensive back propagation techniques

Disadvantages:

- 1) online algorithm so training examples are presented one at a time, instead of mini-batches

- 2) For any training example, you have to wait until the set of hidden units reaches a steady state



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