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ITERATION OF COMPACT HOLOMORPHIC MAPS ON A HILBERT BALL

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ABSTRACT. Given a compact holomorphic fixed-point-free self-map, f, of the open unit ball of a Hilbert space, we show that the sequence of iterates, (f^n), converges locally uniformly to a constant map ξ with $\|\xi\| = 1$. This extends results of Denjoy (1926), Wolff (1926), Hervé (1963) and MacCluer (1983). The result is false without the compactness assumption, nor is it true for all open balls of J^*-algebras.

1. INTRODUCTION

There has been extensive literature on the subject of iterating holomorphic functions since the early works of Julia [14], Fatou [6], [7], Denjoy [3] and Wolff [23], [24]. We refer to [2], [20] for some interesting surveys and references.

Given a fixed-point-free holomorphic map $f: \Delta \to \Delta$ where $\Delta = \{z \in \mathbb{C} : |z| < 1\}$, Wolff's theorem [24] states that there is a boundary point $u \in \partial\Delta$ such that every closed disc internally tangent to Δ at u is invariant under the iterates of f. From this follows the result of Denjoy [3] and Wolff [23] that the iterates, $f^n = \underbrace{f \circ \cdots \circ f}_{n}$, of f converge to u uniformly on compact subsets of Δ. Wolff's theorem has been extended to Hilbert balls [8], and the convergence result of Denjoy and Wolff also extends to the open unit ball of $\mathbb{C}^n$ [13], [15], as well as some other domains in $\mathbb{C}^n$ [1]. Nevertheless, the convergence result fails for infinite dimensional Hilbert balls and Stachura [18] has given an example to show that it fails even for *biholomorphic* self-maps.

Recently, Wolff-type theorems have been established for *compact* holomorphic self-maps of the open unit balls of J^*-algebras (which include C^*-algebras and Hilbert spaces) [6], [25]. A natural question is whether a Denjoy-Wolff-type convergence result for *compact* holomorphic maps on J^*-algebras might also follow from these Wolff-type theorems. We show that this is the case for Hilbert spaces, but not the case even for finite-dimensional C^*-algebras. We prove the following result.

Theorem. *Let H be a Hilbert space with open unit ball B. Let $f: B \to B$ be a compact holomorphic map with no fixed point in B. Then there exists $\xi \in \partial B$ such that the sequence (f^n) of iterates of f converges locally uniformly on B to the constant map taking value ξ.*

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We will give a simple example to show that the above result is false if H is replaced by a C^*-algebra. We also note that it has been shown in [10] that if $f\colon B \to B$ is fixed-point-free and so-called *firmly holomorphic*, then the iterates (f^n) converge *pointwise* to a boundary point $\xi \in \partial B$.

It may be useful to recall the Earle-Hamilton Theorem [5] which states that every holomorphic map $f\colon B \to B$, where B is a bounded domain in a Banach space, has a fixed-point if $f(B)$ is *strictly* contained in B.

2. Preliminaries

All Banach spaces will be complex. Given *bounded* domains D and D' in any Banach spaces, we denote by $H(D, D')$ the space of all holomorphic maps $f : D \to D'$. We write $H(D)$ for $H(D,D)$. Every nonempty open ball B in D induces a norm $\|\cdot\|_B$ on $H(D,D')$ where $\|f\|_B = \sup_{x\in B}\|f(x)\|$ for $f \in H(D,D')$. The *topology of local uniform convergence* on $H(D,D')$ is the topology induced by the norms $\|\cdot\|_B$ where B is an open ball in D satisfying $\operatorname{dist}(B, \partial D) > 0$, ∂D being the boundary of D. Using Hadamard's three circles theorem, it has been shown in [21], [22] (see also [19, Lemma 13.1]) that $\|\cdot\|_{B_1}$ and $\|\cdot\|_{B_2}$ induce the same topology for any open balls B_1, B_2 in D satisfying $\operatorname{dist}(B_1,\partial D) > 0$ and $\operatorname{dist}(B_2,\partial D) > 0$. It follows that a sequence (f_n) in $H(D,\overline{D})$ converges to $f \in H(D,\overline{D})$ locally uniformly if, and only if, for every $x \in D$, (f_n) converges uniformly to f on *some* open ball B containing x and satisfying $\operatorname{dist}(B,\partial D) > 0$. Given any x in a Banach space X, and $r > 0$, we let $B(x,r) = \{y \in X : \|y - x\| < r\}$. A map $f\colon D \to D' \subset X$ is called *compact* if the closure $\overline{f(D)}$ is compact in X.

Lemma 1. *Let B be the open unit ball of a Banach space X and let $f\colon B \to B$ be a compact holomorphic map. Then the sequence (f^n) of iterates of f has a subsequence converging locally uniformly to a function in $H(B,\overline{B})$.*

Proof. Choose a sequence (r_n) in $(0,1)$ such that $r_n \uparrow 1$ and $f(B)\cap B(0,r_1) \neq \emptyset$. We have $f(B) = \cup_{n=1}^{\infty}\big(f(B)\cap B(0,r_n)\big)$. We first find a subsequence of (f^n) converging uniformly on $f(B)\cap B(0,r_1)$. By compactness of $\overline{f(B)\cap B(0,r_1)} \subset \overline{f(B)}$, there is a countable set $\{z_n\}$ in $f(B)\cap B(0,r_1)$, which is dense in $\overline{f(B)\cap B(0,r_1)}$.

Since f is compact, (f^n) has a subsequence, $(f^{(n,1)})$, such that $\big(f^{(n,1)}(z_1)\big)$ converges. Likewise, $(f^{(n,1)})$ has a subsequence $(f^{(n,2)})$ such that $\big(f^{(n,2)}(z_2)\big)$ converges. Proceed to find subsequences $\big(f^{(n,k)}\big)_n$ which converge at $z_1,\ldots,z_k$. We show that the diagonal sequence $\big(f^{(k,k)}\big)$ converges uniformly on $f(B)\cap B(0,r_1)$. It suffices to show that it is uniformly Cauchy on $f(B)\cap B(0,r_1)$. Let $\varepsilon > 0$. Since $\operatorname{dist}(B(0,r_1),\partial B) = 1 - r_1 > 0$, we have

$$\|h(z) - h(w)\| \le \frac{\|z - w\|}{1 - r_1} \tag{1}$$

for $h \in H(B)$ and $z, w \in B(0,r_1)$ (cf. [19, 1.17]). By compactness, there exist $z_{n_1},\ldots,z_{n_l}$ in $\{z_n\}$ such that

$$\overline{f(B)\cap B(0,r_1)} \ \subset\ \bigcup_{i=1}^{l} B\big(z_{n_i}, \frac{\varepsilon}{3}(1-r_1)\big).$$

There exists N such that $j,k > N$ implies

$$\|f^{(j,j)}(z_{n_i}) - f^{(k,k)}(z_{n_i})\| < \frac{\varepsilon}{3}$$

for $i = 1, \dots, l$. Hence, for any $z \in f(B) \cap B(0, r_1)$, we have $z \in B(z_{n_i}, \frac{\varepsilon}{3}(1 - r_1))$ for some i, and

$$\begin{aligned}\|f^{(j,j)}(z) - f^{(k,k)}(z)\| &\le \|f^{(j,j)}(z) - f^{(j,j)}(z_{n_i})\| + \|f^{(j,j)}(z_{n_i}) - f^{(k,k)}(z_{n_i})\| \\ &\quad + \ \|f^{(k,k)}(z_{n_i}) - f^{(k,k)}(z)\| \\ &< \frac{\varepsilon(1 - r_1)}{3(1 - r_1)} + \frac{\varepsilon}{3} + \frac{\varepsilon(1 - r_1)}{3(1 - r_1)} \quad = \varepsilon\end{aligned}$$

whenever $j, k > N$. This shows that $\big(f^{(k,k)}\big)$ is uniformly convergent on $f(B) \cap B(0, r_1)$.

We repeat the diagonal process as follows. Choose a subsequence (f^{n_1}) of (f^n) converging uniformly on $f(B) \cap B(0, r_1)$. Then choose a subsequence (f^{n_2}) of (f^{n_1}) converging uniformly on $f(B) \cap B(0, r_2)$, and so on. The diagonal sequence (f^{n_n}) then converges uniformly on $f(B) \cap B(0, r_k)$ for $k = 1, 2, \dots$.

Finally, we show that (f^{n_n+1}) converges locally uniformly on B. Pick $x \in B$ and choose $r, R > 0$ such that $r + R = 1 - \|x\|$ and $\frac{r}{R} < 1 - \|f(x)\|$. Then $B(x, r)$ and $B(f(x), \frac{r}{R})$ are contained in B. As in (1), $\operatorname{dist}(B(x, r), \partial B) \ge R > 0$ implies

$$f\big(B(x, r)\big) \subset \ B\big(f(x), \frac{r}{R}\big) \cap f(B) \subset \ B(0, r_k) \cap f(B)$$

for some k. It follows that (f^{n_n}) converges uniformly on $f\big(B(x, r)\big)$ and hence (f^{n_n+1}) converges uniformly on $B(x, r)$. $\square$

Remark 1. The above proof implies that every subsequence (f^{n_k}) of the iterates (f^n) has a locally uniformly convergent subsequence.

We need the following version of the maximum modulus principle and we include a proof for completeness (cf. [4, p.95]).

Lemma 2. *Let D be a domain in a Banach space X and let B be the open unit ball of a Hilbert space H. Given any holomorphic function $f\colon D \to \overline{B}$, we have either $f(D) \subset B$ or $f(z) = \xi \in \partial B$ for all $z \in D$.*

Proof. Suppose $f(z_0) = \xi \in \partial B$ for some $z_0 \in D$ where D contains some open ball $B(z_0, r)$ with $r > 0$. We show that $f(z) = \xi$ for all $z \in D$. It suffices to show $f(v) = \xi$ for all $v \in B(z_0, r)$. Fix v arbitrary in $B(z_0, r)$. Define $\varphi_v : \Delta \longrightarrow \mathbb{C}$ by

$$\varphi_v(\lambda) = \langle f(z_0 + \lambda(v - z_0)),\ \xi\,\rangle \qquad\qquad (\lambda \in \Delta)$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product on H. Then $\varphi_v\colon \Delta \to \overline{\Delta}$ and $\varphi_v(0) = 1$ imply $\varphi_v \equiv 1$ by the maximum modulus principle. It follows that $f(z_0 + \lambda(v - z_0)) = \xi$ for all $\lambda \in \Delta$ which gives $f(v) = \xi$ by continuity. $\square$

3. Denjoy-Wolff-type result

In this section, we prove the **Theorem** and give some simple examples. Wolff's theorem has been extended to fixed-point-free holomorphic self-maps f of a Hilbert ball B, in which case there exists $\xi \in \partial B$ such that the "*ellipsoids*"

$$E(\xi, \lambda) = \left\{ x \in B : \frac{|1 - \langle x, \xi \rangle|^2}{1 - \|x\|^2} < \lambda \right\} \qquad\qquad (\lambda > 0)$$

are invariant under f, and further, $\overline{E(\xi, \lambda)} \cap \partial B = \{\xi\}$ (cf. [8]). If $\dim B < \infty$, then the iterates f^n must converge locally uniformly to the "*Wolff point*" ξ (cf. [13, 15]).

As remarked before, the latter result is false in infinite dimensions by Stachura's example [18].

Let $\mathcal{L}(H,K)$ be the Banach space of bounded linear operators between Hilbert spaces H and K. A closed linear subspace $Z \subset \mathcal{L}(H,K)$ is called a J^*-algebra if $TT^*T \in Z$ whenever $T \in Z$, where T^* denotes the adjoint of T (cf. [11, 19]). Every Hilbert space $H = \mathcal{L}(\mathbb{C}, H)$ is a J^*-algebra, and so is every C^*-algebra.

Let B be the open unit ball of a J^*-algebra Z and let $f\colon B \to B$ be a fixed-point-free *compact* holomorphic map. A Wolff-type result has been obtained in [16, Theorem 5] which states that under certain conditions on f, there exist a "*Wolff point*" $\xi \in \partial B$ and circular domains $D_{z,\xi}$ $(z \in B)$ invariant under f (see also [25]). The question of whether the iterates f^n would converge to ξ was unanswered in [16]. The following example gives a negative answer.

Example 1. Let $Z = \mathbb{C} \times \mathbb{C}$ be equipped with the coordinatewise product and norm $\|(z,w)\| = \max(|z|,|w|)$. Then Z is a C^*-algebra with open unit ball $\Delta \times \Delta$. Pick any fixed-point-free $h \in H(\Delta)$. Define $f : \Delta \times \Delta \to \Delta \times \Delta$ by

$$f(z,w) = \big(iz, h(w)\big) \qquad (z, w \in \Delta).$$

Then f is fixed-point-free and we have

$$f^n(z,w) = \big(i^n z, h^n(w)\big),$$

where (h^n) converges locally uniformly on Δ to some $\xi \in \partial\Delta$. The iterates (f^n) clearly do not converge to any boundary point in $\partial(\Delta \times \Delta)$.

Nevertheless, we can still derive a Denjoy-Wolff-type convergence result for *compact* holomorphic maps on Hilbert spaces, by adapting MacCluer's arguments for $\mathbb{C}^n$ in [15]. A crucial step in the proof depends on the fact that the automorphisms of a Hilbert ball map affine sets to affine sets, and consequently that the fixed-point set of a nonconstant holomorphic map is affine. In contrast, the automorphisms of the open unit ball of an arbitrary J^*-algebra may distort the affine sets and the fixed-point set of a nonconstant holomorphic map need *not* be affine, even in the simple case of the bidisc as shown by the example below. This is one reason why a Denjoy-Wolff-type result fails for arbitrary J^*-algebras.

Let B be the open unit ball of a Banach space X. By an *affine subset* of B we mean a nonempty set of the form $(c+L) \cap B$ where $c \in X$ and L is a closed linear subspace of X. If X is a Hilbert space, c can be chosen to be orthogonal to L, and also, for nonconstant $h \in H(B)$, its fixed-point set $\mathrm{Fix}\,(h) = \{x \in B : h(x) = x\}$ is affine by [12] (see also [9, Theorem 23.2]). This was proved in [17] in finite dimensions.

Example 2. Let $Z = \mathbb{C} \times \mathbb{C}$ be as in Example 1, with open unit ball $\Delta \times \Delta$. Fix $a \in \Delta\backslash\{0\}$. Define $h : \Delta \times \Delta \to \Delta \times \Delta$ by

$$h(z,w) = \big(g_a(w), g_{-a}(z)\big) \qquad (z, w \in \Delta)$$

where $g_a(w) = \dfrac{a+w}{1+\overline{a}w}$. Then $\mathrm{Fix}\,(h) = \{(z, g_{-a}(z)) : z \in \Delta\}$ which is *not* affine since $(0,-a)$ and $(a,0)$ are in $\mathrm{Fix}\,(h)$ while $\frac{1}{2}(0,-a) + \frac{1}{2}(a,0) \notin \mathrm{Fix}\,(h)$. We also note that $h^{2n}(z,w) = (z,w)$ and $h^{2n+1}(z,w) = h(z,w)$. So $(h^n(z,w))_n$ does not converge if $(z,w) \notin \mathrm{Fix}\,(h)$.

We are now ready to prove the Theorem.

Proof of the Theorem. We will use the same symbol throughout for both a constant function and its value.

Let $\xi \in \partial B$ be the "*Wolff point*" of f as mentioned in the beginning of this section. Let $\Gamma(f)$ be the set of all subsequential limits of $\{f^n : n = 1, 2, \dots\}$ in $H(B, \overline{B})$ with respect to the topology of local uniform convergence. By Lemma 1, $\Gamma(f) \neq \emptyset$.

We first show that $\Gamma(f)$ consists of constant maps only. Suppose, otherwise, that $\Gamma(f)$ contains a nonconstant map $g \in H(B, \overline{B})$. We deduce a contradiction. By Lemma 2, $g(B) \subset B$. Let (f^{n_k}) be a subsequence of (f^n) converging to g. Let $m_k = n_{k+1} - n_k$. By Remark 1, (f^{m_k}) has a convergent subsequence, and we may assume, without loss of generality, that $f^{m_k} \to h_0 \in H(B, \overline{B})$. Since $f^{n_{k+1}} = f^{m_k} \circ f^{n_k} \to h_0 \circ g$, we have $h_0 \circ g = g$ and h_0 is the identity on $g(B)$. So h_0 is nonconstant, $h_0(B) \subset B$ and $A_0 = \operatorname{Fix}(h_0)$ is an affine subset of B. Since $A_0 \subset \overline{f(B)}$ which is compact, it follows that $\dim A_0 < \infty$. Clearly, $A_0 \subset h_0(B)$. If $A_0 \neq h_0(B)$, then we repeat the above process. Letting $p_k = m_{k+1} - m_k$, we may assume $f^{p_k} \to h_1 \in H(B)$ satisfying $h_1 \circ h_0 = h_0$. So h_1 is the identity on $h_0(B)$ and $h_0(B) \subset A_1 = \operatorname{Fix}(h_1)$. We have $A_0 \subsetneq A_1$ and A_1 is a finite dimensional affine subset of B, with $\dim A_1 > \dim A_0$. If $A_1 \neq h_1(B)$, we repeat the process again. Continuing in this manner, we must eventually find some $h_i \in H(B)$ such that $h_i(B) = A_i = \operatorname{Fix}(h_i)$. For otherwise, we can construct a sequence $(v_j)_j \subset \cup_{i=1}^{\infty} A_i$ with the property that $\|v_i - v_j\| > \delta$ for all $i \neq j$ and some $\delta > 0$. Since $\cup_{i=1}^{\infty} A_i \subset \overline{f(B)}$ which is compact, this is clearly impossible. It follows that $h_i^2 = h_i$. Let (f^{l_k}) converge to h_i locally uniformly. Note that $f(A_i) \subset A_i$. Since A_i is a finite dimensional affine subset of B and the automorphisms act transitively on B, a similar argument to that in [15, p. 98] shows that A_i is biholomorphically equivalent to the open unit ball of $\mathbb{C}^n$ where $n = \dim A_i$. Now $f|_{A_i} \colon A_i \to A_i$ is fixed-point-free and by [13, 15], there exists $\mu \in \partial A_i$ such that $(f|_{A_i})^n$ converges locally uniformly to μ on A_i. So $h_i|_{A_i} = \lim_{k\to\infty} (f|_{A_i})^{l_k} = \mu$ which is impossible as h_i is nonconstant and $h_i(B) = A_i$.

Therefore $\Gamma(f)$ must consist of constant maps only. Now take any $\eta \in \Gamma(f)$. Then $\eta \in \partial B$, for otherwise η would be a fixed point of f in B. There is a subsequence (f^{n_k}) converging to η. Let $\lambda > 0$ and let $z \in E(\xi, \lambda)$. We have

$$\eta = \lim_{k\to\infty} f^{n_k}(z) \ \in \ \overline{E(\xi, \lambda)} \cap \partial B \ = \{\xi\}$$

since $E(\xi, \lambda)$ is f-invariant. Therefore every convergent subsequence of (f^n) converges to the constant map ξ. It follows from Remark 1 that (f^n) must converge locally uniformly to ξ and the proof is complete. □

We end with the following example of a fixed-point-free compact holomorphic map on a Hilbert ball.

Example 3. Let B be the open unit ball of the (complex) Hilbert space l_2. Define $f : B \longrightarrow B$ by

$$\begin{aligned} f(x_1, x_2, \dots) &= \left(\frac{1+x_1}{2}, \ \left(\frac{1-x_1}{2}\right)\frac{x_1}{2}, \ \left(\frac{1-x_1}{2}\right)\frac{x_2}{3}, \dots\right) \\ &= \left(\frac{1+x_1}{2}, 0, 0, \dots\right) \ + \ \frac{1-x_1}{2}\left(0, \ \frac{x_1}{2}, \ \frac{x_2}{3}, \ \dots\right) \end{aligned}$$

for $(x_1, x_2, \dots) \in B$. Then f is fixed-point-free and holomorphic. Moreover, f is compact since it is the sum of two compact maps. Also the *Wolff point* is $(1, 0, 0, \dots)$.

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