P8130_hw2

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Problem1

a)

Let X denote the number of people who develop uveal melanoma in a given year. Since n is very large and p is small, X follows a poisson distribution as followed:

$$X \sim Poi(42.5), \quad \lambda = 42.5$$

Therefore

$$P(X = 30) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{42.5^{30} e^{-42.5}}{30!} = 0.0093431 \approx 0.009$$

b)

The population of Asians, non-Hispanic Whites and Black are 1.19×10^6, 3.638×10^6, 2.0655×10^8, and therefore, X_{Asians},$X_{non-HispanicWhites}$ and X_{Black} separately follow poisson distributions as followed:

$$X_{Asians} \sim Poi(0.464)$$
$$X_{non-HispanicWhites} \sim Poi(21.901)$$
$$X_{Black} \sim Poi(0.640)$$

Therefore

$$P(X_{Asians} = 30) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{0.464^{30} e^{0.464}}{30!} = 2.346209 \times 10^{-43} \approx 2.346 \times 10^{-43}$$

$$P(X_{non-HispanicWhites} = 30) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{21.901^{30} e^{21.901}}{30!} = 0.0190009 \approx 0.019$$

$$P(X_{Black} = 30) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{64.031^{30} e^{64.031}}{30!} = 3.0464263 \times 10^{-39} \approx 3.046 \times 10^{-39}$$

As the result shows, a disparity (between the probability of exactly 30 uveal melanoma cases occurring in a given year) exists for different racial/ethnic groups. The non-Hispanic whites have the highest probability of 0.019, followed by Blacks and Asians, which have rather low probability.

Problem2

a)

Intervention Group

The hypothesis is:

$$H_0 : \mu_1 - \mu_2 = 0 \quad vs \quad H_1 : \mu_1 - \mu_2 \neq 0$$

With the significance level $\alpha = 0.05$, compute the test statistic:

$$t_{stats} = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{-0.76}{1.44/\sqrt{36}} = -3.17$$

Because $|t_{stats}| = 3.17 > t_{35,0.975} = 2.03$, at 5% significance level, we reject H_0 and conclude that there's a difference of BMI between 6-months follow-up and baseline among intervention group.

Control Group

The hypothesis is:

$$H_0 : \mu_1 - \mu_2 = 0 \quad vs \quad H_1 : \mu_1 - \mu_2 \neq 0$$

With the significance level $\alpha = 0.05$, compute the test statistic:

$$t_{stats} = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{0.28}{0.97/\sqrt{36}} = 1.73$$

Because $|t_{stats}| = 1.73 < t_{35,0.975} = 2.03$, at 5% significance level, we fail to reject H_0 and conclude that there's no enough evidence to support a difference of BMI between 6-months follow-up and baseline among control group.

b)

Now perform a test to compare the BMI absolute changes between the two groups.

Test for Equality of Variances

The hypothesis is:

$$H_0 : \sigma_1 = \sigma_2 \quad vs \quad H_1 : \sigma_1 \neq \sigma_2$$

With the significance level $\alpha = 0.05$, compute the test statistic:

$$F_{stats} = \frac{s_1^2}{s_2^2} = \frac{1.44^2}{0.97^2} = 2.204 \sim F_{35,35}$$

Because $F_{stats} = 2.204 > F_{35,35,0.975} = 1.961$, at 5% significance level, we reject H_0 and conclude that there's a difference of variances of BMI change between intervention and control group.

Two-Sample Independent t-test with Unequal Variances

$$H_0 : \mu_1 = \mu_2 \quad vs \quad H_1 : \mu_1 \neq \mu_2$$

With the significance level $\alpha = 0.05$, compute the test statistic:

$$t_{stats} = \frac{\bar{X_1} - \bar{X_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{-0.76 - 0.28}{\sqrt{\frac{1.44^2}{36} + \frac{0.97^2}{36}}} = 3.594$$

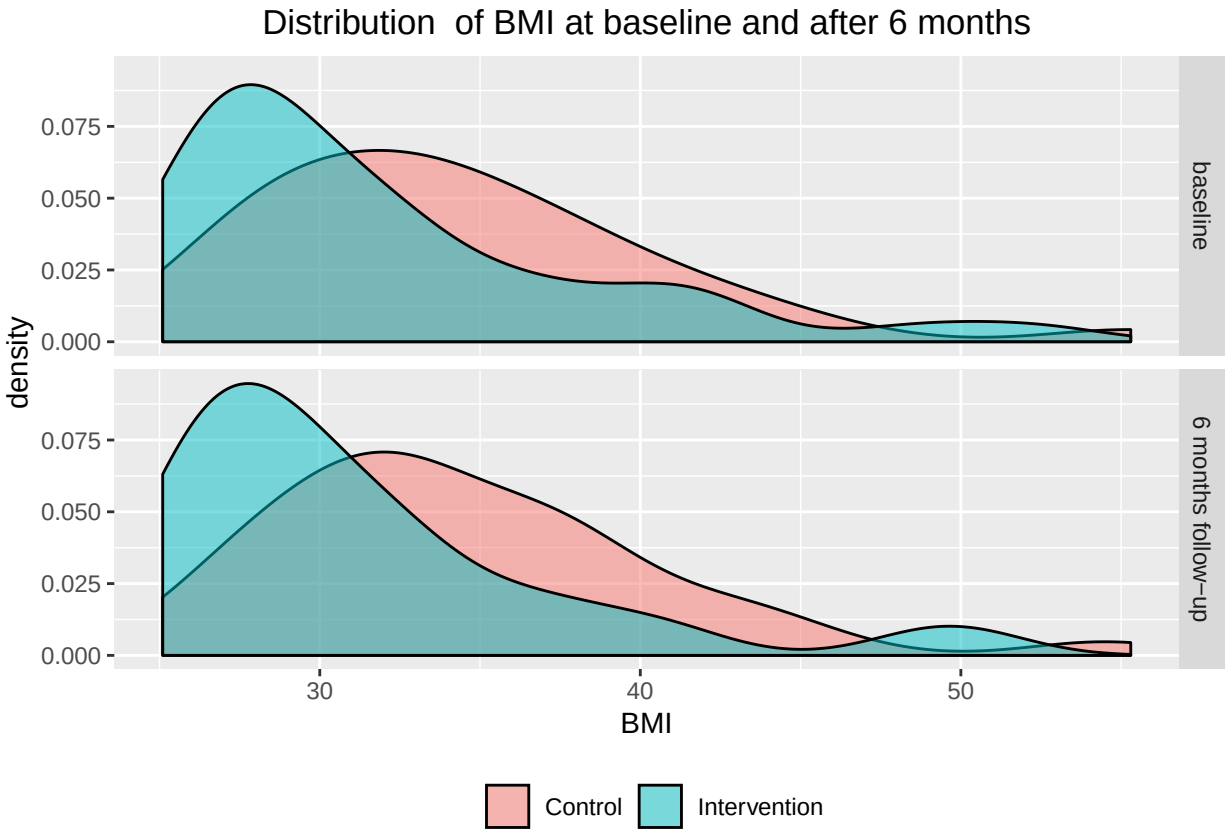
$$d' = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{(\frac{s_1^2}{n_1})^2/(n_1 - 1) + (\frac{s_2^2}{n_1})^2/(n_2 - 1)} = \frac{(\frac{1.44^2}{36} + \frac{0.97^2}{36})^2}{(\frac{1.44^2}{36})^2/(36 - 1) + (\frac{0.97^2}{36})^2/(36 - 1)} = 61.340 = 61$$

Because $t_{stats} = 3.594 > t_{61,0.975} = 2$, at 5% significance level, we reject H_0 and conclude that there's a difference of BMI change between intervention and control group.

c)

The assumption is that the distribution of BMI is normal.

i) Check the normality



As we can see in the plot above, the distribution of BMI is right-skewed. In another word, the normality assumption is (very likely) invalid.

ii) Effect of non-normality and remedies

Our tests are based on the normality assumption, which decide the underlying distribution of BMI. Therefore, non-normality invalidate our tests and may distort the truth. Fortunately, we do have alternatives, such as **Non-parametric test** and **Transformation**.

Problem3

Let X denote the number of restaurants that close by the end of 2019. As we know,

$$X \sim B(20, 0.60)$$

Therefore, use exact method:

$$P(X \geq 10) = 1 - P(X < 10) = 1 - F(9) = 0.872$$

- Poisson approximation: since n=20 is not large and p=0.60 is not small, it's inappropriate to use poisson approximation to binomial.
- Normal approximation: since n(1-p) = 8 < 10, it's also inappropriate use normal approximation.
- According to the result, the probability that more than 10 restaurants will close by the end of 2019 is 87.2%.

Problem4

a)

Paired t-test with Equal Variances

$$H_0 : \mu_{drug1} - \mu_{drug1} = 0 \quad vs \quad H_1 : \mu_{drug1} - \mu_{drug2} < 0$$

With the significance level $\alpha = 0.05$, compute the test statistic:

$$t_{stats} = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{-1.58}{1.23/\sqrt{10}} = -4.062$$

Because $t_{stats} = -4.06 < t_{9,0.05} = -1.833$, at 5% significance level, we reject H_0 and conclude drug2 has a better efficacy than drug1 in increasing sleep time.

b)

According to the formula

$$95\%CI = (-\infty, \bar{d} + t_{1-0.05,9} S/\sqrt{n})$$

$$95\%CI = (-\infty, -1.58 + 1.833 \times 1.230/\sqrt{10}) = (-\infty, -0.867)$$

c)

According to the formula

$$Power = \Phi(Z_\alpha + \frac{|\Delta|}{\sigma/\sqrt{n}}) = \Phi(-1.64 + \frac{1.58}{1.230/\sqrt{10}}) = 0.992$$

- The posterior power is 0.992, meaning that if there's anything significant, the probability that we can detect it is 99.2%.

d) PROs and CONs of using a posteriori/post-hoc power analysis

- PROs: When already having a fixed sample size, we can use posteriori power analysis before conducting the study to see whether or not such study would give us adequate power.
- CONs: Post-hoc power analysis may been misused in interpreting nonsignificant study results, where there will be always a low power. As a result, an observed nonsignificant result may be misinterpreted as the study having inadequate power(underpowered to detect) but as a fact, the result may come from the true nonsignificance.