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Project 6: Model for the Spread of Infectious Diseases

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Inhaltsverzeichnis

1 Introduction

This model uses a quadratic grid of size $L \times L$ to examine the spread of an infectious agent. Each grid node is occupied by an immobile person and can be in one of the states Susceptible S, Infected I or Recovered R. Later, the forth state Vaccinated V is introduced, that permanently excludes the respective people from taking any of the three other states. For a person in one of the three active states S, I and R, the following three transitions are possible,

$$\text{Susceptible } S \xrightarrow{p_1} \text{Infected } I \xrightarrow{p_2} \text{Recovered } R \xrightarrow{p_3} \text{Susceptible } S.$$

With that, we have the three transition probabilities p_1, p_2 and p_3:

- p_1: a susceptible person getting infected by a direct infected neighbor
- p_2: an infected person turning recovered
- p_3: a recovered person returning susceptible

Vaccinated people V are set at the beginning of the simulation, with p_4 being the probability that a spot is occupied by one. The other three states are initializes with the same likelyhood, as the simulation steps progress by each updating L^2 randomly chosen nodes according to the above mentioned rules.

Goal was now to use the model to examine the influence of the transition probabilities on the infection rate averaged over all simulation steps. This included varying the $S \to I$ turover rate p_1 for diffrent combinations of $p_2(I \to R)$ and $p_3(R \to I)$ as well as using different vaccination rates p_4 at initialization. For all simulations, different grid sizes were used to observe potential statistical inaccuracies.

Further, the time evolution of the infection rates was looked at to stress the validity of the preceding time averaging. Therefor a quantity of 20 infection rate samples was generated over simulation time with mean and standard deviation calculated for each timestep.

2 Methodology

2.1 Pseudorandom Number Generator (PRNG) MT19937

For the generation of pseudorandom numbers the 1997 developed Mersenne Twister MT19937 was chosen.[1][2]

3 Implementation

3.1 Usage of the Libraries `cvc_numerics.h` and `cvc_rng.h`

3.2 Structure and Workflow of the Main Program

Generation of Random Numbers by a Static RNG

For the generation of pseudorandom numbers the in Subsection 2.1 described Mersenne Twister MT19937 was used. It was implemented statically and can therefore be accessed globally by the respective functions.

Implementation of the Modelling Grid

The above mentioned grid itself was realized as a $(L + 2) \times (L + 2)$ heap section of integer values, with L being the sidelength of the quadratic grid where the actual spread of the infection takes place. While this section is technically one-dimensional, it will for simplicity reasons be here refered to as a two-dimensional structure of the given shape. Inside the grid, the following integer values were used to model the different states of the people within the simulation:

- `0`: this person is Susceptible S to the infection
- `1`: the person is Infected I
- `2`: the person is Recovered R and currently not susceptible
- `-1`: the person is Vaccinated V and does not participate in the spread

The grid was implemented with a supporting edge of ghosts at the top, bottom, left and right border, that are neither infectious nor subject to any updates of the grid — they will permanently take the value `0` and are irrelevant for the later visualization and analysis of the data.

Functions acting on the Modelling Grid

In the main program all changes in the grid are facilitated by functions outside the main one. They entirely work with or change the grid in-place and all take the grid itself as well as its length $L + 2$ as arguments. Inside the functions, the grid is refered to as `grid`, its legth as `length` while the above probabilities are passed in form of the double array `probabilities`. Excluding the ones to calculate the infection rate and its time average, none of the functions does have a return value. Overall the following functions can be called from the main:

- `void print_grid`: prints the grid
- `void grid_init`: initializes the grid applying the vaccination probability
- `void update_node`:
- `void grid_update_linear`:
- `void grid_update_stochastic`:
- `double ratio_infected`:
- `double average_ratio_infected`:

Structure of the Main Function and General Workflow

The main function Each section of the main function is an enclosed space in order to prevent them from interfering with each other.

4 Results and Discussion

4.1 Model for the Spread of Infectious Diseases

4.2 Expected Ratio of Infected People averaged over Time

4.3 Vaccinated People without Participation in the Spread

4.4 Time Evolution of the Expected Ratio of Infected People

While previously the average of the ratio of infected individuals has been taken over time, the focus should now be layed on the time development of th einfection rate $\langle I \rangle_t$ for $N = 20$ samples. As grid size $L = 64$ was chosen for appropriate balance between running time and accuracy, the number of simulation steps again was set to $T = 1000$.

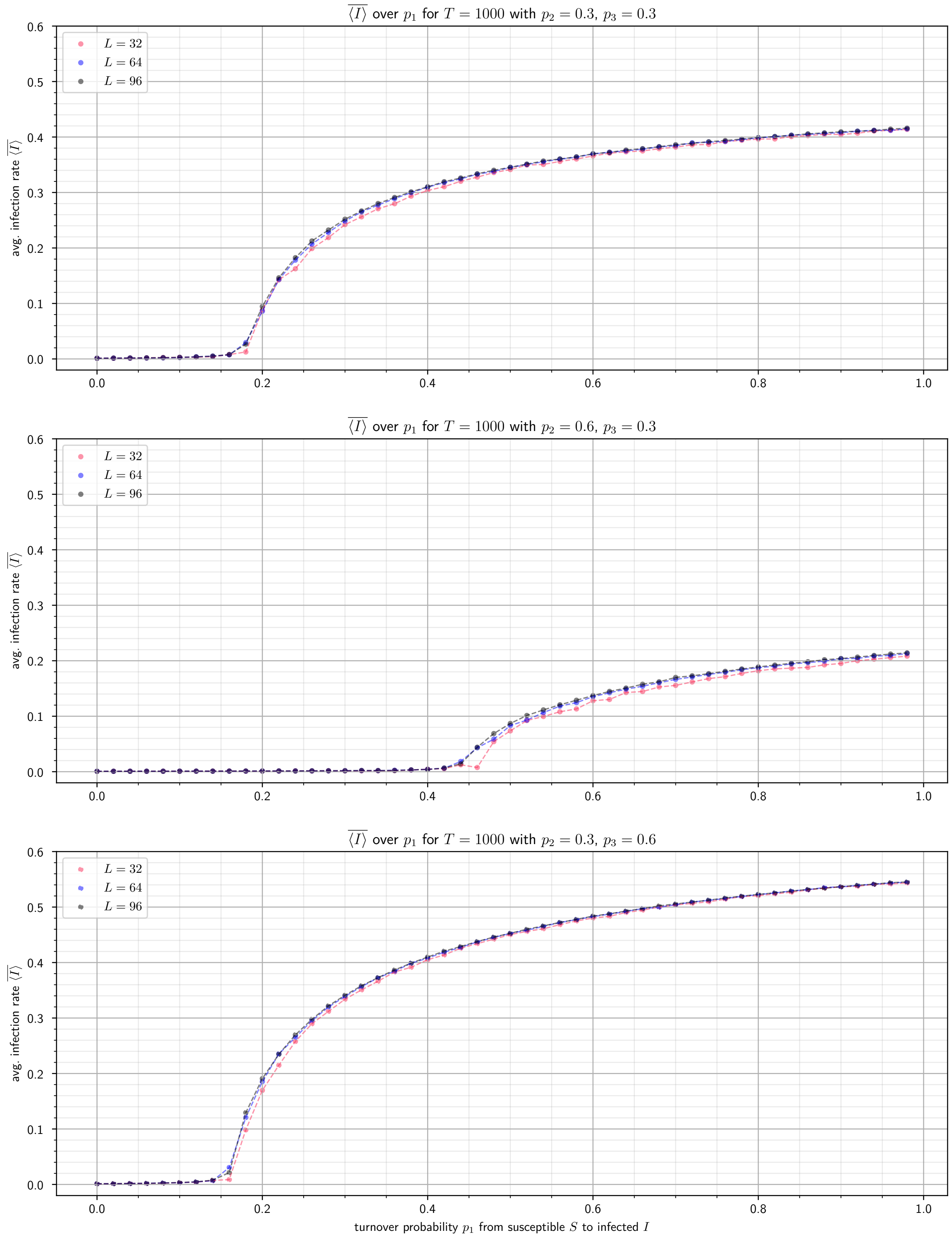

Abbildung 1: Graphic

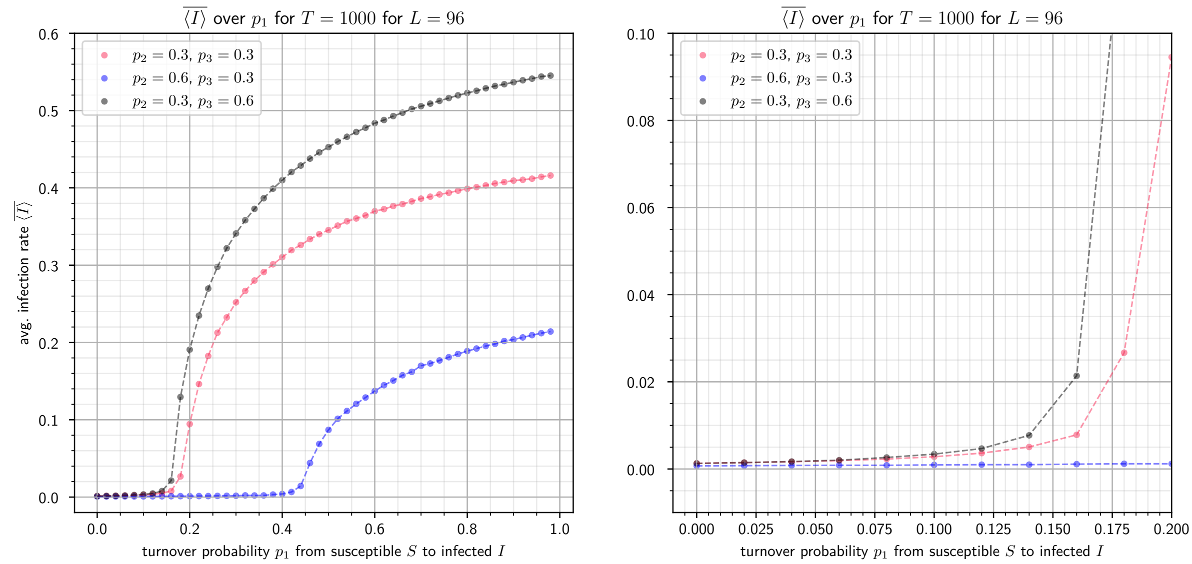

Abbildung 2: Graphic

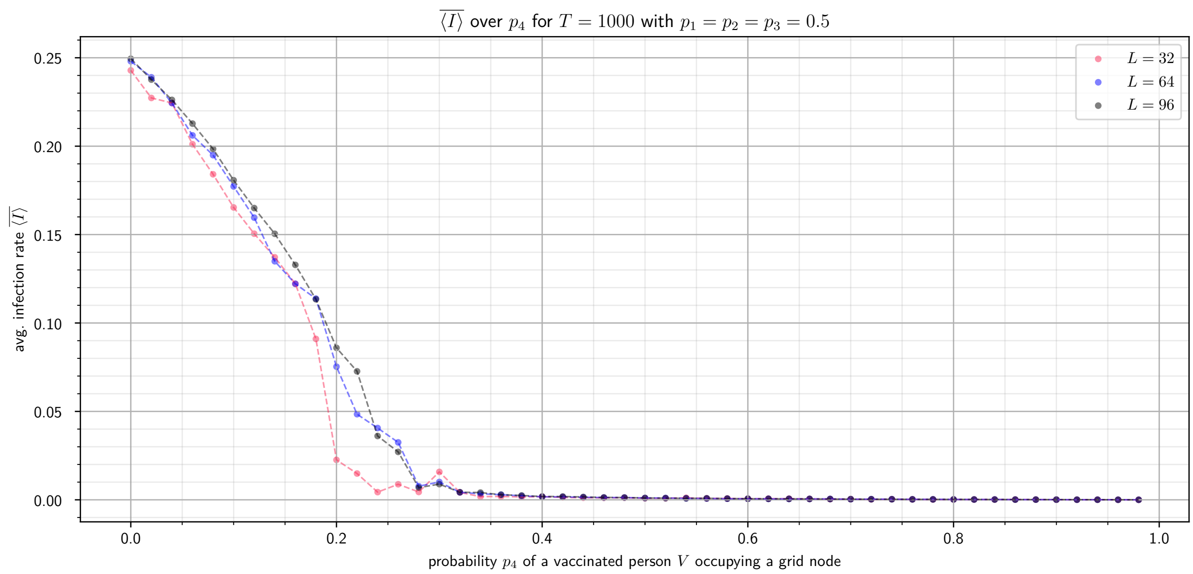

Abbildung 3: Graphic

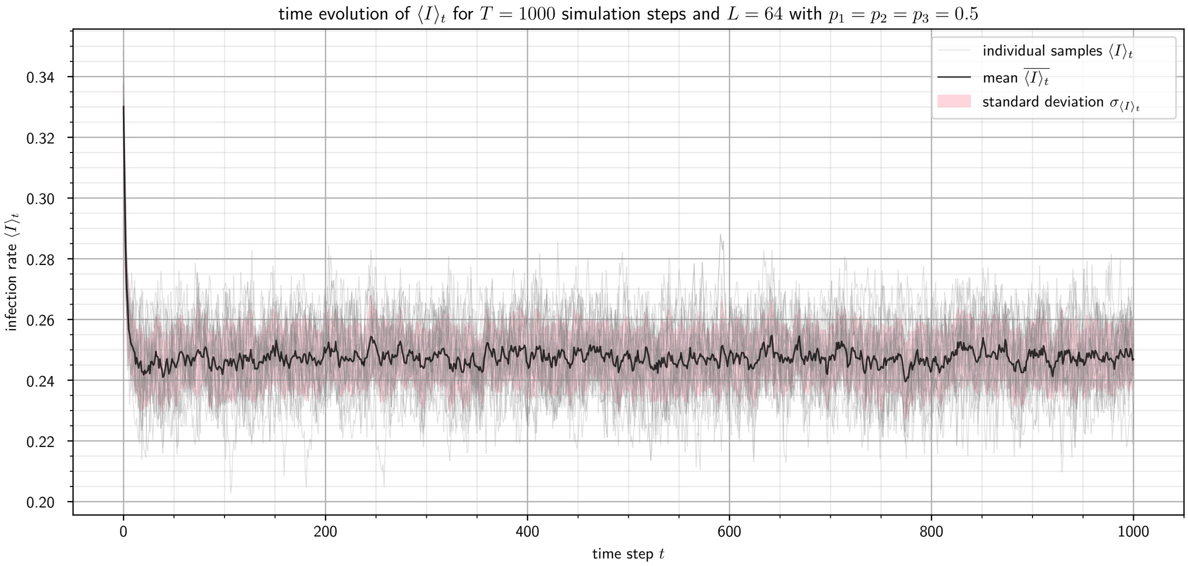

Abbildung 4: Graphic

5 Supplementary Materials

Bibliography

[1] *educative. What is Mersenne Twister?* 29. Juli 2023. URL: https://www.educative.io/answers/what-is-mersenne-twister (besucht am 29. 07. 2023).

[2] Makoto Matsumoto und Takuji Nishimura. „Mersenne Twister: A 623-Dimensionally Equidistributed Uniform Pseudo-Random Number Generator“. In: *ACM Trans. Model. Comput. Simul.* 8.1 (1998). ISSN: 1049-3301. DOI: 10.1145/272991.272995. URL: https://doi.org/10.1145/272991.272995.