Research on Strong Subadditivity of Quantum Information

量子信息的强次可加性的研究

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concepts

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- $H(X:Y) \equiv H(X) + H(Y) - H(X,Y)$

inequalities

independency

- $H(X:Y) \geq 0$ Subadditivity
  $H(X) + H(Y) - H(X,Y) \geq 0$

inequalities

independency

- $H(X:Y)\geq 0$ Subadditivity
  $H(X)+H(Y)-H(X,Y)\geq 0$
- $H(X:Z|Y)\geq 0$ Strong Subadditivity(SSA)
  $H(X,Y)+H(Y,Z)-H(X,Y,Z)-H(Y)\geq 0$

Quantum Information

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- Does SSA holds ?
- Lieb et al.(1973) proof SSA
- Patrick Hayden et al.(2004) proof SSA=0 implies quantum markov state
- Ting-Chun Lin et al.(2023) proof a operator extension of SSA

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operator form

$$\hat{A} = \log\rho_{AB} + \log\rho_{BC} - \log\rho_{A} - \log\rho_{C} \leq 0$$

recover by considering $< \hat{A} >$

operator norm

consider density matrix in AB,BC.

$$\rho_{AB} \otimes \sigma_C^{-1} \leq \rho_A \otimes \sigma_{BC}^{-1}$$

(f(t)=logt operator monotone)

$$(\rho_A^{-\frac{1}{2}} \otimes \sigma_{BC}^{\frac{1}{2}})(\rho_{AB} \otimes \sigma_C^{-1})(\rho_A^{-\frac{1}{2}} \otimes \sigma_{BC}^{\frac{1}{2}}) \leq I_{ABC}$$

$$||(\rho_{AB}^{\frac{1}{2}} \otimes \sigma_C^{-\frac{1}{2}})(\rho_A^{-\frac{1}{2}} \otimes \sigma_{BC}^{\frac{1}{2}})|| \leq 1$$

diagram

proof the below operator has norm ≤ 1.

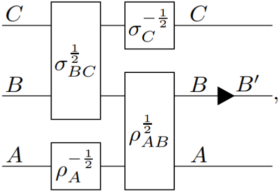

Figure: Caption

Isometry

proof the left and right part has norm 1.

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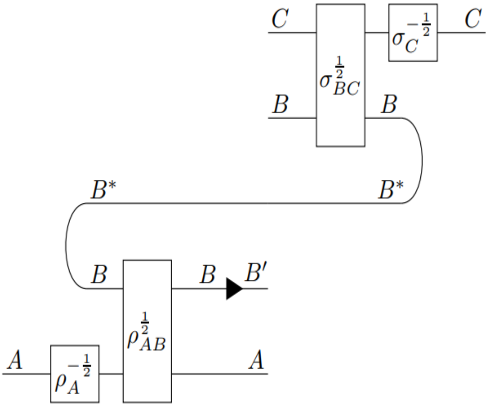

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Quantum markov chain

ABC is a quantum markov chain $\Longleftrightarrow$ H(A:C|B)

Two Characterization

ABC is a quantum markov chain, then

$$\rho_{ABC} = (\mathscr{I}_A \otimes \mathscr{R}_{B\to BC})(\rho_{AB}),$$

Figure: Caption

$$\mathscr{T}_{B\to BC} : X_B \mapsto \rho_{BC}^{\frac{1}{2}}(\rho_B^{-\frac{1}{2}} X_B \rho_B^{-\frac{1}{2}} \otimes \mathrm{id}_C)\rho_{BC}^{\frac{1}{2}}$$

Figure: Caption

Two Characterization

Theorem 6 *A state ρ_{ABC} on $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$ satisfies strong subadditivity (eq. (1)) with equality if and only if there is a decomposition of system B as*

$$\mathcal{H}_B = \bigoplus_j \mathcal{H}_{b_j^L} \otimes \mathcal{H}_{b_j^R}$$

into a direct sum of tensor products, such that

$$\rho_{ABC} = \bigoplus_j q_j \rho_{Ab_j^L} \otimes \rho_{b_j^R C},$$

with states $\rho_{Ab_j^L}$ on $\mathcal{H}_A \otimes \mathcal{H}_{b_j^L}$ and $\rho_{b_j^R C}$ on $\mathcal{H}_{b_j^R} \otimes \mathcal{H}_C$, and a probability distribution $\{q_j\}$.

equivalent condition

- $\hat{A} = 0 \Longrightarrow <\hat{A}> = 0 \Longleftrightarrow \text{SSA} = 0$

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- $\hat{\mathsf{A}} = 0 \Longrightarrow <\hat{\mathsf{A}}> = 0 \Longleftrightarrow \mathsf{SSA} = 0$
- $\hat{\mathsf{A}} = 0 \Longleftrightarrow (\rho_{\mathsf{AB}}^{\frac{1}{2}} \otimes \sigma_{\mathsf{B}}^{-\frac{1}{2}})(\rho_{\mathsf{A}}^{-\frac{1}{2}} \otimes \sigma_{\mathsf{BC}}^{\frac{1}{2}})$ is an isometry

Condition of isometry

recall another form of $(\rho_{AB}^{\frac{1}{2}} \otimes \sigma_{B}^{-\frac{1}{2}})(\rho_{A}^{-\frac{1}{2}} \otimes \sigma_{BC}^{\frac{1}{2}})$

$$\left\| (I_A \otimes V^{\dagger}_{B\to B'} \otimes I_C)(I_A \otimes I_{B'} \otimes V^{\sigma}_{C\to BB^*C}{}^{\dagger})(V^{\rho}_{A\to AB'B^*} \otimes I_B \otimes I_C) \right\|$$

Figure: Caption

denote the three term as A,B*,C, the above can be written as AB*C

Condition of isometry

$A : v_1 -> W, B : v_2 -> W$ A,B is isometry, use Direct Sum Decomposition, we can get

theorem

$$w \notin Im_A \Longleftrightarrow ||A^* w|| < ||w|| \ [Nyc, Wyf]$$

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$$w \notin Im_A \iff ||A^* w|| < ||w|| \ [Nyc, Wyf]$$

corollary

$$A^* B \text{ is isometry} \iff Im_B \subseteq Im_A$$

equality condition

consider dimension of Im_B and Im_C, we get

operator SSA equality condition

$$\hat{A} = 0 \Longleftrightarrow \dim(B) = 0$$

The condition is too strong.

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Future work

- new SSA extension between SSA and operator SSA.

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- new SSA extension between SSA and operator SSA.
- use operator SSA inequality to characterize quantum state when SSA=0.

Thank you

Thank you for listening!

Q&A

Questions?