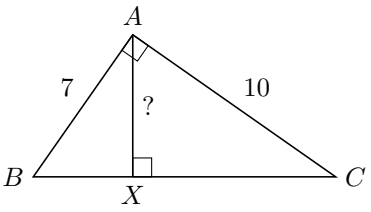
2021 Castro Valley Junior Math Tournament Problems

Mooving Cow

Bessie the Cow is 5 meters north of Farmer Pearson's house. She starts running east at 2 meters per second. After how many seconds will she be exactly 10 meters away from Farmer John's house? Express your answer in exact form.

Cowangle

Bessie the Cow found the following right triangle ABC. The distance from point A to point B is 7, and the distance from point A to point C is 10. $\angle A$ is a right angle, and $\overline{AX}$ is the altitude from point A to side $\overline{BC}$. What is the length of this altitude (the distance from point A to point X)? Express your answer in exact form.



Mooish

Farmer Paul has two types of cows: truthy cows and falsy cows. Physically, they are indistinguishable. They all speak mooish, and they can be distinguished by how they answer questions. The truthy cows always reply truthfully, and the falsy cows always lie. Bessie had a conversation with three of Farmer Paul's cows: Annabelle, Betsie, and Cornelius. Here's a translation of the conversation.

> Bessie: Annabelle, is Betsie truthy or falsy?
> Annabelle: Falsy
> Bessie: Betsie, are the types of Annabelle and Cornelius different?
> Betsie: No.
> Bessie: Talkative lot, aren't they? Cornelius, is Betsie truthy or falsy?
> Cornelius: Truthy.

What is the type of each cow?

Green Cows

Help Bessie the Cow answer the question "How much grass can a green cow chow if green cows can chow grass?" Bessie and Elsie both chow grass at constant rates. If Bessie can chow all the grass in a field in 3 hours and Elsie can chow all the grass in the same field in 4 minutes, how long will it take for them to chow all the grass in this field together? Express your answer in exact form.

My Cow Ate My Homework

Bessie the Cow obtained Chloe's math homework again. While she was enjoying this tasty snack, she noticed one problem that seemed very interesting. Here's the problem statement:

> If two real numbers a and b are generate randomly and uniformly such that $-9 < a < 9$ and $-9 < b < 9$, what is the probability that $(a+b)^2 < 2ab + 9$? Express your answer in exact form.

Bessie is bad at math, so help her by solving this problem.

Look Mom No Proof!

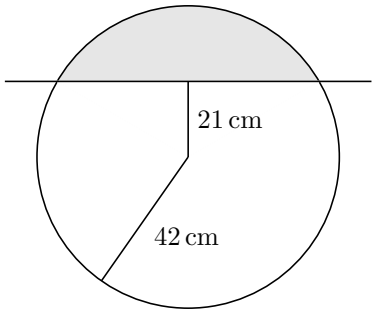
Bessie the Cow found a website called `vixra.org` which mostly publishes scientific nonsense. There's a paper (`https://vixra.org/abs/2008.0229`) which presents a simple method that supposedly checks whether integers greater than 5 are prime. It makes the following statement without proof:

> The answer to whether the given numbers are prime numbers is to check that:
>
> a) The numbers are not even numbers (the last digit is not divisible by 2);
>
> b) The last digit of numbers in not 5;
>
> c) The sum of the digits of each of the remaining numbers is not divisible by 3.
>
> A number that meets the above criteria is either a prime or a power [of a] prime.

Disprove this claim.

Mooshroom Pizza

Farmer John made a mooshroom pizza for the cows. (Mooshroom pizzas are ethically sourced and don't contain mooshroom meat. They're called mooshroom pizzas because they contain mushrooms farmed from mooshrooms.) Bessie tried to slice it, but she failed miserably, resulting in the cut shown in the following figure. If the radius of the pizza is 42 cm and the cut is 21 cm away from the center of the pizza, what is the area of the smaller piece of pizza that was cut off? Express your answer in exact form.

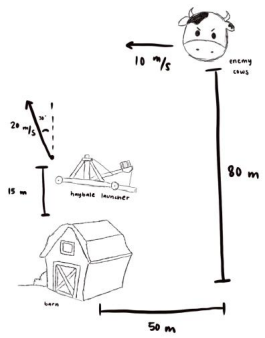


Cowcycles

Bessie the Cow is participating in a bike race! There are 5 cows in the race, and they all bike at a constant speed on a circular track starting from the same position. Bella takes 5 minutes to complete one lap, Bessie takes 9 minutes to complete one lap, Betty takes 3.5 minutes to complete one lap, Betsie takes 9.8 minutes to complete one lap, and Bossy takes 4.7 minutes to complete one lap. The cows have a weird tradition where once cow passes another after the race starts, they will instantaneously elbow bump each other. In other words, each of the $\binom{5}{2} = 10$ possible pairs of cows will elbow bump each other any time they are in the exact same position after the start of the race. For example, Bella and Bessie will elbow bump each other for the first time exactly 11.25 minutes after the start of the race. The cows race for 4 hours. How many elbow bumps will occur in total? Include any elbow bumps that occur exactly 4 hours after the start of the race.

Cowtapult

Cowboy Alex's barn is being attacked!!!!! The enemy cows are in a cart that's currently 80 meters north and 50 meters east of the barn, and it is traveling west at 10 meters per second. Bessie the Cow is operating a cowtapult that is 15 meters north of the barn. The cowtapult launches a hay bale with a horizontal speed of 20 meters per second in a direction that is 30 degrees west from north. Bessie wants to hit the enemy cart using the cowtapult. When should she launch the hay bale? Express your answer as a decimal in seconds, rounded to three digits after the decimal point.

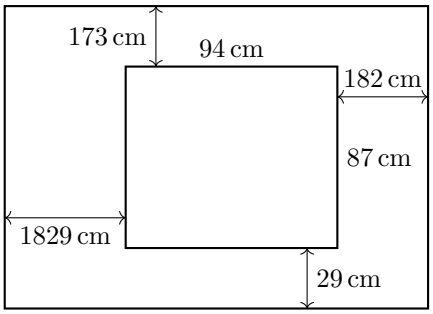


Cowputing

Bessie the Cow's new computing class has N students. If she kicks out 3 students, she can split the students into groups of 8. If she adds 11 students (to the original class of N students), she can evenly split the students into groups of 5. If she adds 5 students (to the original N students), she can split the students into groups of 17. If she adds 1 student, she can divide the students into 4 groups each containing the same number of students. What is the **second** smallest possible value of N?

Moodern Art

Bessie the Cow created a new painting. It is in the shape of a rectangle 87 centimeters tall and 94 centimeters wide. She hangs it on a rectangular wall such that the top edge of the painting is 173 centimeters away from the top edge of the wall, the left edge of the painting is 1829 centimeters away from the left edge of the wall, the right edge of the painting is 182 centimeters away from the right edge of the wall, and the bottom edge of the painting is 29 centimeters away from the bottom edge of the wall. What is the area of the part of the wall that is not covered by the painting?

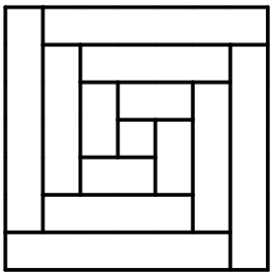


woC

Bessie the Cow chooses an integer randomly and uniformly from 1000 to 8375 inclusive, reverses the digits, then discards any leading zeros. What is the expected value of the result? Express your answer in exact form.

Cowloring

Bessie the Cow found apiece of paper with this figure printed on it and she wants to color it using red, green, and/or blue. In how many ways can she doe this if the orientation of the figure doesn't matter? Each region in the figure must have exactly one of the three colors. Two colorings are considered equivalent if one can be obtained by rotating the other.



Cownt the Calfs

"I see you've got a new calf," said Cowboy Alex, peering probingly into Farmer Julia's cow farm. "Its tail is white too, like my socks. How many calves do you have?"

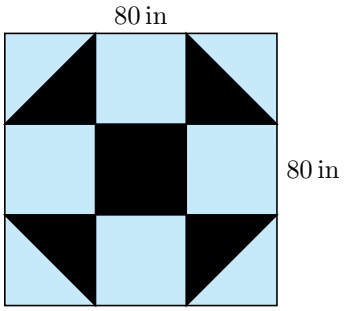
"Not a lot," said Farmer Julia. "Rancher Emily next door has twenty, which is more than what I've got."

"You still haven't told me how many calves you have!" Cowboy Alex exclaimed.

Farmer Julia, who knew that Cowboy Alex was something of an amateur mathematician, said, "Well ... let me put it like this. If you choose two distinct calves of mine at random, the probability that both of them have white tails is exactly one-half."

How may calves does Farmer Julia have in total?

Cowld

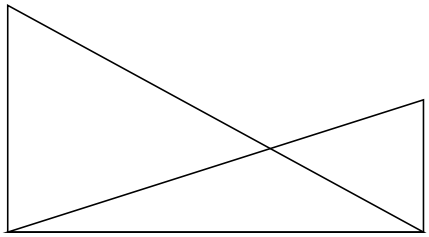


Cowboy Alex wants to make a blanket for his cows with a specific pattern because they are cold. If the blanket uses the pattern 36 times and one side of the pattern is 80 inches, at least how many inches of black fabric will he need to make blankets for 20 cows?

The Great Cow Chase of 2021

Oh no! Kiran the cow is making an escape! Kiran is running at 20 mph. Kiran is currently 10 miles from the end of the fence so Rancher Cedric knows Kiran will make a turn. The left side of the fence is 30 miles long. To catch up with Kiran faster, he decides to ride his horse diagonally across the field. He is riding his horse at 30 mph to catch Kiran. How long will it take for him to catch the cow?

Cowyon

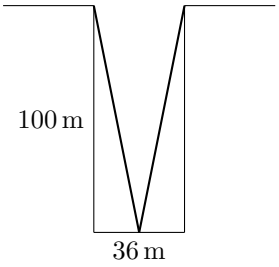


A widely-known bazaar is situated in a great canyon, with vertical cliffs on both sides. One cliff is 700 meters high, while the other is 1200 meters high. A zip line runs from the foot of each cliff to the top fo the other cliff; the zip lines are perfectly straight. At what height above the ground do the two cables meet?

Rowdy Cow

After Rancher Cedric caught Kiran the cow in the Great Cow Chase, Kiran decided to trample the grass and terrorize the chickens. Rancher Cedric had enough, and he decided to lock him in a pen. The pen is in the shape of an equilateral triangle, and each side is 500 meters long. The rowdy cow is tied to one corner, so that the portion of the field it can reach is exactly half of the total area. Assume that Kiran and the rope have zero width. How long is the rope?

The Farmer's Cownundrum



Farmer Robert submits the blueprints for his cow farm to his contractor, but the contractor misreads the information! The Farm is now separated by a 36 meter long divide that is 100 meters deep. Robert has a huge ego, so he believes that the bridge should be shaped in a V because it is the first letter of his last name. Assuming that each side of the V is equal in length and reaches the floor, how long would the bridge be in meters?

Moocraft

Bessie the Cow's new hobby is trying to beat the video game Moocraft as fast as possible. On the first day, Bessie finished Moocraft in 40 minutes. The next day, Bessie finished Moocraft in three-fourths of the time it took the day before. On the third day, Bessie's time is five-sevenths of the time on the second day. How many seconds did it take for Bessie to complete Moocraft on the third day? Round your answer to the nearest integer.

Moss-cow

Bessie the Cow is currently staying in London during a world tour to find the tastiest grass. She prepares to visit Moscow, but wants to know how long it will take to get there. The route from London to Moscow is 1600 miles long. First, Bessie will ride a train moving at 60 miles per hour. If the whole journey took 19.5 hours, how long did Bessie spend on the plane? Assume that the time it takes for Bessie to transfer from the plane to the train is negligible.

Revomootion

Farmer John's cows are planning revolution to overthrow him! The 42 cows plan to distribute themselves among 6 pastures. To make sure Farmer John doesn't get suspicious, there must be at least two cows in each pasture, each pasture must have a different number of cows, and pasture can have exactly 3 or 3 cows. To help the cows calculate their chance of succeeding, find the greatest number of cows that can be in any one pasture.

Bessie Buys Brass Bells by the Big Barn

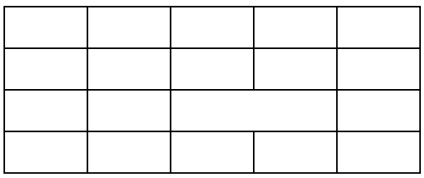
Bessie the Cow is interested in buying new bells for herself and her friends. The Barnyard Bazaar sells three different types of bells: blue bells, brown bells, and burgundy bells. Each type of bell has a constant cost per bell. Three blue bells and one brown bell would cost a total of 35 cents. Two brown bells and four burgundy bells would cost a total of 28 cents. One blue bell and two burgundy bells would cost a total of 9 cents. How much would four blue bells and three brown bells cost?

Moorio Kart

Cowboy Alex gave Bessie the Cow a new racing game called Moorio Kart! She let all one hundred of her cow friends play, and then asked each cow whether they liked each of the three courses. 65 cows said they liked Bowser's Cowstle, 75 cows said they liked Cowconut Mall, and 85 cows said they liked Moo Moo Meadows. What is the fewest number of cows that could have said they liked all three courses?

Cownt the Rectangles

Bessie the Cow found a piece of paper with the following figure printed on it. She wants to know how many rectangles are in the figure. Help her by finding the number of rectangles of any size that is in this figure, including rectangles which contain multiple smaller rectangles.



Cowlifornia

Bessie the Cow and her siblings have always dreamed of living in Cowlifornia. However, Bessie has a lot of siblings, so it'll be hard to find a farm they van all moove to! Bessie has two more sisters than brothers. Her brother, Kiran, has twice as many sisters as brothers. How many siblings does Bessie have?

Moogic

Bessie the Cow wanted to learn how to do magic, but she got distracted on the internet and ended up learning about magic squares. In a magic square, the sum of the numbers in each row, each column, and the two main diagonals are the same. If exactly two numbers are changed in this grid, the result is a magic square. Which two numbers must be changed and what values should they be changed?

4	29	15
27	16	3
17	7	28

Green Cows 2

Bessie the Cow still hasn't figured out how much grass a green cow can chow if green cows can chow grass. If 5 green cows can chow 8 square meters of grass in 20 minutes, how many green cows are needed to chow $10\,\mathrm{m}^2$ of grass in 15 minutes? Assume that all green cows in this problem chow grass at the same constant rate.

Cowkies

When Bessie the Cow was visiting a nearby farm, she noticed that there was a large herd of cows. She brought some cookies to make new cow friends, but she didn't know how many cows there were in the group! She counted a total of 87 black spots 83 white spots. Each cow either has two black spots and three white spots or three black spots and two white spots. How many cows are in the group, excluding Bessie?

Moo York Bagels

Bessie the Cow now lives in Moo York. As a young country-born-and-raised heifer, she feels a bit homesick and misses her local cuisine. Luckily, she discovered Billie's Bagel Business. Every day, she selects a bagel based on her mood. If Bessie is happy, she will spend $\$1,000$ on a toasted blueberry bagel. If Bessie is sad, she will spent \$620 on a strawberry cream cheese bagel. On any given day, Bessie has a 68% chance of being happy. On average, how much does Bessie spend on the bagels each day? Round your answers to the nearest cent.

On the Moove

Bessie the Cow and her sister Bailey the Cow leave their barn at the same time, running at a constant speed on a narrow and straight road. Bessie went northbound traveling at 42 miles per hour. After 2 hours, Bessie and Bailey are 30 miles apart. What was Bailey's speed in miles per hour?

Completely Organic Watermelons (C.O.W.)

Bessie the Cow was looking through the Barnyard Bazaar for some fresh produce when she spotted some Completely Organic Watermelons! Bella the Watermelon Cow has 18 *different* watermelons for sale: 5 are large, 6 are medium, and 7 are small. Bessie wants to buy three watermelons that aren't all the same size. In other words, she wants to buy three watermelons of at least two different sizes. In how many ways can this be done? The order in which she buys them doesn't matter, but each watermelon is unique and different from others of the same size category.

Soccow Tournament

Rancher Cedric decided to hold a soccow tournament for his cows. Bessie the Cow's team, the Tactful Tauruses, lost 7 or their first 9 games. However, Bessie decided that she wasn't going to accept losing anymore, and encouraged her team to work harder Bessie's team ended up winning 75% of their remaining

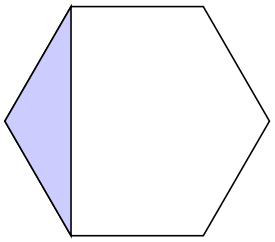
games. They had victories in exactly $\frac{2}{3}$ of all their games. In all, how many games did the Tactful Tauruses win?

Cowculus

Bessie the Cow really enjoys doing math, so she decided to order two cowculus books from the Barnyard Bazaar: the first book is a cowculus textbook, and the second is a cowculus workbook. When they arrived, Bessie noticed that the books were really heavy! The textbook's weight is 20% less than the workbook's weight. The workbook weighs 10 pounds more than the textbook. What is the combined weight of the two cowculus books?

Holy Cow, A New Pen!

Bessie the Cow has been doing some home improvement, and she has decided that she wants a new outdoor pen in the shape of a regular hexagon. She wants to reserve the shaded region of her new pen for tasty grass to grow. The area of this region is $80\,\text{cm}^2$. What is the area of Bessie's whole pen in cm^2?



Crack the Cowde

Bessie the Cow isn't very good at cybersecurity! She made the password to her Moocraft account only three numerical digits long. No digit is 0. The hundreds digit is a multiple of 4, the tens digit is a perfect square, and the ones digit is a multiple of 3. The digits are in decreasing order. What is Bessie's Moocraft password?

Farmer Pearson's Pears

Cowboy Alex just ran out of fresh fruit! He decided to visit Farmer Pearson for some of his world-famous pears. Farmer Pearson puts together 18 pears in one pack and he only sells whole packs. If Cowboy Alex wants to buy at least five dozen pears, how many packs does he need to buy at the minimum?