Intelligent Control Using Neural Networks and Multiple Models

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Abstract

In this paper a new framework for intelligent control is established to adaptively control a class of nonlinear discrete time dynamical systems while assuring boundedness of all signals. A linear robust adaptive controller and multiple nonlinear neural network based adaptive controllers are used, and a switching law is suitably defined to switch between them, based upon their performance in predicting the plant output. Boundedness of all the signals is established regardless of the parameter adjustment mechanism of the neural network controllers, and thus neural network models can be used in novel ways to better detect changes in the system and provide starting points for adaptation. The effectiveness of the proposed approach is demonstrated by simulation studies.

1 Introduction

Intelligent control is a very active and multi-disciplinary field [1, 2, 3, 4, 5, 6]. A search in IEEE Xplore database yields 5476 records with the phrase "intelligent control", among which 405 papers have it in their titles. In this paper we view intelligent control as one stage beyond adaptive control, which focuses on fast adaptation when an abrupt change has occurred to the system being controlled. It is of great importance to problems such as fault detection and reconfigurable control. Multiple models, switching and tuning as proposed in [7] provides a natural framework for intelligent control, in which many models characterizing different modes of operation of the plant are used to predict the behavior of the system, and the one that best describes the plant is used to initialize new adaptation and/or generate new control input. Thus when a fault occurs, adaptation will start from a situation that is similar to it, hence providing faster response and better performance. For linear systems, stability was established in [7] for continuous-time systems when using multiple models, switching and tuning, and extended in [8] to discrete-time systems.

It is desirable to extend the above results to the case when multiple *nonlinear* models are used. There are two issues involved: (1) For a *single* nonlinear model and the asso-

ciated controller (e.g.neural network based), what are the stability properties? (2) Multiple models are used to better prepare for sudden changes in the plant, but it has be established first that for a fixed plant, controlling with multiple models does not lead to instability. The problem becomes even more challenging when these models are nonlinear.

Neural networks have been used as effective system identifiers and controllers in nonlinear adaptive control. Their stability properties have been studied extensively, and many results have been obtained [9]. Recently, a boundedness result was established for using neural networks to adaptively control a class of nonlinear systems where nonlinearity is assumed to be bounded but is not assumed to be affine in the control input u [10]. The result is based on combining linear robust adaptive control and neural network based adaptive control in the framework of multiple models, switching and tuning, and it places little constraint on the architecture or tuning algorithm of the neural networks used. This not only solves a problem in neuro-control, but also gives rise to a framework for intelligent control using multiple nonlinear models. The purpose of this paper is to rigorously establish boundedness of all signals when using such models for a class of nonlinear discrete-time systems, and to illustrate through simulation examples the improvement in performance that can be achieved with such an approach.

The paper is organized as follows. A statement of the problem is given in Section 2, together with the mathematical tools that will be used throughout the paper. Linear robust adaptive control is reviewed in Section 3 with proof of stability. The key result of the paper is presented in Section 4, where it is shown that the proof of boundedness is reduced to that of a linear robust adaptive controller when multiple linear and/or nonlinear models and suitable switching rules are used. A simulation study is described in detail in Section 5, which shows the effectiveness of the proposed framework. Finally conclusions are drawn in Section 6.

2 Statement of the Problem

Let a single-input-single-output discrete-time nonlinear system be described by

$$\Sigma: x(k+1) = F(x(k), u(k))$$

$$y(k) = H(x(k))$$
(1)

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where $u(k), y(k) \in \mathbb{R}$, $x(k) \in \mathbb{R}^n$ and F and H are smooth nonlinear functions such that the origin is an equilibrium state. In view of the role played by the linearization around the equilibrium state, (1) is also equivalently represented as

$$\Sigma: x(k+1) = Ax(k) + bu(k) + \bar{f}(x(k), u(k))$$
$$y(k) = cx(k) + \bar{h}(x(k))$$
(2)

where $A \in \mathbb{R}^{n \times n}$, $b, c^T \in \mathbb{R}^n$ and the triple (c, A, b) represents the linearization Σ_L of Σ . The nonlinear functions \bar{f} and \bar{h} are consequently obtained by subtracting the linear terms from the functions F and H in equation (1), and are said to belong to the class \mathcal{H} of Higher Order Functions. In a neighborhood of the origin, (2) can also be represented as $\{11, 12\}$

$$y(k+d) = a_0 y(k) + \dots + a_{n-1} y(k-n+1) + b_0 u(k) + \dots + b_{n-1} u(k-n+1) + f\left(y(k), \dots, y(k-n+1), u(k), \dots, u(k-n+1)\right)$$

$$\stackrel{\triangle}{=} \theta^T \omega(k) + f(\omega(k))$$
(3)

where d is the delay of the system,

$$\omega(k) \stackrel{\Delta}{=} [y(k), \cdots, y(k-n+1), u(k), \cdots, u(k-n+1)]^T$$

is referred to as the regression vector,

$$\theta \triangleq [a_0, \dots, a_{n-1}, b_0, \dots, b_{n-1}]^T, b_0 \ge b_{min} > 0$$

is the (linear) parameter vector, and f is a smooth nonlinear function that (locally) consists of higher order terms of $\omega(k)$. It is assumed that the order of the system n and the relative degree d are specified while the parameter vector θ and the nonlinear function f are unknown. A bounded signal $y^*(k)$ represents the desired output of the system, and the value $y^*(k+d)$ is known to the controller at time k. The objective of adaptive control is to generate a bounded control signal u(k) such that the output y(k) asymptotically approaches the specified bounded signal $y^*(k)$, i.e.,

$$\lim_{k \to \infty} |y(k) - y^*(k)| = 0$$

When the nonlinearity f is small, it can be treated as a bounded disturbance, and the problem becomes one of linear robust adaptive control. In order to achieve better performance, efforts have to be made to compensate for the nonlinearity f, e.g., by using a neural network. However, boundedness of all signals has to be guaranteed when such efforts are made. Moreover, in view of the need for intelligent control, running multiple nonlinear models is desirable in order to capture the new characteristics of the system after it undergoes an abrupt change. This makes it even more challenging to maintain the boundedness of all signals in the system. In this paper, we are interested in a general framework for the adaptive control of system (3) that has the following properties:

- (i) (Linear Robust Control) It guarantees boundedness of all signals.
- (ii) (Neural Network Compensator) It allows for the use of neural networks to compensate for nonlinearities and improve performance, but the constraint on the neural networks should be minimal, i.e., the particular structure or tuning algorithm used should not be critical to establishing boundedness of all signals.
- (iii) (Multiple Models) It allows for the use of multiple linear/nonlinear models in various ways to provide fast adaptation.

To establish such a framework, we make the following assumptions about the class of nonlinear systems we consider in this paper:

Assumptions:

- (i) The system under consideration has a *global* representation (3).
- (ii) The nonlinearity $f(\cdot)$ is globally bounded, i.e., $|f(\cdot)| \leq \Delta$, and the bound Δ is known.
- (iii) The system has a globally uniformly asymptotically stable zero dynamics, so that an input sequence never grows faster than the output sequence.
- (iv) It is known a priori that the linear parameter vector θ lies in a compact region B.

Growth rates of signals

One of the principal tools used in proving boundedness of signals in an adaptive system is the relationship between different signals in the system and their rates of growth. We provide some definitions used in this paper. For more details, see [13, 10].

Let x(k) and y(k) (scalar or vector) be two discrete time signals defined for all $k \in \mathbb{N}^+$ where \mathbb{N}^+ is the set of all non-negative integers. Let $|\cdot|$ denote a norm.

Definition 1 (large order) Denote y(k) = O[x(k)] if there exist positive constants M_1 , M_2 and k_0 such that $|y(k)| \le M_1 \max_{\tau \le k} |x(\tau)| + M_2$, $\forall k \ge k_0$

Definition 2 (small order) Denote y(k) = o[x(k)] if there exists a discrete-time function $\beta(k)$ with the property that $\lim_{k\to\infty} \beta(k) = 0$, and a constant k_0 such that $|y(k)| \le \beta(k) \max_{\tau \le k} |x(\tau)|$, $\forall k \ge k_0$

Definition 3 (equivalence) If x(k) = O[y(k)] and y(k) = O[x(k)], we refer to x(k) and y(k) as being equivalent and denote it as $x(k) \sim y(k)$. If follows directly that this equivalence relation is reflexive, symmetric and

transitive, so that the symbol \sim represents an equivalence class.

3 Linear robust adaptive control

In this section, results from linear robust adaptive control will be stated, and the proof that all the signals are bounded will be given. This is the basis for using multiple models to improve performance while maintaining stability.

Let the system under consideration be described by

$$y(k+1) = \theta^T \omega(k) + f(\omega(k)) \tag{4}$$

where θ , $\omega(k)$ and f are defined as in Section 2, $|f(\cdot)| \leq \Delta$, and the component b_0 in θ satisfies

$$b_0 \ge b_{min} > 0 \tag{5}$$

Here only unit delay is considered for the sake of brevity. A model is set up as follows:

$$\hat{y}(k+1) = \hat{\theta}^T(k)\omega(k)$$

The parameter $\hat{\theta}(k)$ is updated in two steps. In the first step, an unconstrained value is calculated. In the second step, if the component b_0 of that value is less than b_{min} , it is set to b_{min} so that (5) always holds. For the former, the parameter adaptation law is

$$\hat{\theta}(k) = \hat{\theta}(k-1) - \frac{a(k)e(k)\omega(k-1)}{1 + |\omega(k-1)|^2}$$
 (6)

where

$$e(k) \stackrel{\Delta}{=} \hat{y}(k) - y(k) = (\hat{\theta}^T(k-1) - \theta^T)\omega(k-1) - f(\omega(k-1))$$

and

$$a(k) = \begin{cases} 1 & \text{if } |e(k)| > 2\Delta \\ 0 & \text{otherwise} \end{cases}$$

Define $\phi(k) = \hat{\theta}(k) - \theta$ and we have

$$\phi(k) = \phi(k-1) - \frac{a(k)e(k)\omega(k-1)}{1 + |\omega(k-1)|^2}$$

and therefore [14]

$$|\phi(k)|^2 \le |\phi(k-1)|^2 - \frac{a(k)(e^2(k) - 4\Delta^2)}{2(1 + |\omega(k-1)|^2)}$$
(7)

Two conclusions can be drawn from inequality (7):

- (i) $\{|\phi(k)|^2\}$ is a non-increasing sequence, hence $\hat{\theta}(k)$ is bounded. Moreover,
- (ii) $\lim_{k\to\infty}\frac{a(k)(e^2(k)-4\Delta^2)}{2(1+|\omega(k-1)|^2)}\to 0$. It then follows that $e(k)=o[\omega(k-1)]$ if $\omega(k-1)$ is unbounded.

The proof that all signals are bounded is by contradiction.

Proof: Assume that y is unbounded. Then

(i) By certainty equivalence, u(k) is always chosen such that $\hat{y}(k+1) = y^*(k+1)$, and therefore

$$e(k) = \hat{y}(k) - y(k) = y^*(k) - y(k)$$

Since $y^*(k)$ is bounded, we have

$$e(k) \sim y(k) \tag{8}$$

or, e(k) grows at the same rate as y(k).

(ii) By the assumption that the system has an asymptotically stable zero dynamics (or minimum phase, for linear systems), i.e., any input sequence u(k-1) cannot grow faster than the output sequence y(k), we have

$$u(k-1) = O[y(k)]$$

Since

$$\omega(k-1) = [y(k-1), \dots, y(k-n), u(k-1), \dots, u(k-n)]^T$$

it follows that

$$\omega(k-1) = O[y(k)] \tag{9}$$

or, $\omega(k-1)$ does not grow faster than y(k).

(iii) By the adaptation law (6),

$$e(k) = o[\omega(k-1)] \tag{10}$$

or, e(k) grows slower than $\omega(k-1)$.

(iv) Therefore, from (8), (9) and (10),

$$y(k) = o[y(k)]$$

or, y(k) grows slower than itself. This cannot happen if y(k) is assumed to be unbounded. Therefore y(k) is bounded, and the boundedness of other signals follows in a straightforward fashion.

It is seen from the proof that the property of the identifier, $e(k) = o[\omega(k-1)]$, or, the identification error grows at a lower rate than the regression vector, plays a central role. When multiple models are used, the objective of the switching scheme is to maintain this relationship, even while the control inputs can be generated by different controllers. This will be demonstrated in the next section.

4 Adaptive control using multiple models

In this section, the problem of adaptive control using multiple models is considered. The general architecture of such an approach is shown in Figure 1, where m models are used to predict the future output of the plant. At each time instant, depending on the performance criterion for the models, one of them, e.g., Model i, will be chosen as the best, and the

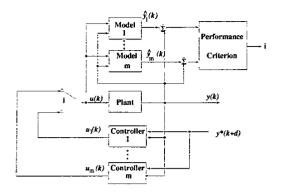


Figure 1: Multiple models: switching between a linear controller and a neural network controller

corresponding Controller i will be used to generate the control input to the plant. A special case of the plant will be discussed in detail below, and the proof of boundedness of all signals will be given. Due to space limitation interested readers are referred to [10] for general cases.

Let the system be described by

$$y(k+1) = \theta^{T} [y(k), \dots, y(k-n+1), u(k), \dots, u(k-n+1)]^{T} + f\left([y(k), \dots, y(k-n+1), u(k-1), \dots, u(k-n+1)]^{T}\right)$$

$$\stackrel{\triangle}{=} \theta^{T} \omega(k) + f(\bar{\omega}(k))$$

$$\stackrel{\triangle}{=} b_{0} u(k) + \bar{\theta}^{T} \bar{\omega}(k) + f(\bar{\omega}(k))$$
(11)

where

$$\theta \stackrel{\Delta}{=} [a_0, \cdots, a_{n-1}, b_0, \cdots, b_{n-1}]^T$$

$$\bar{\theta} \stackrel{\Delta}{=} [a_0, \cdots, a_{n-1}, b_1, \cdots, b_{n-1}]^T$$

$$\omega(k) \stackrel{\Delta}{=} [y(k), \cdots, y(k-n+1), u(k), \cdots, u(k-n+1)]^T$$

$$\bar{\omega}(k) \stackrel{\Delta}{=} [y(k), \cdots, y(k-n+1), u(k-1), \cdots, u(k-n+1)]^T$$

The purpose of these definitions is to isolate terms with u(k). Note that it is assumed that the delay is unity, and that the current control input u(k) affects the output y(k+1) only through a linear term $b_0u(k)$. In such a case, the determination of a certainty equivalence control input u(k) will be straightforward.

Let Model 1 be defined as

$$\hat{y}_1(k+1) = \hat{\theta}_1^T(k)\omega(k) \tag{12}$$

where parameter $\hat{\theta}_1(k)$ is updated as

$$\hat{\theta}_1(k) = \hat{\theta}_1(k-1) - \frac{a(k)e_1(k)\omega(k-1)}{1 + |\omega(k-1)|^2}$$
 (13)

where

$$e_1(k) \stackrel{\Delta}{=} \hat{y}_1(k) - y(k)$$

$$a(k) = \begin{cases} 1 & \text{if } |e_1(k)| > 2\Delta \\ 0 & \text{otherwise} \end{cases}$$

and the parameter $\hat{b}_{1,0}(k)$ associated with u(k) in $\hat{\theta}_1(k)$ is always constrained to be greater than $b_{min} > 0$, as discussed earlier.

Controller 1 (i.e., control input $u_1(k)$ at time k) is determined as:

$$u_1(k) = \frac{1}{\hat{b}_{1,0}(k)} \left(y^*(k+1) - \hat{\theta}_1(k)\bar{\omega}(k) \right)$$

Let Model $j, j = 2, \dots, m$, be defined as

$$\hat{y}_j(k+1) = \hat{\theta}_j^T(k)\omega(k) + \hat{f}_j(\bar{\omega}(k), W_j(k))$$
 (14)

where $\hat{f}_j(\cdot)$ is a bounded continuous nonlinear function parameterized by a vector $W_j(k)$ (e.g., a neural network with "weights" vector W_j). No restriction is made on how the parameters $\hat{\theta}_j(k)$ or $W_j(k)$ are updated except that they always lie inside some pre-defined compact region S_j :

$$\hat{\theta}_i(k), \hat{W}_i(k) \in \mathcal{S}_i, \ \forall k$$
 (15)

and in this region the linear parameter $b_{j,0}$ associated with u(k) is always greater than b_{min} .

Controller j is determined as:

$$u_j(k) = \frac{1}{\hat{b}_{j,0}(k)} \left(y^*(k+1) - \hat{\bar{\theta}}_j^T(k) \bar{\omega}(k) - \hat{f}_j(\bar{\omega}(k), W_j(k)) \right)$$

The performance criterion is defined as

$$J_{i}(k) = \sum_{l=1}^{k} \frac{a_{i}(l)(e_{i}^{2}(l) - 4\Delta^{2})}{2(1 + |\omega(l-1)|^{2})} +$$

$$c \sum_{l=1}^{k} (1 - a_{i}(l))e_{i}^{2}(l), \quad i = 1, 2, \dots, m$$
(16)

where $e_i(l) \stackrel{\Delta}{=} \hat{y}_i(l) - y(l)$ and

$$a_i(l) = \left\{ egin{array}{ll} 1 & \mbox{if } |e_i(l)| > 2\Delta \ 0 & \mbox{otherwise} \end{array}
ight.$$

N is an integer and $c \ge 0$ is a constant.

At every instant k, the controller i that corresponds to the model i that yields the smallest performance criterion $J_i(k)$ is used to determine the control input $u(k) = u_i(k)$ to the system.

Theorem 1 For the system (11) with identifiers (12) and (14) together with their adaptation laws (13) and (15), and performance index (16), all the signals in the closed-loop switching system described above are bounded.

Proof: We will prove the theorem by contradiction, as in the single model case. Assume that y(k) is unbounded. Hence

the regression vector $\omega(k)$ is unbounded. Note that the second term in (16) is always bounded. By the properties of the linear robust adaptive identifier (13), $J_1(k)$ is bounded, and

$$e_1(k) = o[\omega(k-1)]$$

By the use of certainty equivalence, at every instant k,

$$e_c(k) = e_i(k), i \in \{1, 2, \cdots, m\}$$

where $e_c(k) \stackrel{\Delta}{=} y^*(k) - y(k)$ is the control error, since we are assuming unit delay. Let e(k) be the "combined" identification error that corresponds to the control error $e_c(k)$. There can be two cases:

- (i) All models produce bounded performance criterion. In such a case, it follows immediately that $e(k) = o[\omega(k-1)]$.
- (ii) If for some $j \in \{2, 3, \dots, m\}$, $J_j(k)$ is unbounded, then there exists a constant k_j such that $J_j(k) > J_1(k)$, $\forall k \geq k_j$, since J_1 is bounded. By the switching rule, Controller j will never be used again after a finite time k_j . Continuing the argument for other models, it follows that there exists a finite time k_0 such that for $k > k_0$, switching occurs only between the models with bounded performance criterion, which reduces case (ii) to case (i), and hence $e(k) = o[\omega(k-1)]$.

Since $e(k) = o[\omega(k-1)]$, the rest of the proof follows along the same lines as in the single model case, and is omitted here. \blacksquare .

Comment 1: The performance index (16) consists of two parts. The first part, $\sum_{l=1}^k \frac{a_i(l)(e_i^2(l)-4\Delta^2)}{2(1+|\omega(l-1)|^2)}$, is used to distinguish between signals with different growth rates, so that boundedness of all the signals can be established. The second part, $c\sum_{l=k-N+1}^k (1-a_i(l))e_i^2(l)$, is a measure of the prediction error over a finite window and is included to improve performance.

Comment 2: The significance of the above theorem is that it decouples the stability issue from the performance issue. No restriction (except the obvious one that parameters are not allowed to become arbitrarily large) on the architecture, number of nodes, or training method of the neural network is needed for the boundedness result to hold. This provides a convenient framework in which any type of neural network and training method can be attempted. The specific neural networks that the authors frequently use will be specified later with the simulation example. From the perspective of intelligent control, the theorem also means that the following different kinds of models can be used without compromising the boundedness of all signals:

- (i) Fixed models: The parameters of the models are determined off-line and are fixed on-line.
- (ii) Adaptive models: The parameters are adaptive as in conventional adaptive control.
- (iii) Re-initialized adaptive models: The parameters are

adaptive, but each time a fixed model is chosen, the parameter of one of the adaptive models in this category is re-initialized using the parameter of that fixed model, and adaptation continues from this point.

The above ideas were first introduced in [7] for linear models, and have found wide applications in many practical problems.

5 A Simulation of Intelligent Control

To illustrate the effectiveness of the proposed framework for intelligent control using neural networks and multiple models, a simulation study is described in this section. The objective is to control a plant so that it follows the reference trajectory given by

$$r(k) = 1.5 \left(\sin \left(\frac{2\pi k}{10} \right) + \sin \left(\frac{2\pi k}{25} \right) \right)$$

The simulation is carried out from k = 1 to k = 400. At k = 200, an abrupt change is introduced to the plant so that it switches from P_1 to P_2 , where P_1 is described by

$$y(k+1) = 2.6y(k) - 1.2y(k-1) + u(k) + 0.5u(k-1) + \sin(u(k-1) + y(k) + y(k-1)) - \frac{u(k-1) + y(k) + y(k-1)}{1 + u^2(k-1) + y^2(k) + y^2(k-1)}$$

and P_2 is described by

$$\begin{split} y(k+1) &= y(k) - 2y(k-1) + u(k) + 0.5u(k-1) + \\ &\sin\left(.9u(k-1) + y(k) + y(k-1)\right) - \frac{.9u(k-1) + y(k) + y(k-1)}{1 + .81u^2(k-1) + y^2(k) + y^2(k-1)} \end{split}$$

Note that for illustration purposes, both P_1 and P_2 are of a special case, *i.e.*, the delay is chosen as unity, and the nonlinear terms do not contain the current control input u(k). The more general cases have also been simulated by the authors, with similar results. Since the nonlinear terms are bounded, a linear robust adaptive controller is first used to control the plant. The performance is not very good, even though the input and the output of the plant are always kept bounded.

After input-output data for the plant is collected while the linear robust adaptive controller is running, the simulation proceeds as follows. First the data is separated into two sets that correspond to k < 200 and $k \ge 200$ respectively. For each set, a linear model and a nonlinear model are extracted. The former is of the form

$$\hat{y}(k+1) = a_0 y(k) + a_1 y(k-1) + b_0 u(k) + b_1 u(k-1)$$

where the parameters are obtained simply by solving a matrix equation. The latter is of the form

$$\hat{y}(k+1) = a_0 y(k) + a_1 y(k-1) + b_0 u(k) + b_1 u(k-1) + NN(y(k), y(k-1), u(k-1))$$

where the linear parameters are the same as the former, and the neural network parameters are obtained by extensive training. The multi-layer perceptron neural network that we use has two hidden layers with 10 and 8 nodes respectively. (The whole neural network structure is sometimes denoted by $\mathcal{N}_{3,10,8,1}$.) After this, the following models are constructed and used in the control of the plant: (i) Free running linear robust adaptive controller, which will be called C_1 ; (ii) Two linear controllers with fixed parameters, which will be called C_2 and C_3 respectively; (iii) Two nonlinear controllers with fixed parameters, which will be called C_4 and C_5 respectively; (iv) Two nonlinear adaptive controllers whose initial parameters are the same as in C_4 and C_5 respectively. They are called C_8 and C_7 .

Simulations have been conducted using (i) C_1 only; (ii) C_1 , C_2 and C_3 ; (iii) C_1 , C_2 , C_3 , C_4 and C_5 ; (iv) C_1 , C_2 , C_3 , C_4 , C_5 , C_6 and C_7 . The addition of a model usually improves the performance to some extent. When all 7 models are used, the performance is shown in Figure 2. The win-

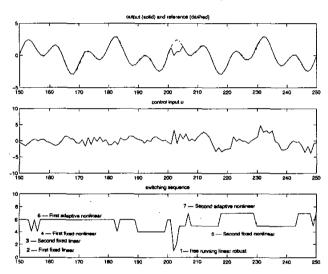


Figure 2: Performance when using all 7 models

dow size N is chosen as 4 in the simulations. It can be seen that, due to the success of extensive off-line training, switching occurs mostly between the fixed nonlinear model C_4 and the adaptive nonlinear model C_6 when the plant is P_1 , and between C_5 and C_7 when the plant is P_2 , and tracking error is very small, even when the plant undergoes an abrupt change.

6 Conclusions

In this paper a new framework for intelligent control is established to adaptively control a class of nonlinear discrete time dynamical systems while assuring boundedness of all the signals. A linear robust adaptive controller and multiple nonlinear neural network based adaptive controllers are used, and a switching law is suitably defined to switch between them, based upon their performance in predicting the plant output. Boundedness of all the signals is established regardless of the parameter adjustment mechanism of the

neural network controllers, and thus neural network models can be used in novel ways to better detect changes in the system and provide starting points for adaptation. Simulation studies show the effectiveness of the proposed approach, and the authors believe that the method will find wide application in fault detection and intelligent control problems.

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