

$$\begin{array}{l}
20\times \\
20 \\
?? \\
20400\times \\
400? \\
?CC= \\
WW^T, W \in \\
R^{d\times k}, rank(W) \leq \\
k \ll \\
dW \\
? \\
?? \\
? \\
? \\
? \\
? \\
\Sigma\Sigma= \\
U\Lambda U^T U k < \\
d \\
? \\
?? \\
\text{Grass-} \\
\text{mann} R^n k
\end{array}$$

$$(1) \quad Gr(k,n) = \{V \subset R^n, V \text{ is a linear subspace with } \dim V = k\}$$

$$\begin{array}{l}
? \\
\text{Stiefel(non-} \\
\text{compact)} \\
^1 n \times \\
k, (0 < \\
k < \\
n)
\end{array}$$

$$(2) \quad St(k,n) = \{A \in R^{n \times k}; rank(A) = k\} = \{A = (a_1, \dots, a_k) \in R^{n \times k}; a_1, \dots, a_k \text{ are linearly independent}\}$$

$$\begin{array}{l}
\text{Stiefel(compact)} \\
^1 AA^T A = \\
I_k
\end{array}$$

$$(3) \quad St^*(k,n) = \{A \in R^{n \times k}; A^T A = I_k\}$$

$$\begin{array}{l}
\text{Gen-} \\
\text{eral} \\
\text{Lin-} \\
\text{ear} \\
\text{Group} n n \times \\
n
\end{array}$$

$$(4) \quad GL(n) = \{A \in R^{n \times n}; \det(A) \neq 0\}$$

$$\begin{array}{l}
?? \\
\text{Stiefel(non-} \\
\text{compact)} \\
\text{V.S} \\
\text{Stiefel(compact)} GS(\cdot)
\end{array}$$

$$(5) \quad GS : St(k,n) \rightarrow St^*(k,n)$$

$$\begin{array}{l}
GS(\cdot) GS(\cdot) \\
\text{Stiefel(non-} \\
\text{compact)} \\
\text{V.S} \\
\text{Grass-} \\
\text{mann} \pi
\end{array}$$

$$(6) \quad \pi : St(k,n) \rightarrow Gr(k,n), A = (a_1, \dots, a_k) \rightarrow span(A)$$

$$\begin{array}{l}
\pi \pi \\
\pi A \in \\
St(k,n)
\end{array}$$

$$(7) \quad \pi^{-1}[\pi(A)] = \{AP; P \in GL(k)\}$$

$$\begin{array}{l}
\pi \\
\text{Stiefel(compact)} \\
\text{V.S} \\
\text{Grass-} \\
\text{mann} ?? \\
\pi
\end{array}$$

$$(8) \quad \bar{\pi} : St^*(k,n) \rightarrow Gr(k,n); A = (a_1, \dots, a_k) \rightarrow span(A), A^T A = I_k$$

$$\begin{array}{l}
\bar{\pi} \pi \pi = \\
\bar{\pi} \circ \\
GS \\
St(k,n)/GL(k,R) = \{[A][A]A[GL(k)]; A \in St(k,n)\}
\end{array}$$

$$(9) \quad St^*(k,n)/\mathcal{O}(k) = \{[A][A]A[\mathcal{O}(k)]; A \in St^*(k,n)\} \mathcal{O}(k) = \{U|U \in R^{k \times k}; U^T U = I_k\}$$