EM算法解高斯混合模型

迭代流程：

1) E步

$$\gamma_{ik}^{(t+1)} = \frac{\pi_k^t N(x_i \mid \mu_k^t, (\sigma_k^2)^t)}{\sum_{j=1}^{k} \pi_j^t N(x_i \mid \mu_j^t, (\sigma_j^2)^t)}$$

$$= \frac{\pi_k^t \cdot ((\sigma_k^2)^t)^{-\frac{1}{2}} \cdot \exp\left\{-\frac{1}{2} \cdot \frac{(x_i - \mu_k^t)^2}{(\sigma_k^2)^t}\right\}}{\sum_{j=1}^{k} \pi_j^t \cdot ((\sigma_j^2)^t)^{-\frac{1}{2}} \cdot \exp\left\{-\frac{1}{2} \cdot \frac{(x_i - \mu_j^t)^2}{(\sigma_j^2)^t}\right\}}.$$

$$\therefore \gamma_{i1}^{(t+1)} = \frac{N_1^t \cdot \sigma_2^t \cdot \sigma_3^t}{N_1^t \cdot \sigma_2^t \cdot \sigma_3^t + N_2^t \cdot \sigma_1^t \cdot \sigma_3^t \cdot \exp\{x_2^t - x_1^t\} + N_3^t \cdot \sigma_2^t \cdot \sigma_1^t \cdot \exp\{x_3^t - x_1^t\}}$$

其中 $x_1^t = -\frac{(x_i - \mu_1^t)^2}{2(\sigma_1^2)^t}$，$x_2^t = -\frac{(x_i - \mu_2^t)^2}{2(\sigma_2^2)^t}$

$x_3^t = -\frac{(x_i - \mu_3^t)^2}{2(\sigma_3^2)^t}$，$N_k^t = N \cdot \pi_k^t$

$$\gamma_{i2}^{(t+1)} = \frac{N_2^t \cdot \sigma_1^t \cdot \sigma_3^t}{N_2^t \cdot \sigma_1^t \cdot \sigma_3^t + N_1^t \cdot \sigma_2^t \cdot \sigma_3^t \cdot \exp\{x_1^t - x_2^t\} + N_3^t \cdot \sigma_1^t \cdot \sigma_2^t \cdot \exp\{x_3^t - x_2^t\}}$$

$$\gamma_{i3}^{(t+1)} = 1 - \gamma_{i1}^{(t+1)} - \gamma_{i2}^{(t+1)}$$

2) M步

$$N_k^{(t+1)} = \sum_{i=1}^{N} \gamma_{ik}^{(t+1)}, \quad \mu_k^{(t+1)} = \frac{1}{N_k^{(t+1)}} \sum_{i=1}^{N} \gamma_{ik}^{(t+1)} x_i$$

$$\sigma_k^{(t+1)} = \frac{1}{N_k^{(t+1)}} \sum_{i=1}^{N} \gamma_{ik}^{(t+1)} x_i^2 - \left(\mu_k^{(t+1)}\right)^2$$